Reflections and maximal quotients in Coxeter groups

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Abstract

We present a formula relating the set of left descents of an element of a Coxeter group with the sets of left descents of its projections on maximal quotients indexed by simple right descents.

1 Introduction

A reflection t in a Coxeter system (W, S) is a conjugate of a generator, and t is a left descent of an element $w \in W$ if $\ell(tw) < \ell(w)$, where ℓ is the length function. Donoting by $T_L(w)$ the set of left descents of w, it is well known that the right weak order \leq_R on W is isomorphic to the set $\{T_L(w) : w \in W\}$ ordered by inclusion. In this note we prove the following formula:

$$T_L(v) = T_L(u) \cup \bigcup_{s \in D_R(u^{-1}v)} T_L(v^{S\setminus\{s\}}),$$

for all $u, v \in W$ such that $u \leq_R v$. Here $D_R(w)$ is the set of right simple descent (right descents of length one) of w, and w^J is the element of minimal length in the coset wW_J , for any $J \subseteq S$. We then deduce some corollaries; for example our result implies that the set $\{w^{S\setminus\{s\}}: s \in D_R(w)\}$ admits a join and it is precisely w (Corollary 2.4), i.e.

$$w = \bigvee_{s \in D_R(w)} w^{S \setminus \{s\}},$$

for all $w \in W$. We are regarding the poset (W, \leq_R) as a complete meet-semilattice, for which the joins may not exist.

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We now fix notation and recall some definitions useful for the rest of the paper. We refer to the book of Björner and Brenti [1] for Coxeter groups.

Let (W, S) be a Coxeter system, i.e. a presentation of the group W given by a set S of involutive generators and relations encoded by a Coxeter matrix $m: S \times S \to \{1, 2, ..., \infty\}$. A Coxeter matrix over S is a symmetric matrix which satisfies the following conditions for all $s, t \in S$:

- 1. m(s,t) = 1 if and only if s = t;
- 2. $m(s,t) \in \{2,3,...,\infty\}$ if $s \neq t$.

The presentation (W, S) of the group W is then the following:

$$\begin{cases} \text{generators} : S; \\ \text{relations} : (st)^{m(s,t)} = e, \end{cases}$$

for all $s,t \in S$, where e denotes the identity in W. The Coxeter matrix m attains the value ∞ at (s,t) to indicate that there is no relation between the generators s and t. The class of words expressing an element of W contains words of minimal length; the length function $\ell:W\to\mathbb{N}$ assigns to an element $w\in W$ such minimal length. The identity e is represented by the empty word and then $\ell(e)=0$. A reduced word or reduced expression for an element $w\in W$ is a word of minimal length representing w. The set of reflections of (W,S) is defined by $T:=\{wsw^{-1}:w\in W,s\in S\}$. If $J\subseteq S$ and $v\in W$, we let

$$W^{J} := \{ w \in W : \ell(w) < \ell(ws) \ \forall \ s \in J \},$$

$${}^{J}W := \{ w \in W : \ell(w) < \ell(sw) \ \forall \ s \in J \},$$

$$D_{L}(v) := \{ s \in S : \ell(sv) < \ell(v) \},$$

$$D_{R}(v) := \{ s \in S : \ell(vs) < \ell(v) \}.$$

With W_J we denote the subgroup of W generated by $J \subseteq S$; in particular, $W_S = W$ and $W_\varnothing = \{e\}$. We let \leq_R and \leq be the right weak order and the Bruhat order on W, respectively. The covering relations of the right weak order are characterized as follows: $u \triangleleft_R v$ if and only if $\ell(u) < \ell(v)$ and $u^{-1}v \in S$. For $w \in W$, define

$$T_L(w) := \{ t \in T : \ell(tw) < \ell(w) \},$$

 $T_R(w) := \{ t \in T : \ell(wt) < \ell(w) \}.$

The right weak order has the following property (see [1, Proposition 3.1.3]):

$$u \leqslant_R v \Leftrightarrow T_L(u) \subseteq T_L(v).$$

The covering relations of the Bruhat order are characterized as follows: $u \triangleleft v$ if and only if $\ell(u) = \ell(v) - 1$ and $u^{-1}v \in T$. The posets (W, \leq_R) and

