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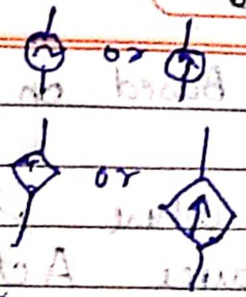
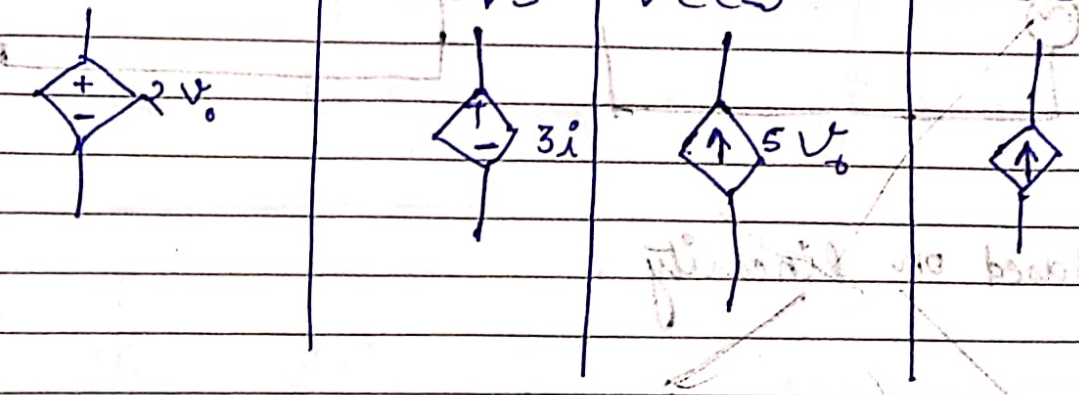
Energy Sources — Independent
Dependent.



VCVS | CCVS | VCCS | CCCS



~~Energy Sources Classification~~ –

Network Classification

1.) Based on direction of Current.

Unilateral → circuits connected with diode

Bilateral → circuits connected with R, L & C.

2.) Based on Seprability.

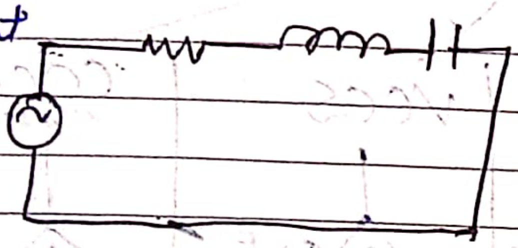
Lumped → [circuits which are not distributed. particular value of R, L & C is given)

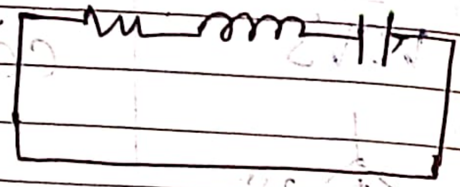
Distributed. → [R, L & C are connected throughout length in the circuit] e.g. transmission lines.

3. Based on presence or absence of source,

the circuit where energy source is present — Active

Passive, energy source is not present





4. Based on linearity,

Linear

Non Linear,

• principle of homogenity & principle of Superposition followed.

Linearity Test eqn's –

① $F(x_1 + x_2) = F(x_1) + F(x_2)$.

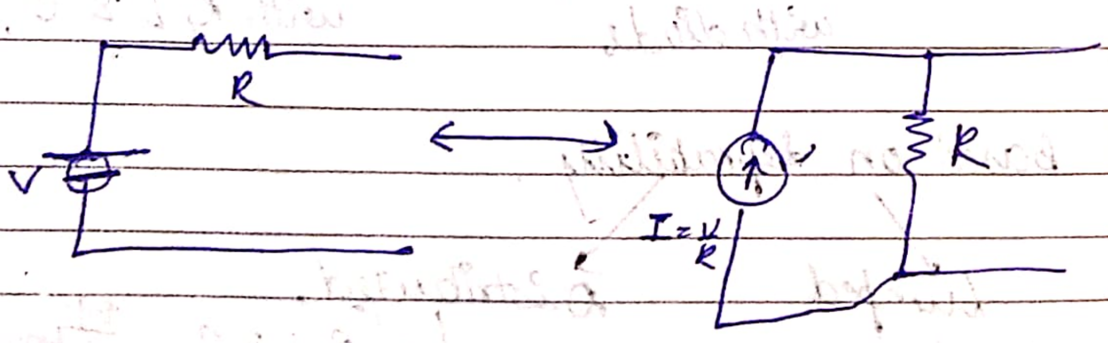
② $F(ax) = aF(x)$

Source Transformation –

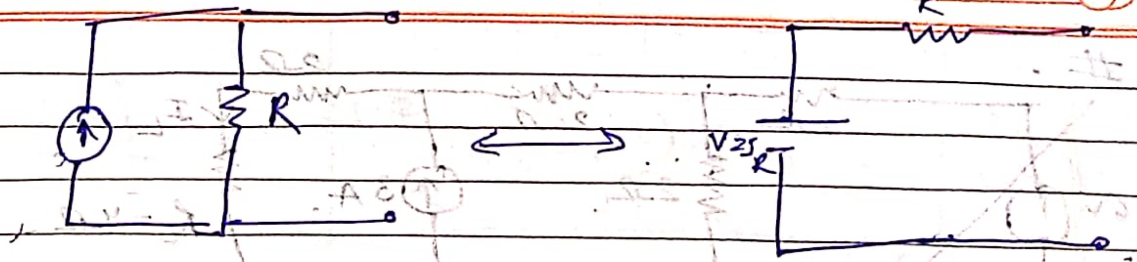
Voltage Source can be converted into Current Source & vice-versa.

