



# A quantitative model of work-related fatigue: background and definition

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Fatigue has been identified as a major risk factor for shiftworkers. However, few organizations or governments currently manage work-related fatigue in any systematic or quantitative manner. This paper outlines an approach to managing fatigue that could improve shiftwork management. Using shift start and finish times as an input, the outlined model quantifies work-related fatigue on the basis of its known determinants; that is shift timing and duration, work history and the biological limits on sleep length at specific times of day. Evaluations suggest that work-related fatigue scores correlate very highly with sleep-onset latency, neurobehavioural impairment and subjective sleepiness. The model is useful in that it allows comparisons to be made between rosters independent of shift length and timing or the total number of work hours. Furthermore, unlike many models of sleepiness and fatigue, individual's sleep times are not required as hours of work are used as the input. It is believed the model provides the potential quantitatively to link the effects of shiftwork to specific organizational health and safety outcomes. This simple approach may be especially critical at a time when many organizations view longer and more flexible hours from their employees as an immediate productivity gain.

## 1. Introduction

Many of the occupational health and safety risks associated with shiftwork are attributable, in part, to the effects of sleepiness and fatigue (Torsvall and Åkerstedt 1987, Lauber and Kayton 1988, Mitler *et al.* 1994, Rosekind *et al.* 1999). Previous studies have concluded that shiftwork, and nightwork in particular, significantly reduces the duration of sleep (Tilley *et al.* 1981, Torsvall *et al.* 1981, 1989, Åkerstedt *et al.* 1990, 1991, Folkard and Barton 1993). This reduction is associated with increased sleepiness (Gander and Graeber 1987, Kecklund *et al.* 1994), reduced alertness (Borbely 1982, Åkerstedt and Folkard 1990, Dijk *et al.* 1992, Czeisler *et al.* 1994), impaired neurobehavioural performance (Folkard and Monk 1979, Tepas *et al.* 1981, Tilley *et al.* 1982, Porcu *et al.* 1998) and a potentially higher risk of fatigue-related accident and injury (Lauridsen and Tonnesen 1990, Laundry and Lees 1991, Gold *et al.* 1992, Åkerstedt 1994). However, few policy-makers or organizations currently attempt to manage work-related fatigue in a systematic or quantitative manner.

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The scarcity of systematic and quantitative attempts to manage fatigue may reflect the lack of simple, yet functional, tools available for use in the workplace. Such tools may not currently be in high use because it is difficult to generalize the results of laboratory-based studies to a workplace environment. Experimental studies typically oversimplify the complex psycho-social context in which shiftwork occurs. These studies frequently preclude many of the typical social activities that compete with rest and recovery. Similarly, relatively few shift schedules have been exhaustively modelled in experimental settings. Given the lack of consistent findings for specific shift schedules, it can be difficult for organizations or individuals to risk extrapolating specific findings to different shift schedules.

Not surprisingly, the difficulty in generalizing from experimental to applied settings constitutes a significant problem for shiftwork policy-makers trying to improve workplace health and safety. An applied modelling approach that enables organizations to estimate the work-related fatigue *in situ* would, therefore, provide an invaluable tool to improve shiftwork management. It would be useful in evaluating potential rosters prior to their implementation. It would also be useful for organizations attempting to delineate the cost consequences and statistical relationships between hours-of-work and health and safety outcomes for their own organizations.

This paper illustrates an initial step in developing the framework of such a modelling approach. While it is acknowledged that no model can ever hope to predict work-related fatigue completely, it is believed that there is currently sufficient theoretical understanding to produce a systematic quantitative model.

## 2. Model description

### 2.1. Theoretical considerations

Any attempt to develop a general predictive model must rely on the broad conceptual determinants of work-related fatigue. At the broadest level, hours-of-work can be viewed as a time-varying function. At any point in time the individual exists in one of two states: (work or non-work). From this perspective, any shift schedule can be expressed as a square wave function oscillating in a continuous manner between work and non-work. It is possible to consider each individual employee as a 'black box' into which a specific shift 'signal' is input and from which a continuously varying 'signal' is output. Using this approach, the shift schedule can be viewed as the 'input signal' and fatigue as the 'output signal'. Similarly, an organizational perspective of shiftwork can be seen as the signal processing of the aggregated group of black boxes.

The mathematical function that links the input and output functions of a signal processing system is commonly referred to as the 'transfer function'. The modelling approach to shiftwork and work-related fatigue outlined in this paper is, in essence, an attempt formally to articulate the transfer function that links the input and output to the 'black box' conceptualization of an employee (or organization). The methodology outlined is not theoretically novel, but rather an attempt to develop a utilitarian approach on the basis of current theoretical understanding of work-related fatigue. (For a mathematical description of the model, see Appendix A.)

Using the conceptual approach outlined above, the fatigue in an individual at a specific point of time can be thought to reflect the balance between two competing forces, i.e. forces producing 'fatigue' and forces reversing the effects of fatigue, leading to 'recovery'. Fatigue (and recovery) are likely to increase as a function of

the duration of the work (or non-work) period but are also dependent on the amounts and timing of wake (or sleep) periods in the previous week. For the purposes of this exercise, the duration, circadian timing and recency of work periods will be considered as 'fatiguing' forces while the duration, circadian timing and recency of non-work periods will be considered as 'recovery' forces.

The amount of sleep predicted to occur within a specific recovery period is based on a statistical distribution of sleep. Given a certain work/non-work pattern, the sleep which an individual will achieve can be predicted with surprising accuracy. However, if sleep does not occur across a period in which it is expected, then the actual work-related fatigue score would be higher than the model output.

Quite obviously, the processes outlined above are a simplification of the interacting components of fatigue. It is clear from recent work that many factors contribute and interact to influence fatigue as a broad construct (Matthews and Desmond 1998). There has been considerable discussion in the literature of fatigue and also on the importance of specific task factors such as automation (Desmond 1998). Such task factors are becoming increasingly important as developments in technology allow for work environments that require less direct control by humans.

Recent research into driving indicated that the performance of vehicle operators and, by inference, operators of any equipment or task, can alter effort in response to changing task requirements (Desmond and Matthews 1997). However, the ability to increase effort in response to challenging task requirements may become more difficult at higher levels of fatigue. Therefore, it is possible to argue that task demands such as automation do not specifically influence fatigue; they interact with fatigue to affect the overall performance of an individual.