 (W, \leq) are graded with rank function ℓ and $(W, \leq_R) \hookrightarrow (W, \leq)$. For $J \subseteq S$, each element $w \in W$ factorizes uniquely as $w = w^J w_J$, where $w^J \in W^J$, $w_J \in W_J$ and $\ell(w) = \ell(w_J) + \ell(w^J)$; see [1, Proposition 2.4.4]. We consider the idempotent function $P^J : W \to W$ defined by

$$P^J(w) = w^J,$$

for all $w \in W$. This function is order preserving for the Bruhat order (see [1, Proposition 2.5.1]). We let

$$P^{(s)} := P^{S \setminus \{s\}}.$$

for all $s \in S$. In a similar way, one defines an order preserving function $Q^J: (W, \leq) \to (W, \leq)$ by setting $Q^J(w) = {}^Jw$, where $w = w'_J{}^Jw$ with $w'_J \in W_J$, ${}^Jw \in {}^JW$ and $\ell(w) = \ell(w'_J) + \ell({}^Jw)$. Moreover we have that P^J and Q^I commute, for all $I, J \subseteq S$ (see [5, Lemma 2.6]). In the following lemma we state some known results about W^J useful in the sequel (see [2, Sec. 3]).

Lemma 1.1. Let $s \in S$ and $w \in W^J$. Then exactly one of the following three possibilities occurs:

- 1. $s \in D_L(w)$. In this case $sw \in W^J$.
- 2. $s \notin D_L(w)$ and $sw \in W^J$.
- 3. $s \notin D_L(w)$ and $sw \notin W^J$. In this case sw = wr for a unique $r \in J$.

For the following proposition see [1, Corollary 2.6.2].

Proposition 1.2. Let $u, v \in W$. Then

$$u \leqslant v \Leftrightarrow P^{(s)}(u) \leqslant P^{(s)}(v) \ \forall s \in D_R(u).$$

Now we recall some results about the standard geometric representation of a Coxeter group. Let (W,S) be a Coxeter system with Coxeter matrix m. Let V be the free \mathbb{R} -vector space on the set $\{\alpha_s:s\in S\}$ and $\sigma_s\in \operatorname{End}(V)$ defined by $\sigma_s(v)=v-2(\alpha_s|v)\alpha_s$, where the bilinear form $(\cdot|\cdot)$ is defined by setting $(\alpha_s|\alpha_t):=-\cos\left(\frac{\pi}{m(s,t)}\right)$, for all $s,t\in S$; then the assignment $w\mapsto\sigma_w:=\sigma_{s_1}\cdots\sigma_{s_k}$, where $s_1\cdots s_k$ is any reduced word of $w\in W$, provides a faithful representation $W\to\operatorname{GL}(V)$ (see, e.g. $[1,\operatorname{Ch}, 4]$). Then we let $w(v):=\sigma_w(v)$, for all $v\in V, w\in W$. Let $\Phi:=\{w(\alpha_s):w\in W, s\in S\}, V^+:=\{\sum_{s\in S}a_s\alpha_s:a_s\geqslant 0\,\forall\,s\in S\}\setminus\{0\}$ and $V^-:=\{\sum_{s\in S}a_s\alpha_s:a_s\leqslant 0\,\forall\,s\in S\}\setminus\{0\}$; then it is well known that $\Phi=\Phi^+\uplus\Phi^-=\{\alpha_t:t\in T\}\uplus\{-\alpha_t:t\in T\}, \Phi^+\subseteq V^+ \text{ and }\Phi^-\subseteq V^-$. It holds that $t(v)=v-2(\alpha_t|v)\alpha_t$ and $w(\alpha_t)\in\Phi^-$ if and only if wt< w, for all $w\in W, t\in T$ (see, e.g. $[1,\operatorname{Sec},4.4]$). Moreover, if $s\in S$ and $t\in T\setminus\{s\}$, then $s(\alpha_t)=\alpha_{sts}$.

2 Reflections and maximal quotients

In this section we prove the main result of this paper, i.e. the formula mentioned in the introduction. The next lemma is needed for the proof of the theorem.