Voltage Source with Series

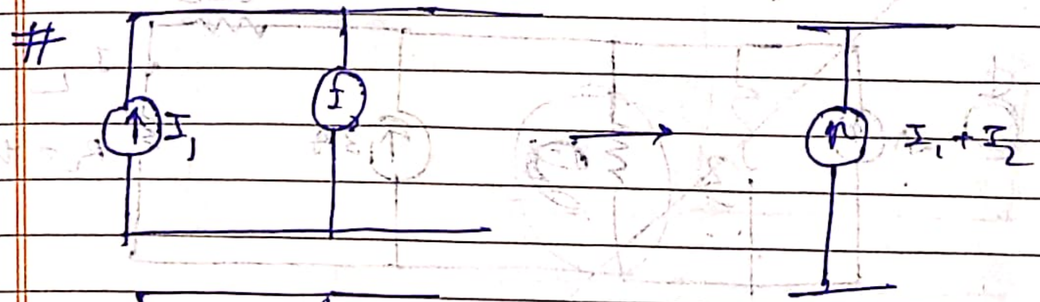
Current Source with Parallel



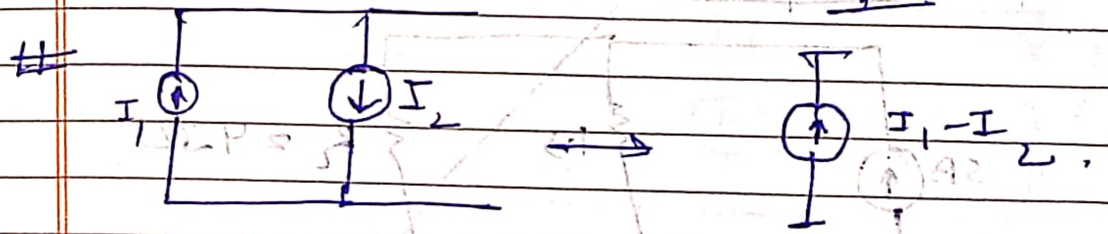
7.5

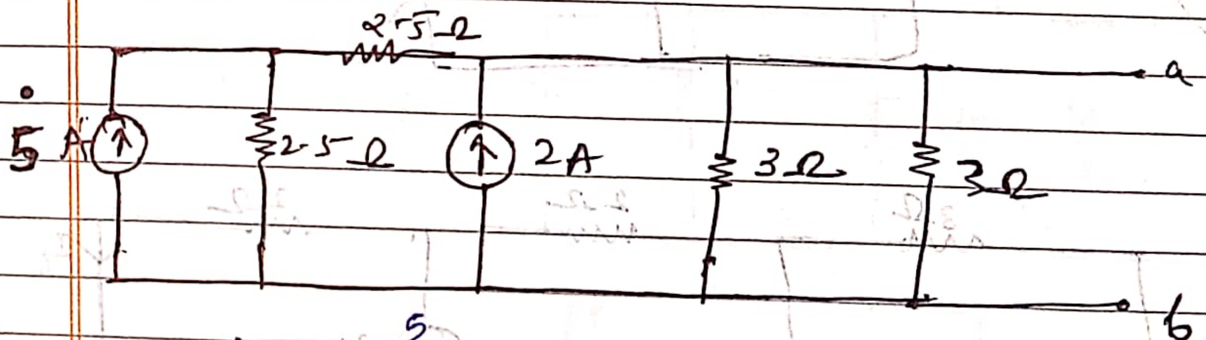

R
R
V=IR
R

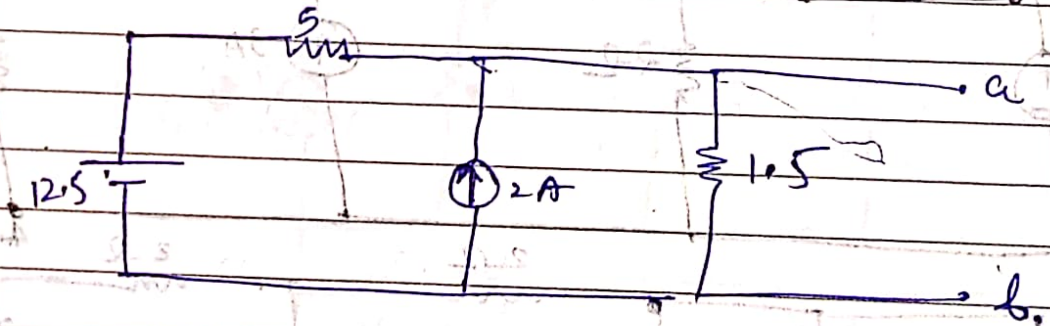
#

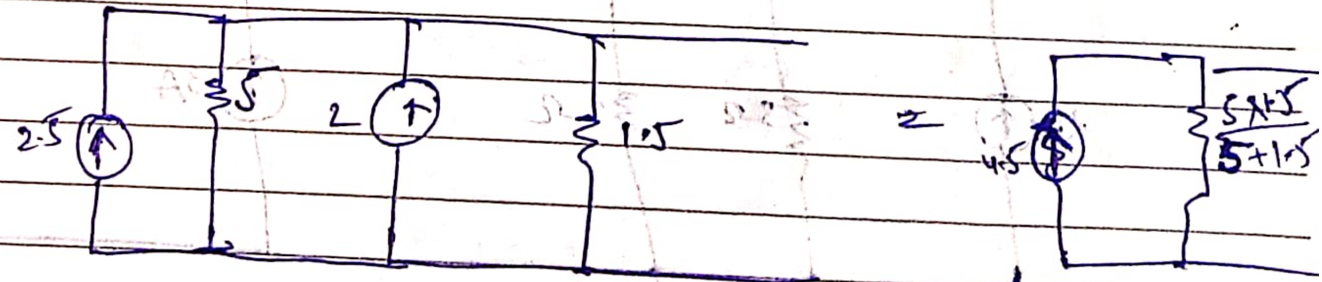

I_1
I_2
I_1+I_2

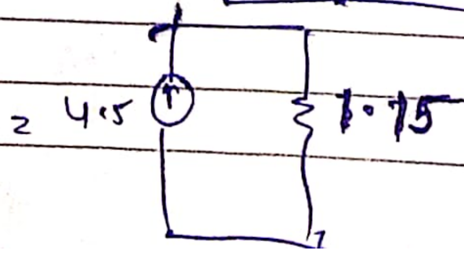
#


I_1
I_2
I_1-I_2

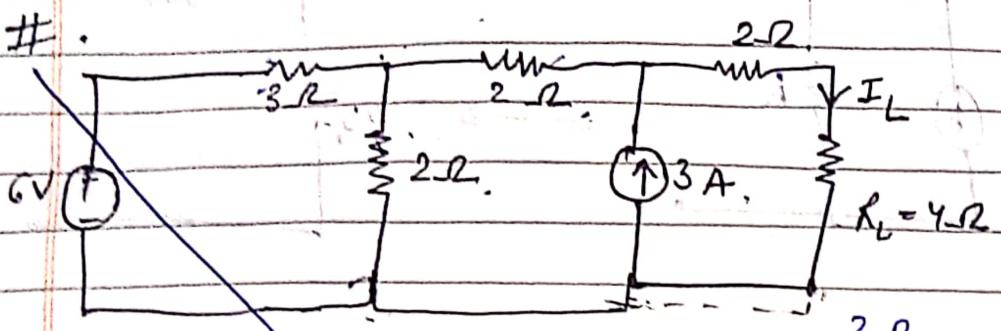

2.5 Ω
5 A
2.5 Ω
2A
3 Ω
3 Ω
a
b

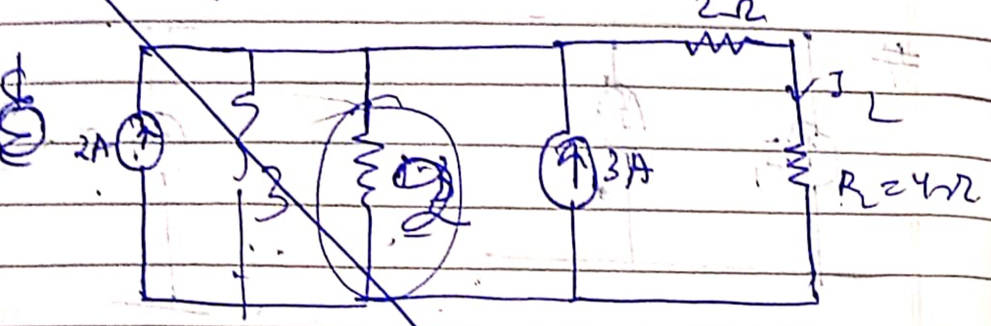

5
12.5
2A
1.5
a
b

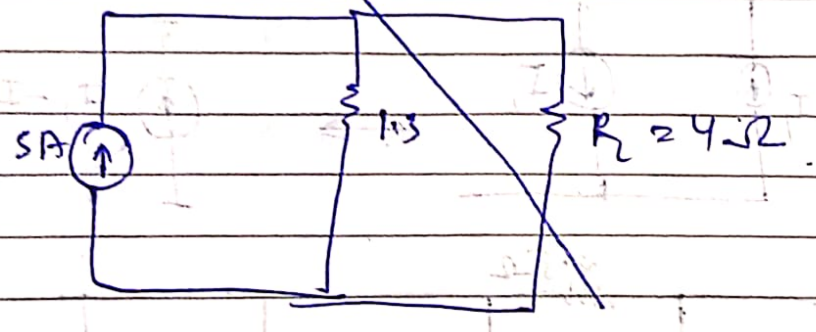

2.5
5
2
1.5
=
4.5
$\frac{5\times1.5}{5+1.5}$


= 4.5
1.15

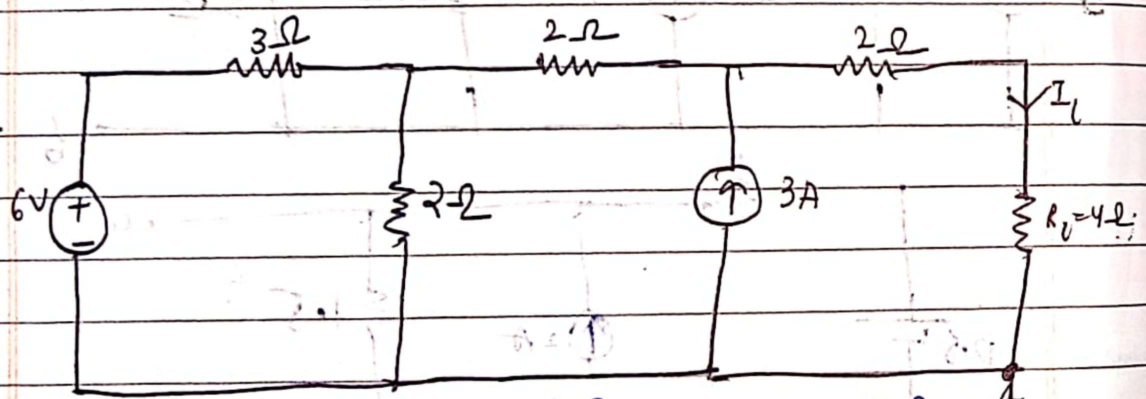
#

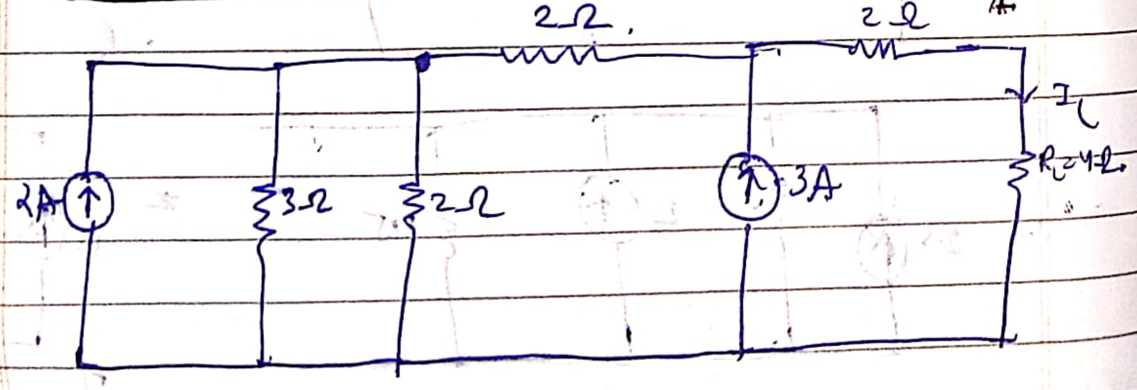

2Ω
3Ω
2Ω
6V
2Ω
3A
IL
RL = 4Ω


2Ω
2A
3
2
3A
IL
RL = 4Ω

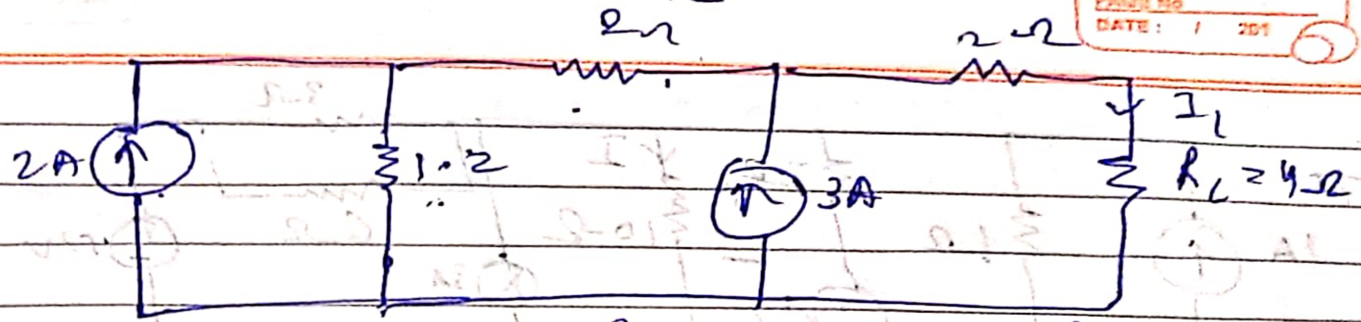

5A
1.3
RL = 4Ω

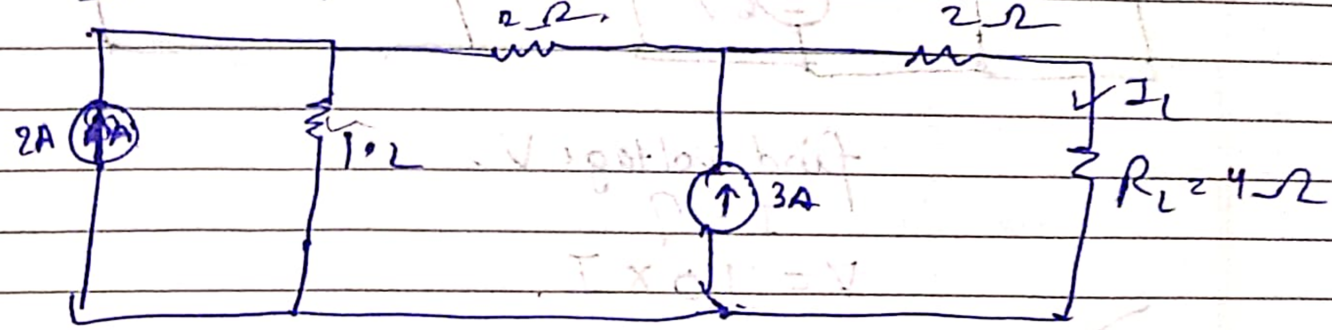
#

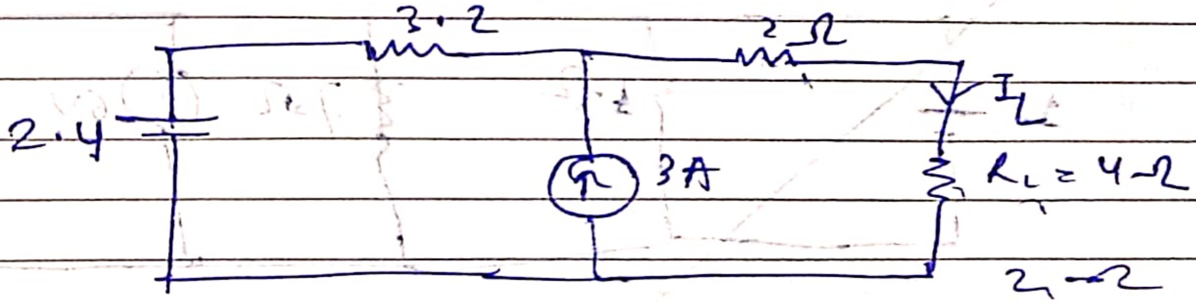

3Ω
2Ω
2Ω
6V
2Ω
3A
IL
RL=4Ω

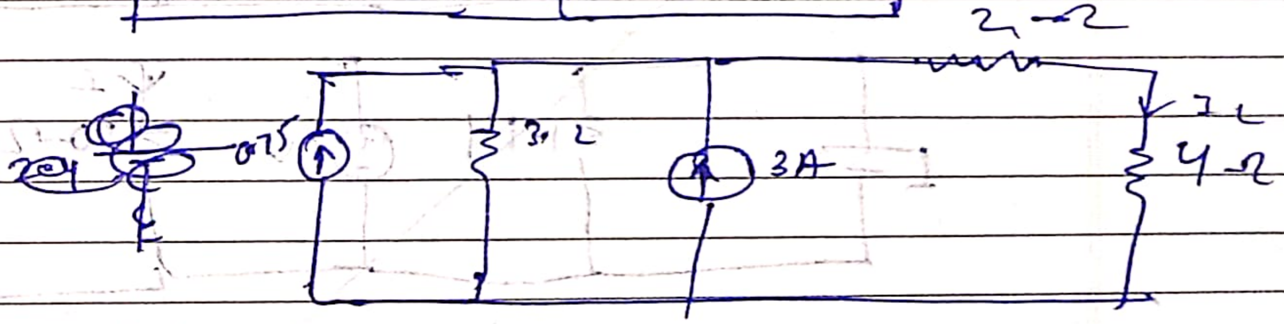

2Ω
2Ω
2A
3Ω
2Ω
3A
IL
RL=4Ω

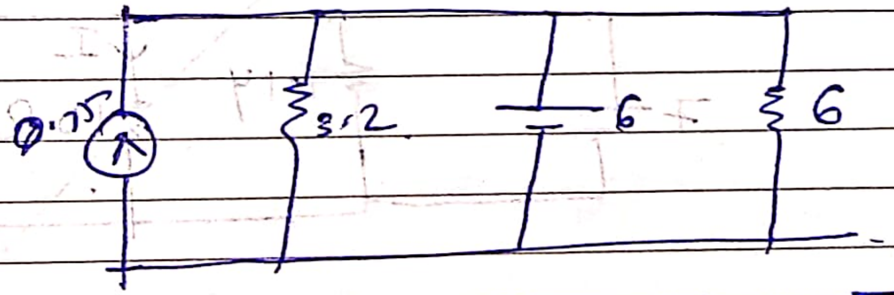
$\frac{1}{R}$



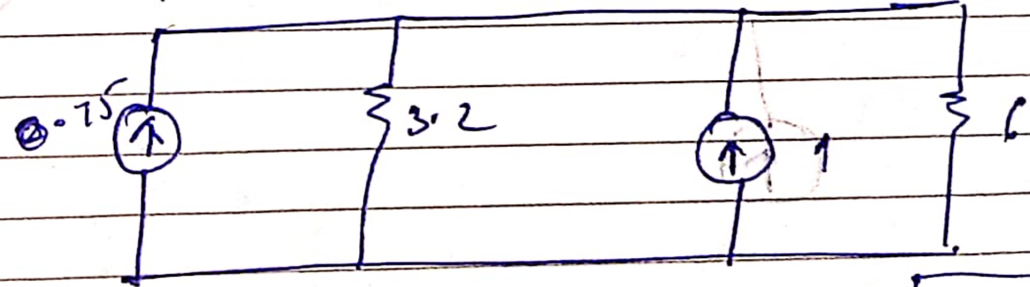


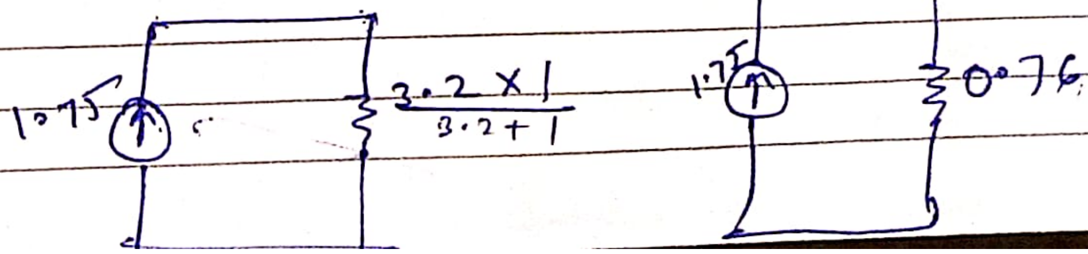






$I = \frac{V}{R}$





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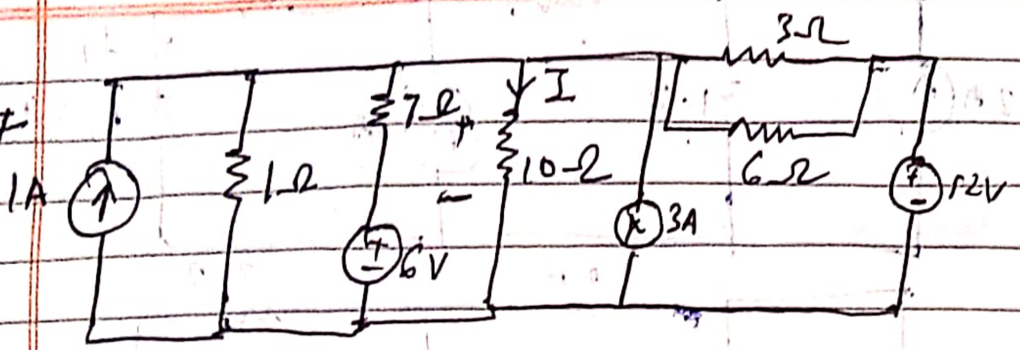
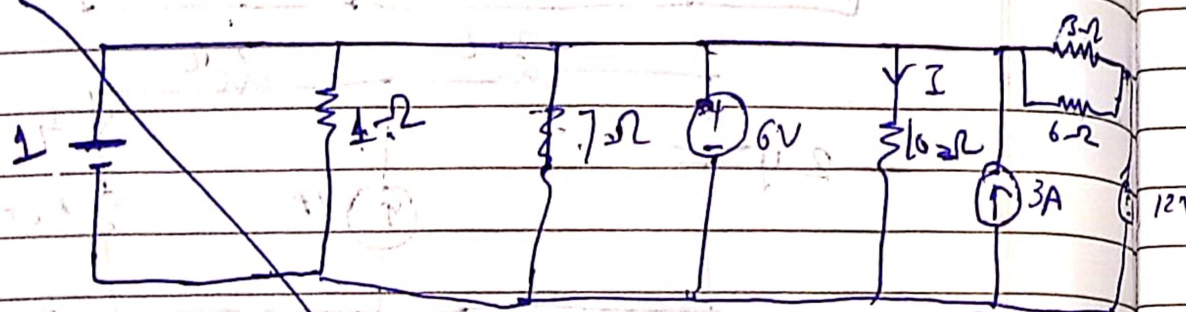
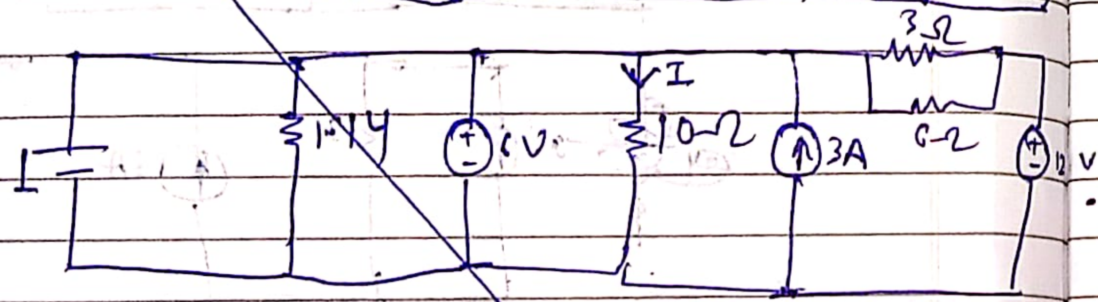


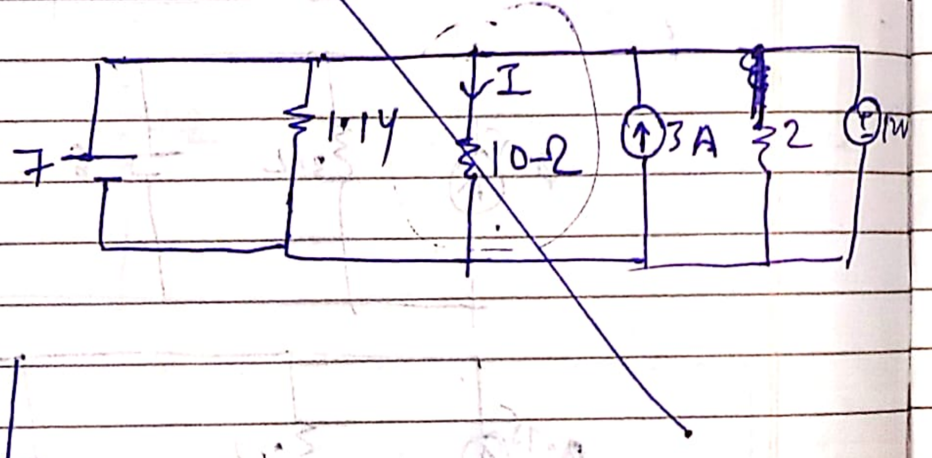
find voltage V.

