#### **Dynamic Programming Patterns**



# **Patterns**

Minimum (Maximum) Path to Reach a Target

**Distinct Ways** 

Merging Intervals

**DP** on Strings

**Decision Making** 

# Minimum (Maximum) Path to Reach a Target

Problem list: https://leetcode.com/list/55ac4kuc

Generate problem statement for this pattern

Statement

Given a target find minimum (maximum) cost / path / sum to reach the target.

## Approach

Choose minimum (maximum) path among all possible paths before the current state, then add value for the current state.

```
routes[i] = min(routes[i-1], routes[i-2], ..., routes[i-k]) + cost[i]
```

Generate optimal solutions for all values in the target and return the value for the target.

# Top-Down

```
for (int j = 0; j < ways.size(); ++j) {
    result = min(result, topDown(target - ways[j]) + cost/ path / sum);
}
return memo[/*state parameters*/] = result;</pre>
```

#### Bottom-Up

```
for (int i = 1; i <= target; ++i) {
    for (int j = 0; j < ways.size(); ++j) {
        if (ways[j] <= i) {
            dp[i] = min(dp[i], dp[i - ways[j]] + cost / path / sum);
        }
    }
}</pre>
return dp[target]
```

#### Similar Problems

746. Min Cost Climbing Stairs Easy

#### Top-Down

```
int result = min(minCost(n-1, cost, memo), minCost(n-2, cost, memo)) + (n ==
cost.size() ? 0 : cost[n]);
return memo[n] = result;
```

## Bottom-Up

```
for (int i = 2; i <= n; ++i) {
    dp[i] = min(dp[i-1], dp[i-2]) + (i == n ? 0 : cost[i]);
}
return dp[n]</pre>
```

64. Minimum Path Sum Medium

#### Top-Down

```
int result = min(pathSum(i+1, j, grid, memo), pathSum(i, j+1, grid, memo)) +
grid[i][j];
return memo[i][j] = result;
```

#### Bottom-Up

```
for (int i = 1; i < n; ++i) {
    for (int j = 1; j < m; ++j) {
        grid[i][j] = min(grid[i-1][j], grid[i][j-1]) + grid[i][j];
    }
}
return grid[n-1][m-1]</pre>
```

322. Coin Change Medium

# Top-Down

```
for (int i = 0; i < coins.size(); ++i) {
   if (coins[i] <= target) { // check validity of a sub-problem
        result = min(ans, CoinChange(target - coins[i], coins) + 1);
   }
}
return memo[target] = result;</pre>
```

## Bottom-Up

```
for (int j = 1; j <= amount; ++j) {
    for (int i = 0; i < coins.size(); ++i) {
        if (coins[i] <= j) {
             dp[j] = min(dp[j], dp[j - coins[i]] + 1);
        }
    }
}</pre>
```

- 931. Minimum Falling Path Sum Medium
- 983. Minimum Cost For Tickets Medium
- 650. 2 Keys Keyboard Medium
- 279. Perfect Squares Medium
- 1049. Last Stone Weight II Medium
- 120. Triangle Medium
- 474. Ones and Zeroes Medium
- 221. Maximal Square Medium
- 322. Coin Change Medium
- 1240. Tiling a Rectangle with the Fewest Squares Hard
- 174. Dungeon Game Hard
- 871. Minimum Number of Refueling Stops Hard

# **Distinct Ways**

Problem List: https://leetcode.com/list/55ajm50i

Generate problem statement for this pattern

#### Statement

Given a target find a number of distinct ways to reach the target.

#### **Approach**

Sum all possible ways to reach the current state.

```
routes[i] = routes[i-1] + routes[i-2], ..., + routes[i-k]
```

Generate sum for all values in the target and return the value for the target.