Lemma 2.1. Let $w \in W$, $t \in T_R(w)$ and $r \in S \setminus D_R(w)$. Then

$$wrw^{-1} \in S \implies rtr \in T_R(w).$$

Proof. Let $t \in T_R(w)$, $r \in S \setminus D_R(w)$, and $r' := wrw^{-1} \in S$. We have that $w(\alpha_t) \in \Phi^-$ and $w(\alpha_{rtr}) = wr(\alpha_t) = r'w(\alpha_t)$. Assume $w(\alpha_{rtr}) \in \Phi^+$; hence $r'(-w(\alpha_t)) \in \Phi^-$. Since $-w(\alpha_t) \in \Phi^+$, we have that $t' \in T_R(r')$, where $\alpha_{t'} := -w(\alpha_t)$. This implies that t' = r' and then $-w(\alpha_t) = \alpha_{r'}$, i.e. $w^{-1}(\alpha_{r'}) \in \Phi^-$, which is a contradiction, since r'w = wr > w. Therefore $w(\alpha_{rtr}) \in \Phi^-$, i.e. $rtr \in T_R(w)$.

We are ready to give a proof of our theorem, which states a formula for the set of descents of elements of any Coxeter system.

Theorem 2.2. Let (W,S) be a Coxeter system and $u,v \in W$ such that $u \leq_R v$. Then

$$T_L(v) = T_L(u) \cup \bigcup_{s \in D_R(u^{-1}v)} T_L(v^{S\setminus \{s\}}).$$

Proof. We first prove the result for u=e. If we consider the factorization $v=v^Jv_J$, the inclusion $\bigcup_{s\in D_R(v)}T_L(v^{S\setminus\{s\}})\subseteq T_L(v)$ is straightforward. We prove the opposite inclusion by induction on $\ell(v)$. If v=e the result is obvious. Let $\ell(v)>0$ and $r\in D_L(P^{(z)}(v))$, for some $z\in D_R(v)$. In particular, $r\in D_L(v)$. If $s\in D_R(v)$ we have that

$$\begin{split} P^{(s)}(rv) &= P^{(s)}(Q^{\{r\}}(v)) = \\ Q^{\{r\}}(P^{(s)}(v)) &= \left\{ \begin{array}{ll} P^{(s)}(v), & \text{if } rP^{(s)}(v) > P^{(s)}(v); \\ rP^{(s)}(v), & \text{if } rP^{(s)}(v) < P^{(s)}(v). \end{array} \right. \end{split}$$

Hence, by our inductive hypothesis

$$T_L(rv) \subseteq \bigcup_{s \in D_R(rv)} T_L(P^{(s)}(rv))$$

$$= \bigcup_{\substack{s \in D_R(rv) \\ rP^{(s)}(v) > P^{(s)}(v)}} T_L(P^{(s)}(v)) \quad \cup \bigcup_{\substack{s \in D_R(rv) \\ rP^{(s)}(v) < P^{(s)}(v)}} T_L(rP^{(s)}(v)).$$

If $rP^{(s)}(v) > P^{(s)}(v)$ we have that $rP^{(s)}(v) = P^{(s)}(v)r'$, for some $r' \in S$, otherwise $rP^{(s)}(v) \in W^{S\setminus\{s\}}$ and rv > v, a contradiction. Hence, by

the left version of Lemma 2.1, $rP^{(s)}(v) > P^{(s)}(v)$ implies $rT_L(P^{(s)}(v))r = T_L(P^{(s)}(v))$. The relation $rP^{(s)}(v) < P^{(s)}(v)$ implies that $rT_L(rP^{(s)}(v))r = T_L(P^{(s)}(v)) \setminus \{r\}$. Therefore

$$T_{L}(v) = \{r\} \cup rT_{L}(rv)r$$

$$\subseteq \{r\} \cup \bigcup_{\substack{s \in D_{R}(rv) \\ rP^{(s)}(v) > P^{(s)}(v)}} rT_{L}(P^{(s)}(v))r \cup \bigcup_{\substack{s \in D_{R}(rv) \\ rP^{(s)}(v) < P^{(s)}(v)}} rT_{L}(rP^{(s)}(v))r$$

$$= \{r\} \cup \bigcup_{\substack{s \in D_{R}(rv) \\ s \in D_{R}(rv)}} T_{L}(P^{(s)}(v)) \subseteq \bigcup_{\substack{s \in D_{R}(v) \\ s \in D_{R}(v)}} T_{L}(P^{(s)}(v)),$$

since $D_R(rv) \subseteq D_R(v)$ and $r \in D_L(P^{(z)}(v))$ for some $z \in D_R(v)$. This proves the case u = e.

