MANIPAL UNIVERSITY JAIPUR

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INSPIRED BY LIFE

Name	**Ankit Shaw**
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Set-I:

Answer-1:

Inclusion-exclusion principle:

The inclusion-exclusion principle is a fundamental idea in combinatorics, a branch of mathematics concerned with combination, counting, and organization. This approach makes it easier to calculate the size of a union of many sets, particularly when such sets overlap. It is particularly useful because it takes into consideration the fact that just combining the set sizes will count the elements in their intersections more than once.

For two sets, A and B, the principle states:

$|A \cup B| = |A| + |B| - |A \cap B|$

1. $|A \cup B|$ is the number of elements in either A or B.
2. $|A|$ and $|B|$ are the numbers of elements in A and B, respectively.
3. $|A \cap B|$ is the number of elements in both A and B.

The inclusion-exclusion concept is used in many disciplines, including computer science, probability, statistics, and combinatorial problems where you must count the number of things that have a specific property without going overboard.

Let's take A as a set of students passed in mathematics.

Let’s take B as a set of students passed in data structure.

$|A| = 625$,{ “ Students who passed Mathematics” }

$|B| = 525$, { “Students who passed Data Structure” }

$|A \cup B|$ = represents the total number of students in the class 1000 who passed in at least one of the students.

By using the inclusion-exclusion principle in this case, we can determine the proportion of students who succeed in both data structure and mathematics. “$|A \cap B|$” :

$|A \cap B| = |A| + |B| - |A \cup B| = 625 + 525 – 1000 = 150$

Hence we got the number of students who passed in both of the subjects is 150 here.

Students passed only in Mathematics: $|A| - |A \cap B| = 625 – 150 = 475$

Students passed only in Data structure: $|B| - |A \cap B| = 525 – 150 = 375$

So, here we found 475 students only pass in mathematics and 375 in data structure.

Solutions-2: (a)

$\int \log log\ x\ dx$,

For $\int \log log\ x\ dx$ **,** by method of I.L.A.T.E rule.

Let s = log(x) , it is a lo function

Let dt = dx

Here doing the integration by parts formula $\int s\ dt\ =$ s*t - $\int t\ ds$,

=> $\int \log log\ (x)\ dx = x.\log(x) - \int x.\frac{1}{x}\ dx =$

=> $x.\log(x) - \int 1dx\ = x.\log(x) - x + C$

So, $\int \log log\ x\ dx\ =\ x.log(x) - x\ +\ C$

Answer: 2(b)

(i) $\frac{2+n+n^2}{2+3n+4n^2}$

Putting n = ∞, $\frac{2+n+ n^2}{2+3n+4n^2}$ => $\frac{2+\infty+ \infty^2}{2+3\infty+4\infty^2} = \frac{\infty}{\infty}$

hence we are not getting any solution here.

Now, here take common 'n^2' from equation, $\frac{n^2(\frac{2}{n^2} + \frac{1}{n} + 1)}{n^2(\frac{2}{n^2} + \frac{3}{n} + 4\ )}$

Now, we can cancel 'n^2' from equation, $\frac{(\frac{2}{n^2} + \frac{1}{n} + 1)}{(\frac{2}{n^2} + \frac{3}{n} + 4\ )}$

Then we put ∞ in equation, $\frac{(\frac{2}{\infty^2} + \frac{1}{\infty} + 1)}{(\frac{2}{\infty^2} + \frac{3}{\infty} + 4\ )}$, here 2 upon ∞ , is approaching 0 "zero", so we can take it as complete 0 "zero" here.

=> $\frac{(0 + 0 + 1)}{(0 + 0 + 4\ )}\ = \frac{1}{4}$

Hence, $\frac{2+n+n^2}{2+3n+4n^2}\ =\ \frac{1}{4}$

(ii) $\frac{2x^2-3x-2}{x-2}$,

If we directly put n = 2 in equation it gives 0 "ZERO" in denominator, $\frac{(2*2^2)-(3*2)-2}{(2-2)}$

So we now factorize the numerator ($2x^2 - 3x - 2$) in ($2x^2 + 1)(x - 2)$.

