(ii) Bisection Method vs Regula-Falsi Method

Point	Bisection Method	Regula-Falsi Method
Type of Method	Cuts the interval in half each time.	Uses a smarter guess with interpolation.
Speed	Slow but steady.	Faster than Bisection.
Accuracy	Always finds the root (guaranteed).	Faster but may not always converge.
Complexity	Very simple to understand and use.	Slightly more complex.
Best For	Beginners and simple problems.	Faster solutions with fewer steps.

(i) Gauss Elimination Method vs Gauss-Seidel Iterative Method

Point	Gauss Elimination	Gauss-Seidel
Type of Method	Direct: Solves in fixed steps.	Iterative: Improves guess repeatedly.
Speed	Slow for large systems.	Faster for large systems.
Memory	Needs more memory to store data.	Needs less memory.
Convergence	Always works (if no division by zero).	Works only for special matrices.
Best For	Small systems with exact answers.	Large systems needing approximate answers.

Inverse interpolation
ka matlab hai, jab humein kisi function ki result (output) milti hai, aur hum usse uska input (value) dhundhte hain.
Ex. Inverse interpolation ka ek asaan example hai: "Agar aapko kisi given time par kisi object ki speed pata karni ho, aur aapko alag-alag time par speed ke data points mile ho, toh aap inverse interpolation se woh speed nikaal sakte hain."

$$x = x_1 + \frac{(y - y_1)(x_2 - x_1)}{(y_2 - y_1)}$$

Briefly discuss the terms accuracy, precision and significant digits with suitable example of each.

Accuracy

1. Definition: aapka measurement asli value ke mukable kitna sahi hai
2. Example: Agar thermometer paani ko 100.1°C show karta h
   jabki asli mein wo 100°C pe boil ho raha hai, to yeh accurate hai

Precision

1. aapka measurement ek dusre element se kitna close hai
2. Example: Agar scale baar baar kisi object ka weight 50.123 kg, 50.125 kg, aur 50.124 kg show karta h
   to yeh precise hai kyunki sab measurements ek dusre ke bohot paas hain.

Significant Digits (Significant Figures)

1. Definition: Significant digits number ki precision dikhata hai leading aur trailing zeros ko chod kar.
2. Example: Number 0.004560 mein significant digits hain 4, 5, 6, 0, h Total 4 significant digits hain. Starting ke zeros count nahi hote.

Differentiate between direct methods and iterative methods for the solutions of linear algebraic equation with the help of suitable examples. 5

OR

Compare direct methods and iterative methods of solving linear algebraic equations. Give merits and demerits of each. Give *one* name of the methods for each category i.e. direct and indirect methods.

Direct Methods vs. Iterative Methods

1. Direct Methods:

- Kya hai: Direct methods ek fixed number of steps mein problem ko solve karte hain, jisse exact answer milta hai (agar rounding errors na ho).
- Example: Gaussian Elimination method.

Faayde:

- Yeh exact solution dete hain (rounding errors ke bina).
- Chhoti aur medium problems ke liye yeh achhe hote hain.

Nuksaan:

- Badi problems ke liye yeh method slow ho sakta hai.
- Zyada memory chahiye hoti hai kyunki yeh pura data use karta hai.

2. Iterative Methods:

- Kya hai: Iterative methods ek initial guess (andaaza) se shuru karte hain aur dheere-dheere answer ko improve karte hain, jab tak humara required answer nahi milta.
- Example: Jacobi Method

Faayde:
- Badi aur complex problems ke liye yeh method fast aur useful hota hai.
- Kam memory use karta hai

Nuksaan:
- Kabhi-kabhi yeh method answer tak pahuch nahi pata ya bohot slow hota hai.
- Answer approximate hota hai aur yeh shuru ke guess par depend karta hai.

https://chatgpt.com/c/670fcdd5-a82c-8006-87f8-1c5fe87709fc

Use the Euler's method to obtain the approximate value of $y(0.5)$ for the solution of the initial value problem $y' = 1 + y^2$, $y(0) = 0$. Take $h = 0.1$.

Here's how to use Euler's method to approximate y(0.5) for the given initial value problem:

Given:

- Differential equation: `y' = 1 + y²`
- Initial condition: `y(0) = 0`
- Step size: `h = 0.1`
- Target value: `y(0.5)`

Steps:

1. **Initialize:**
   - `x0 = 0` (starting point)
   - `y0 = 0` (initial value)
   - `h = 0.1` (step size)
2. **Iterate:**
   - We want to find `y(0.5)`, so we'll need 5 iterations (since `0 + 5 * 0.1 = 0.5`).

   Iteration 1:

   - `x1 = x0 + h = 0 + 0.1 = 0.1`
   - `y1 = y0 + h * (1 + y0²) = 0 + 0.1 * (1 + 0²) = 0.1`

   Iteration 2:

   - `x2 = x1 + h = 0.1 + 0.1 = 0.2`
   - `y2 = y1 + h * (1 + y1²) = 0.1 + 0.1 * (1 + 0.1²) = 0.201`

   Iteration 3:

   - `x3 = x2 + h = 0.2 + 0.1 = 0.3`
   - `y3 = y2 + h * (1 + y2²) = 0.201 + 0.1 * (1 + 0.201²) = 0.304`

   Iteration 4:

   - `x4 = x3 + h = 0.3 + 0.1 = 0.4`
   - `y4 = y3 + h * (1 + y3²) = 0.304 + 0.1 * (1 + 0.304²) = 0.412`

   Iteration 5:

   - `x5 = x4 + h = 0.4 + 0.1 = 0.5`
   - `y5 = y4 + h * (1 + y4²) = 0.412 + 0.1 * (1 + 0.412²) = 0.524`

Result:

The approximate value of `y(0.5)` using Euler's method with `h = 0.1` is **0.524**.

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Forward and backward difference and central difference

First derivative:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

or

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

or

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second derivative:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

newton forward difference ya interpolation

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \dots$$

where

$$u = \frac{x - x_0}{h}$$

eurlers method

$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$

central difference

First derivative:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second derivative:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

bisection

$$x_{n+1} = \frac{a_n + b_n}{2}$$

$$f(x_{n+1}) = f\left(\frac{a_n + b_n}{2}\right)$$

difference table or dividend difference

$$\Delta f(x_0) = f(x_1) - f(x_0)$$

$$\Delta^2 f(x_0) = \Delta f(x_1) - \Delta f(x_0)$$

$$\Delta^3 f(x_0) = \Delta^2 f(x_1) - \Delta^2 f(x_0)$$

$$\Delta^n f(x_0) = \Delta^{n-1} f(x_n) - \Delta^{n-1} f(x_{n-1})$$

Rangula

falsi and secant

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

shift operator

$$Ef(x) = f(x+h)$$

$$E^n f(x) = f(x+nh)$$

$$E^{-1} f(x) = f(x-h)$$

absolute, relative

Absolute Error:

$$\text{Absolute Error} = |x_{\text{true}} - x_{\text{approx}}|$$

Relative Error:

$$\text{Relative Error} = \frac{|x_{\text{true}} - x_{\text{approx}}|}{|x_{\text{true}}|}$$

newton raphson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

maclaurins series

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \dots$$

in