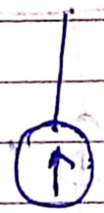
I = ?

V = 10 × I.









6.23V

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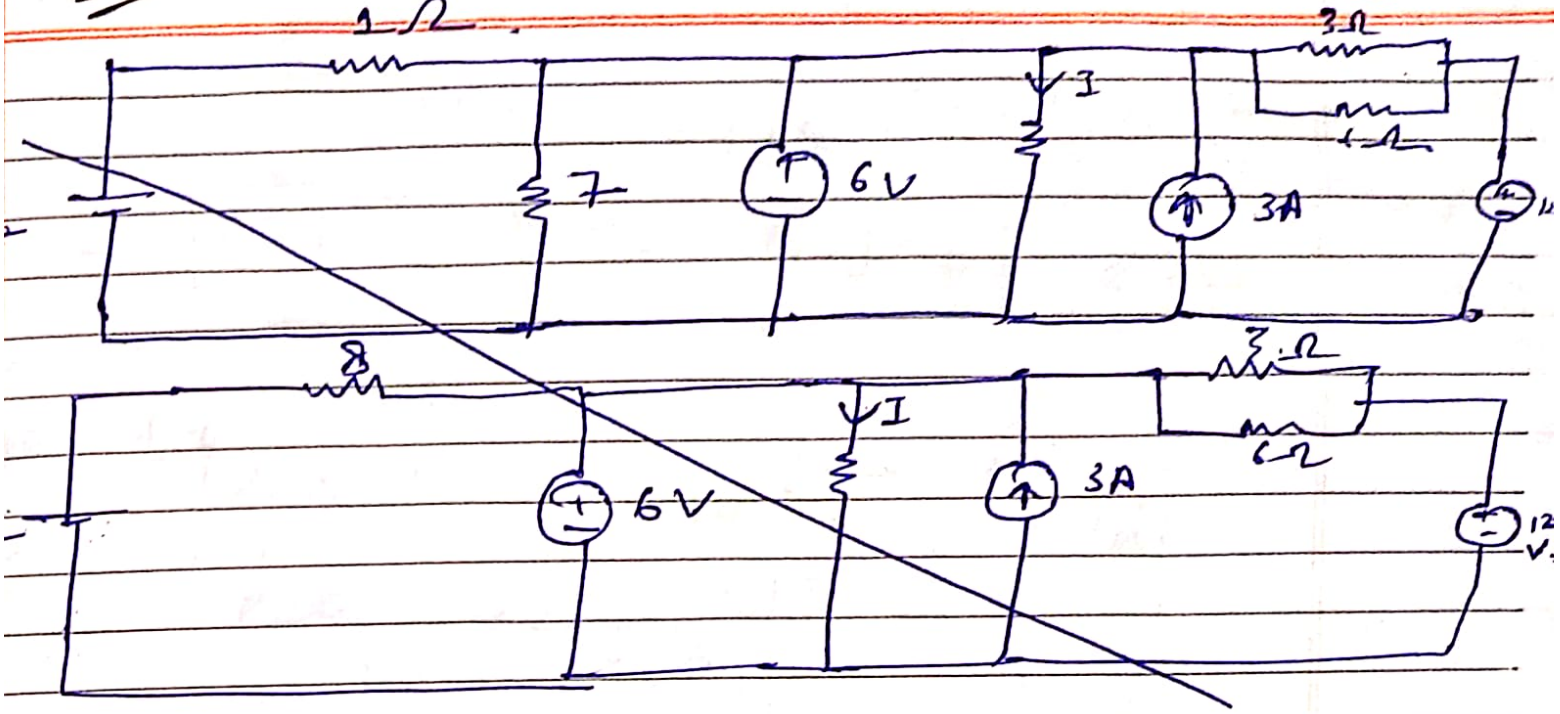
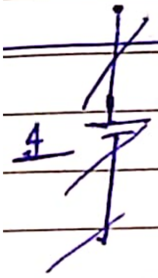
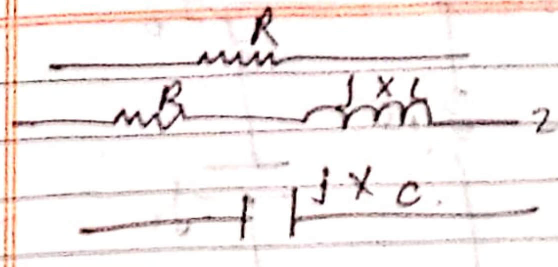






$Z = R$

$Z = R + jX_L$

$Z = R - jX_C$

$\vec{E} = a + jb \longrightarrow E\angle\theta$

$R \longrightarrow P$

$R \longleftrightarrow E\cos\theta + jE\sin\theta$

final ans has to be in polar form.

e.g.

Rec $E_1 = 3 + j4 \longrightarrow 5\angle 53.13$ Polar.

↓

Pol

Pol(3, 4)
= 5

RCL
tan
(53.13).

Rec(5, 53.13) ⟶ 3 ⟶ Real

RCL tan ⟶ 3.99 ≃ 4 ⟶ img.

add, subtract ⟶ Rec.
multiply + divide ⟶ Pole.

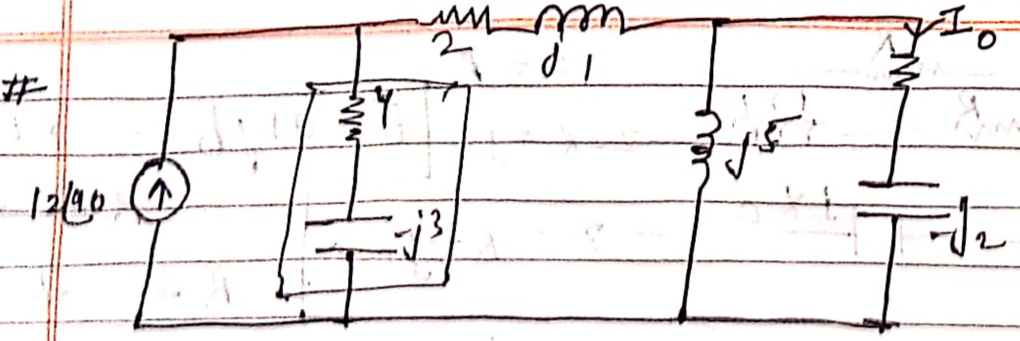
$2 + 5j + 6 + 1j = 8 + 6j$

$4 + 3j - (2 + 1j) = 2 + 2j$

$4\angle 50 \times 2\angle 10 = 8\angle 60$

$\dfrac{4\angle 50^\circ}{2\angle 10} = 2\angle 40.$

#



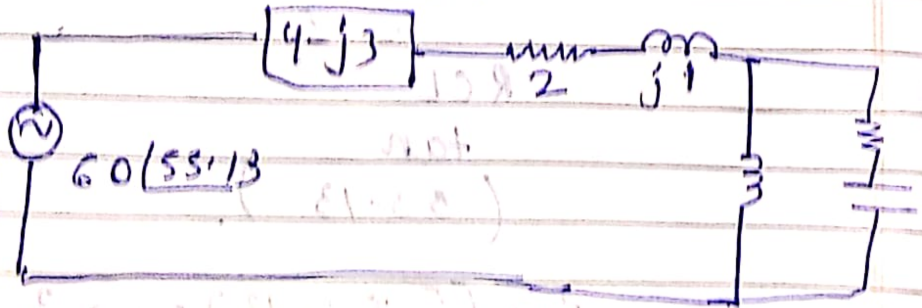
$I_0 = ?$

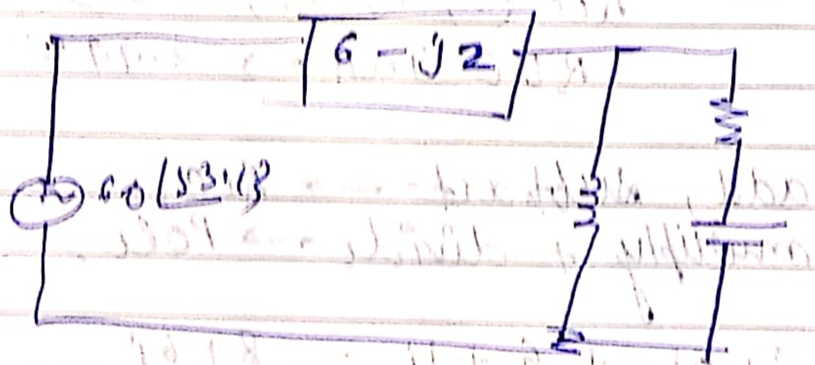
$Z = 4 - j3$

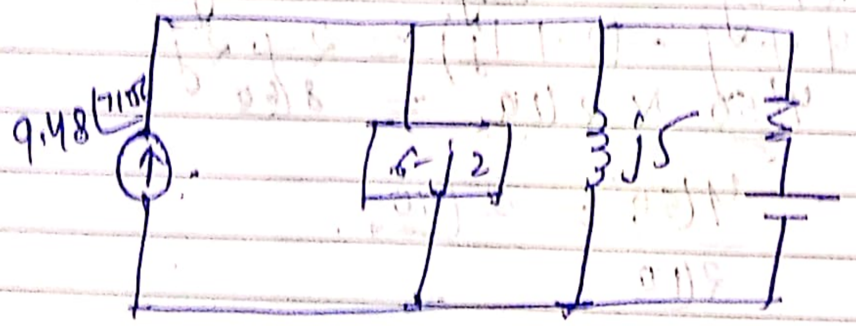
$Pol(4,3) = 5\angle -36.86$

$V = I\,Z$

$12\angle 90 \times (4 - j3)$







$z = \frac{z_1 z_2}{z_1 + z_2}$ $(6 - j2) \parallel (0 + j5)$

$$\frac{(6-j2)(0+j5)}{(6-j2)+(0+j5)}$$

$6.32\angle -18.4$ $12\angle 90$

$= 5$

$$Z = \cancel{\frac{6.32\angle -18.4 \quad 5\angle 90}{6.32\angle -18.4 + 5\angle 90}}$$

$$= \cancel{\frac{31.6\angle 71.6}{11.32\angle 71.6}}$$

$= 2.79$

$$= \frac{(6 - j2)(0 + j5)}{(6 + j3)}$$

$$= \frac{6.32\angle -18.4 \cdot 5\angle 90}{\cancel{6.32}\ 3\angle 90}$$

$$= \frac{31.6\angle 71.6}{3\angle 90}$$

$= 10.5\angle -18.4$

$= 21.1$

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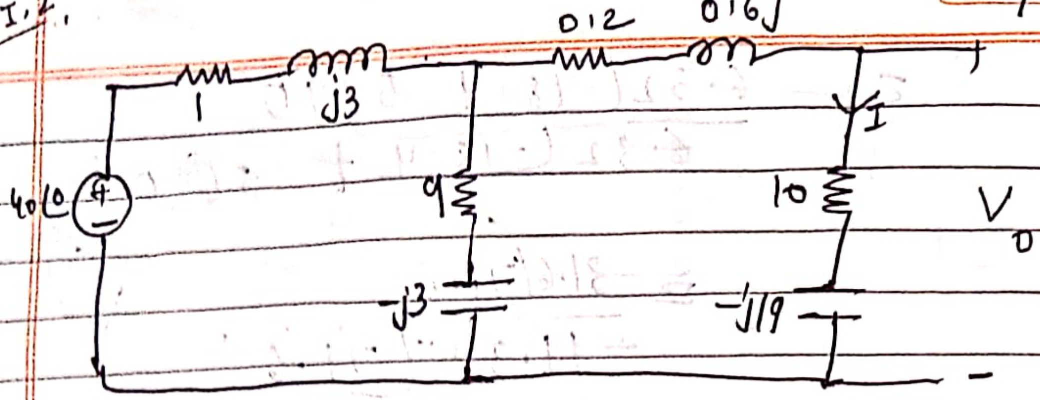
Vc I.Z.



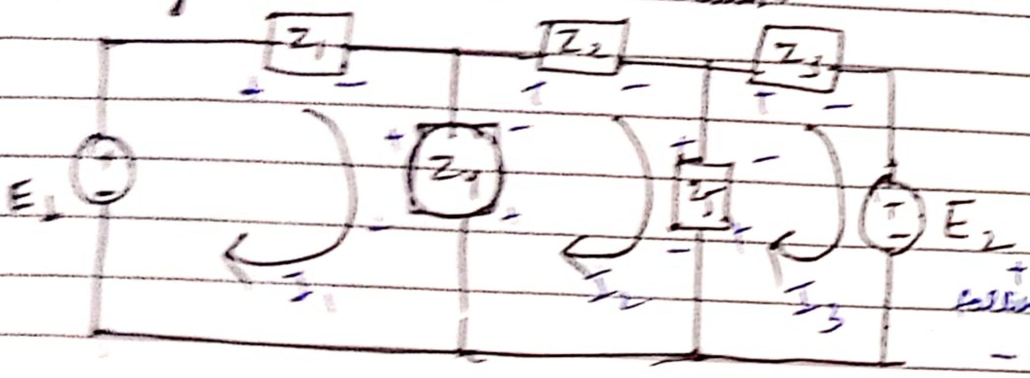
Find $V_0 = I(10 - j19)$.

Ans
↓
$14.74 \angle -27.05°$

* Cramer's Rule, will use.

(KVL) , $I_1 =$; $I_2 =$

* <u>Mesh Analysis</u> - find branch currents.



