

$$p(y|x) = \mathcal{N}(\mu = x^T \omega, \sigma^2)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - x^T \omega)^2}{2\sigma^2}\right)$$

For n independent observations, joint log-likelihood function will be:

$$L(\omega, \sigma^2) = \sum_{i=1}^n \log \left[\frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left(-\frac{(y^{(i)} - x^{(i)T} \omega)^2}{2\sigma^2}\right) \right]$$

$$= \sum_{i=1}^n \left[\log(2\pi\sigma^2)^{-1/2} + \log \exp\left(-\frac{(y^{(i)} - x^{(i)T} \omega)^2}{2\sigma^2}\right) \right]$$

$$= -\frac{n}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^n (y^{(i)} - x^{(i)T} \omega)^2$$

Deriving log-likelihood function w.r.t ' ω ':

$$\frac{\partial L}{\partial \omega} = -\frac{1}{2\sigma^2} \sum_{i=1}^n 2(y^{(i)} - x^{(i)T} \omega) \times (-x^{(i)})$$

$$= \frac{1}{\sigma^2} \sum_{i=1}^n x^{(i)} (y^{(i)} - x^{(i)T} \omega)$$

Set derivative to zero (maximization condition)

$$0 = \frac{1}{\sigma^2} \sum_{i=1}^n x^{(i)} (y^{(i)} - x^{(i)T} \omega)$$

$$0 = \sum_{i=1}^n x^{(i)} y^{(i)} - \sum_{i=1}^n x^{(i)} x^{(i)T} \omega$$

$$\sum_{i=1}^n x^{(i)} x^{(i)T} \omega = \sum_{i=1}^n x^{(i)} y^{(i)}$$

$$\therefore \omega = \left[\sum_{i=1}^n (x^{(i)} x^{(i)T}) \right]^{-1} \times \sum_{i=1}^n x^{(i)} y^{(i)}$$

Now, deriving log-likelihood w.r.t ' σ^2 '

$$\frac{\partial L}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n (y^{(i)} - x^{(i)T} \omega)^2$$

Set derivative to zero (maximization condition)

$$0 = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n (y^{(i)} - x^{(i)T} \omega)^2$$

$$\frac{n}{2\sigma^2} = \frac{1}{2\sigma^4} \sum_{i=1}^n (y^{(i)} - x^{(i)T} \omega)^2$$

$$\therefore \sigma^2 = \frac{1}{n} \sum_{i=1}^n (y^{(i)} - x^{(i)T} \omega)^2$$