Let $u \leq_R v$ and define $w := u^{-1}v$. For $s \in D_R(w)$ we claim that $uT_L(P^{(s)}(w))u^{-1} \subseteq T_L(P^{(s)}(uw))$. We prove the claim by induction on $\ell(u)$. If $\ell(u) = 0$ the claim is obvious. Let u > e and $r \in D_L(u)$. Then $uT_L(P^{(s)}(w))u^{-1} \subseteq rT_L(P^{(s)}(ruw))r$ by the inductive hypothesis. We have that $rP^{(s)}(ruw) > P^{(s)}(ruw)$, otherwise $r \in D_L(ruw)$, a contradiction. Hence $rT_L(P^{(s)}(ruw))r = T_L(rP^{(s)}(ruw)) \setminus \{r\}$. As in the previous case, we have that

$$P^{(s)}(ruw) = \begin{cases} P^{(s)}(uw), & \text{if } rP^{(s)}(uw) > P^{(s)}(uw); \\ rP^{(s)}(uw), & \text{if } rP^{(s)}(uw) < P^{(s)}(uw). \end{cases}$$

Therefore

$$T_L(rP^{(s)}(ruw)) \setminus \{r\} = \begin{cases} rT_L(P^{(s)}(uw))r, & \text{if } rP^{(s)}(uw) > P^{(s)}(uw); \\ T_L(P^{(s)}(uw)) \setminus \{r\}, & \text{if } rP^{(s)}(uw) < P^{(s)}(uw). \end{cases}$$

As before, we have that $rT_L(P^{(s)}(uw))r = T_L(P^{(s)}(uw))$ if $rP^{(s)}(uw) > P^{(s)}(uw)$. Then $uT_L(P^{(s)}(w))u^{-1} \subseteq rT_L(P^{(s)}(ruw))r \subseteq T_L(P^{(s)}(uw))$; this concludes the proof of the claim.

Assume that u > e. By the case u = e and our claim we obtain

$$T_L(v) = T_L(u) \cup uT_L(w)u^{-1}$$

$$= T_L(u) \cup \bigcup_{s \in D_R(w)} uT_L(P^{(s)}(w))u^{-1}$$

$$\subseteq T_L(u) \cup \bigcup_{s \in D_R(w)} T_L(P^{(s)}(uw)).$$

Since $u \leq_R v$ implies $T_L(u) \subseteq T_L(v)$, we have also the inclusion $T_L(u) \cup \bigcup_{s \in D_R(u^{-1}v)} T_L(P^{(s)}(v)) \subseteq T_L(v)$ and this concludes the proof of the theorem.

In the following example we consider a Coxeter system of type B_4 . The set of generators is $S = \{s_0, s_1, s_2, s_3\}$; the Coxeter matrix takes the values $m(s_0, s_1) = 4$ and $m(s_1, s_2) = m(s_2, s_3) = 3$.

Example 2.3. In type B_4 , let $u = s_2 s_3 s_2$ and $v = s_2 s_3 s_2 s_1 s_0 s_2 s_3$. Then $u^{-1}v = s_1 s_0 s_2 s_3$, $D_R(v) = \{s_0, s_2, s_3\}$ and $D_R(u^{-1}v) = \{s_0, s_3\}$. We have that