+ to - falling potential

- to + rising potential

Assume three mesh currents I_1, I_2, I_3.

mesh 1

$$E_1 - I_1 Z_1 - Z_4(I_1 - I_2) = 0. \quad — (1)$$

Mesh 2,

$$-Z_4(I_2 - I_1) - Z_2 I_2 - Z_5(I_2 - I_3) = 0.$$

Mesh 3,

$$-Z_5(I_3 - I_2) - Z_3 I_3 - E_2 = 0$$

$I_1 =$

$I_2 =$

$I_3 =$

$$\left.\begin{aligned} I_{z1} &= I_1 \\ I_{z2} &= I_2 \\ I_{z3} &= I_3 \\ I_{z4} &= I_1 - I_2 \text{ or } I_2 - I_1 \\ I_{z5} &= I_2 - I_3 \text{ or } I_3 - I_2. \end{aligned}\right\}$$

[Matrix method]

$$IZ = E$$

$$[I][Z] = [E].$$

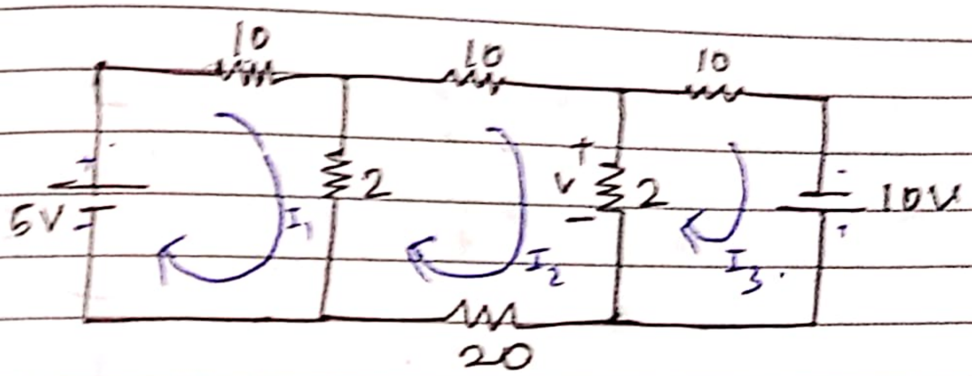
self impedances.

mutual impedance

$$\begin{bmatrix} Z_{11} & -Z_{12} & -Z_{13} \\ -Z_{21} & Z_{22} & -Z_{23} \\ -Z_{31} & -Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_1 + Z_4 & -Z_4 & 0 \\ -Z_4 & Z_4 + Z_2 + Z_5 & -Z_5 \\ 0 & -Z_5 & Z_5 + Z_3 \end{bmatrix}$$

Ques:



find V.

$$5 + 10 I_1 + 2(I_1 - I_2) = 0 \quad \text{— (i)}$$

$$20 I_2 + 2(I_2 - I_1) + 10 I_2 - 2(I_2 - I_3) = 0 \quad \text{— (ii)}$$

$$2 I_3 + 10 I_3 + 10 = 0 \quad \text{— (iii)}$$

$$5 + 10 I_1 + 2(I_1 - I_2) = 0$$

$$5 + 10 I_1 + 2 I_1 - 2 I_2 = 0$$

$$5 + 12 I_1 - 2 I_2 = 0.$$

$$5 - 10I_1 - 2(I_1 - I_2) = 0 \quad \text{—} \; 1$$

$$-2(I_2 - I_1) - 20I_2 - 2(I_2 - I_3) = 0$$

$$-2(I_3 - I_2) - 10I_3 + 10 = 0$$

$6.3\angle 51.3$ $(1+j2)$ $(3+j3)$ $A_{11} = (1+j2)(3+j3)$

A_{21}

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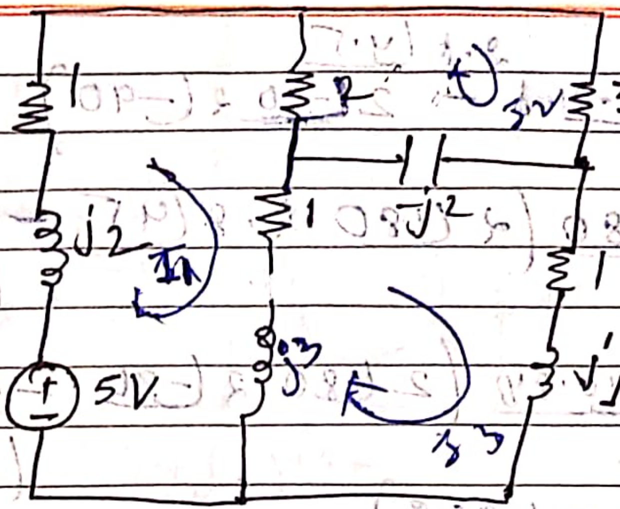


find voltage across capacitor.

$$\begin{bmatrix} (1+j2)+(3+j3) & -2 & (1+j3) \\ -2 & 5-j2 & -(-j2) \\ -(j3+1) & -(-j2) & j2+2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4+j5 & -2 & -1-j3 \\ -2 & 5-j2 & -(-j2) \\ -(j3+1) & -(-j2) & j2+2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \Big[4+j5 \left[(5-j2)(j2+2) - (j2 - j2)\right] + -2\left[-2(j2+2) + (j2)(j3+1)\right]$$

$$+ (-1-j3)\left[-2j2 + (5-j2)(j3+1)\right] \Big]$$

$$= \begin{bmatrix} 6.3\angle 51.3 & 2\angle 180 & 3.16\angle -108.43 \\ 2\angle 180 & 5.3\angle -21.8 & 2\angle -90 \\ -3.16\angle 18.4 & 2\angle -90 & 2.8\angle 45 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$2.8\angle 45$$

$$\Big[6.3\angle 51.3\,\big(5.3\angle -21.8 - 2\angle -90 \cdot 2\angle -90\big)$$

$$-\,2\angle 180\,\big(2\angle 180 \cdot 2.8\angle 45 - 2\angle -90 \cdot (-3.16\angle 18.4)\big)$$

$$+\,3.16\angle -10.84\,\big(2\angle 180 \cdot 2\angle -90 - 5.3\angle 21.8 \cdot (-3.16\angle 18.4)\big)$$

$$\cancel{\Big[6.3\angle 51.3\,\big(4.92 - 1.965\big)}$$

$$\Rightarrow \Big[6.3\angle 51.3\,\big(14.84\angle -17.3 - 4\angle -180\big)$$

$$-\,2\angle 180\,\big(5.6\angle 225 - (6.32)(\angle -71\ldots$$

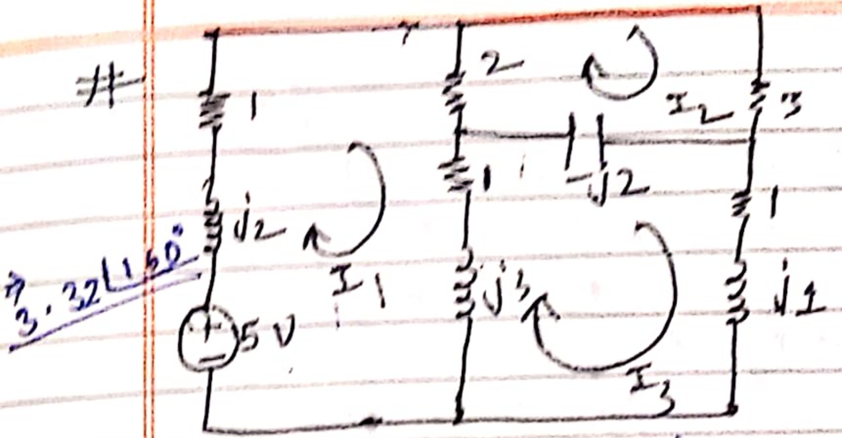
$$+\,3.16\angle -10.843\,\big(4\angle 90 - 16.74\angle 40.2\big)$$

$$Z \cdot \Big[$$

$\Rightarrow 0.902\angle$ ...

$0.42 \ominus 0.49j$

#



$I_2 = \frac{\Delta 2}{\Delta}$

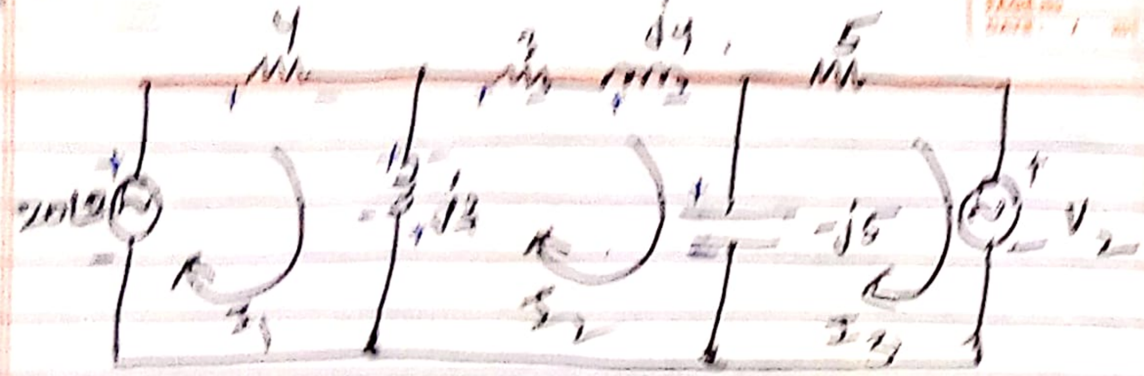
$$\begin{bmatrix} (1+j2)(3+j3) & -2 & (-1-j3) \\ -2 & 5-j2 & -(-j2) \\ -(j3+1) & -(-j2) & j2+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4+j5 & -2 & -1-j3 \\ -2 & (5-j2) & -(-j2) \\ -(j3+1) & -(-j2) & j2+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \Big[4+j5\,[(5-j2)(j2+2) - (j2-j2)] + 2\,[-2(j2+2) - (j2 + j3+1)] + (-1-j3)\,[-2(j2) - (5-j2)(-j3-1)] \Big]$$

$=$

Ans:



Determine value of V_2 such that current in $(3+j4)$ is zero using mesh analysis.

Mesh 1,

$$20\angle 0 - 4I_1 - j3(I_1 - I_2) = 0 \quad \text{--- (1)}$$

Mesh 2,

$$-j3(I_2 - I_1) - 3I_2 - j4I_2 - (-j5)(I_2 - I_3) = 0 \quad \text{--- (II)}$$

$$-(j5)(I_3 - I_2) - 5I_3 - V_2 = 0 \quad \text{--- (3)}$$

or

$$\Delta_2 = \begin{bmatrix} 4+j3 & 20\angle 0 & 0 \\ -j3 & 0 & j5 \\ 0 & -V_2 & 5-j5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+j3 & 20 & 0 \\ -j3 & 0 & j5 \\ 0 & -V_2 & 5-j5 \end{bmatrix}$$

$$= \left[4+j3\left(0 + V_2\, j5\right) - 20(-j3)5-j5 - 0 + 0(- \; -) \right]$$

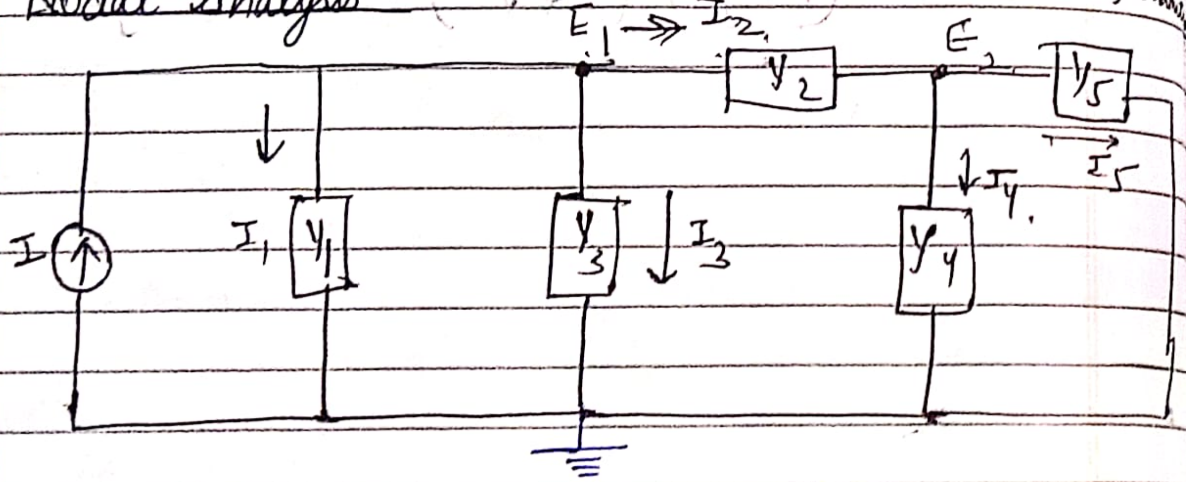
$$= (4+j3)(V_2\, j5) - 20(j3)(5-j5)$$

$$= (4+j3)(V_4 j5) - 20(-5j3 - 15)$$

$Y = \frac{1}{Z}$ $Z = \frac{1}{Y}$

Y → admittance

Nodal Analysis -



Node 1 ⇒ $I = I_1 + I_2 + I_3$ or $I_1 + I_2 + I_3 = I$

$E_1 Y_1 + (E_1 - E_2) Y_2 + E_1 Y_3 = I$

Node 2 - $I_2 = I_4 + I_5$ or $I_2 - I_4 - I_5 = 0$

$(E_1 - E_2) Y_2 - E_2 Y_4 - E_2 Y_5 = 0$

$(Y_1 + Y_2 + Y_3) E_1 + (-Y_2) E_2 = I$ — (1)

$(-Y_2) E_1 + (Y_2 + Y_4 + Y_5) E_2 = 0$ — (2)

$$\begin{bmatrix} Y_1 + Y_2 + Y_3 & -Y_2 \\ -Y_2 & Y_2 + Y_4 + Y_5 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$$[Y][E] = [I]$$

$$\begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

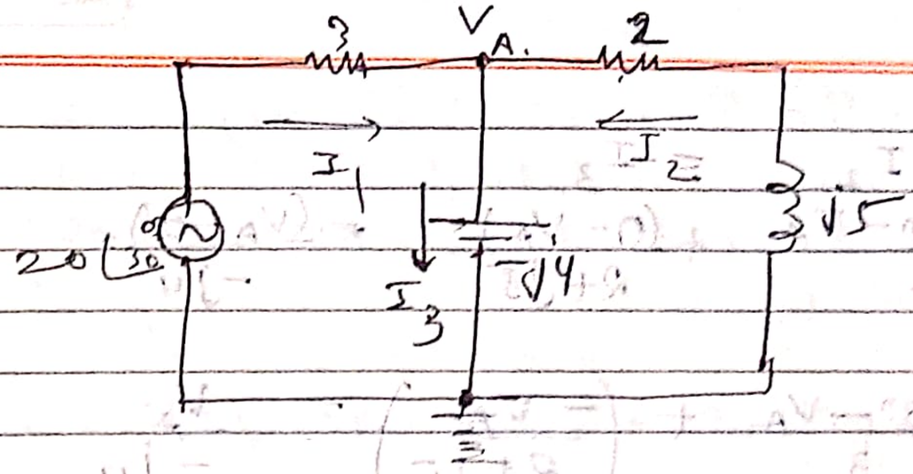
mutual admittance

self admittance

16.27 17.32 ↳ angle caution nahi.

17.32

Ques.



Find the power dissipated in each resistance.

