Subject

SNA Assignment

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Exercise (chapter 16)

1.

a) As the first person has no prior information, & only his own private signal, the person believes in it and takes decision supporting his private signal

Person 2 will know person 1's decision. Person 2 gets two signals. If the 2 signals are same, the decision is easy. (As only person ahead 2 is 1 ($i-1=1$)).

If the 2 signals are opposite, person 2 will be indifferent between accepting & rejecting. Here person 2 will follow it's own private signal.

Thus, either way 1 & 2 follow their private signal irrespective of the change that they can see signal of $i-1^{th}$ person.

b) Individual 3 can only observe 2 signals, one private & other the previous one's i.e. 2nd person's signal.

This is all over like the case of 2nd person. There is just 1 prior signal & thus for any person his private signal will be decisive as he will be indifferent. If the signals are same, the choice is easy. If they are opposite, the choice is indifferent.

So the inference from 2nd person's action is that the private signal of 2nd person is the same as it's action.

c) NO, 3 can't infer anything about 1's signal from 2's action as 2's action is independent of 1's signal. It solely depends on his private signal –

d) If 3's signal is high & he knows that 2 is accepted, it makes the decision easy for him; to accept.

But, if 3's signal is low & he knows 2 is accepted, it makes the choice indifferent and independent. It rather subsides to his own private information.

Just like the case of 2nd person who knows the action / signal of 1st person

c) No, a cascade cannot form in this world, as every case is a replication of the case of 2^{nd} person who is indifferent to prior signal & decisive based on private signal.

For eg:- 3^{rd} person will know only about 2^{nd} person's signal, which we know is true, replicating the case of 2^{nd} person. giving true output

Illy, for 4^{th} person, knowing 3^{rd}'s true output, 4^{th} will again replicate same case & be indifferent. Will give true output based on private signal.

$\therefore$ No cascade formed as never will the difference between 2 signals be ≥ 2.

Q.

a) Let the positive payoff be $= P$

Let the negative payoff be $= N$

Let the positive payoff given tech is good $= q > 1/2$

$P[P|G] = q > 1/2$

$P[N|G] = (1-q) < 1/2$

$P[P/B] = (1-q) < 1/2$

$P[N|B] = q > 1/2$

(i) First person will have no prior information, therefore will make independent choice based on private signal

(ii) Second person will have 2 signals. Private & payoff of 1st person. But it won't affect as we have seen previously, it will be independent & dependent on private signal

(iii) From 3rd person, if first two people get a random payoff positive even though the tech is bad & the private signal of the ~~p~~ 3rd person is low. He will ignore the private info & cascade will start as the avg. payoff is +ve.

so if $n(P) > n(N)$, for any person, the cascade will begun.

given: Tech is bad

cases: First person rejects it as has low signal & second person accepts & gets negative payoff.

$\therefore$ 3rd person now starts cascade of rejecting if he gets a low signal.

If first two get a random positive payoff. the cascade of acceptance is started.

b) If the tech is actually good & first two people accept it & get a random payoff negative, the cascade of rejections will start from 3rd person for the new tech.

3. given: $\Pr[G/A] = 1/2 = p$

$\Pr[H|G] = \Pr[L|B] = q = 3/4$

good is actually true.

a) (i) $\Pr[\text{Accept}] = \Pr[H]$ & as G is true

$\therefore \Pr[\text{Accept}] = 3/4 = q$

$\Pr[\text{Reject}] = \Pr[L] = 1/4 = 1-q$

b) $Pr[A, A] = Pr[H|G] * Pr[H|G] = 3/4 * 3/4$

$= 9/16$

$Pr[A, R] = Pr[H|G] * Pr[L|G] = q * (1-q)$

$= \frac{3}{4} * \frac{1}{4} = \frac{3}{16}$

$Pr[R, A] = Pr[L|G] * Pr[H|G] = (1-q)q$

$= \frac{1}{4} \times 3/4 = 3/16$

$Pr[R, R] = Pr[L|G] * Pr[L|G] = (1-q)^2$

$= 1/4 * 1/4 = 1/16$

c) A cascade will only happen if $n(A) - n(R) \geq 2$, i.e. both have same decision i.e. (A, A) or (R, R).

