# Operations Research

Prerequisite: Basic Engineering Mathematics and Probability Theory
L-T-P: 4-0-0, Credits: 4
Type: Core Essential Subjects (CES)

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# Course Description

This course module on Operations Research aims to introduce students to use quantitative methods and techniques for effective decisions–making; model formulation and applications that are used in solving business decision problems.

## Course objective

| Students undergoing this course are expected to: | |
|---|---|
| 1 | Understand some common operations research models and algorithms. |
| 2 | Refers to the science of informed decision making. |
| 3 | To provide rational basis for decision-making by analyzing and modeling complex situations, and to utilize this understanding to predict system behavior and improve system performance. |
| 4 | Applying OR involves problem formalization, model construction and validation; other activities include a computational part, analysis of solutions, arriving at conclusions, and implementation of the decision. |
| 5 | Use the concepts of mathematical modeling, statistical analysis and optimization techniques on operations research. |
| 6 | To understand the emphasis is on applications rather than the details of methodology. |
| 7 | To know that operation research act as a tool to the courses namely data mining, business intelligence and decision support systems. |

## Course Outcomes

| CO Numbers | Course Outcomes (Action verb should be in italics) | Bloom's taxonomy |
|---|---|---|
| CO-1 | **Identify** and develop operational research models from the verbal description of the real system. | Identify/Knowledge |
| CO-2 | **Analyze** any real-life system with limited constraints and depict it in a model form. | Analyzing |
| CO-3 | **Understand** a variety of real-life problems like assignment, transportation, a traveling salesman, inventory management, project planning, etc. | Understanding/Comprehension |
| CO-4 | **Design** and **solve** operational research models using appropriate methods. | Applying |
| CO-5 | **Learn** to propose the best strategy in any organization using decision-making methods in the domains of cost, time, and benefit. | Creating |

# Syllabus

**UNIT 1:** Introduction to OR Models, Graphical Method for LPP, Convex sets, Simplex Method, Big M Method, Two Phase Method, Multiple solutions of LPP, Unbounded solution of LPP, Infeasible solution of LPP, Revised Simplex Method, Case studies, Primal Dual Construction, Duality Principle, Primal-Dual relationship of solutions, Dual Simplex Method, Sensitivity Analysis.

**UNIT II:** The Transportation Model, Definitions, Formulation, North-West Corner Method, Vogel's Approximation Method, u-v method, Post optimality Analysis, The Assignment Model: Definition and mathematical representation, The Hungarian method Network Optimization Models: Sequencing and Scheduling (Critical Path Methods and PERT)

**UNIT III:** Dynamic Programming: Formulation, Optimal subdivision problem, Solution of L.P.P. by dynamic programming, applications, Deterministic and probabilistic dynamic programming.

**UNIT IV:** Integer Programming: Non-Linear Programming Constrained and Unconstrained optimization, Multi-variable unconstrained optimization, Kuhn-Tucker Conditions, Queuing Theory.

**Unit V:** Decision Theory, The Theory of Games, Pure and Mixed strategies Games, n-person zero sum games, Basic Queuing Models, Inventory Models, Simulation Modeling

# Books

**Text Books**

1. Taha, Hamdy A. Operations research: an introduction. Pearson Education India, 2013.
2. Hillier, Frederick S., and Gerald J. Lieberman. "Introduction to Operations Research, Burr Ridge, IL." 10th Edition (2017)
3. Operations Research, P K Gupta and D S Hira, S. Chand and Company LTD. Publications, New Delhi – 2007
4. Operations Research, Theory and Applications, Sixth Edition K Sharma, Trinity Press, Laxmi Publications Pvt. Ltd. 2016.

**Reference Books**

1. Churchman, C. West, Russell L. Ackoff, and E. Leonard Arnoff. "Introduction to operations research." (1957).
2. 2. Hillier, Frederick S. Introduction to operations research. Tata McGraw-Hill Education, 2012.

# Time Table

Motilal Nehru National Institute of Technology Allahabad
Time Table [Odd Semester 2023-24)

## Computer Science and Engineering 3rd Semester

## CSN13402 = Operation Research

| | 08:00-09:00 Hrs. | 09:00-10:00 Hrs. | 10:00-11:00 Hrs. | 11:00-12:00 Hrs. | 12:00-13:00 Hrs. | 13:00 – 14:00 Hrs. | 14:00-15:00 Hrs. | 15:00 -16:00 Hrs. | 16:00 -17:00 Hrs. |
|---|---|---|---|---|---|---|---|---|---|
| **Monday** | | | | | | | CSN13402 (L) CSC **SEW1** | CSN13402 (L) CSC **SEW1** | CSN13402 (L) CSD **GS8** |
| **Tuesday** | | CSN13402 (L) CSC **GS7** | | | | | | | |
| **Wednesday** | CSN13402 (L) CSD **GS8** | CSN13402 (L) CSA **FEW1** | | CSN13404 DS Lab | CSN13404 DS Lab | | | | |
| **Thursday** | | | CSN13402 (L) CSB **SEW10** | CSN13402 (L) CSB **SEW10** | | | | CSN13402 (L) CSA **SEW1** | |
| **Friday** | CSN13402 (L) CSB **SEW10** | | CSN13402 (L) CSA **SEW10** | | | CSN13402 (L) CSD **NLH2** | | | |

# OPERATIONS RESEARCH

The term Operations Research (OR) was coined by **J.F. McCloskey and F.N. Trefethen** in 1940 in Bowdsey in the United Kingdom.