From this perspective, different tasks are likely to be differentially susceptible to a given level of fatigue or alertness. For example, at a given level of fatigue, a task with low cognitive demands such as photocopying may create less impairment than one with high cognitive demands task such as air traffic control. It is therefore the interaction of impaired neurological function (fatigue) with the fatigue sensitive task-factors that determine performance. From this perspective, it may be appropriate to view the fatigue score as an indicator of neurological function and to assign a threshold fatigue level based on the sensitivity of the task to fatigue-related impairment. Thus, in the case of a photocopier operator, a maximum level of fatigue may be set and for an air traffic controller the set maximum may be considerably lower.

Overall, work-related fatigue and task factors interact to affect performance. This model does not attempt to incorporate all of the relevant task factors that interact with fatigue. This current model simply attempts to account for the basic contributors to work-related fatigue, which are discussed below.

## 2.2. Duration and timing of work periods

Previous research has established that fatigue increases as a function of hours of prior wakefulness (Borbely 1982, Daan *et al.* 1984). This increase is not, however, a simple monotonic function of hours of wakefulness. Rather, the function that links hours-of-wakefulness to fatigue levels shows a complex relationship in which there are significant linear (hours of prior wakefulness) and sinusoidal (circadian) components (Borbely 1982, Folkard and Åkerstedt 1991). On the basis of previous work (Czeisler *et al.* 1980a, Zulley and Wever 1982, Johnson *et al.* 1992) it is assumed that the circadian component of fatigue mapped closely to the core temperature

rhythm, that is it was approximately sinusoidal with 24 h and was allocated an arbitrary amplitude of 1.0 unit. To simplify the calculations, we further assumed a maximum fatigue rate of 2.0 units per h occurring at 05:00 hours and a minimum value of 1.0 unit per h at 17:00 hours.

In general, it is probably reasonable to argue that the fatigue value of a work period varies as a function of the duration (Rosa *et al.* 1989, Folkard 1997) and circadian timing (Folkard and Åkerstedt 1991) of the work period. In simple terms, the longer the work period the more fatiguing it is likely to be. In addition, it is inferred that the rate at which fatigue accumulates is likely to be greater when the work period occurs during the subjective night than during the subjective day. The increase in fatigue across a work period is therefore not linear but also dependent on the time of day that the work is occurring.

Similarly, the recovery value of a non-work period is also likely to vary as a function of the duration and time at which it occurs, since the duration and quality of sleep (Czeisler *et al.* 1980a, for a review, see Strogatz 1986) and, by inference, the 'recovery' value show a strong circadian component; thus, the recovery value of a given rest period is likely to vary according to the time at which it occurred. For example, since the amount and quality of sleep varies as a function of the time of sleep onset (Czeisler *et al.* 1980a, Zulley *et al.* 1981, Monk 1987), amount of sleep and, therefore, the recovery value of a 12-h break during subjective night is likely to be greater than during subjective day.

Knowing the circadian timing and duration of work and non-work, the model will enable prediction of the amount of sleep an individual is able to obtain. It is assumed that the recovery function was proportional to the amount of sleep that would be obtained. It is therefore assumed that the recovery function closely follows the sleep propensity curves derived from free-run and forced-desynchrony protocols (Czeisler *et al.* 1980a, b). The recovery function was, therefore, also approximately sinusoidal in shape with 24 h and an amplitude of 1 unit. It is further assumed that the maximum rate of 2.0 units per h occurred at 05:00 hours and the minimum value 1.0 unit per h at 17:00 hours.

This approach, in turn, provides the 'fatigue' and 'recovery' values for a specific work or non-work period. Given that the fatigue level of an individual can be viewed as an algebraic function of the 'fatigue' and 'recovery' functions, it is then possible to calculate the relative fatigue level for an individual on the basis of the shift history of work and non-work periods. By recording only an individual's hours-of-work, it should be possible to determine the work-related fatigue level at any particular point.

### 2.3. Recency of shifts

The timing and duration of work and non-work periods are not the only determinants of work-related fatigue. It is conceptually important to note that the relative contribution of each prior work and non-work period is unlikely to be equivalent. This assumption of the model dictates that more immediate work (or non-work) periods carry greater weight in determining the fatigue level than those that occurred further in the past.

The nature of the function describing the relationship between prior work and non-work periods has, by definition, an empirically observable value. For the purposes of this paper, this function has been arbitrarily defined as having a linear decay from a peak weighting of 1 for the most recent hour to 0 after 7 days or 168 h.

That is, over the period of a week, the value of work or non-work periods reduces linearly and periods that occurred more than 7 days prior do not contribute to the work-related fatigue score calculation.

#### 2.4. *Saturation*

The linear decay function allows the model to give more value to the work and non-work periods that are most important, that is, those which are most recent. The model also incorporates another function to produce more realistic outputs. The saturation function limits the total value of recovery that can be accumulated at any time. In practice, this saturation function does not let recovery be stored beyond full recovery. That is, individuals can only recover from fatigue that has been accumulated and cannot store recovery to offset against potential future fatigue. This saturation of recovery reflects the fact that sleep and recovery cannot be stored because individuals find it difficult to extend sleep beyond 10–11 h, irrespective of the amount of prior wakefulness (Strogatz 1986).

#### 2.5. *A 'token economy' analogy*

The general principles of the model can therefore be explained by conceptualizing a 'token economy' in which employees acquire fatigue or recovery tokens. The token 'value' of a single work or non-work period is dependent on both the duration and timing of that period. In addition, the fatigue or recovery 'value' of tokens that are held will decline over time because recently acquired tokens carry greater value than those gained previously. Furthermore, there is a limit to the total 'value' of recovery tokens held at any point in time. Owing to this limit on the 'value' of recovery tokens held, recovery is said to saturate when this limit is reached.

Therefore, the relative measure of the fatigue level for any individual at any point in time is the 'net worth' of the tokens that an individual holds. Similarly, the aggregate value of fatigue and recovery tokens for all individual employees reflects the total fatigue level of an organization.

### 3. **Illustrated outputs**

The fatigue model attempts to illustrate work-related fatigue as a continuous output function derived from the interaction of a two state (work and non-work) input function. This is accomplished using a transfer function incorporating duration, circadian timing and recency of hours-of-work. By creating a stationary output function for the standard work week, a 'benchmarking' approach can be utilized to compare work-related fatigue scores produced by other shift schedules. It is also useful to assess the effects of changes to rosters by comparing the two rosters as a 'before' and 'after' scenario.