$\therefore$ Pr for Accept cascade to start from 3rd person is $= Pr[A, A] = \underline{9/16}$

$\therefore$ Pr for Reject cascade to start from 3rd person is $= Pr[R, R] = \underline{1/16}$

∴ the probability of cascade starting from 3rd person is

$Pr[A,A] + Pr[R,R] = 9/16 + 1/16 = \underline{\underline{\frac{10}{16}}}$

4. $Pr(G) = P = 1/2$

$Pr[H|G] = q = 2/3 = Pr[L|B]$

given: we 10th person, all 9 choose R

R - cascade

a) $Pr[\text{incorrect cascade}] = Pr[R,R]$
(As everyone has R)

As, it is a Reject cascade, we know that first two people have rejected it. given it is good. The prob. it is incorrect is:

$$Pr[R,R] = Pr[L|G] \cdot Pr[L|G]$$
$$= 1/3 \cdot 1/3 = \underline{\underline{1/9}}$$

b) As, we know first two will have decision as per their private signal; we take their decision to be true & now we know 9^{th} person's true signal that is High.

Therefore 3 true signals that we have are

L, L & H.

$\therefore$ If we, as a 10^{th} person get a High signal, & thus $n(H) - n(L) = 0$

It is a tie of signals. Therefore 10^{th} person will be indifferent to prior signals & give decision based on private signal i.e. H & thus Accepts breaking the Reject cascade.

If 10^{th} person gets a low signal

$\| n(H) - n(L) \| = 2$,

$\therefore$ we again fall into reject cascade & reject it.

$\therefore$ What decision we make, A or R depends on which signal we recieve. Here we make decision directly proportional to our signal.

c) Choice of 11^{th} person if :

<u>Case 1 :</u> If 10^{th} person R.

If all people prior to 11 have made choice R, person 11 knows 10^{th} knows 4 true signals. & it's a Reject

$\therefore$ irrespective of 11^{th} person's private signal, he will Reject. following the cascade.

case 2: If person 10 chooses A.

As, 11th person knows, you did know 4 true signals & it led to A, ∴ 5 signals that 11th person has is

R, R, R, A & his private signal

if its High, the signals will

L L L H H

$|n(H) - n(L)| = 1 \leq 2$

∴ He will be indifferent to prior signals & make decision as per private signal i.e. Accept. (cascade broken by 10th person)

If 11th person get signal L, & he knows 10th knew 9th person's sig the cascade is broken & he knows 9th lied, ∴ He will ignore prior signals & decide as per private signal i.e. Reject.

5.

a) It is due to information cascade. The first two people chose candidate A which started the cascade & led to all ignoring their private signal & support the candidature of candidate A.

b) Instead of asking them to speak out their opinion, we can let them write out their opinion on a paper & submit. This will ensure there is no information cascade.

6.

(a) If all experts knowing each others recommendation recommend with this prior knowledge. we know that information cascade has happened as nearly a fraction close to 1 experts recommend product A.

We can't be confident about this choice even though majority recommend it.

We don't know the order of recommend-ation too, so we can't see where the cascade started hence cannot be sure with any decision as it can be both correct or an incorrect cascade.

(b) As we have to hire them so they have no information about the other expert's recommendation, there will be no information cascade & we will get genuine results & correct data to make decision on.

when you hire 5 of them and—

(i) use procedure I, i.e. ask them to sequentially tell their recommendation, it will lead to information cascade for any product x, if first two experts say in favour of x. This is not a correct method. Announcing the results will lead to next expert having two signals & lead to cascade & they ignoring their private signal.

(ii) If we use procedure II, we are asking each expert individually in private, which leads to experts having their own genuine private signal & no public signal to decide upon. There will therefore be no information cascade. The results will be true & genuine, and no expert will ignore their private signal, rather base their result solely on it.

Therefore, procedure II will be a better procedure & provide us with most correct information to base our decision on.