This innovative science was **discovered during World War II** for a specific military situation, when military management sought decisions based on the optimal consumption of limited military resources with the help of an organized and systematized scientific approach.

Hence, Operation Research can be associated with '***an art of winning the war without actually fighting it***'.

# Definitions

*Operation Research* is the scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources. **H.A. Taha**

*Operation Research* is a scientific approach to problem solving for executive management. **H.M. Wagner**

*Operation Research* is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. **Morse and Kimball (1946)**

*Operation Research* is art of wining wars without actually fighting them. **Authar Clark**

*Operation Research* may be described as a scientific approach to decision making that involves the operations of organization system. **F. Hiller and G.J. Lieberman**

# Scope (Applications) of Operations Research

OR is useful in various important fields like:

**(i) Agriculture**: Resource optimization, production planning of crops, transportation, manufacturing, purchasing, and selling.

**(ii) Finance**: Cash flow analysis, long range capital requirements, Investment decision, Profit plan for the company, dividend policies, investment portfolios

**(iii) Industry**: production planning and control; quality control; inventory, equipment, warehouse, and materials management; plant layout

**(iv) Organization Behaviors**: Selection of personnel, determination of retirement age and skills, Recruitment policies and assignment of jobs, Recruitment of employees, Scheduling of training programs

**(v) Marketing**: Product selection, number of salesmen, advertising media, finding the best time to launch a product

**(vi) Personnel Management**: Selection of personnel on minimum salary, Mixes of ages and skills, recruitment policies, Manpower Scheduling.

**(vii) Production Management**: Physical distribution, facilities planning, manufacturing, maintenance and project scheduling

**(viii) Research and Development**: Determination of areas of concentration of R&D, project selection, determination of time, cost of development projects

**(ix) Transportation:** Routing of Vehicles, Scheduling of dispatches

**(x) Government:** Policy planning, Resource allocation, Costing, Taxation

# Phases of Operations Research

The procedure to be followed in the study of OR generally involves the following major phases:

i. Definition of the problem

ii. Construction of a mathematical model

iii. Deriving the solution of the model

iv. Validation or Testing the model and its solution

v. Implementation of the solution

# Tools of Operation Research

In any area of human endeavor, whether it is a production system, business system or service system where an objective is to be optimized, the problem falls into the domain of operation research. Some of the commonly used techniques of operation research are as follows:

- Linear programming
- Waiting line theory or queuing theory
- Inventory control models
- Replacement problems
- Network Analysis
- Dynamic programming
- Assignment problems
- Decision theory
- Integer Programming
- Transportation Problems
- Simulation
- Goal Programming
- Markov Analysis
- Game Theory
- Heuristic Models
- Routing Models

# Classification of Models of Operation Research

The various models of operation research have been classified under five major categories:

- Solution method
- Time or Behavior
- Certainty or Environment
- Function
- Structure

**Classification based on solution:** *In this category, the models of operations research are classified* ***according to the methodology of solution procedure****, which are described as follows:*

**Analytical models:** When the model has clearly specified mathematical structure and the clear mathematical technique could be applied. For example, a general linear programming model, as well as the specially structured transportation and assignment models are analytical models.

**Simulation models:** When the solution of models cannot be achieved by simple mathematical technique, it requires computer-assisted algorithms procedure under the environment of some assumed factors.

**Heuristic models:** When the method of solving the model is completely unknown, one has to put some rules, which correlate the given situation with the present and generate the solutions that are not necessarily optimal, yet applicable.

**Classification based on time (behavior):** *In this category, the models of operations research are classified as per their representation of time constraints, which are described as follows:*

**Static models:** Only one decision is needed for the duration of a given time period. They are used only for a single time. It represents the system at a specified time and do not consider changes in the system over a change in the time period. Example: Economic Order Quantity Model

**Dynamic models:** It can be for multiple times on similar problems. It used in a situation where time plays important role and used for optimization of multistage decision problems. Example: Dynamic Programming model

**Classification based on Certainty (Environment):** *In this category, the models of operations research are classified as per their certainty factor, which are described as follows:*

**Deterministic models:** Models which have all known values and the outcomes are certain. Example: Linear Programming model, Transportation Problem etc.

**Probabilistic models:** It has at least one parameter or decision variable is a random variable and surety of solution is uncertain. These models are also known as ***stochastic models***. Example: It used in the conditions of uncertain demand to decide the economic ordering quantity (EOQ).

**Classification based on Function:** *In this category, the models of operations research are classified as per their utilities, which are described as follows:*

**Descriptive models:** It explains various operations in non-mathematical language. Example: Observation, Survey, Questionnaires etc.

**Predictive models:** It explain or predict the behavior of a system. Example: Forecast model predicts future, sale or future demand.

**Prescriptive models (Normative):** It used to prescribe certain optimal solutions for problems. These models are also known as optimization models because they provide the best solutions called optimal or optimum solutions under the restrictions of pre-determined constraints. Example: Linear Programming models

**Classification based on Structure:** *In this category, the models of operations research are classified based on their structure, which are described as follows:*

**Physical models (Iconic):** Physical representation of a real situation with a different scale. Example: Map, Images, Paintings, model of a car etc.