Figures 1 and 2 show standard raster plots of the work-related fatigue scores associated with each of the specific shift schedules outlined. Figure 1 shows the work-related fatigue scores associated with a range of continuous shift patterns commonly worked in Australia. Similarly, figure 2 shows the scores for five international commercial pilot rosters. In both figures the 'A' plot represents the standard work week.

Figure 3 illustrates work-related fatigue histograms for the continuous shift schedules in relation to the proportions of the roster spent at standard, moderate and high work-related fatigue scores. Figure 4 illustrates work-related fatigue histograms for the aviation schedules in relation to the proportions of the roster spent at

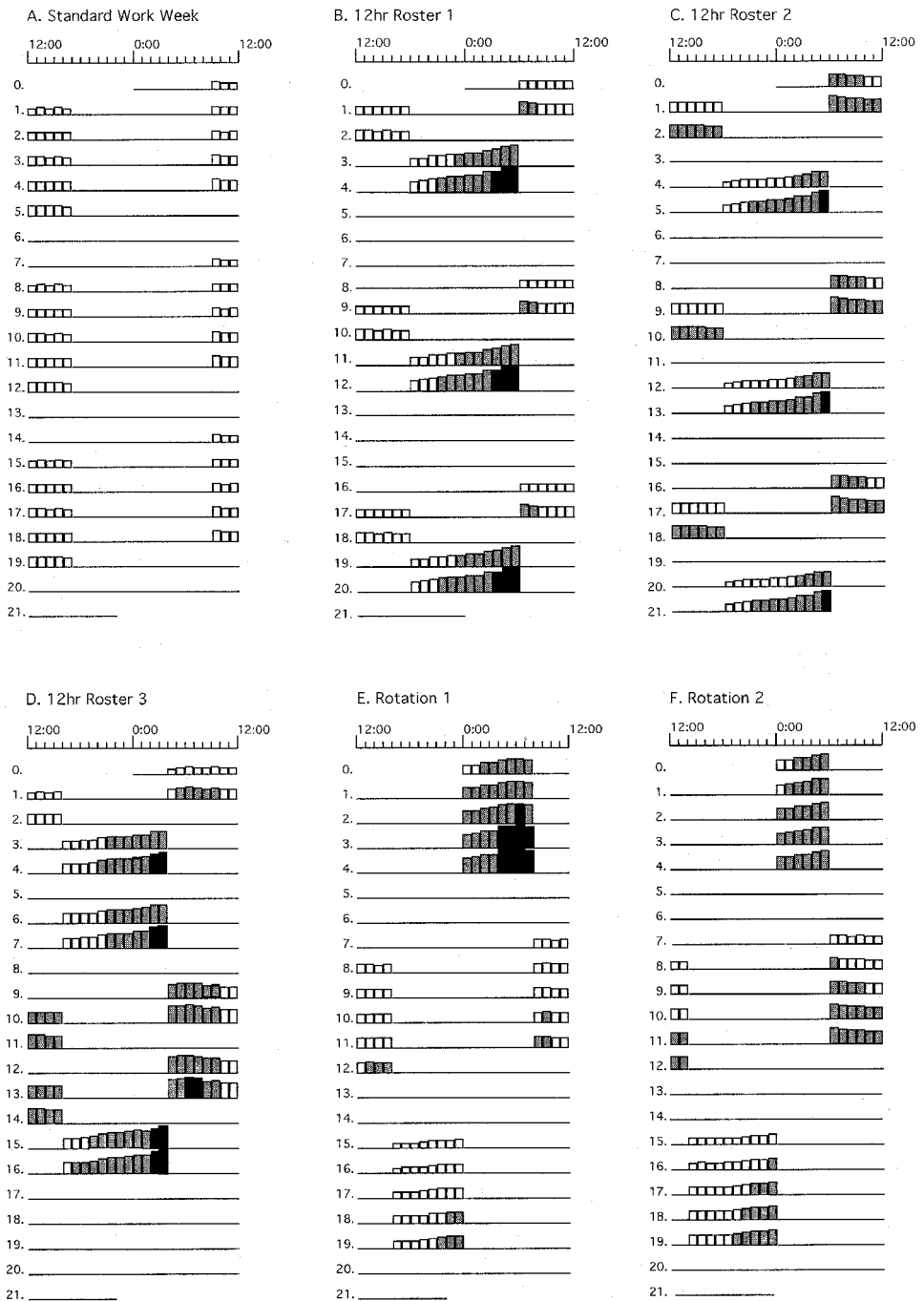


Figure 1. Standard raster plots of a standard work week (Monday–Friday, 09:00–17:00 hours) and five continuous rosters. The white, grey and black bands represent hours of work spent at ‘standard’, ‘moderate’ and ‘high’ levels of work-related fatigue respectively.

standard and moderate scores. Table 1 gives key summary statistics derived from each of the rosters when compared using the model.