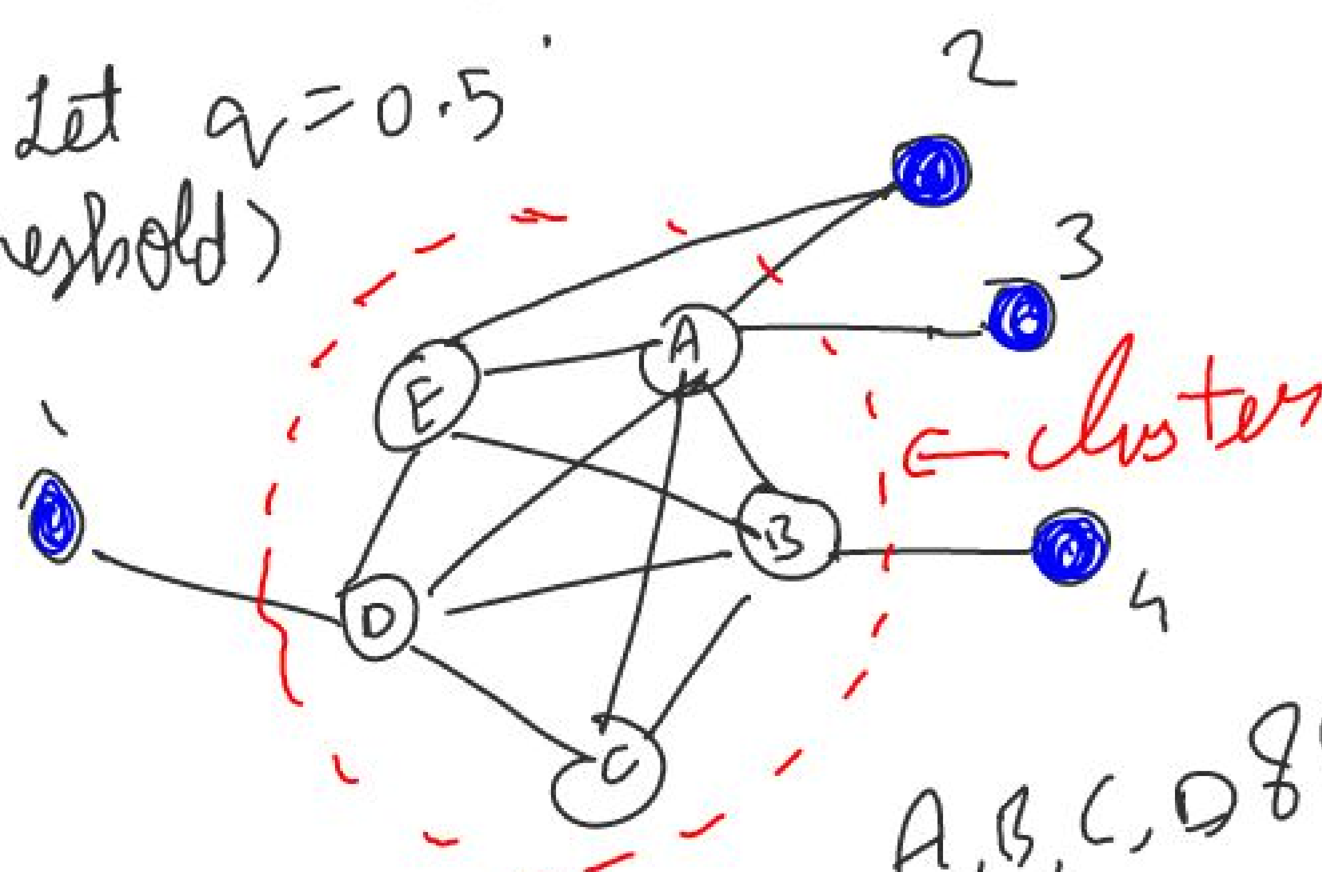
(2) Theorem: A complete cascade can not occur on a network having cluster of density greater than $1-q$, where q is the threshold of adoption.

Ans: If, the density of the cluster is $d > 1-q$, then the maximum fraction of connections of any node outside the cluster with that of nodes inside is $< q$.

As, the fraction of connections outside the cluster for any node in cluster is $< q$, even if every neighbours of node outside it's cluster are all part of cascade, the node in cluster will not adopt the cascade as the threshold won't be met i.e. $\geq q$.

$\therefore$ No complete cascade possible

eg:- Let $q = 0.5$ (threshold)



A, B, C, D & E cluster

density $d = 0.67$

If nodes 1, 2, 3 & 4 are part of cascade, for $a \Rightarrow 2/6 = 0.3 < 0.5$

for $B \Rightarrow 1/5 = 0.2 < 0.5$

for $E \Rightarrow 1/4 = 0.25 < 0.5$

for $D \Rightarrow \frac{1}{5} = 0.2 < 0.5$

As, for no node v the fraction in cluster of neighbors part of cascade are greater than threshold, it cannot be accepted into the cluster, hence no complete cascade.

Here, $q = 0.5$

$1 - q = 0.5$

density $= d = 0.67 > 0.5 = 1 - q$

Hence Proved