#### Top-Down

```
for (int j = 0; j < ways.size(); ++j) {
    result += topDown(target - ways[j]);
}
return memo[/*state parameters*/] = result;</pre>
```

# Bottom-Up

```
for (int i = 1; i <= target; ++i) {
    for (int j = 0; j < ways.size(); ++j) {
        if (ways[j] <= i) {
            dp[i] += dp[i - ways[j]];
        }
    }
}</pre>
```

#### Similar Problems

```
70. Climbing Stairs Easy
```

# Top-Down

```
int result = climbStairs(n-1, memo) + climbStairs(n-2, memo);
```

```
return memo[n] = result;
```

## Bottom-Up

```
for (int stair = 2; stair <= n; ++stair) {
   for (int step = 1; step <= 2; ++step) {
      dp[stair] += dp[stair-step];
   }
}</pre>
```

62. Unique Paths Medium

## Top-Down

```
int result = UniquePaths(x-1, y) + UniquePaths(x, y-1);
return memo[x][y] = result;
```

#### Bottom-Up

```
for (int i = 1; i < m; ++i) {
   for (int j = 1; j < n; ++j) {
      dp[i][j] = dp[i][j-1] + dp[i-1][j];
   }
}</pre>
```

1155. Number of Dice Rolls With Target Sum Medium

```
for (int rep = 1; rep <= d; ++rep) {
  vector<int> new_ways(target+1);
  for (int already = 0; already <= target; ++already) {
    for (int pipe = 1; pipe <= f; ++pipe) {
        if (already - pipe >= 0) {
            new_ways[already] += ways[already - pipe];
            new_ways[already] %= mod;
        }
    }
  }
  ways = new_ways;
}
```

#### Note

Some questions point out the number of repetitions, in that case, add one more loop to simulate every repetition.

- 688. Knight Probability in Chessboard Medium
- 494. Target Sum Medium
- 377. Combination Sum IV Medium
- 935. Knight Dialer Medium
- 1223. Dice Roll Simulation Medium
- 416. Partition Equal Subset Sum Medium
- 808. Soup Servings Medium
- 790. Domino and Tromino Tiling Medium
- 801. Minimum Swaps To Make Sequences Increasing
- 673. Number of Longest Increasing Subsequence Medium
- 63. Unique Paths II Medium
- 576. Out of Boundary Paths Medium
- 1269. Number of Ways to Stay in the Same Place After Some Steps Hard
- 1220. Count Vowels Permutation Hard

# Merging Intervals

Problem List: https://leetcode.com/list/55aj8s16

Generate problem statement for this pattern

#### Statement

Given a set of numbers find an optimal solution for a problem considering the current number and the best you can get from the left and right sides.

## Approach

Find all optimal solutions for every interval and return the best possible answer.

```
// from i to j
dp[i][j] = dp[i][k] + result[k] + dp[k+1][j]
```

Get the best from the left and right sides and add a solution for the current position.

#### Top-Down

```
for (int k = i; k <= j; ++k) {
    result = max(result, topDown(nums, i, k-1) + result[k] + topDown(nums, k+1, j));
}
return memo[/*state parameters*/] = result;</pre>
```

#### Bottom-Up

```
for(int l = 1; l < n; l + +) {
    for(int i = 0; i < n - 1; i + +) {
        int j = i + 1;
        for(int k = i; k < j; k + +) {
            dp[i][j] = max(dp[i][j], dp[i][k] + result[k] + dp[k + 1][j]);
        }
    }
}
return dp[0][n - 1];

for(int l = 1; l < n; l + +) {
    for(int i = 0; i < n - 1; i + +) {
        int j = i + 1;
    }
}</pre>
```

#### Similar Problems

1130. Minimum Cost Tree From Leaf Values Medium

```
for (int l = 1; l < n; ++l) {
    for (int i = 0; i < n - 1; ++i) {
        int j = i + 1;
        dp[i][j] = INT_MAX;
        for (int k = i; k < j; ++k) {
            dp[i][j] = min(dp[i][j], dp[i][k] + dp[k+1][j] + maxs[i][k] *
        maxs[k+1][j]);
        }
    }
}</pre>
```

- 96. Unique Binary Search Trees Medium
- 1039. Minimum Score Triangulation of Polygon Medium
- 546. Remove Boxes Medium
- 1000. Minimum Cost to Merge Stones Medium
- 312. Burst Balloons Hard