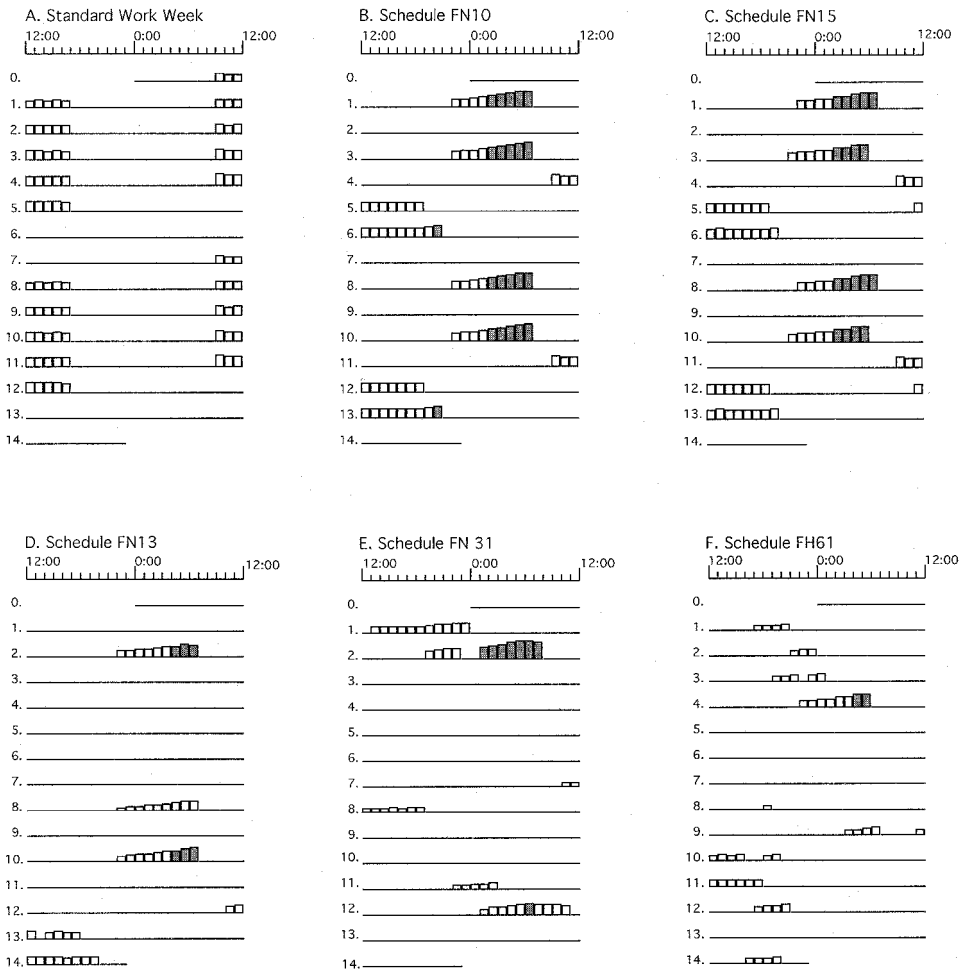


Figure 2. Standard raster plots of a standard work week (Monday–Friday, 09:00–17:00 hours) and five commercial aviation schedules. The white and grey bands represent hours of work spent at ‘standard’ and ‘moderate’ levels of work-related fatigue respectively.

Overall, the work-related fatigue scores for the rosters show considerable variation in the average and peak work-related fatigue scores. Figure 1(b–f) shows that all of the continuous shift systems produce work-related fatigue scores greater than the standard work week and all but 1(f) produces work-related fatigue scores more than twice the peak score of a standard work week at some point in the roster. Similarly, all the aviation schedules produce peak work-related fatigue scores greater than the standard work week despite the relatively small number of hours worked.

### 3.1. Continuous shift schedules

Figure 1 illustrates the work-related fatigue scores associated with five continuous shift patterns. Work-related fatigue scores are represented by three shades: white, grey and black. These three shades reflect the work-related fatigue score for each hour at relative scores of 0–100, 100–200 or >200% of the peak work-related

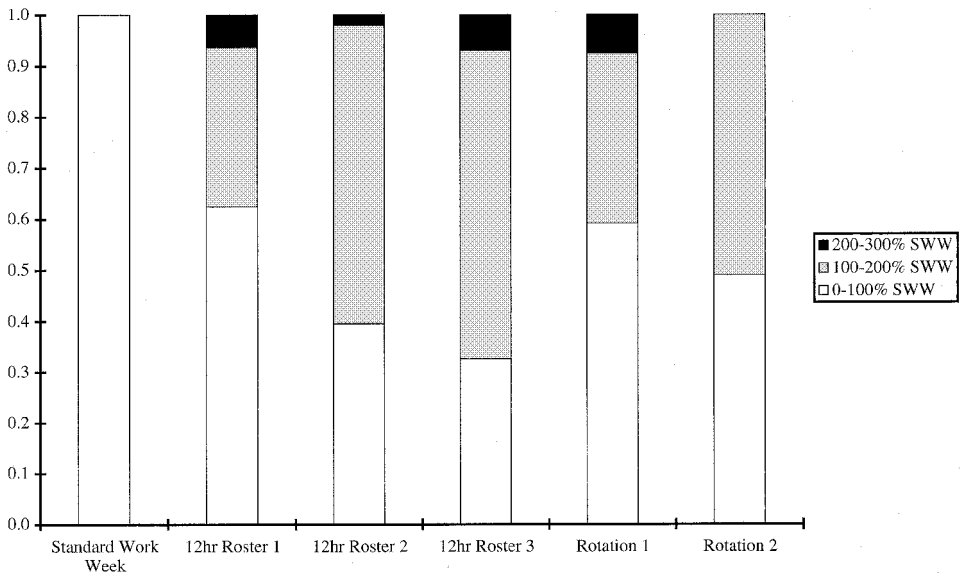


Figure 3. Relative amounts of time spent at 'standard', 'moderate' and 'high' fatigue scores for each roster in figure 1.

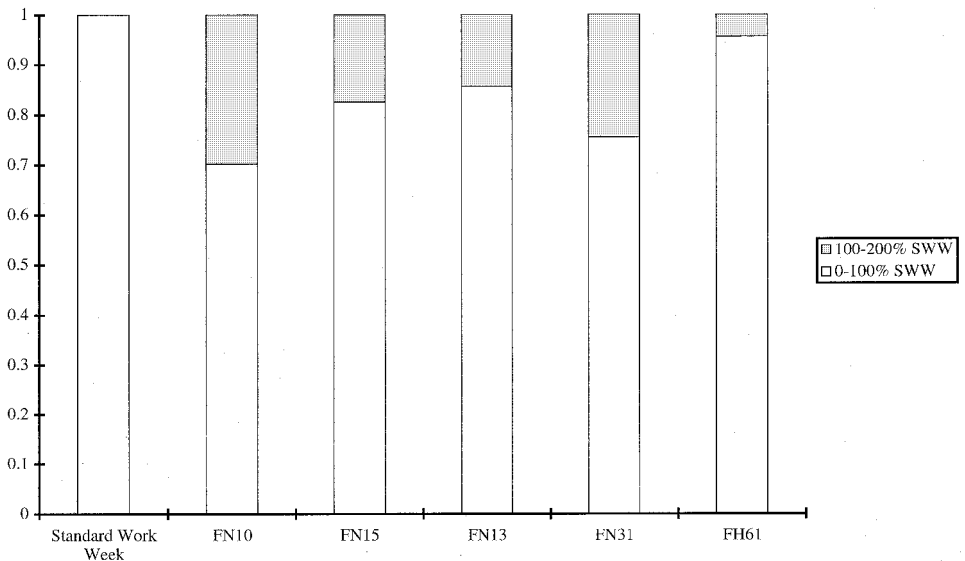


Figure 4. Relative amounts of time spent at 'standard' and 'moderate' fatigue scores for each roster in figure 2.

fatigue score produced by a standard work week. These bands have been assigned as representing standard, moderate and high scores of work-related fatigue.

