

# Arrays 1D

1) Prefix Sum

2) Kadane's Algo.  $\rightarrow$  max subarray Sum.

$$\text{arr}[5] = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 10 & -4 & 6 \\ \hline \end{array}$$

$$\text{Prefix\_Sum}[5] = \begin{array}{|c|c|c|c|c|} \hline 1 & 3 & 13 & 9 & 15 \\ \hline \end{array}$$

$$\hookrightarrow \text{Pr}[i] \Rightarrow \text{Sum of } a[0] \dots a[i]$$

$$\# \quad l=1, r=4 \quad \text{Sum} \Rightarrow \text{Pr}[4] - \text{Pr}[0]$$

# arr[3][4]

	0	1	2	3
0				
1				
2				

Q1 Given a matrix of size  $n \times m$ . Given  $q$  queries  
Find the sum of the submatrix. You  
are given Top left & Bottom right  
coordinates.

Submatrix  $\Rightarrow$  Part of a matrix

$\Rightarrow$  Defined by two 2 points.

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	7	2	3
4	3	2	6	9

TL: (2,1)

BR: (4,2)

Sum = 26

Brute force: Traverse through the submatrix  
and find sum for each query.

TC for each query:  $O(M \times N)$

$$N \leq 10^3$$

$$M \leq 10^3$$

$$Q \leq 10^3$$

$$TC: O(QMN)$$

$$SC: O(1)$$

$DP[n][m] = \text{Prefix Sum matrix}$

$DP[i][j] \Rightarrow$  Submatrix sum where  
TL  $\Rightarrow (0,0)$  & BR  $= (i,j)$

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

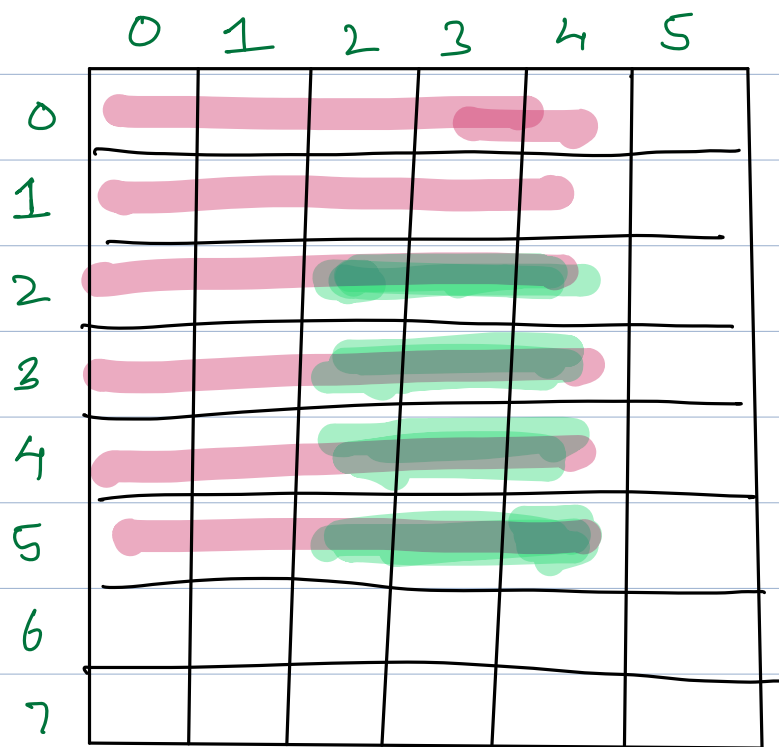
$DP[2][1] \Rightarrow 25$

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

$DP[3][2]$

$DP[0][1]$

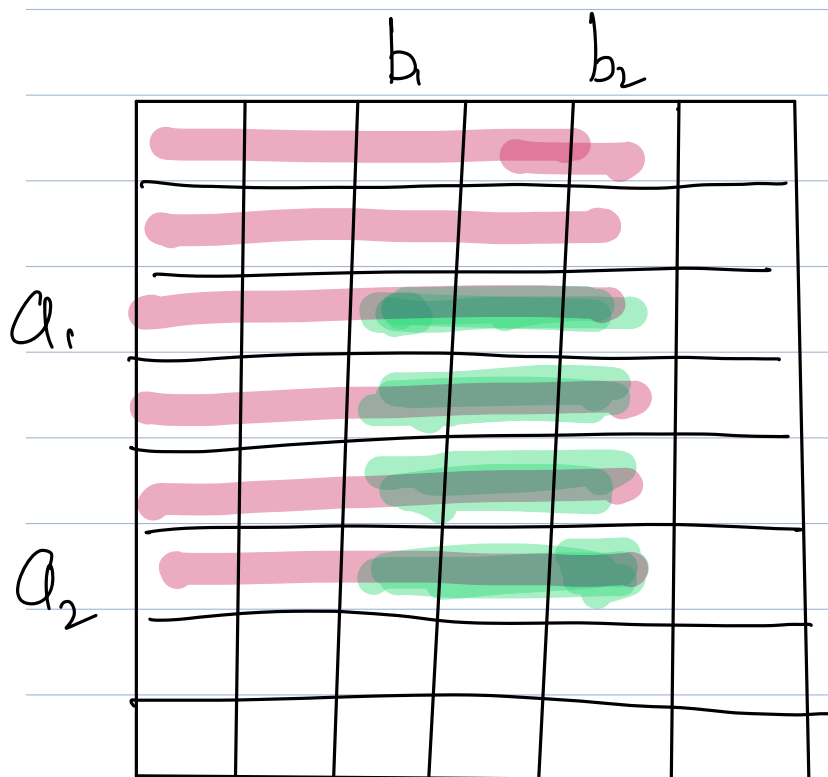
	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9