**Symbolic models (Mathematical):** It represent the original system by using mathematical symbols or relations such as numbers, letters etc. Example: Resource allocation model, Newspaper boy problem, transportation model etc.

# Advantages of Operation Research

Along with larger applicability of operations research, following are the added advantages of operations research theory**:**

**Better Coordination:** Maintaining proper balance and coordinating all the activities for desired results, which provides greater control.

**Flexibility:** Accurate measures and controlled environment provide the flexibility to the models.

**Better Planning:** A realistic approach of operations research models towards the constraints and other factors provides a better and strong plan to apply which involves optimum short-term and long-term plans.

**Scientific approach:** Pre-defined deterministic methods or algorithm ensures the accurate results generation.

# Limitations of Operation Research

Along with several advantages, it has some limitations too, which are discussed below:

**Required trained people:** Nearly all the models and solution methods of OR are highly complex and difficult to understand. Hence, they require a skilled and trained person to deal.

**Lengthy procedure:** The methodology of operations research has to take too many factors into consideration. Therefore, this procedure takes long time and enormous calculations.

**Complex:** Due to high range of value into consideration and lengthy calculations, the solution generation turned out to be very complex and highly sensitive even with a single small calculation mistakes.

# Format of OR Model

OR model can be organized in the following general format:

Maximize or Minimize **Objective Function**

subject to

**Constraints**

A solution of the model is **feasible** if it satisfies all the constraints. It is **optimal** if, in addition to being feasible, it yields the best (max./min) value of the objective function.

# Problem 1

Imagine that you have a 5-week business commitment between India (IND) and United States of America (USA). You fly out of India on Mondays and return on Wednesdays. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

# Solution

It is a decision-making problem whose solution requires answering three questions:

1. What are the decision alternatives?

2. Under what restrictions is the decision made?

3. What is an appropriate objective criterion for evaluating the alternatives?

# Problem 1

Imagine that you have a 5-week business commitment between India (IND) and United States of America (USA). You fly out of India on Mondays and return on Wednesdays. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

**1. What are the decision alternatives?**

Three alternatives are considered:

1. Buy five regular IND-USA-IND for departure on Monday and return on Wednesday.
2. Buy one IND-USA, four USA-IND-USA that span weekends, and one USA-IND.
3. Buy one IND-USA-IND to cover Monday of the first week and Wednesday of the last week and four USA-IND-USA to cover the remaining legs. All tickets in this alternative span at least one weekend.

# Problem 1

Imagine that you have a 5-week business commitment between India (IND) and United States of America (USA). You fly out of India on Mondays and return on Wednesdays. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

**2. Under what restrictions is the decision made?**

The restriction on these options is that you should be able to leave IND on Monday and return on Wednesday.

**3. What is an appropriate objective criterion for evaluating the alternatives?**

An obvious objective criterion for evaluating the proposed alternative is the price of the tickets. The alternative that yields the smallest cost is the best.

Three alternatives are considered:

1. Buy five regular IND-USA-IND for departure on Monday and return on Wednesday of the same week.
2. Buy one IND-USA, four USA-IND-USA that span weekends, and one USA-IND.
3. Buy one IND-USA-IND to cover Monday of the first week and Wednesday of the last week and four USA-IND-USA to cover the remaining legs. All tickets in this alternative span at least one weekend.

Specifically, we have

Alternative 1 cost = 5 X 400 = $2000

Alternative 2 cost = .75 X 400 + 4 X (.8 X 400) + .75 X 400 = $1880

Alternative 3 cost = 5 X (.8 X 400) = **$1600**

Thus, you should choose **alternative 3**.

## Problem 2

Consider forming a maximum-area rectangle out of a piece of wire of length $L$ inches. What should be the width and height of the rectangle?

## Solution:

Here, the number of alternatives is not finite; namely, the width and height of the rectangle can assume an infinite number of values.

To formalize this observation, the alternatives of the problem are identified by defining the width and height as continuous (algebraic) variables.

Let

$w$ = width of the rectangle in inches

$h$ = height of the rectangle in inches

The restrictions of the situation can be expressed verbally as:

Width of rectangle + Height of rectangle = Half the length of the wire

Width and height cannot be negative

These restrictions are translated algebraically as

$$2(w + h) = L$$
$$w >= 0, h >= 0$$

The objective of the problem; namely, maximization of the area of the rectangle. Let $z$ be the area of the rectangle, then the complete model becomes

$$\text{Maximize } z = w\,h$$

subject to:

$$2(w + h) = L$$

$$w, h >= 0$$

The optimal solution of this model is $\boldsymbol{w = h = L/4}$, which calls for constructing a square shape.

# Linear Programming (LP)

Linear programming (LP) is designed by George B. Dantzig in 1947 to optimize operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities

**General Form:** The general form of linear programming is given as

$$\text{(Max or Min) } Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n \ (\leq / \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n \ (\leq / \geq)\ b_2$$
.
.
.
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots + a_{mn}x_n \ (\leq / \geq)\ b_m$$

and $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$

where

$Z$ = value of on the whole measure of performance

$x_j$ = level of activity (for $j = 1, 2, \ldots, n$)

$c_j$ = increase in Z that would result from each unit enhance in level of activity j

$b_i$ = amount of resource i that is available for allocation to activities (for $i = 1,2, \ldots, m$)

$a_{ij}$ = amount of resource i used by each unit of activity j

# Example 1

Reddy Mikks produces both interior and exterior paints from two raw material M1, M2. The following table provides the basic data of problem:

| Tons of raw material per ton | | | |
|---|---|---|---|
| | Exterior paint | Interior paint | Max daily availability |
| *Raw material, M1* | 6 | 4 | 24 |
| *Raw material, M2* | 1 | 2 | 6 |
| profit per ton ($1000) | 5 | 4 | |

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit?