Although the mean number of hours worked per week for the continuous rosters are similar to that for the standard work week, the range of values produced for the



fatigue measures is broad. The summary statistics from table 1 indicate that a standard 12-h roster, as illustrated in figure 1(b), produces a daily mean work-related fatigue score 57% higher than that for the standard work week. The mean work-related fatigue scores accumulated during each hour worked is 38% higher, the peak work-related fatigue score is 143% higher and there is 6.25% of the roster that is in the high work-related fatigue band. By altering the timing of work and non-work periods but otherwise keeping roster constraints the same, another standard roster as illustrated in figure 1(c) can be observed. As can be seen in table 1, the mean work-related fatigue scores accrued in those hours worked in figure 1(c) is further increased by 6.7% when compared with figure 1(b). However, even though the mean work-related fatigue scores increased due to the redistribution of non-work periods, the maximum peak dropped by 18% and the proportion of the roster producing high work-related fatigue scores dropped to 2.1%.

By observing a 12-h shift system that is commonly used by tanker truck drivers in figure 1(d), it can be seen that further redistribution of work and non-work periods can produce work-related fatigue scores much higher than those illustrated in figure 1(b) and (c). As can be seen in table 1, the roster in figure 1(d) produces a further increase in the mean work-related fatigue scores accrued per day of 36% when compared with 1(c). Additionally, an increase of 18% is observed in the mean work-related fatigue scores accrued per hour of work, the peak work-related fatigue score is raised 12% and the percentage of time spent over the 200% maximum standard work week score is increased to 6.9%.

Figure 1(e) illustrates an 8-h shift system with 24-h coverage. The shifts in the example illustrated are: 00:00–08:00 hours for night shift, 08:00–16:00 hours for day shift and 16:00–00:00 hours for afternoon shift. The number of hours worked per week is therefore identical to the standard work week; however, the timing of the work period rotates 8 h on a weekly basis so that a roster cycle is completed every 3 weeks. Table 1 indicates that the roster illustrated in figure 1(e) produces a 44% increase in accrued daily work-related fatigue scores. Also, the hourly mean of work-related fatigue scores rises by 44%, the peak work-related fatigue score increases by 140% and the percentage of time in the 'high' portion of the roster is 7.5% as opposed to zero. The roster illustrated in figure 1(f) is essentially a variation of the roster in figure 1(e) in which shift length better reflects capacity to tolerate work at particular times of day. That is, shift length is shorter during the subjective night (6 versus 8 h) and longer during the subjective day (10 versus 8 h). As seen in table 1, the roster illustrated in figure 1(f) produces daily accrued work-related fatigue scores 2.5% lower than the roster illustrated in figure 1(e). Additionally, the hourly mean of work-related fatigue scores accrued while working decreases by 2.3%, the peak work-related fatigue score reduces by 23% and the proportion of the roster in the 'high' band is returned back to zero.

### 3.2. Commercial aviation schedules

Figure 2 illustrates the work-related fatigue scores associated with five aviation schedules. Work-related fatigue scores are represented by two shades: white and grey. These two bands reflect the work-related fatigue score for each hour at relative scores of 0–100 or 100–200% of the peak work-related fatigue score produced by a standard work week. These bands have been assigned as representing standard and moderate scores of work-related fatigue.

The number of hours worked per week for all of the aviation schedules is less than for the standard work week. In particular, the schedules illustrated in figure 2(d–f) contain only just over half the hours per week compared with the standard work week, yet the model indicates that these schedules still show moderate level work-related fatigue scores for significant proportions of the shifts. As can be seen in table 1, the rosters illustrated in figure 2(b) and (c) produce higher scores on all summary statistics even considering they have less mean hours per week worked than the standard work week. The level of work-related fatigue scores accumulated per day is 15–20% higher, the hourly mean accumulation for hours worked is 25–28% higher and the peak work-related fatigue score is increased by 51–53% as compared with those produced by the standard work week.

The two schedules above are relatively demanding in terms of the density of hours normally worked over 2 weeks by commercial pilots. However, figure 2(d–f) illustrates that even schedules comprising nearly half the hours of a standard work week can produce comparable hourly means and peak scores due to the structure and timing of work and non-work hours. As can be seen in table 1, the work hours for figures 2(d–f) accumulate daily work-related fatigue scores at levels  $\leq 55\%$  than for the standard work week. However, the mean of hourly work-related fatigue scores accrued for each hour worked are still similar to that for a standard work week due to a high proportion of the hours worked being late at night or early in the morning. Similarly, the peak work-related fatigue scores for these schedules range from 24–72% higher than the maximum for the standard work week.

### 3.3. *Proportionate work-related fatigue scores*

Figure 3 further illustrates the relative work-related fatigue scores produced by the continuous rosters in figure 1. Each roster has been divided into the number of hours spent working at 0–100, 100–200 and 200–300% of the maximum work-related fatigue produced by a standard work week. The number of hours spent at each of the standard, moderate and high work-related fatigue levels were added up and converted to percentages of time spent in each of the three levels.

Figure 3(b) illustrates a standard continuous 12-h roster in which standard, moderate and high work-related fatigue levels are respectively represented by 62.5, 31.25 and 6.25% of total hours worked. By simply utilizing two 2-day breaks as opposed to one 4-day break every 8 days, the 12-h roster illustrated in figure 1(c) produces decreases in scores of work-related fatigue to 39.6, 58.2 and 2.2%. Figure 1(d) allows us further to observe the timing of work and non-work hours. As can be seen in table 1, without altering the shift length or the number of shifts worked in the 3-week period the model outputs increased proportions of both the moderate and high work-related fatigue scores when compared with the rosters illustrated in figure 1(b) or (c). The roster illustrated in figure 1(d) produces 32.6, 60.4 and 7.2% of hours worked at standard, moderate and high scores. The roster illustrated in figure 1(e) shows that for a standard 8-h shift system with one 8-h shift rotation each week, standard, moderate and high scores of work-related fatigue are proportioned at 59.2, 33.3 and 7.5% respectively. By changing the shift lengths at particular portions of the day so that the timing of work and non-work better reflect the effects of work-related fatigue at different times, it can be observed that the high portion of the roster is reduced to 0% and the moderate portion increases to 50.8%.