Here we put new substitute in numerator, $\frac{(2x^2+1)(x-2)}{x-2}$

Now we cancel the $(x - 2)$ from numerator and denominator, $(2x^2 + 1)$.

Here we put n = 2 now , hence in this way get the solution.

$(2 * 2^2 + 1) = 5$

Hence, $\frac{2x^2-3x-2}{x-2} = 5$.

Solution-3:

Given system of equations are,

3x + y + 2z = 3

2x – 3y – z = -3

x + 2y + z = 4

Now we form a coefficient matrix A and constant matrix B from given equation:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

Using Cramer's rule to solve the system of equation, we proceed further with,

$x = \frac{det(A_x)}{det(A)}$, $y = \frac{det(A_y)}{det(A)}$, $z = \frac{det(A_z)}{det(A)}$ …equ(I)

=> det(A) = 3((-3)(1) – (-1)(3)) – 1((2)(1)-(-1)(1)) + 2((2)(2) – (-3)(1))

=> det(A) = -3 -3 + 14 = 8

=> $det(A_x)$ =3((−3)(1) − (−1)(2)) −1((−3)(1) − (−1)(4)) + 2((−3)(2) − (−3)(4))

=> $det(A_x)$ = - 3 – 3 + 12 = 8

=> $det(A_y)$ = 3((−3)(1) − (−1)(4)) − 3((2)(1) − (−1)(1)) + 2((2)(4) − (−3)(1))

=> $det(A_y)$ = 3 – 9 + 12 = 16

=> $det(A_z)$ = 3((−3)(4) − (−3)(2)) − 1((2)(4) − (−3)(1)) + 3((2)(2) − (−3)(1))

=> $det(A_z)$ = - 18 – 11 + 21 = -8

Therefore,

determinant of matrix A is 8

determinant of matrix A_x is 8

determinant of matrix A_y is 16

determinant of matrix A_z is -8

Substituting the value of the determinants in the equation(I)

=> x = $\frac{det(A_x)}{det(A)} = \frac{8}{8} = 1$

=> y = $\frac{det(A_y)}{det(A)} = \frac{16}{8} = 2$

=> z = $\frac{det(A_z)}{det(A)} = \frac{-8}{8}$ = -1

So, the solution of the equation is x = 1 , y = 2 , z = 1.

Set-II:

Answer-4:

(i) (p V q) V (~p)

p	q	p V q	~p	(p V q) V (~p)
F	F	F	T	T
F	T	T	T	T
T	F	T	F	T
T	T	T	F	T

Since (p V q) V (~p) is true , hence (p V q) V (~p) tautology.

(ii) ~[p V (~p)]

p	~p	p V (~p)	~[p V (~p)]
F	T	T	F

T	F	T	F

Since ~[p V (~p)] is false, hence ~[p V (~p)] is a contradiction.

Answer-5:

(i) $\sqrt{3} + i$

Here in modulus of complex no. z = a – bi is as $|z| = \sqrt{a^2 + b^2}$.

In given equation $\sqrt{3} + i$, a $= \sqrt{3}$ and b = 1

Modulus of $|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$

For a $= \sqrt{3}$ and b = 1

Argument θ $= (\frac{b}{a}) = \frac{1}{\sqrt{3}}$

θ $= \frac{\pi}{6}$ or 30 degrees, then $\tan(30^o) = \frac{1}{\sqrt{3}}$.

Polar form of complex no. is as r(θ $+ i \sin sin$ θ) , where 'r' is the modulus and θ is the argument.

For $\sqrt{3} + i$, polar form is: 2($30^o + i \sin sin\, 30^o$)

Hence the complex no. $\sqrt{3} + i$ with the polar form is 2($30^o + i \sin sin\, 30^o$).

(ii) $1 - i$

Here in modulus of complex no. z = a – bi is as $|z| = \sqrt{a^2 + b^2}$.

In given equation $1 + i$, a = 1 and b = -1

Modulus of $|z| = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$

For a = 1 and b = 1

Argument θ $= (\frac{b}{a}) = \frac{-1}{1}$ = -1

θ $= -\frac{\pi}{4}$ or (- 45) degrees, then $\tan(-45^o) = -1$.