$\Rightarrow \quad I_1 + I_2 = I_3$

$$\frac{20\angle 30 - V_A}{3} + \frac{(0 - V_A)}{2+j5} = \frac{(V_A - 0)}{-j4}$$

$I_1 =$

$I_2 =$

$P_{3\Omega} = I_1^2 \times 3 =$

$P_{2\Omega} = I_2^2 \times 2$

$$\Rightarrow \frac{(17.32+10j - V_A)}{3} + \frac{(0 - V_A)}{2+j5} = \frac{V_A - 0}{-j4}$$

$$\Rightarrow \frac{(17.32+10j - V_A)(2+j5) + (-3V_A)}{3\cdot(2+j5)} = \frac{V_A}{-j4}$$

$$\frac{(17.32+10j - V_A)(2+j5) + (-3V_A)}{3\cdot(2+j5)} - \frac{V_A}{-j4} = 0$$

$$\Rightarrow \frac{(-j4)\left[(17.32+10j - V_A)(2+j5) - 3V_A\right] - 3V_A(2+j5)}{3\cdot(2+j5)(-j4)} = 0$$

$$\Rightarrow \frac{(-j4)\left[(19.99\angle 30° - V_A)(5.3\angle 68.1) - 3V_A\right](5.38\angle 68.1)}{-12j\,(2+j5)} = 0$$

$19.99\angle 30°(-j4)$

$\Rightarrow$

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$$I_1 + I_2 = I_3$$

$$\frac{20\angle 30 - V_A}{3} + \frac{(0 - V_A)}{2+j5} = \frac{(V_A - 0)}{-j4}$$

$$\frac{20\angle 30 - V_A}{3} + \left(\frac{-V_A}{2+j5}\right) = \frac{V_A}{-j4}$$

$$\frac{20\angle 30 - V_A}{3} = \frac{V_A}{-j4} + \frac{V_A}{(2+j5)}$$

$$\frac{20\angle 30}{3} = \frac{V_A}{-j4} + \frac{V_A}{2+j5} + \frac{V_A}{3}$$

$$\frac{20\angle 30}{3} = \frac{V_A\left(3(2+j5) - 3j4 + j4(2+j5)\right)}{-3j4(2+j5)}$$

$$= \cancel{\frac{V_A\left(3(5.38\angle 68.1) -\right.}{-3j4(5.38\angle 68.1)}}$$

$$\frac{20\angle 30}{3} = V_A\left(\right.$$

$$\frac{(17.32 + 10j)(-3j4(2+j5))}{3\cdot(3(2+j5) - 3j4 + j4(2+j5))} = V_A$$

$$\cancel{(17.32 + 10j)(-12j(2+j5))}$$

$$\frac{1279.2 + 184.32j}{-42 + 33i}$$

$$-16.69 - 17.50j = V_A$$

$$I_1 + I_2 = I_3$$

$$\frac{20\angle 30 - V_A}{3} + \frac{(0 - V_A)}{2+j5} = \frac{V_A - 0}{-j4}$$

$$\frac{(17.32 + 10j}{3} - \frac{V_A)}{3} - \frac{V_A}{2+j5} = \frac{V_A}{-j4}$$

$$\frac{17.32 + 10j}{3} = -\frac{V_A}{j4} + \frac{V_A}{3} + \frac{V_A}{2+j5}$$

$$\frac{17.32 + 10j}{3} = (0.402 + 0.077j)\,V_A$$

$$15.38 + 5.34j = V_A$$

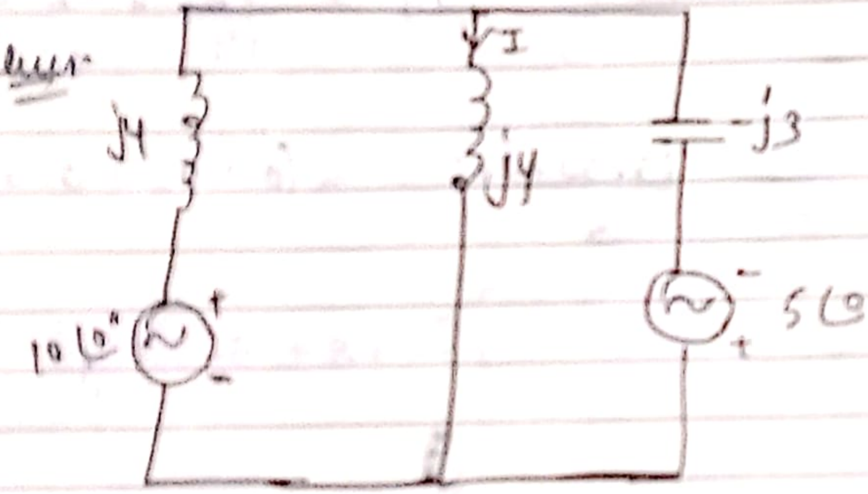
Superposition Theorem

In an active linear network containing several sources, the overall response in any branch is equal to the algebraic sum of the response of each source considered separately with all sources replaced by their internal impedances.

* internal impedance of ideal voltage source
↓
short ckt the ckt or impedance = 0.

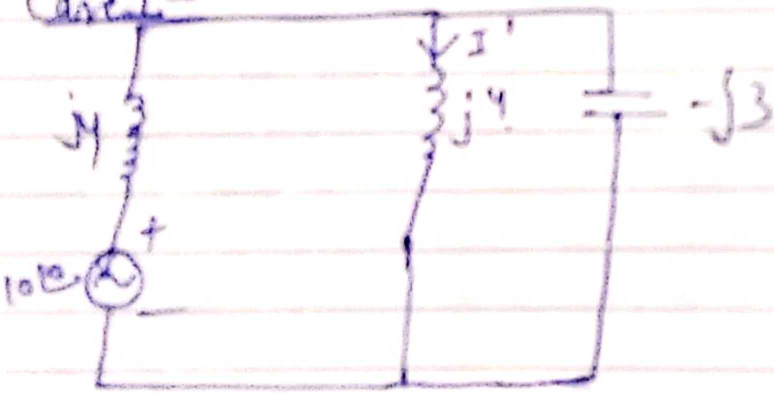
Internal impedance of ideal current source
↓
open ckt the ckt or impedance = ∞.

Case:



find I.

Case I:-



$$Z_{eq} = [j4 \,||\, -j3] + j4$$

$$= \frac{(j4)(-j3)}{j4 + (-j3)} + j4$$

$$= -j8.$$

$12j$

$8\angle -90$ $3\angle -90$

PAGE No. DATE $3\angle -90$

$$I_{AB} = \frac{V}{Z}$$

$$= \frac{10\angle 0}{0 - j8} = \frac{10\angle 0}{8\angle -90}$$

$$= 1.25\angle 90$$

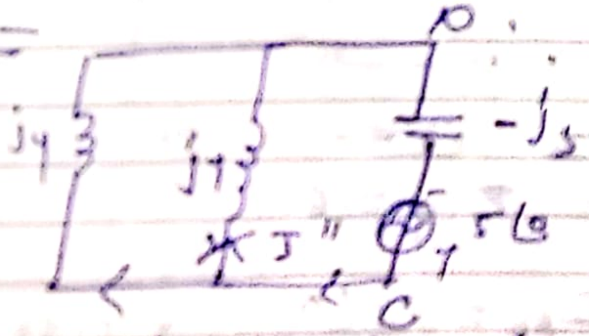
$$I' = 1.25\angle 90 \times \frac{-j3}{j4 + (-j3)}$$

$$I' = 1.25\angle 90 \times \frac{3\angle -90}{j}$$

$$= \frac{1.25\angle 90 \times 3\angle -90}{1\angle 90}$$

$$I' = 3.75\angle -90$$

Case 2 –



$$Z_{eq} = (j4 \parallel j4) + (-j3) = -j$$

$$I_{CD} = \frac{5\angle 0}{0 - j} = 5\angle 90 \text{ or } j5$$

$$I'' = j5 \times \frac{j4}{j4 + j4}$$

$$I'' = 2.5j$$

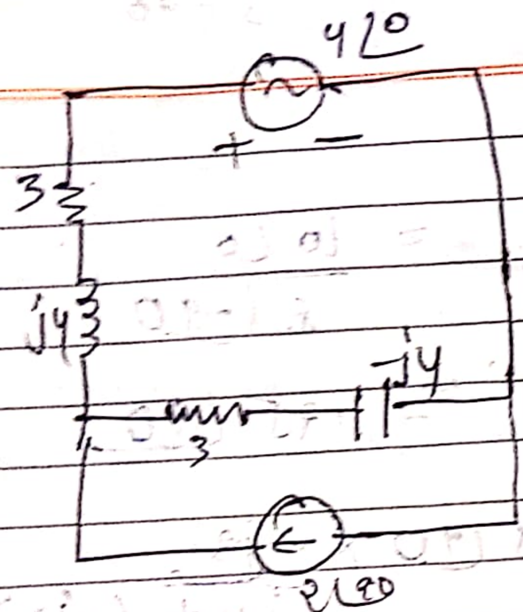
$$I = I + (-I)''$$

$$I = (-j3.75 + -j2.5) = -j6.25$$

$$= 6.25\angle 90$$

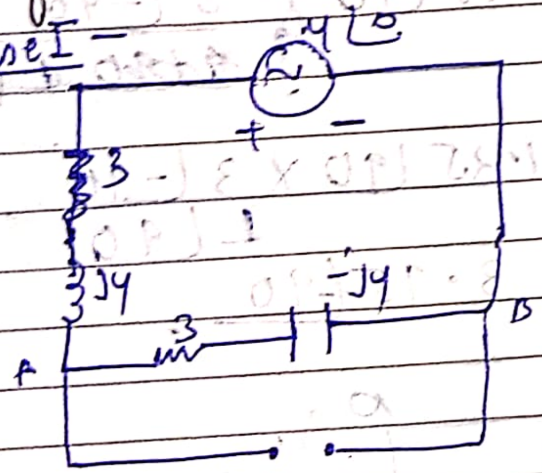
$4.12\angle -75.9$

Ans



find current in capacitor.

Case I –

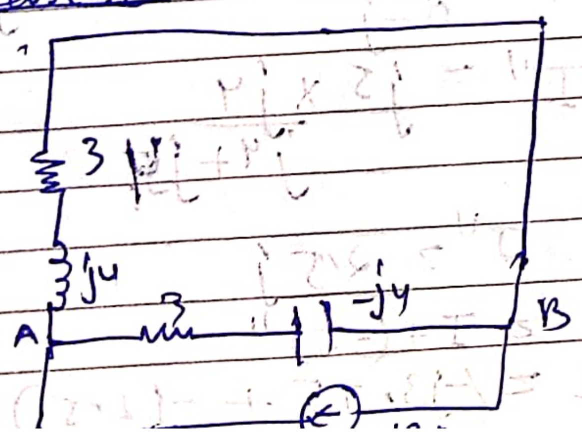


$Z_{eq} = \cancel{30}$

$Z_{eq} = 6.$

$I_{AB} = \frac{4\angle 0}{6} = 0.66\angle 0.$

Case II –



$V = IZ$

$Z_{eq} = 2\angle 90 \times \frac{3+j4}{3+\cancel{j4}+3-\cancel{j4}}$

$= 2\angle 90 \times \frac{3+j4}{6}$

$= 2\angle 90 \times \frac{5\angle 53.13}{6}$

$= 2\angle 90 \times 0.83\angle 53.13$

$= 1.66\angle 143.13$

$I = Z_{eq} + Z_{eq}$

$= 0.66\angle 0 + 1.66\angle 143.13$

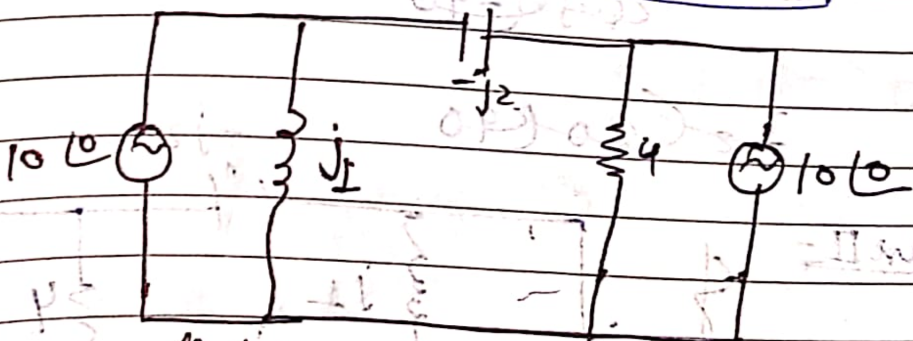
$= 0.66 + 0j + (-1.32 + 0.99j)$

$= -0.66 + j$

$\boxed{I = 1.19\angle 123.4}$

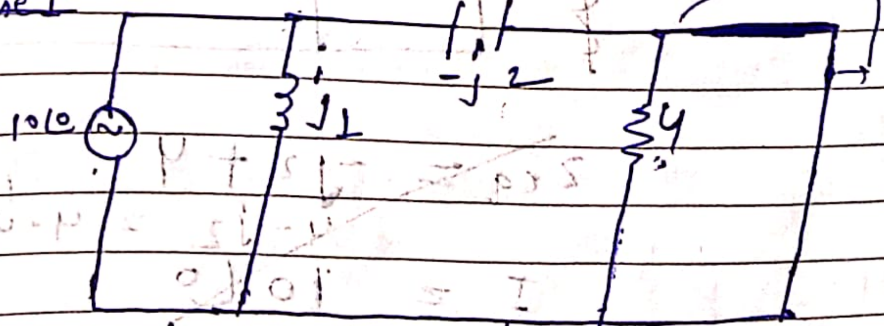
15/09/23

Ques.



find current through inductor as well as capacitor.