Figure 4 further illustrates the relative work-related fatigue scores produced by the commercial aviation schedules illustrated in figure 2. Each roster has been

Table 1. Key summary statistics for the 11 rosters outlined in figures 1 and 2 as calculated according to the model.

ROSTER (Fig)	Shift Duration (h)	Description	Start time(s) (h)	Mean h/weeks	Fatigue points/day	Hourly mean	Peak level	High fatigue (%)
1(a)	8	Monday – Friday 09:00 – 17:00 hours	09:00	40	169.7	29.7	40.9	0
1(b)	12D – 12N	2D2N4O	06:00 – 18:00	42	264	41.4	99.25	6.25
1(c)	12D – 12N	2D2O2N2O	06:00 – 18:00	42	264	44.2	81.2	2.08
1(d)	12D – 12N	2D1O2N1O2N1O2D1O2D1O2N5O	04:00 – 16:00	48	358.9	52.3	91.2	6.94
1(e)	8N – 8D – 8A	5N2O5D2O5A2O	00:00 – 08:00 – 16:00	40	244.3	42.7	98.2	7.50
1(f)	6N – 8D – 10A	5N2O5D2O5A2O	00:00 – 06:00 – 14:00	40	238.1	41.7	75.8	0
2(a)	8	Monday – Friday 09:00 – 17:00 hours	09:00	40	169.7	29.7	40.9	0
2(b)	irregular	irregular	irregular	37	201.4	38.1	62.6	0
2(c)	irregular	irregular	irregular	37	196.4	37.2	61.7	0
2(d)	irregular	irregular	irregular	21	80.1	26.7	52.3	0
2(e)	irregular	irregular	irregular	23	93.6	28.5	70.4	0
2(f)	irregular	irregular	irregular	23	69.7	21.2	50.9	0

Shift duration represents the length of each shift worked for each specific roster. Where roster timing changes throughout the roster, specific shifts are represented as either dayshift (D), nightshift (N), afternoon shift (A) or irregular where shift lengths do not clearly follow a pattern. The roster description outlines the roster repeated for specific cycles but represents the entire cycle for 1(d). Roster description is defined as irregular for the pilot flight regimes. Start time(s) correspond to the shift start time for the standard work week, the day- and nightshifts for 1(b–d) and the night-, day- and afternoon shifts for 1(e–f). Start times vary for figure 2. Mean hours per week represents the average number of hours per week worked across the roster cycle. Fatigue scores per day represents the average number of fatigue scores accumulated per day. Hourly mean represents the average number of fatigue scores accumulated for each hour of work. Peak score indicates the maximum fatigue score at any point across the roster. High fatigue proportion indicates the percentage of time worked at fatigue scores more than double the peak value for the standard work week.

divided into the number of hours spent working at 0–100 and 100–200% of the maximum work-related fatigue produced by a standard work week. The number of hours spent at each of the standard and moderate work-related fatigue levels were summed and converted to percentages of time spent in each of the two work-related fatigue levels.

There are large differences between the proportions of time spent in the standard and moderate work-related fatigue score levels for the aviation schedules. As the peak work-related fatigue score for the standard work week is set as the upper limit for standard work-related fatigue level, the standard work week only produces standard work-related fatigue levels. By contrast, the aviation schedules all produced moderate work-related fatigue levels. The values from these schedules range from 4.4% for FH61 (figure 2f), 14.3% for FN13 (figure 2d), 17.4% for FN31 (figure 2e), 24.3% for FN15 (figure 2c) and 29.7% for FN10 (figure 2b).

#### **4. Discussion**

By constructing fatigue as a simple input–output model of hours-of-work that is modified by circadian, recovery and recency-of-work factors, it has been possible to quantify work-related fatigue. Using only the timing and duration of work as an input, it is possible to calculate work-related fatigue scores that discriminate between a variety of shift schedules. Moreover, by benchmarking non-standard schedules against a standard Monday–Friday work week it is possible to gain a perspective on the relative work-related fatigue scores associated with specific roster systems.

The work-related fatigue scores calculated using the model are based only on the timing and duration of work and non-work periods in the roster schedules outlined above. This is a significant advantage from an organizational perspective since hours of work are usually trapped as part of the normal management information process. Using this approach, it is possible for organizations to implement a modelling approach without significantly increasing data capture within the workplace. While the current model is based on a theoretical distribution of the recovery function, commercial applications of this approach use recovery functions derived from specific individuals in specific workplaces under real operating conditions. Specific details of this methodology can be obtained from the authors.

If the results from the model calculations for each of the shifts are compared, a number of important differences can be observed. For example, comparisons of scores can be made between schedules with different numbers of work hours, different timing of work hours or a combination of the two. In addition, global evaluations of potential work-related fatigue associated with different work schedules can be deduced from the key performance indicators shown in table 1.

The major advantage of this approach is that it enables theoretical insights gained in laboratory studies to be applied to specific or potential roster systems. Furthermore, it provides comparative indices that enable organizations to evaluate any putative relationship between work-related fatigue and organizational performance indicators (e.g. work performance, health and safety costs, absenteeism, etc.) in a straightforward manner.

Although most regulations are based on the number of hours per shift or week, the distribution of work hours seems to have a large impact on the work-related fatigue scores. By keeping the number of hours worked per week relatively stable, yet distributing hours in different ways, observations can be made regarding the importance of shift length and timing of hours of work.

Figure 1 illustrates the relative work-related fatigue scores produced by the five continuous roster systems and the standard work week. It also illustrates the proportions of time spent at standard, moderate and high work-related fatigue scores for these schedules as indicated by the relative numbers of white, grey and black bands. The 12-h shift schedules illustrated in figure 1(b–d) illustrate that, even though there are various ways to distribute hours-of-work in terms of work-related fatigue scores produced, it is very difficult to create a 12-h shift schedule that does not produce work-related fatigue scores in the range of 80–100.

However, the 8 h shift schedules outlined in figure 1(e) and (f) illustrate that rosters that cover the entire 24 h work window do not necessarily produce high work-related fatigue scores. Therefore, it seems that it is the compressed nature of work in the 12-h shift systems that make it difficult to eliminate work-related fatigue scores over 80 points. The roster in figure 1(e) is similar to the standard work week in that the number of hours worked per week is the same, however, the start times of the 8-h roster changes by 8 h each week so that the 24-h work window is covered by an individual in 3 weeks. The roster illustrated in figure 1(f) is similar to the roster in 1(e), but the hours of work have been redistributed so that more hours are worked in a row during times in which it is easier to perform and less hours are worked in a row at times when it is biologically more difficult. Thus, the model indicates that it is the timing and distribution of the hours worked in addition to the total number of hours worked that impact on the relative work-related fatigue associated with different schedules. In addition, it may be that the timing and distribution of the hours worked is more important than the total number of hours worked.