## Solution:

Variables of model are defined as:

$x_1$=tons produced daily of exterior paint
$x_2$=tons produced daily of interior paint

**Objective function:**

To construct objective function, note that the company wants to maximize (i.e. increase much as possible) the total daily profit of both paints.

Given that the profit per ton of exterior and interior paints are 5 and 4 (thousand) dollars, respectively, it follows that

Total profit from exterior paint$=5\mathbf{x_1}$

Total profit from exterior paint$=4\mathbf{x_2}$

Letting z represent the total daily profit, the objective of the company is

$$\text{Maximize } z=5\mathbf{x_1}+4\mathbf{x_2}$$

# constrains:

To construct the constrains that restrict raw material usage and product demand.

**(usage of a raw material by both paint) $\leq$ (max. raw material availability)**

*The daily usage of raw material M1 is 6 ton of exterior paint and 4 ton of interior paint. Thus*

Usage of raw material M1 by exterior paint=$6x_1$ tons/day
Usage of raw material M1 by interior paint=$4x_2$ tons/day

**Usage of raw material M1 by both paints=$6x_1+4x_2$ tons/day**

*The daily usage of raw material M2 is 1 ton of exterior paint and 2 ton of interior paint. Thus*

Usage of raw material M2 by exterior paint=$1x_1$ tons/day
Usage of raw material M2 by interior paint=$2x_2$ tons/day

**Usage of raw material M2 by both paints=$x_1+2x_2$ tons/day**

Because the daily availabilities of raw materials M1 and M2 are limited to 24 and 6 ton, respectively, the associated restriction are given as

$$6x_1 + 4x_2 \le 24 \text{ (Raw material M1)}$$

$$x_1 + 2x_2 \le 6 \text{ (Raw material M2)}$$

The first demand restriction stipulate that excess of daily production of interior over exterior paint, $x_2$-$x_1$, should not exceed 1 ton, which translates to

$$x_2 - x_1 \le 1 \text{ (market limit)}$$

The second demand restriction stipulate that the max. daily demand of interior paint is limited to 2 tons, which translates to

$$x_2 \le 2 \text{ (demand limit)}$$

An implicit (or "understood to be") restriction is that variable $x_1$ and $x_2$ cannot assume negative values.

$$x_1, x_2 \ge 0$$

**The complete Reddy Mikks model is**

$$\text{Maximize } z = 5x_1 + 4x_2$$

subject to:

$$6x_1 + 4x_2 \le 24$$
$$x_1 + 2x_2 \le 6$$
$$x_2 - x_1 \le 1$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0$$

# Properties of Linear Programming (LP) Solution

***Feasible Solution:*** If all the constraints of the given linear programming model are satisfied by the solution of the model, then that solution is known as *feasible solution*.

***Optimal Solution:*** If there is no other superior solution to the solution obtained for a given linear programming model, then the solution obtained is treated as the *optimal solution*.

***Alternate Optimal Solution:*** For some LPP, there may be more than one combination of values of the decision variables yielding the best objective function value. Such combinations of the values of the decision variables are known as *alternate optimum solutions*.

***Unbounded Solution:*** For some LP model, the objective function value can be increased/decreased infinitely without any limitation. Such solution is known as *unbounded solution*.

***Infeasible Solution:*** If there is no combination of the values of the decision variables satisfying all the constrained of the LP model, then that model is said to have *infeasible*.

***Degenerate Solution:*** In LPP, intersection of two constraints will define a corner point of the feasible region. But if more than two constraints pass through any one of the corner points of the feasible region, excess constraints will not serve any purpose, and therefore they act as redundant constraints. Under such situation *degeneracy* will occur.

# Graphical Method

*If the number of variables in any LPP is only two, one can use graphical method to solve it. These are the practical steps involved in solving LPP by Graphical method:*

**Step 1** →Consider each inequality constraint as equation.

**Step 2** →Take one variable (say x) in a given equation equal to zero and find the value of other variable (say y) by solving that equation to get one co-ordinate [say (0, y)] for that equation.

**Step 3** →Take the second variable (say y) as zero in the said equation and find the value of first variable (say x) to get another co-ordinate [say (x, 0)] for that equation.

**Step 4** →Plot both the co-ordinates so obtained [i.e., (0, y) and (x, 0)] on the graph and join them by a straight line. This straight line shows that any point on that line satisfies the equality and any point below or above that line shows inequality. Shade the feasible region which may be either convex to the origin in the case of less than type of inequality (<) or opposite to the origin in case of more than type of inequality (>).