Case I -



resultant will be zero

$Z_{eq} = j1 \parallel -j2$

$\frac{1}{Z_{eq}} = \frac{1}{j1} + \frac{1}{-j2} \Rightarrow \frac{-j2 + j1}{-j2 j1}$

$-j + 4$

$-2j$

$-j + 4$

$$= \frac{-j2 + j1}{-j2\,j1}$$

$$= \frac{-j}{-j2 \cdot j1}$$

$$= \frac{1\angle -90}{2\angle -90 \cdot 1\angle 90}$$

$$= \frac{1\angle -90}{2\angle 0}$$

$\cancel{Z_{eq} = 0.5\angle -90}$, $\boxed{Z_{eq} = 2\angle -90}$

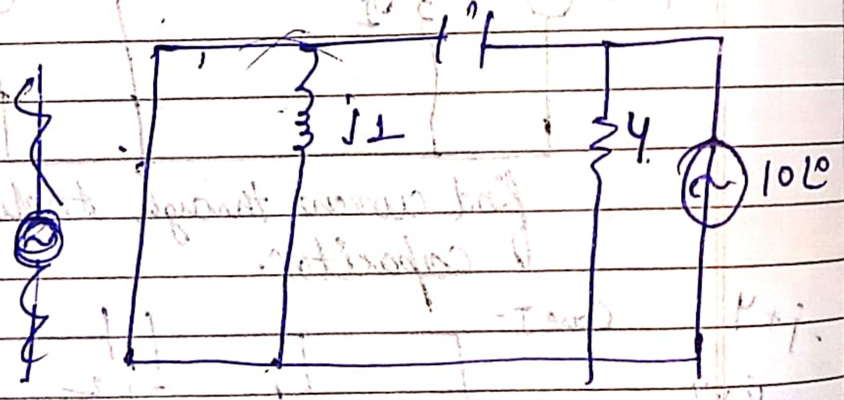
Case I –

$$I = \frac{V}{Z_{eq}}, \qquad I = \frac{10\angle 0}{2\angle -90}$$

$$\cancel{I = \frac{10\angle 0}{0.5\angle -90}} \qquad I = 5\angle 90$$

$$\cancel{I = 20\angle 90}$$

Case II –



$$Z_{eq} = \frac{-j2 + 4}{4 - j2}, \quad V = 10\angle 0$$

$$= 4.47\angle -26.5$$

$$I = \frac{10\angle 0}{4.47\angle -26.5}$$

$$I = 2.23\angle 26.5$$

$10\angle 0$

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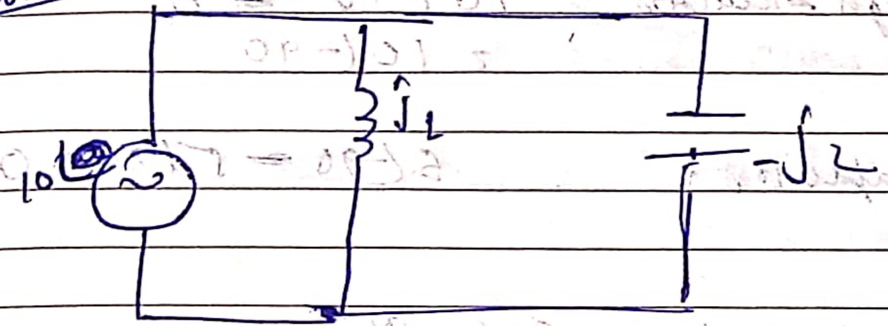
$$\frac{1}{Z_{eq}} = \frac{1}{-j2} + \frac{1}{4}$$

$$= \frac{4 \cdot -j2}{4 \cdot -j2}$$

Case I.



$$I \text{ through } j1 \text{ (inductor)} = \frac{10\angle 0}{j1}$$

$$= \frac{10\angle 0}{1\angle 90}$$

$$= 10\angle -90$$

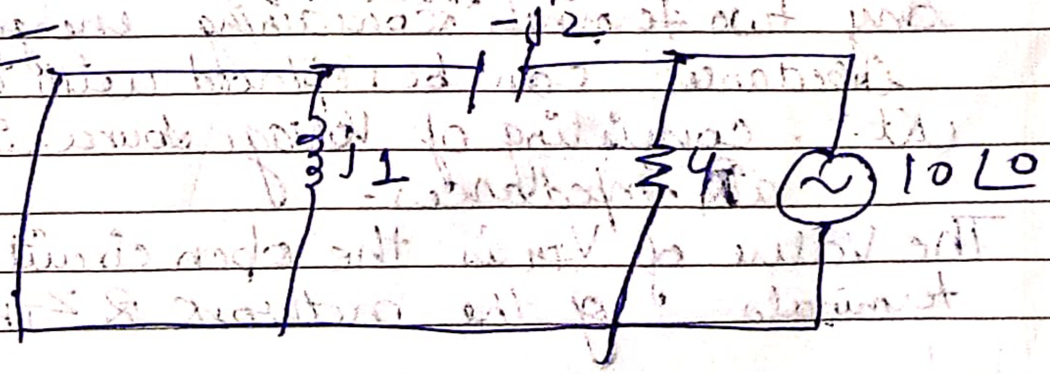
$$I \text{ through } -j2 \text{ (capacitor)} = \frac{10\angle 0}{-j2}$$

$$= \frac{10\angle 0}{2\angle -90}$$

$$= 5\angle 90$$

$$= 10\angle -90 + 5\angle 90$$

Case II

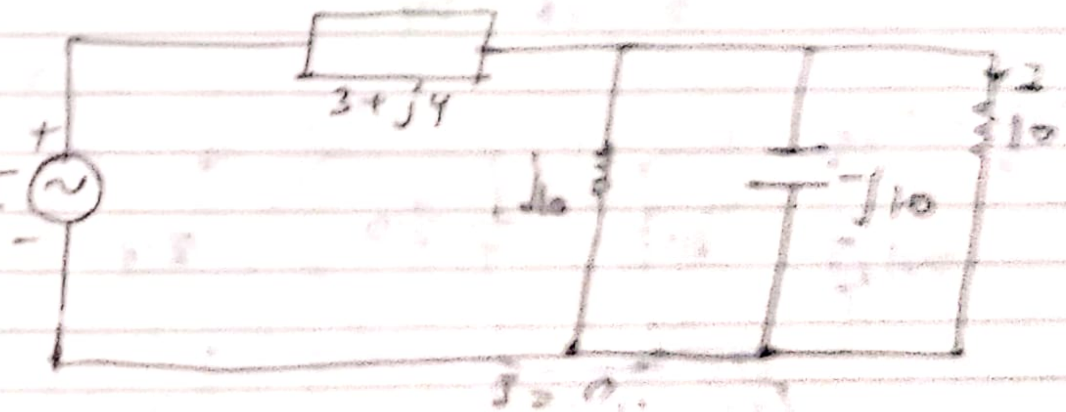


measured b/w the terminals of the network with all energy sources replaced by their internal impedances.

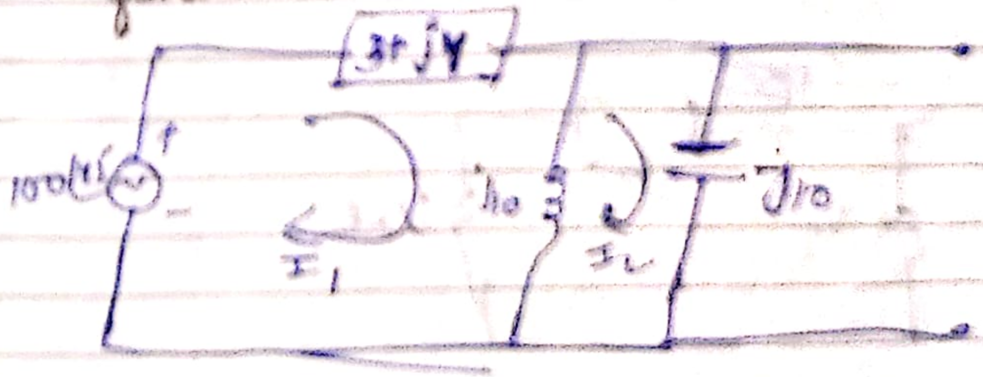
Limitations of Thevenin's theorem.-

1. Not applicable to circuits consists unilateral elements like diodes.
2. Not applicable to circuits consisting of non-linear elements like diode, transistor.
3. Not applicable to circuits consisting of controlled or dependent sources.
4. Not applicable to circuits consisting of magnetic coupling between load & any other ckt element.

eg:



find V_{TH} or V_{OC}



$j2 \,||\, -j2 \,||\, 10\angle 0$

Capacitor $= \frac{10\angle 0}{-j2}$, inductor $= 0$

$= 5\angle 90$

current through inductor $= 10\angle -90 = 0$

$= 10\angle -90$

∴ capacitor : $5\angle 90 = 5\angle 90 = 0$

Limitation of superposition theorem –

1. Applicable only to linear ckt., not applicable to circuits consisting non-linear elements like diode, transistor etc.

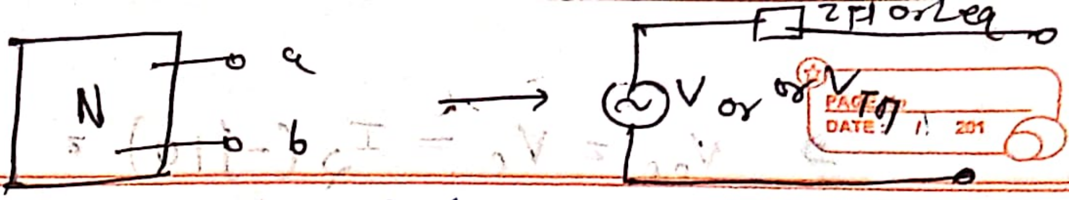
2. Not valid for power Relationships.

3. Not applicable to ckt consisting of only dependent sources, not useful to ckts consisting of less than two independent sources.

Thevenin's Theorem –

Any two terminals electric ckt containing energy sources & impedances can be replaced with an equivalent ckt consisting of voltage source in series with an impedances.

The value of V_{TH} is the open circuit voltage between terminals of the network & Z_{TH} is the impedance



measured b/w the terminals of the network with all energy sources replaced by their internal impedances.

Limitations of Thevenin's theorem -

1. Not applicable to circuits consists unilateral elements like diode.
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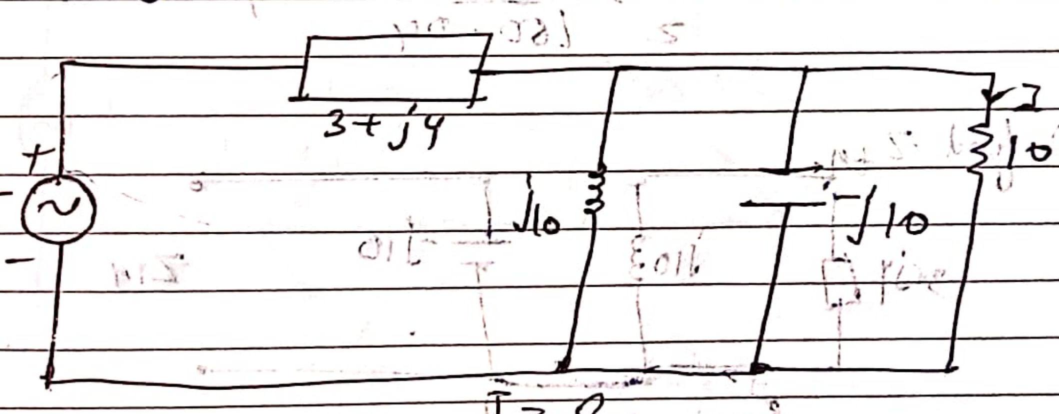
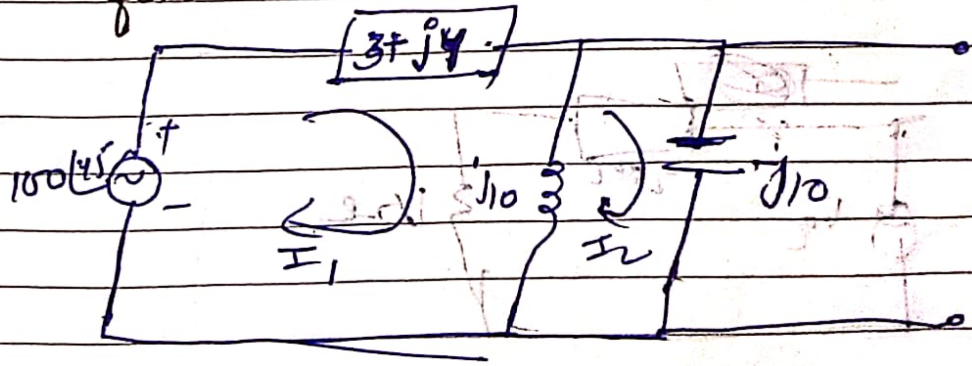
4. Not applicable to circuits consisting of magnetic coupling between load & any other CKT element.

Ques 2



find V_{TH} or V_{OC}



$\Rightarrow V_{oc} = V_c = I_2(-j10)$

$$\begin{bmatrix} 3+4j & -j10 \\ -j10 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100\angle 45 \\ 0 \end{bmatrix}$$

$\Delta = 100\angle 0$

$\Delta_1 = 0$, $\Delta_2 = 1000\angle 135$

$I_2 = \frac{\Delta 2}{\Delta} = \frac{1000\angle 135}{100\angle 0}$

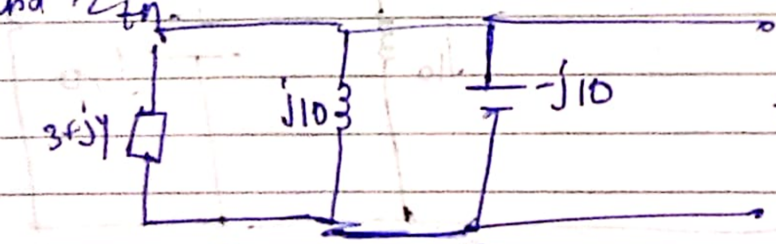
$= 10\angle 135$,

$V_{oc} = V_c = 10\angle 135\,(-j10)$

$= 10\angle 135 \quad 10.04\angle -84.2$

$= 100.04$

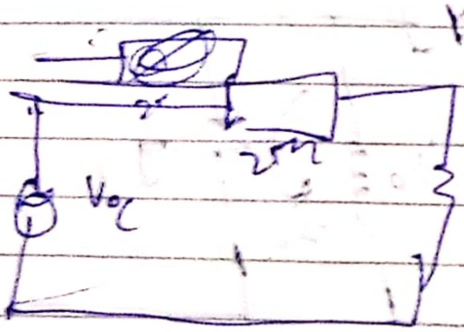
(2) find Z_{th}.



$[3+j4 \,||\, j10] \,||\, (-j10) = Z_{th} =$

$I = \frac{V_{oc}}{Z_{th}+10}$

(3)



70.7 + 70.71 j

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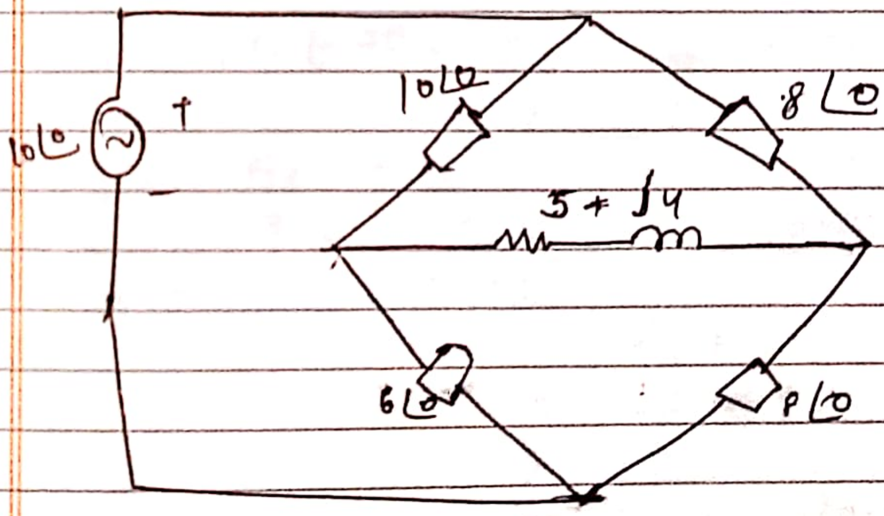
#



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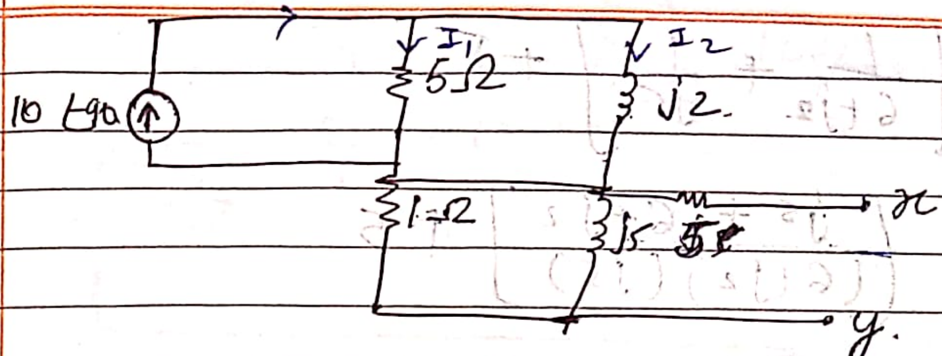
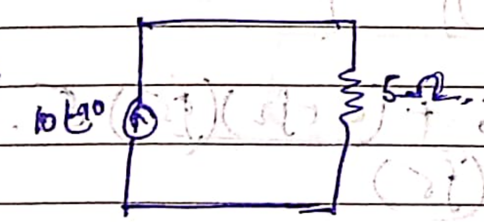
Ckt



Find V_{oc} + Z_{th} at terminals $x-y$.