- $T_L(u) = \{s_2, s_3, s_2s_3s_2\},\$
- $T_L(v^{S\setminus\{s_3\}}) = T_L(s_1s_2s_3) = \{s_1, s_1s_2s_1, s_1s_2s_3s_2s_1\},$
- $T_L(v^{S\setminus\{s_0\}}) = T_L(s_3s_2s_1s_0) = \{s_3, s_2s_3s_2, s_1s_2s_3s_2s_1, s_3s_2s_1s_0s_1s_2s_3\},$
- $T_L(v) = \{s_1, s_2, s_3, s_1s_2s_1, s_2s_3s_2, s_1s_2s_3s_2s_1, s_3s_2s_1s_0s_1s_2s_3\}.$

An element $w \in W$ is uniquely determined by its descent set; hence, by Theorem 2.2 for u = e, we have that w is uniquely determined by the set $\{w^{S\setminus\{s\}}: s\in D_R(w)\}$. This fact can be also deduced from Proposition 1.2. An additional information provided by Theorem 2.2 is the following one, which concerns the join operation in the right weak order.

Corollary 2.4. Let
$$w \in W$$
. Then $w = \bigvee_{s \in D_R(w)} w^{S \setminus \{s\}}$.

Proof. Clearly $T_L(w^J) \subseteq T_L(w)$, for all $J \subseteq S$, or, equivalently, $w^J \leq_R w$, for all $J \subseteq S$. Hence the set $\{w^{S\setminus \{s\}}: s \in D_R(w)\}$ is bounded and then it admits a join w' (see [3, Theorem 1.5]). Moreover, by [4, Proposition 2.14] we have that $T_L(w') = T_L(w)$ and this concludes the proof.

By repeated use of Theorem 2.2, we obtain the following result.

Corollary 2.5. Let $s_1s_2\cdots s_k$ be a reduced expression for $w\in W$. Then

$$T_L(w) = \bigcup_{i=0}^{k-1} T_L((s_1 \cdots s_{k-i})^{S \setminus \{s_{k-i}\}}).$$

We provide now a version of Theorem 2.2 involving double quotients.

Corollary 2.6. Let $w \in W$. Then

$$T_L(w) = \bigcup_{s \in D_R(w)} \bigcup_{r \in D_L(w^{S \setminus \{s\}})} w_{r,s} T_L(S \setminus \{r\} w^{S \setminus \{s\}}) w_{r,s}^{-1},$$

where, for $r, s \in S$, $w_{r,s} \in W_{S \setminus \{r\}}$ and $w^{S \setminus \{s\}} = w_{r,s}(^{S \setminus \{r\}}w^{S \setminus \{s\}})$.

Proof. For $r, s \in S$ we set ${}^{(r)}w^{(s)} := {}^{S\setminus\{r\}}w^{S\setminus\{s\}}$. By Theorem 2.2, for u = e, and its right version, we have that

$$T_{L}(w) = \bigcup_{s \in D_{R}(w)} \{w^{S \setminus \{s\}} t(w^{S \setminus \{s\}})^{-1} : t \in T_{R}(w^{S \setminus \{s\}})\}$$

$$= \bigcup_{s \in D_{R}(w)} \bigcup_{r \in D_{L}(w^{S \setminus \{s\}})} \{w^{S \setminus \{s\}} t(w^{S \setminus \{s\}})^{-1} : t \in T_{R}({}^{(r)}w^{(s)})\}$$

$$= \bigcup_{s \in D_{R}(w)} \bigcup_{r \in D_{L}(w^{S \setminus \{s\}})} \{w_{r,s} t w_{r,s}^{-1} : t \in T_{L}({}^{(r)}w^{(s)})\},$$

because
$$t \in T_R({}^{(r)}w^{(s)})$$
 if and only if $({}^{(r)}w^{(s)})t({}^{(r)}w^{(s)})^{-1} \in T_L({}^{(r)}w^{(s)})$, and $w^{S\setminus\{s\}}({}^{(r)}w^{(s)})^{-1} = w_{r,s}$.

In the following example we consider a Coxeter system of type H_3 . The set of generators is $S = \{s_1, s_2, s_3\}$; the Coxeter matrix takes the values $m(s_1, s_2) = 3$, $m(s_2, s_3) = 5$ and $m(s_1, s_3) = 2$.