**Step 5** → Repeat Steps 2 to 4 for other constraints.

**Step 6** → Find the common shaded feasible region and mark the co-ordinates of its corner points.

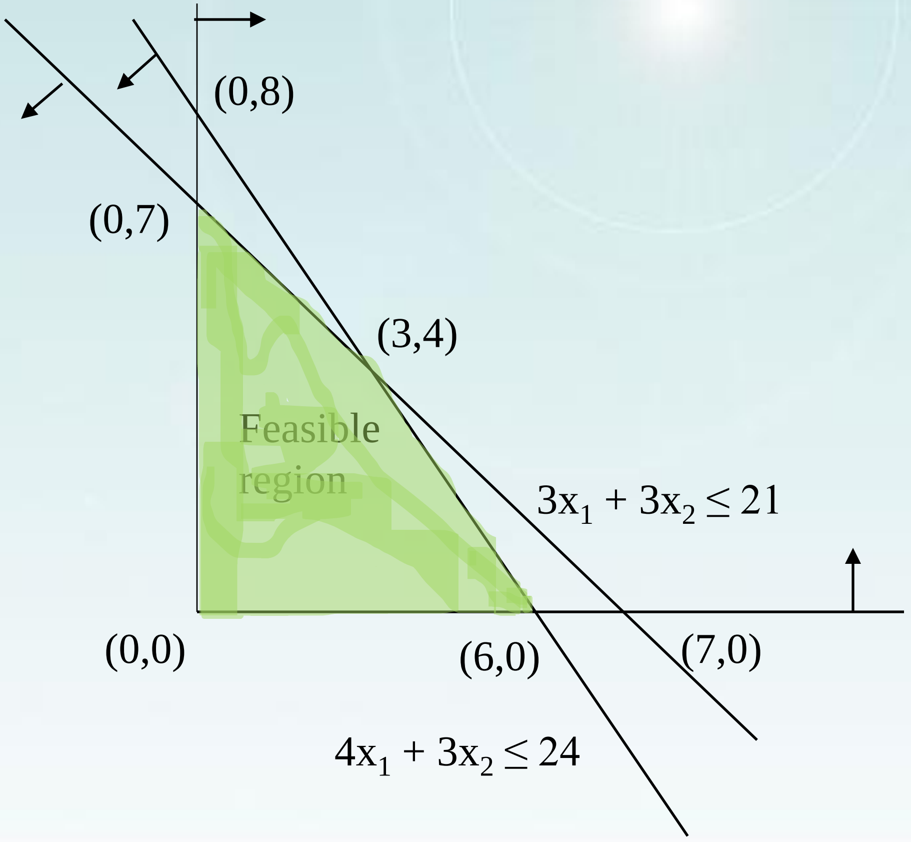
**Step 7** → Put the co-ordinates of each of such vertex in the objective function. Choose that vertex which achieves the most optimal solution.

# Example 1

*Maximize* $Z = 10x_1 + 9x_2$

*Subject to*

$$3x_1 + 3x_2 \le 21$$
$$4x_1 + 3x_2 \le 24$$
$$x_1, x_2 \ge 0$$




There are total 4 corner points:

At (0,0): Z=0

At (0,7): Z=63

At (6,0): Z= 60

At (3,4): Z= 66 ⇨ Best Solution or Optimal Solution

## Example 2

Maximize $z=5\,x_1 + 4x_2$

subject to:

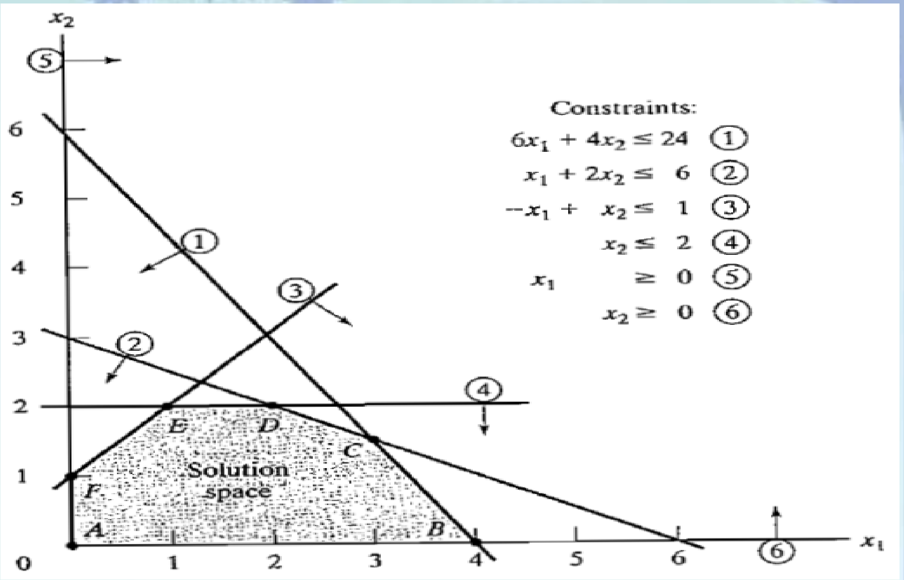
$6\,x_1 + 4x_2 \le 24$

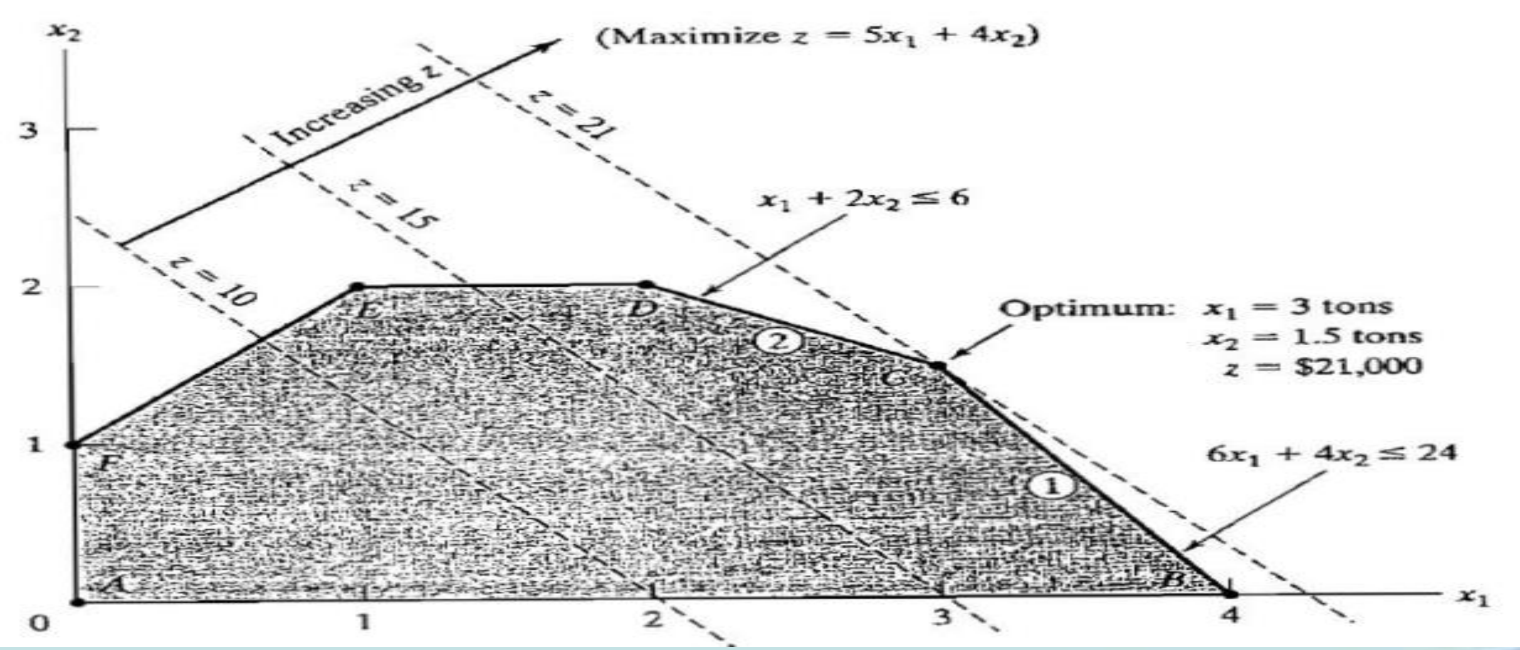
$x_1 + 2x_2 \le 6$

$x_2 - x_1 \le 1$

$x_2 \le 2$

$x_1, x_2 \ge 0$



## Example 3

Ozark farms uses at least 800 lb of special feed daily. The special feed is mixture of corn and soybean meal with the following composition:

| Lb per lb of feedstuff | | | |
|---|---|---|---|
| Feedstuff | Protein | Fiber | Cost($/lb) |
| Corn | 0.09 | 0.02 | 0.30 |
| Soybean meal | 0.60 | 0.06 | 0.90 |

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. Ozark farms wishes to determine the daily minimum-cost feed mix

## Solution

**Variables:**

The feed mix consists of corn and soybean meal, the decision variables of the model are define as:

X1=lb of corn in the daily mix

X2=lb of soybean meal in the daily mix

**Constraints:**

The constraints of the model reflect the daily amount needed and the dietary requirements because Ozark farms needs at least 800 lb of feed a day, the associated constrain can be expressed as

$$x1 + x2 \geq 800$$

As for the protein dietary requirement constraint, the amount of protein included in x1 lb of corn and x2 lb of soybean meal is ($0.09x1 + 0.6x2$)lb. This quantity should equal at least 30% of the total feed mix (x1+x2) lb :

$$0.09x1 + 0.6x2 \geq 0.3(x1+x2)$$

or

$$0.21x1-0.3x2 \leq 0$$

As for the fiber dietary requirement constraint, the amount of fiber included in x1 lb of corn and x2 lb of soybean meal is ($0.02x1+ 0.06x2$)lb. This quantity should equal at most 5% of the total feed mix (x1+x2) lb :

$$0.02x1 + 0.06x2 \leq 0.05(x1+x2)$$

or

$$0.03x1-0.01x2 \geq 0$$

**Objective function:**

The objective function seeks to minimize the total daily cost (in dollars) of the feed mix and is thus expressed as