⇒



$V_{oc} = V_{j5} = I_2(j5) =$

$I_2 = \dfrac{10\angle{-90} \times 5}{5+(j2+j5+1)}$

$= 10\angle{-90} \times \dfrac{5}{5+j7+1}$

$= 10\angle{-90} \times \dfrac{5}{6+j7}$

$= 10\angle{-90} \times \dfrac{5}{9.2\angle 49.3}$

$\boxed{\cancel{\dfrac{50\angle{-90}}{5.95\angle 0.73}} \quad = \cancel{8.40\angle{-90.73}}}$

$= \dfrac{50\angle{-90}}{9.2\angle 49.3}$

$= 5.43\angle{-139.3}$

$V_{oc} = I_2 \times j5$

$= \cancel{8.40\angle{-90.73} \times}$

$= 5.43\angle{-139.3} \times j5$

$= 5.43\angle{-139.3} \times 5\angle 90$

$\boxed{V_{oc} = 27.15\angle{-49.3}\ V.}$

$Z_{TH} = [(5+1+j2) \,||\, j5] + 5$

$= [(6+j2) \,||\, j5] + 5$

$v = J2$

$Z_{TH} = 7.3 \angle 5 \quad \angle 23.5$

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$$= \left[\frac{1}{6+j2} + \frac{1}{j5}\right] + 5$$

$$= \left[\frac{j5 + 6 + j2}{(6+j2)(j5)}\right] + 5$$

$$= \left[\frac{j7 + 6}{(6+j2)(j5)}\right] + \frac{5}{1}$$

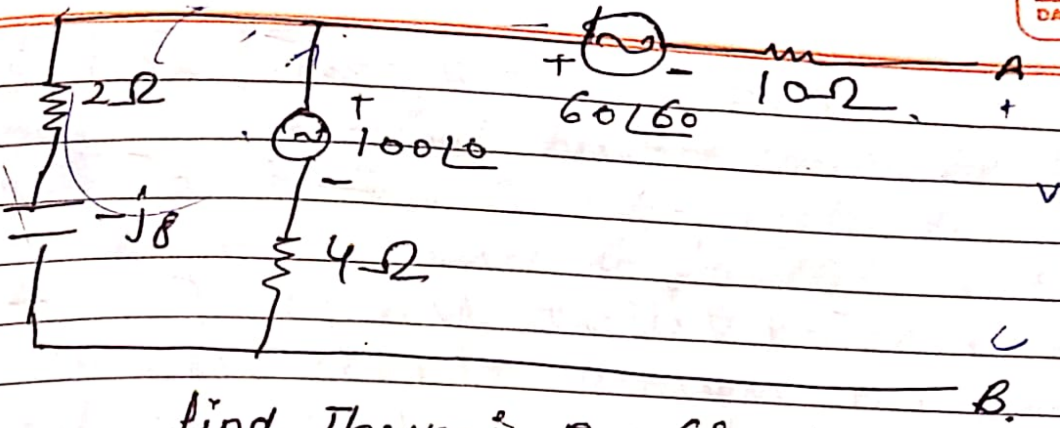
$$= \frac{j7 + 6 + (6+j2)(j5)\,5}{(6+j2)(j5)}$$

$$= \frac{}{6.32 \angle 18.4 \quad 5 \angle 90}$$

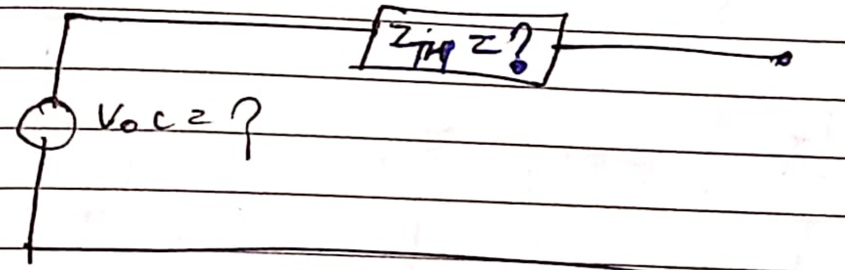
V_{TH}

$= 96 \angle 6$

TH = (13 − j(1·28) jΩ



find Thevenin Eq. Circuit



~~RTH~~

$R_{TH} = 2 - j8 + 4$

$= 6 - j8$

~~$I_1 = 100\angle 0 \times 6 - j8$~~

~~$= 100\angle 0 \times 10\angle -53.13$~~

~~$= 1000\angle 53.13$~~

$I = \dfrac{100\angle 0}{10\angle -53.13}$

$= 10\angle 53.13$

Voltage drop across 4Ω : $= 10\angle 53.13 \times 4$ $= 24 + j32$

$= 40\angle 53.13$

∠113°V

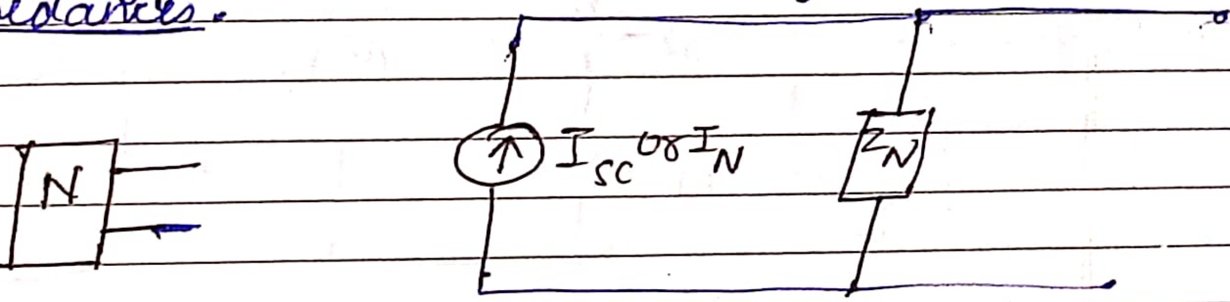
applying KVL, ~~$-(24+j32)$~~

$-40\angle 53.13 + 100\angle 0 - 60\angle 60 - V_{oc} = 0$

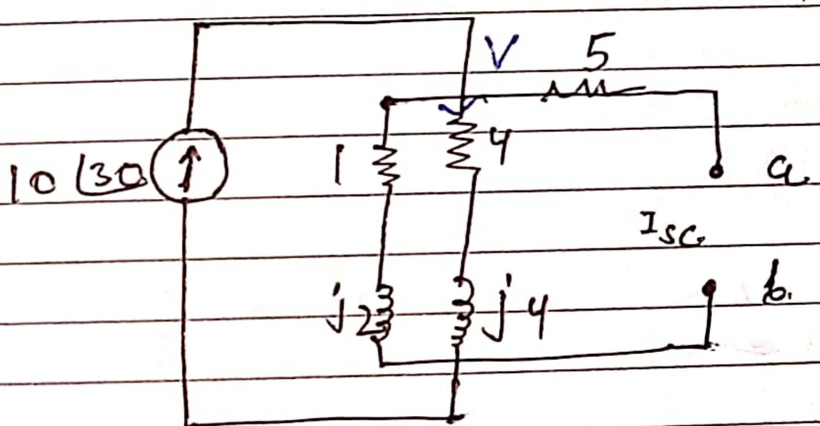
$-40\angle 53.13 + 100\angle 0 - 60\angle 60 = V_{oc}$

Norton's Theorem –

Any two terminal network containing energy sources and impedances can be replaced by an equivalent source in parallel with an impedance i.e., The value of current source is thus, short circuit current between the terminals with all energy sources replaced by their internal impedances.



Norton's Eqⁿ Circuit.



Draw norton's Eq. circuit

By nodal,
apply KCL eqⁿ –

$$10\angle 30 = \frac{V}{1+j2} + \frac{V}{4+j4} + \frac{V}{5}.$$

$$10\angle 30 = V\left[\frac{(4+j4)(5)+5(1+j2)+(1+j2)(4+j4)}{(1+j2)(4+j4)(5)}\right]$$

$$10\angle 30 = V\left[\frac{5.65\angle 45\ 5\angle 0 + (5)2.23\angle 63.4 + 2.23\angle 63.4\ 5.65\angle 45}{2.23\angle 63.4 \cdot 5.65\angle 45\ 5\angle 0}\right]$$

$V = 12.664$, $Z_n = 1.09 \angle 18.1$ $Z = [(1+j2) \parallel (4+j4)] + 5$

$10 \angle 30 \times 2.23 \angle 63.4 \times 5.65 \angle 45 \times 5 \angle 0$

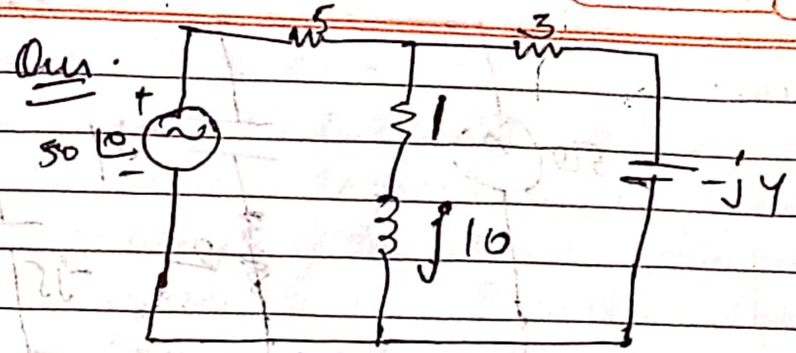
$= V(5.65 \angle 45 \times 5 \angle 0 + 5 (2.23$

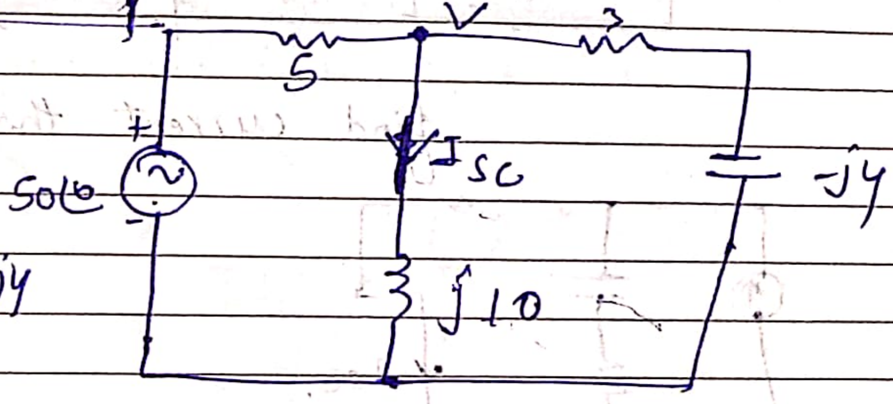
$\Rightarrow 629.975 \angle 138.4 = V(28.25 \angle 45 + 11.15 \angle 63.$ $+ 12$

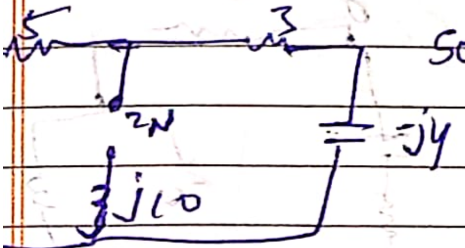
$V(19.975j$

$$\frac{(3.4 + (2.23 \angle 63.4)}{(5.65 \angle 45)} \quad .5995 \angle 108.4$$

Ans.







$$\frac{V - 50\angle 0}{5} + \frac{V}{j10} + \frac{V}{3 - j4} = 0$$

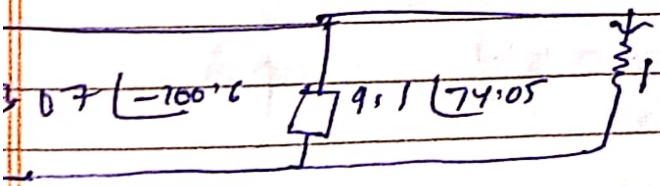
$$\cancel{V = 30.7 \angle -100.6}$$

$$\boxed{V = 30.7 \angle -10.6}$$

$$\frac{V}{j10} = I_{sc} = \frac{30.7 \angle -10.6}{j10 = 10\angle 90} = 3.07 \angle -100.6$$

$$Z_N = j10 + \frac{5(3 - j4)}{5 + 3 - j4}$$

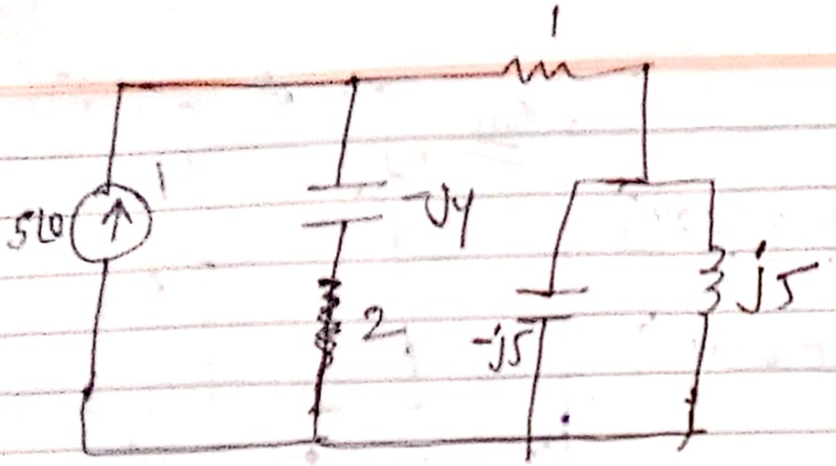
$$= 9.1 \angle 74.05$$



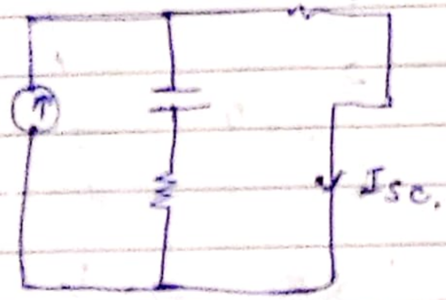
$$I = 3.07 \angle -100.6 \times \frac{9.1 \angle 74.05}{9.1 \angle 74.05 + 1}$$

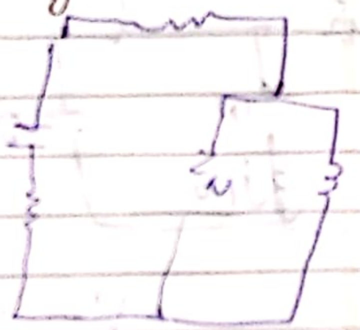
$=$

Ans.



find current through $-j5$.