Polar form of complex no. is as r(θ $+ i \sin sin$ θ) , where 'r' is the modulus and θ is the argument.

For 1 - i , polar form is: $\sqrt{2}\,((-\ 45^{o})\ +\ i\sin sin\,(-\ 45^{o}))$

Hence the complex no. 1- i with the polar form is

$\sqrt{2}\,((-\ 45^{o})\ +\ i\sin sin\,(-\ 45^{o}))$

<u>Answer-6(a):</u>

Given x = a(θ + sin sin θ) and y = a(1 - cos cos θ),

Differentiating y with respect to θ :

=> y = a(1 - cos cos θ)

=> $\frac{dy}{d\theta} = a.\frac{d}{d\theta}(1 - \cos cos\,(\theta)) = a.\sin sin\,(\theta)$

Differentiating x with respect to θ:

=> x = a(θ + sin sin θ)

=> $\frac{dx}{d\theta} = a.\frac{d}{d\theta}(\theta + \sin sin\,\theta) = a(1 + \cos cos\,\theta)$

Now we do calculate $\frac{dy}{dx}$ through the chain rule method:

=> $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\theta}{a(1+\cos cos\,\theta)}$

Now here put the values in equation:

When $\theta = \frac{\pi}{2}$, sin $sin\,\frac{\pi}{2}$ = 1 and cos $cos\,\frac{\pi}{2} = 0.$

=> $\frac{dy}{dx}_{\theta=\frac{\pi}{2}}$ => $\frac{a.1}{a(1+0)} = \frac{a}{a} = 1.$

Hence, $\frac{dy}{dx}\ at\ \theta = \frac{\pi}{2}\ is\ 1.$

<u>Answer 6(b):</u>

Given,

$\vec{f} = x.y^{2}\hat{i} + 2x^{2}.y.z\hat{j} + 3y.z^{2}\hat{k}$ at point (1, -1,1).

Divergence of vector field $\vec{f} = P\hat{i} + Q\hat{j} + R\hat{k}$ is as

=> div($\vec{f}$) = $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

here $\vec{f}$: P = xy^{2} , Q = $2x^{2}yz$, R = $-\ 3yz^{2}$

partial derivatives :

=> $\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(x.y^2\right) = y^2$

=> $\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}\left(2x^2yz\right) = 2x^2z$

=> $\frac{\partial R}{\partial z} = \frac{\partial}{\partial z}\left(-3yz^2\right) = -6yz$

Putting values at (1, -1,1):

=> $\frac{\partial P}{\partial x} = (-1)^2 = 1$

=> $\frac{\partial Q}{\partial y} = 2(1)^2(1) = 2$

=> $\frac{\partial R}{\partial z} = -6(-1)(1) = 6$

Now the divergence at (1, -1,1) is:

=> div($\vec{f}$) = 1 + 2 + 6 = 9

Here now curl of vector div($\vec{f}$) = $P\hat{i} + Q\hat{j} + R\hat{k}$ is as:

=> curl($\vec{f}$) = ($\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$) $\hat{i}$ + ($\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}$) $\hat{j}$ + ($\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$) $\hat{k}$

partial derivatives :

=> ($\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$) = $\frac{\partial}{\partial z}\left(-3yz^2\right)$ - $\frac{\partial}{\partial y}\left(2x^2yz\right) = -3 + 2 = -1$

=> ($\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}$) = $\frac{\partial}{\partial x}\left(x.y^2\right)$ - $\frac{\partial}{\partial z}\left(-3yz^2\right)$ = 0 + 0 = 0

=> ($\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$) = $\frac{\partial}{\partial y}\left(2x^2yz\right) - \frac{\partial}{\partial x}\left(x.y^2\right) = 4xyz - 2xy$

Again we put at (1, -1,1):

=> ($\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$) = -3$(1)^2 - 2(1)^2(-1) = -3 + 2 =-1$

=> ($\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}$) $= 0$

=> ($\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$) = 4(1)(-1)(1) - 2(1)(-1) = - 4 + 2 = -2

Hence, the curl($\vec{f}$) = (-1) $\hat{i}$ + (0) $\hat{j}$ + (-2) $\hat{k}$.