TL  $\Rightarrow (2, 2)$

BR  $\Rightarrow (5, 4)$

$$\text{Sum} \Rightarrow P_f[5][4] - P_f[1][4] - P_f[5][1] + P_f[1][1]$$



$$Q = \begin{matrix} & \text{TL} & \text{BR} \\ (a_1, b_1) & (a_2, b_2) \end{matrix}$$

$$\text{Sum} = P_f[a_2][b_2] - P_f[a_1-1][b_2] - P_f[a_2][b_1-1] + P_f[a_1-1][b_1-1]$$

#  $a_1 = 0$

TL  $\Rightarrow (0, 2)$

B  $\Rightarrow (3, 4)$

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
6						
7						

if  $(a_1 == 0)$   
don't need

need to deduct  
the top  
sub-matrix

or

add top

#  $b_1 = 0$

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
6						
7						

TL  $\Rightarrow (4, 0)$

BR  $\Rightarrow (5, 2)$

$$\# \text{ ans} \Rightarrow \text{Pf } \underline{[a_2]} [b_2]$$

$$\text{Pf } \underline{[a_{i-1}]} [b_2] \Rightarrow \text{check if } (a_1 \neq 0)$$

$$\text{Pf } [a_2] [b_{i-1}] \Rightarrow \text{check if } (b_1 \neq 0)$$

$$\text{Pf } \overset{+}{[a_{i-1}]} [b_{i-1}] \Rightarrow \text{check if } (a_1 \neq 0 \text{ or } b_1 \neq 0)$$

$$\underline{\underline{\text{TC: } O(1)}}$$

$$\text{SC: } O(M \times N)$$

$a_0$	$b_0$	$c_0$
$a_1$	$b_1$	$c_1$
$a_2$	$b_2$	$c_2$

↓  
Column wise Prefix-Sum

$a_0$	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

$a_0$	$b_0$	$c_0$
$a_0 + a_1$	$b_0 + b_1$	$c_0 + c_1$
$a_0 + a_1 + a_2$	$b_0 + b_1 + b_2$	$c_0 + c_1 + c_2$

→

$a_0$	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

↓  
Row wise prefix Sum

TC : Column wise Prefix Sum + Row-wise Prefix sum

$$O(m \times n) + O(n \times m) = O(mn)$$

TC of Original Ques  $\Rightarrow O(mn + 9)$



Q2 Given a matrix  $[N \times M]$ , find max submatrix sum, where submatrix ~~starts~~ starts from row=0 & ends at row=n-1

	0	1	2	3	4	5
0	-3	2	3	4	-6	4
1	5	5	-5	2	2	-7
2	-4	-3	1	-1	1	4

↓

-2	4	-1	5	-3	1
----	---	----	---	----	---

⇒ Apply Kadane's

⇒ [8]

Tc:  $O(m \times n)$   
 Sc:  $O(m)$

Q2 Given a matrix  $[N \times M]$ , find max submatrix sum, where submatrix ~~starts~~ starts from row=0 & end anywhere.

	0	1	2	3	4	5
0	2	-4	1	3	-1	2
1	1	3	2	-7	3	3
2	0	-1	1	3	4	-7
3	1	-2	-6	4	-4	6

# Start end

row 0 0 

2	-4	1	3	-1	2
---	----	---	---	----	---

  
=5

row 0 1 

3	-1	3	-4	2	5
---	----	---	----	---	---

  
⇒ 8

row 0 2 

3	-2	4	-1	6	-2
---	----	---	----	---	----

  
⇒ 10

row 0 3 

4	-3	-2	3	2	4
---	----	----	---	---	---

  
⇒ 9

Pseudo

$S \Rightarrow 0$

$Sum[m] \Rightarrow 0$ ;  $max\_sum = Indices.min;$

for (int end = 0; end < n; end++) {

$O(m)$  for (int j = 0; j < m; j++) {

$Sum[j] = Sum[j] + arr[end][j];$   
}

$O(m)$   $\Leftarrow$  int curr\_sum  $\Rightarrow$  Kadane(Sum, m);  
 $max\_sum \Rightarrow \max(max\_sum, curr\_sum);$   
}  
return max\_sum;

Tc:  $(nm)$   
Sc:  $O(m)$

Q2 Given a matrix  $[N \times M]$ , find max submatrix sum.

	0	1	2	3	4	5
0	2	-4	1	3	-1	2
1	1	3	2	-7	3	3
2	0	-1	1	3	4	-7
3	1	-2	-6	4	-4	6

Start

end

1

1

1	3	2	-7	3	3
---	---	---	----	---	---

$\Rightarrow 6$

1

2

1	2	3	-4	7	-4
---	---	---	----	---	----

$\Rightarrow 9$

1

3

Pseudo

max\_sum = Integer.MIN;

for (int s = 0; s < n; s++) {

sum[m] => 0;

for (int end = s; end < n; end++) {

O(m) for (int j = 0; j < m; j++) {

} sum[j] = sum[j] + arr[end][j];

O(m) if int curr\_sum >= kadane(sum, m);  
max\_sum => max(max\_sum, curr\_sum);

}

return max\_sum;

TC:  $O(n \times n \times m) = O(n^2 m)$

SC:  $O(m)$