## Top-Down

```
for (int k = i; k <= j; ++k) {
    result = max(result, topDown(nums, i, k-1, memo) + (i-1 >= 0 ? nums[i-1] :
1) * nums[k] * (j+1 < nums.size() ? nums[j+1] : 1) + topDown(nums, k+1, j, memo));
}
return memo[i][j] = result;</pre>
```

#### Bottom-Up

```
for(int l = 1; l < n; l++) {
    for(int i = 0; i < n-l; i++) {
        int j = i+l;
        for(int k = i; k <= j; k++) {
            dp[i][j] = max(dp[i][j], (((k>i && k>0) ? dp[i][k-1] : 0) + (i>0 ? nums[i-1] : 1) * nums[k] * (j<n-1 ? nums[j+1] : 1) + ((k<j && k<n-1) ? dp[k+1][j] : 0)));
        }
    }
} return dp[0][n-1];</pre>
```

375. Guess Number Higher or Lower II Medium

# **DP** on Strings

Problem List: https://leetcode.com/list/55afh7m7

General problem statement for this pattern can vary but most of the time you are given two strings where lengths of those strings are not big

#### Statement

Given two strings s1 and s2, return some result.

# Approach

Most of the problems on this pattern requires a solution that can be accepted in  $O(n^2)$  complexity.

```
// i - indexing string s1
// j - indexing string s2
for (int i = 1; i <= n; ++i) {
   for (int j = 1; j <= m; ++j) {
      if (s1[i-1] == s2[j-1]) {
        dp[i][j] = /*code*/;
}</pre>
```

If you are given one string s the approach may little vary

```
for (int l = 1; l < n; ++l) {
   for (int i = 0; i < n-l; ++i) {
      int j = i + l;
      if (s[i] == s[j]) {
         dp[i][j] = /*code*/;
      } else {
        dp[i][j] = /*code*/;
    }
}</pre>
```

#### 1143. Longest Common Subsequence Medium

```
for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= m; ++j) {
        if (text1[i-1] == text2[j-1]) {
            dp[i][j] = dp[i-1][j-1] + 1;
        } else {
            dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }
    }
}</pre>
```

#### 647. Palindromic Substrings Medium

```
for (int l = 1; l < n; ++l) {
    for (int i = 0; i < n-l; ++i) {
        int j = i + l;
        if (s[i] == s[j] && dp[i+1][j-1] == j-i-1) {
            dp[i][j] = dp[i+1][j-1] + 2;
        } else {
            dp[i][j] = 0;
        }
    }
}</pre>
```

- 516. Longest Palindromic Subsequence Medium
- 1092. Shortest Common Supersequence Medium
- 72. Edit Distance Hard
- 115. Distinct Subsequences Hard
- 712. Minimum ASCII Delete Sum for Two Strings Medium
- 5. Longest Palindromic Substring Medium

# **Decision Making**

Problem List: https://leetcode.com/list/55af7bu7

The general problem statement for this pattern is forgiven situation decide whether to use or not to use the current state. So, the problem requires you to make a decision at a current state.

#### Statement

Given a set of values find an answer with an option to choose or ignore the current value.

# Approach

If you decide to choose the current value use the previous result where the value was ignored; vice-versa, if you decide to ignore the current value use previous result where value was used.

```
// i - indexing a set of values
// j - options to ignore j values
for (int i = 1; i < n; ++i) {
   for (int j = 1; j <= k; ++j) {
      dp[i][j] = max({dp[i][j], dp[i-1][j] + arr[i], dp[i-1][j-1]});
      dp[i][j-1] = max({dp[i][j-1], dp[i-1][j-1] + arr[i], arr[i]});</pre>
```

```
}
```

198. House Robber Easy

```
for (int i = 1; i < n; ++i) {
   dp[i][1] = max(dp[i-1][0] + nums[i], dp[i-1][1]);
   dp[i][0] = dp[i-1][1];
}</pre>
```

- 121. Best Time to Buy and Sell Stock Easy
- 714. Best Time to Buy and Sell Stock with Transaction Fee Medium
- 309. Best Time to Buy and Sell Stock with Cooldown Medium
- 123. Best Time to Buy and Sell Stock III Hard
- 188. Best Time to Buy and Sell Stock IV Hard