Example 2.7. In type H_3 , let $w = s_3s_1s_2s_3s_1s_2s_1$. Then $D_R(w) = \{s_1, s_2\}$, $w_1 := w^{S\setminus\{s_1\}} = s_3s_1s_2s_3s_2s_1$ and $w_2 := w^{S\setminus\{s_2\}} = s_3s_1s_2s_3s_1s_2$. We have that $D_L(w_1) = D_L(w_2) = \{s_1, s_3\}$ and the double quotients are

- $\bullet \ ^{(s_1)}w^{(s_1)} = s_1s_2s_3s_2s_1, \quad ^{(s_3)}w^{(s_1)} = s_3s_2s_3s_2s_1,$
- $\bullet \ ^{(s_1)}w^{(s_2)} = s_1s_2s_3s_2, \quad ^{(s_3)}w^{(s_2)} = s_3s_2s_3s_1s_2.$

Then $w_{s_1,s_1} = s_3$, $w_{s_3,s_1} = s_1$, $w_{s_1,s_2} = s_3 s_2$, $w_{s_3,s_2} = s_1$ and

$$T_L(w) = s_3\{s_1, s_1s_2s_1, s_1s_2s_3s_2s_1, s_1s_2s_3s_2s_1, s_1s_2s_3s_1s_2s_1s_3s_2s_1\}s_3$$

$$\cup s_1\{s_3, s_2s_3s_2, s_3s_2s_3, s_2s_3s_2s_3s_2, s_3s_2s_3s_1s_2s_1s_3s_2s_3\}s_1$$

$$\cup s_3s_2\{s_1, s_1s_2s_1, s_1s_2s_3s_2s_1, s_1s_2s_3s_2s_3\}s_2s_3$$

$$\cup s_1\{s_3, s_3s_2s_3, s_2s_3s_2s_3s_2, s_3s_2s_3s_1s_2s_1s_3s_2s_3\}s_1.$$

We now deduce from Theorem 2.2 the statement of [6, Lemma 4.2]. Such result has inspired our main formula and it is related to the shellability of Coxeter complexes.

Corollary 2.8. Let $u, v, w \in W$ be such that $u \leq_R v$ and $u \leq_R w$. If $P^{(s)}(v) = P^{(s)}(w)$ for all $s \in D_R(u^{-1}v)$, then $v \leq_R w$.

Proof. Since $u \leq_R w$, we have that $T_L(u) \subseteq T_L(w)$. Moreover $T_L(v^{S\setminus\{s\}}) = T_L(w^{S\setminus\{s\}}) \subseteq T_L(w)$ for all $s \in D_R(u^{-1}v)$. Hence by Theorem 2.2 we obtain $T_L(v) \subseteq T_L(w)$, which is equivalent to $v \leq_R w$.

The next proposition states that the union in Theorem 2.2 is minimal in respect to its indexing set.

Proposition 2.9. Let $w \in W$ and $r, s \in D_R(w)$. Then

$$wrw^{-1} \in T_L(w^{S\setminus\{s\}}) \Rightarrow r = s.$$

Proof. Let $r, s \in D_R(w)$, $r \neq s$ and $J_s := S \setminus \{s\}$. Consider the factorization $w = w^{J_s}w_{J_s}$. Then $r \in D_R(w_{J_s})$, because $r \in J_s$, i.e. $w_{J_s}(\alpha_r) \in \Phi^-$. Moreover, $w(\alpha_r) \in \Phi^-$ and, if $t_r := wrw^{-1}$, then $\alpha_{t_r} = -w(\alpha_r)$. We have that

$$(w^{J_s})^{-1}(\alpha_{t_r}) = -(w^{J_s})^{-1}w(\alpha_r)$$

= $-w_{J_s}(\alpha_r) \in \Phi^+,$

i.e.
$$wrw^{-1} \notin T_L(w^{S\setminus\{s\}})$$
.

In Example 2.7 we note that the union in Corollary 2.6 may fail to be minimal in respect to its indexing set $\{(s,r) \in S \times S : s \in D_R(w), r \in D_L(w^{S\setminus\{s\}})\}$. In fact $s_3s_2T_L(^{(s_1)}w^{(s_2)})s_2s_3 \subseteq s_3T_L(^{(s_1)}w^{(s_1)})s_3$.

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