Although the number of hours worked in the aviation schedules illustrated by figure 2(b–f) are less than those for the standard work week, the output measures again reflect the importance of the timing and distribution of these hours. None of these schedules produce work-related fatigue scores over 80 points; however, even those schedules with many less hours per week (figure 2(d–f)) can produce peak work-related fatigue scores higher than the standard work week as seen in table 1.

The hours between midnight and 07:00 hours are those in which recovery occurs at the fastest rate. One possible approach for reducing the peak levels of relative work-related fatigue is to reduce the number of hours worked in this period and thus increase the amount of quality time utilized for recovery. If this approach is not possible then the simplest alternative is to attempt to optimize the timing of work blocks so that they distribute work-related fatigue evenly across the roster. For example, the roster illustrated in figure 1(f) contains the same number of hours and covers the same amounts of each hour in the 24-h window as the roster illustrated in figure 1(e). However, there are no work-related fatigue scores > 80 points as the hours of work have been redistributed to better reflect an even distribution of work-related fatigue across the shifts.

It is interesting to note that the cost of reducing high work-related fatigue scores is an increase in moderate work-related fatigue scores. In essence, the peak work-related fatigue score levels are reduced by distributing the work-related fatigue scores more evenly across the roster to reduce the amplitude of work-related fatigue score output. This is clearly illustrated in figure 1(e) and (f) by comparing rotation 2 to rotation 1. The redistribution of the hours-of-work eliminates high work-related fatigue scores, yet the overall amount of moderate work-related fatigue scores is largely increased to compensate for this.

Again using these aviation schedules as an example, it is possible to produce large work-related fatigue scores without working very long hours. All of these schedules have weekly hours of work less than that of the standard work week; however, they all produce peak work-related fatigue scores higher than for the standard work week.

It is possible for the model to produce standard work-related fatigue scores in work hours following moderate work-related fatigue scores, or moderate work-related fatigue scores in work hours following a high work-related fatigue score. This is an artefact of the in-built assumptions of the model; that is, the model weights the importance of time-of-day higher than the maintenance of a continuously increasing work-related fatigue score during work hours. This is specifically the case when the weighting for time-of-day is most rapidly declining, usually about 05:00 hours or after. This is illustrated in figure 2(e) on day 12 where there is an hour of moderate work-related fatigue scored, but the hours following are at standard work-related fatigue scores. This event occurs because the increase in the work-related fatigue score due to having worked another hour is not sufficiently large enough to compensate for the decline in the time-of-day weighting for that particular hour. It is difficult to rationalize that work-related fatigue could go down during a shift; however, what the model is actually creating is an output which suggests that an individual can be relatively less fatigued at 08:00 hours than they were at 04:00 hours.

Anecdotally this is true; many shiftworkers, even those who work 12-h shifts or longer, frequently report that they feel much more alert and functional once the sun has come up in the morning. The timing of 'sun-up' corresponds to the timing of the occurrences of fatigue scores changing from moderate to standard or from high to moderate in consecutive hours. Therefore, although it was not expected that fatigue scores could 'decline' over consecutive hours, this outcome can be explained in terms of many bodily processes initiating at this time-of-day which increases alertness, cognitive performance and vigilance.

Future evaluations of the model will assess the possibility of scores 'declining' over consecutive hours. There are other possible developments to be examined so that the model incorporates more factors that influence fatigue. For example, it is possible that recovery functions can be tailored to specific industries, facilities or worksites based on aggregate data from individuals. Information to be sought would, at a minimum, include work history and sleep history. This information would enable us to forecast the value of different lengths of recovery time taken at different times of day by predicting the timing and duration of sleep during specific non-work periods. Essentially, such a development would allow the recovery component of the model to more accurately reflect the relative recovery value.

The work-related fatigue model outlined in this paper is obviously a simplification of the actual processes that interact to modulate fatigue. Work-related fatigue is a multi-dimensional construct and it is obvious that no modelling methodology could ever incorporate all of the relevant factors. However, the information that this model provides allows simple comparisons of different rosters irrespective of amounts, duration or timing of hours-of-work. This model is useful in the sense that organizations can utilize information that is probably already in existence, that is hours-of-work, and assess various outputs without undertaking a complex or expensive process.

Ultimately, such a modelling approach may also be useful as a part of reporting processes for accidents, incidents, etc. Measuring work-related fatigue scores in

relation to occupational health and safety data, absenteeism levels, employee illness days or other organizationally meaningful data would allow a clearer illustration of the relationship between hours-of-work and its related costs. It could be foreseen that one part of a reporting process would require the collection of a 1-week work history so that relative work-related fatigue levels prior to an incident can be assessed. To what extent work-related fatigue is a factor in incidents can also be critical in relation to legal issues where prior hours-of-work may influence outcomes and allocation of accountability.

#### 4.1. *Circadian versus clock time*

In this model, circadian time is defined in a fixed phase relationship to clock-time. From previous research (Czeisler *et al.* 1990, 1994) it is clear that the phase of the circadian system regulating fatigue and alertness has been linked to the core temperature rhythm and assigned a nominal minimum between 04:00 and 06:00 hours with a maximum 12 h later. For the purpose of this model, we have assumed that circadian time shifts in response to variations in hours-of-work do not have a significant impact on the accuracy of the outputs. It is likely however, that this assumption is not the case in practice. The scientific literature indicates that the circadian system can phase-shift in response to changes in hours-of-work. However, small shifts of 1–2 h are unlikely to have a significant impact on the model outputs.

The shift schedules modelled in this paper include the hours-of-work for pilots working in a civil aviation setting. These pilots typically fly <300 h per year and generally fly one 'tour of duty' of 3–4 days duration per fortnight. The schedules illustrated in this paper are all flown in a single time zone, that is in a north–south direction. This is to avoid the results being confounded by pilots crossing time zones and having non-work-related circadian disruption. In practice, the mean phase shift between circadian and clock time is in the order of up to 1–2 h without individuals having crossed time zones. On the basis of this model, corrections of this magnitude would have little effect on model values.