$$\text{Minimize } z=0.3x1+0.9x2$$

The complete model thus become

Minimize $z=0.3x1+0.9x2$

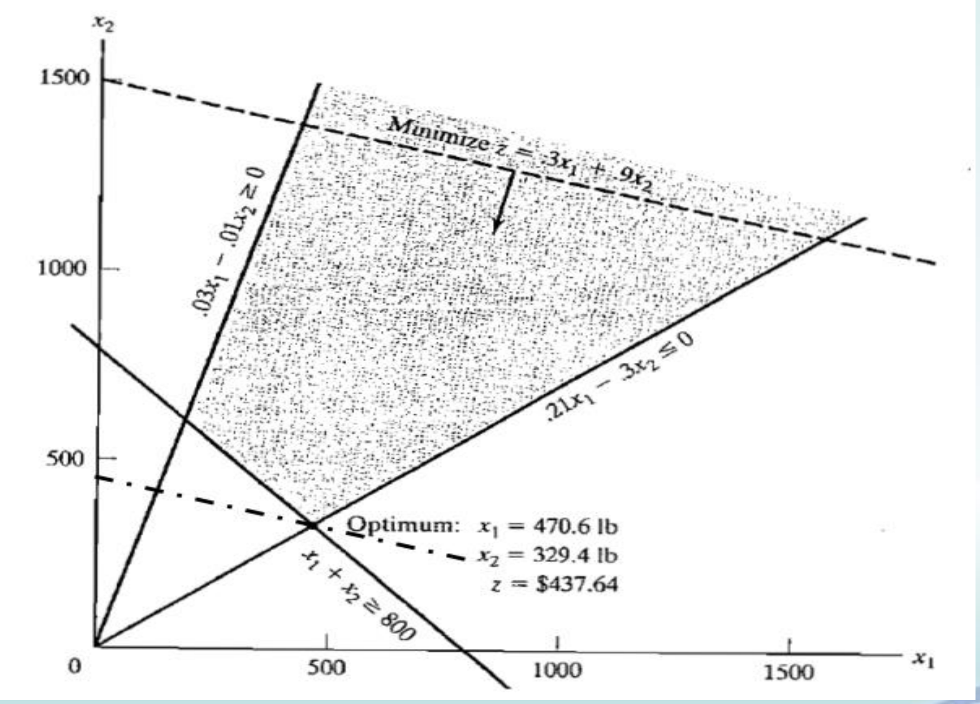
Subject to

$x1 + x2 \geq 800$

$0.21x1-0.3x2 \leq 0$

$0.03x1-0.01x2 \geq 0$

$x1, x2 \geq 0$



**Example 4**

A company buying scrap metal has two types of scrap available to them. The first type of scrap metal has 20% of metal A, 10% of impurity and 20% of metal B by weight. The second type of scrap has 30% of metal A,10% of impurity and 15% of metal B by weight. The company requires at least 120 kg. of metal A, at most 40 kg. of impurity and at least 90 kg. of metal B. The price for the two scraps are Rs. 200 and Rs. 300 per kg. respectively. Determine the optimum quantities of the two scraps to be purchased so that the requirements of the two metals and the restriction on impurity are satisfied at minimum cost.

**Solution:**

Introduce the decision variable x1 and x2 indicating the amount of scrap metal (in kg.) to be purchased. The problem can be formulated as

$$\text{Minimize } z = 200x1 + 300x2$$

Subject to

$$0.2x1 + 0.3x2 \geq 120$$
$$0.1x1 + 0.1x2 \leq 40$$
$$0.2x1 + 0.15x2 \geq 90$$
$$x1, x2 \geq 0$$

Minimize z = 200x1 + 300x2

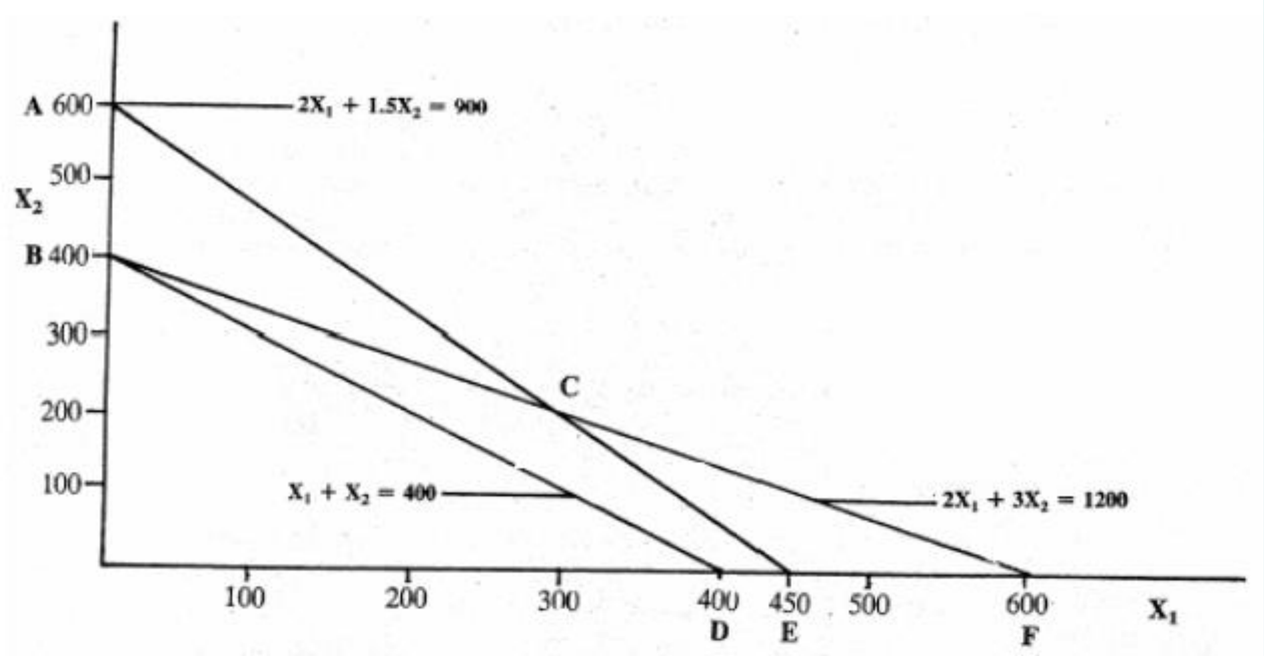
Subject to

$0.2x1 + 0.3x2 \geq 120$

$0.1x1 + 0.1x2 \leq 40$

$0.2x1 + 0.15x2 \geq 90$

$x1, x2 \geq 0$




The region right of the boundary ACF includes all solutions which satisfy the first and the third constraints. The region located on the left of BD includes all solutions which satisfy the second constraint. Hence, there is no solution satisfying all the three constraints. Thus, the problem is **infeasible**.

**Example 5 (Packaging)**

A manufacturer of packing material, manufactures two types of packing tins, round and flat. Major production facilities involved are cutting and joining. The cutting 'department can process 300 round tins or 500 flat tins per hour. The joining department can process 500 round tins or 300 flat tins per hour. If the contribution towards profit for a round tin is the same as that of a flat tin what is the optimum production level?

**Solution**

Let us introduce decision variables x1 No. of round tins per hour, x2 = No. of flat tins per hour. Since the contribution towards profit is identical for both the products the objective function can be expressed as x1 + x2. Hence the problem can be formulated as

Maximum z = x1 + x2

Subject to

$$(1/300)x1 + (1/500)x2 \leq 1$$
$$(1/500)x1 + (1/300)\ x2 \leq 1$$
$$x1,\ x2 \geq 0$$

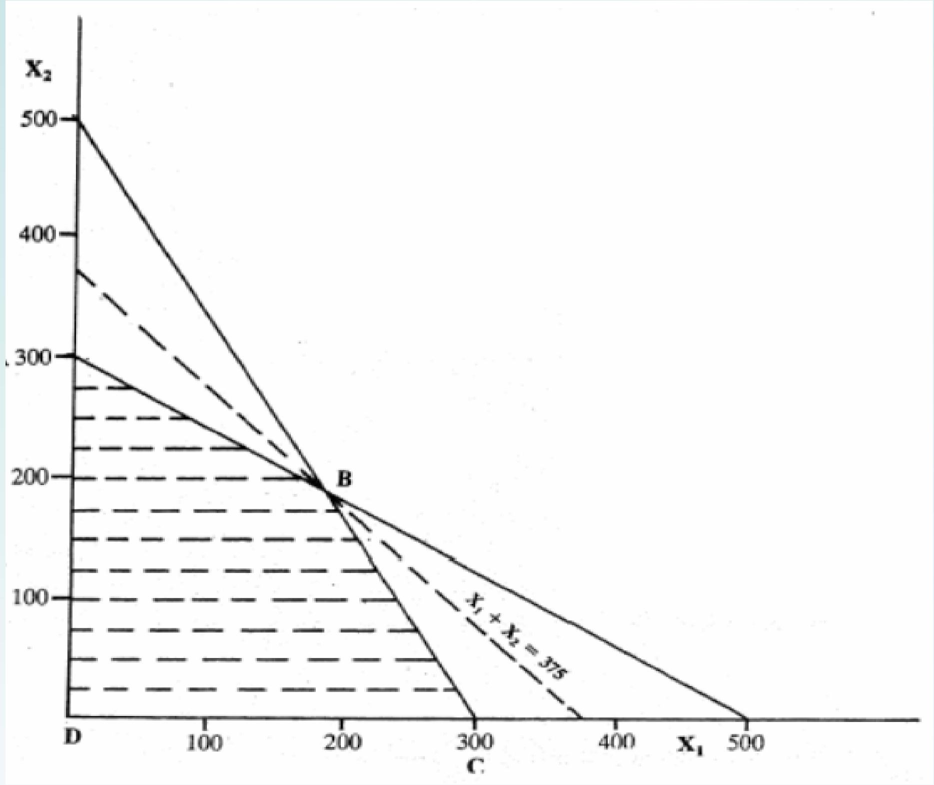
or

$$5x1 + 3x2 \leq 1500$$
$$3x1 + 5x2 \leq 1500$$
$$x1,\ x2 \geq 0$$

Maximum $z = x1 + x2$

Subject to

$5x1 + 3x2 \leq 1500$
$3x1 + 5x2 \leq 1500$
$x1, x2 \geq 0$




Point B is (1500/8, 1500/8).

The optimum value of x1+x2 :

$x1+x2 = 1500/8 + 1500/8$

$= 375.$

**Problem**

A nutrition scheme for babies is proposed by a committee of doctors. Babies can be given two types of food (I and II) which are available in standard sized packets weighing 50 grams. The cost per packet of these foods is Rs.2 and Rs.3, respectively. The vitamin availability in each type of food per packet and the minimum vitamin requirement for each type of food are summarized below. Develop a LP model to determine the optimal combination of food types with the minimum cost such that the minimum requirement of vitamin in each type is satisfied.

| Vitamin | Vitamin availability per packet | | Minimum daily required vitamin |
|---|---|---|---|
| | Food Type I | Food Type II | |
| 1 | 1 | 1 | 6 |
| 2 | 7 | 1 | 14 |
| Cost/Packet(Rs) | 2 | 3 | |

**Table:** Details of Food Types

**Solution**

Minimize $z = 2x1 + 3x2$

Subject to

$x1 + x2 \geq 6$

$7x1 + x2 \geq 14$

$x1, x2 \geq 0$