There is little doubt that a more generalized development of the model should incorporate circadian phase shifts due to shiftwork. Computationally, this is a relatively straightforward process. However, predicting the direction and magnitude of the phase shift in a given individual in response to a specific shift schedule may be more challenging. It is acknowledged that some schedules may not adhere to the model's assumption of a fixed clock time/circadian time relationship; however, it seems that even in night workers, the amount of adaptation may not be that great (Folkard 1990, Reid *et al.* 1997). This difficulty should not, however, detract from the potential utility of a more simple approach, since the average phase shift in response to a broad range of shift schedules is typically in the order of 2–4 h for night shift and less for afternoon shift (Rutenfranz *et al.* 1977, Dawson *et al.* 1995). Pragmatically, even phase shifts in this range are unlikely to carry a significant error for this model where circadian variations in fatigue and recovery are broadly sinusoidal.

#### 4.2. *Potential benefits from a modelling approach*

By quantifying work-related fatigue, it is possible to gain considerable insight into how employees rate different shift schedules and organizations manage shiftwork. If valid, such a model might provide organizational policy-makers with a powerful tool. Occupational health and safety outcomes could be statistically linked to the

fatigue economies of an individual, organization or industry. Accident and injury reporting systems that could record prior shift history would enable organizations to show statistical linkages between hours-of-work and specific organizational costs associated with shiftwork. Using this approach it would also be possible to explore the equity issues associated with the allocation of hours-of-work. This is particularly important in organizations where there is considerable diversity in the hours-of-work that specific individuals perform. Such an approach would also enable organizations to ensure that shifts and overtime were equitably allocated and OH&S risks minimized.

It is believed that the development of the capacity to statistically link the effects of shiftwork to specific outcomes in specific organizations is a critical step in helping organizations appreciate the full impact of inappropriate hours-of-work policies. It is also believed that until organizations have the empirical tools to understand the quantitative relationship between fatigue and organizational costs there is little incentive to implement coherent policies with respect to hours-of-work. In the absence of good empirical tools, organizations will continue to develop fatigue policies that only reflect the day-to-day variations in organizational demands and ignore objective scientific information. This is especially important at a time when many organizations and industries uncritically view longer and more flexible hours from their employees as an immediate productivity gain.

Copies of the work-related fatigue model are available for research purposes at <http://www.interdyne.com.au>

### Appendix 1: Mathematical description of the model

#### 1.1. *Intuitively fatigue and recovery functions would be*

$$\text{Fatigue } (x) = \text{Circadian } (x) \quad (1)$$

$$\text{Recovery } (x) = -\text{Scalar} \times \text{Circadian } (x), \quad (2)$$

where Circadian ( $x$ ) is the score or measure of the relative difficulty of working during hour ' $x$ '. The 'Scalar' determines the rate of fatigue discharge compared with the rate of fatigue accumulation. Note that 'Scalar' is a positive number.

#### 1.2. *Workload model*

The workload model is an intermediate process in the prediction of work-related fatigue and assumes that fatigue accumulates throughout the day and throughout the week of work. If the current fatigue level could be given a score based upon what has been worked previously, then the following recurrence relation can be derived:

$$\begin{aligned} \text{Score } (x) &= \text{Score } (x-1) + \text{Fatigue } (x) \text{ (if hours 'x' worked)} \\ \text{Score } (x) &= \text{Score } (x-1) + \text{Recovery } (x) \text{ (otherwise),} \\ \text{with Score } (0) &= 0. \end{aligned} \quad (3)$$

This means that a person's working times can determine a score or fatigue profile. For a standard work week of 09:00 to 17:00 hours Monday–Friday, with a normal Saturday and Sunday off:

$$\text{Score } (0) = \text{Score } (168) = 0. \quad (4)$$



Equation 4 states that under normal conditions the person working the above shift will return the same fatigue level at the end of the week as they were at the beginning of the week.

A comparison of different shifts under the 'Workload' model will show the rate which the body's fatigue is accumulated and the differences in the final fatigue 'Score'.

### 1.3. *Fatigue model*

The fatigue model does not accumulate in the same way as the workload model. Instead a weighted moving average of previous fatigue and recovery scores is used. The calculation of the weights is performed independently for the fatigue and for the recovery. The weight for the fatigue is a function of the times and the hours worked in the previous week, and the weight for the recovery is a function of the times and hours of recovery in the previous week.

The functions (for fatigue and recovery) were taken as linearly weighted combinations of the worked or the non-worked hours.

Let:  $W(x) = \text{Fatigue}(x)$  (if hour 'x' worked)

$W(x) = 0$  (otherwise)

$R(x) = -\text{Recovery}(x)$  (if hour 'x' not worked)

$R(x) = 0$  (otherwise)

Then:  $F\text{Weight}(x) = (168.W(x) + 167.W(x-1) + 166.W(x-2) + \dots + 2.W(x-166) + 1.W(x-167))/168$

$R\text{Weight}(x) = (168.R(x) + 167.R(x-1) + 166.R(x-2) + \dots + 2.R(x-166) + 1.R(x-167))/168$ .

The score for the particular period is determined simply by multiplying the weight by the fatigue or recovery score for the period:

$\text{Score}(x) = \text{Fatigue}(x) \times F\text{Weight}(x)$  (if hours 'x' worked)

$\text{Score}(x) = \text{Recovery}(x) \times R\text{Weight}(x)$  (otherwise).

### 1.4. *Model comparisons*

The profiles for the two models for the same shift pattern are quite different in their appearances and characteristics. The workload model accumulates fatigue and what is of most interest is the highest, lowest and average levels of fatigue, combined with both the speed of accumulation and the maintenance of the levels of fatigue (area under the profile). The fatigue model concentrates mainly on the levels of fatigue, where they occur, and the normality of the height of the peaks. Scores can be output as being less than zero due to the fact that the 'floor' that values saturate at is arbitrarily defined.

### 1.5. *Summary*

The models derived previously show two alternate functions converting work shift patterns into workload and fatigue profiles, each model determining different information about how shift characteristics, combined with human circadian rhythms, influence the performance of the body during work and leisure.

This approach will allow comparisons between different shifts, and therefore provide guidelines for maintaining equality for employees working different times of

the day, while also providing a measure of normal and fatigue levels. In this respect, shifts which show high fatigue levels once identified can be minimized or eliminated.

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