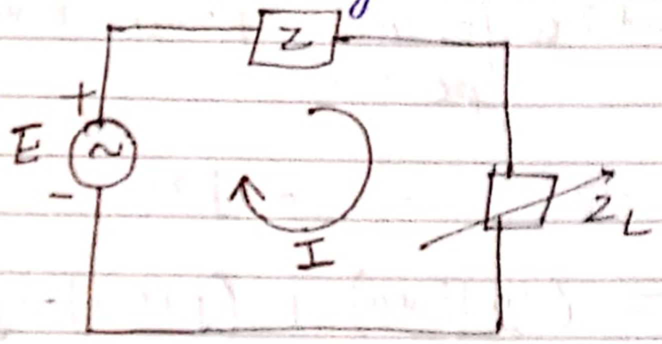




V. Imp.

Maximum Power Triangle Theorem -

Maximum power will be delivered by the network when the ~~load~~ ~~load~~ impedance is equal to the complex conjugate of the internal impedance of the network measured looking back into the terminals of the network.



X_L ↳ Reactance

Z_L → Impedance

$$I = \frac{E}{Z + Z_L} \qquad P = I^2 R_L$$

where $Z = R + jX$

$Z_L = R_L + jX_L$

$$P = \left(\frac{E}{Z+Z_L}\right)^2 R_L = \frac{E^2}{(R+R_L)^2 + (X+X_L)^2} R_L \quad \text{— ①}$$

for maximum power $\frac{\partial P}{\partial x}$ must be zero.

$$\frac{\partial P}{\partial x} = 0. \quad \Rightarrow \quad \boxed{X_L = -X}$$

Again, for maximum power $\frac{\partial P}{\partial R}$ must be zero.

$$\frac{\partial P}{\partial R} = 0$$

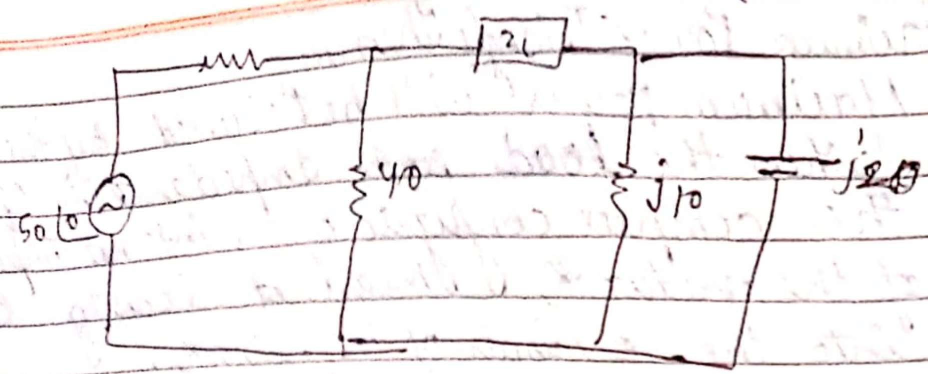
$$\boxed{R_L = R}$$

$$\boxed{P_{max} = \frac{E^2}{4R_L}}$$

$$\left(\frac{1}{10}+\frac{1}{40}\right)+\left(\frac{1}{10}-\frac{1}{20}\right)$$

$$\cancel{0.125+\left(\frac{1}{10j}-\frac{1}{j20}\right)}$$

Imp



Find Z_L for max power & value of max power.

$Z_L = Z^* = 8 - j20$

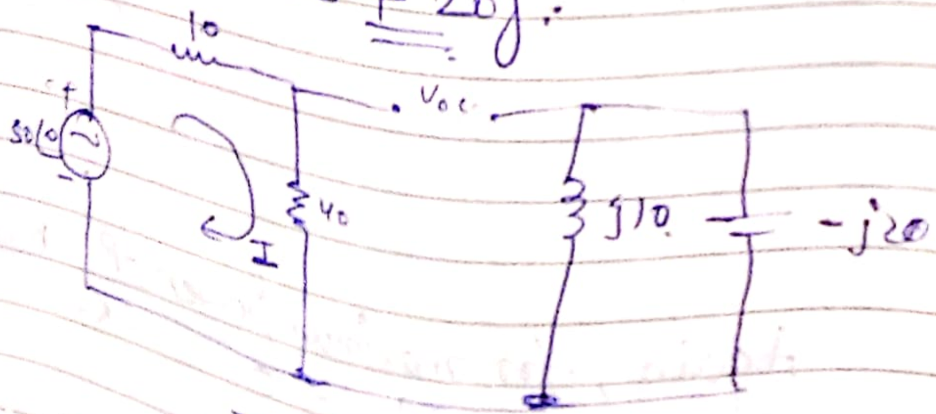
$$Z = (10 \| 40) + (j10 \| -j20)$$

$$= \frac{10\times 40}{10+40} + \frac{10j \times -20j}{10j - 20j}$$

$$= \frac{400}{50} + \left(\frac{-200j^2}{-10j}\right)$$

$$= 8 + \left(\frac{200}{-10j} \times \frac{10j}{10j}\right)$$

$$= 8 \pm 20j$$



$$I = \frac{50\angle 0}{50} = 1$$

$$V_{oc} = 40 \times 1 = 40$$

$$P_{max} = \frac{E^2}{4R_L} = \frac{(V_{oc})^2}{4R_L}$$

$$= \frac{(40\angle 0)^2}{4(8+20j)}$$

$$= \frac{(40\angle 0)^2}{(32+80j)} = \frac{(40\angle 0)^2}{86.16\angle 68.19}$$

$$= \frac{1600\angle -68.19}{86.16} = 18.57\angle -68.1$$

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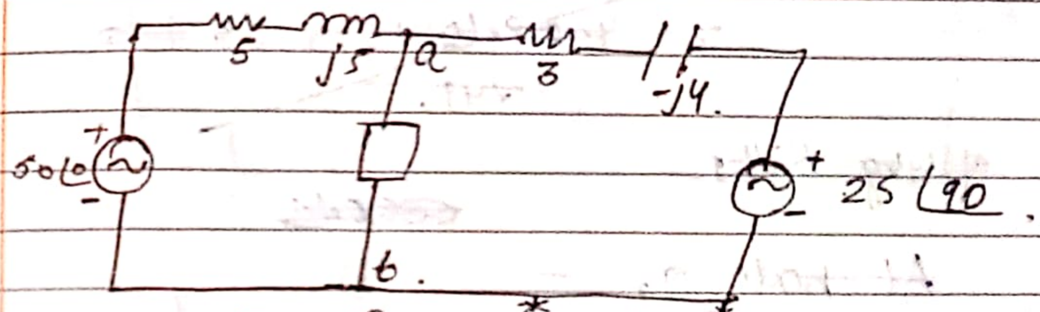
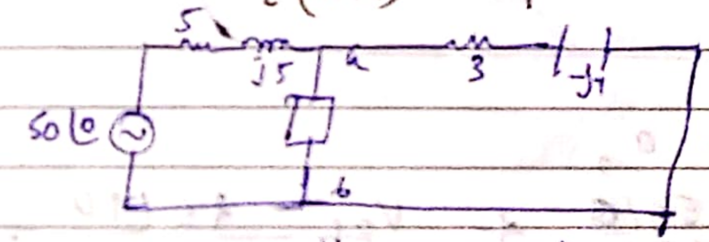


$Z_L = ?$ Z_i^* or Z_{TH}^*

$P_{L(max)} = ?$

⇒



$$Z_{TH} = 5+j5 \parallel 3-j4$$

$$= \frac{5+j5 \times 3-j4}{5+j5+3-j4}$$

$$= \frac{(5+j5)(3-j4)}{(5+j5)+(3-j4)}$$

$$= \frac{7.07\angle 45 \times 5\angle -53.1}{8+j}$$

$$= \frac{35.35\angle -8.1}{8+j} = \frac{35.35\angle -8.1}{8.06\angle 7.12}$$

$$= 4.38\angle -15.22$$

$$= 4.22 + 1.14j \quad \text{Ans}$$

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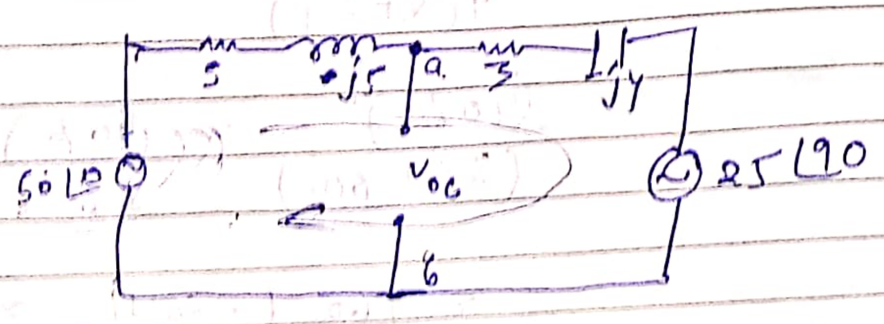
$R_L = 4.22$,

$\Rightarrow$ V_{oc}



at node.

~~At node a,~~

~~$V_{oc} = (5+j5)(50\angle 0)$,~~

~~$= 4.48\angle 0.43 \times 50\angle 0$~~

~~249.~~

~~applying KVL,~~

~~node.~~

~~At node a,~~

~~$5+j5 = 50\angle 0$~~

~~applying KVL,~~

~~$-50\angle 0 -$~~

at node a,

$$\frac{V_{oc} - 50\angle 0}{5+j5} + \frac{V_{oc} - 25\angle 90}{3+j4} = 0$$

$$\frac{3V_{oc} - 150\angle 0 - 4jV_{oc} + 5V_{oc} - 125\angle 90 + 5jV_{oc}}{\cancel{(5+j5)}}$$

$$8V_{oc} - 150\angle 0 - 125\angle 90 + jV_{oc} + 200\angle 90 - 125\angle 180 = 0$$

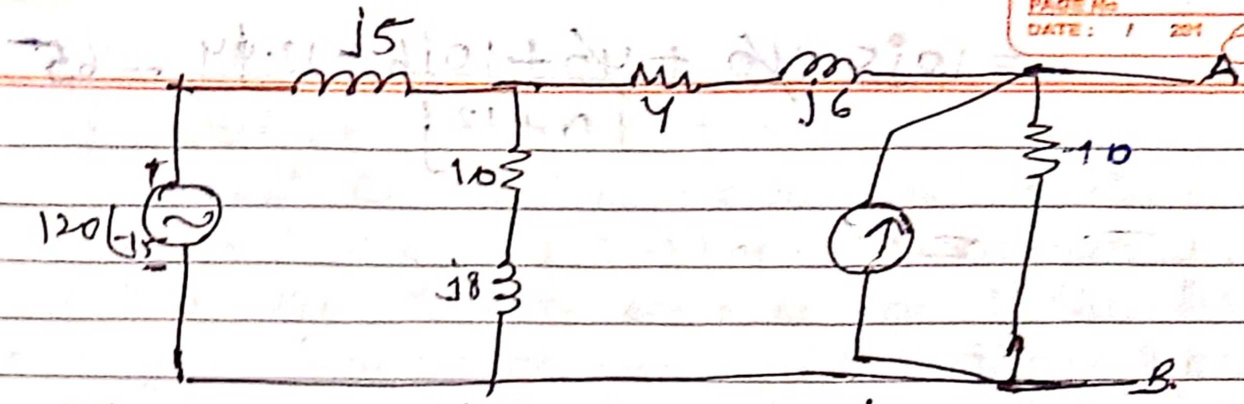
$$V_{oc}(8+j) = 150 + 125j - 200j - 125$$

$$V_{oc} = \frac{25 - 75j}{8+j} = 9.8\angle 75.5$$

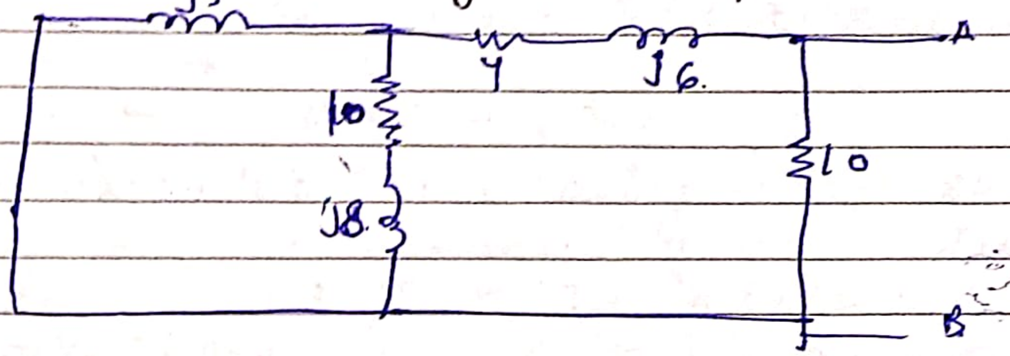
$$P_{L\,max} = \frac{(9.8)^2}{4 \times 4.23}$$

6.21 $\angle 30^\circ$

Ques



Find Z across AB for max power transfer.

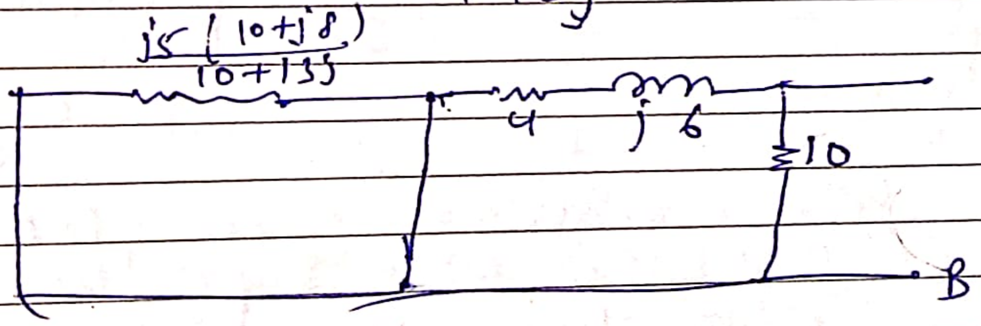


$\Rightarrow$ ~~j5~~ ~~100+j8~~

$$j5 \parallel 10+j8$$

$$Z_1 \Rightarrow \frac{j5 \times 10+j8}{j5+10+j8}$$

$$= \frac{j5(10+j8)}{10+13j}$$



$$z = \frac{j5(10+j8)}{10+13j} + \frac{4+j6}{1}$$

$$z = \frac{j5(10+j8) + (10+13j)(4+j6)}{10+13j}$$

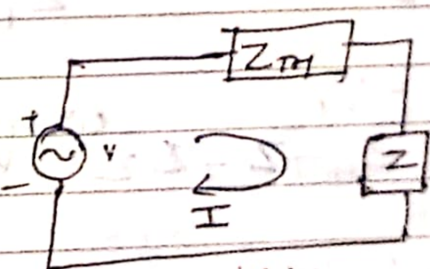
$$z = \frac{10j5 - 40 + 40 + 60 \times j6 + 13 \times 4j - 65}{10+13j}$$

9/11/22

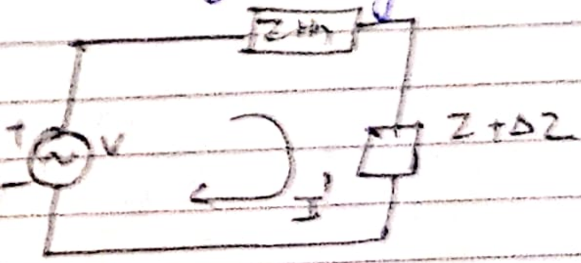
Compensation Theorem –

If a ~~circuit~~ circuit has certain distribution of voltages & currents across & through various branches and due to some reason if the impedance of one of the branch changes, the voltage & currents in other branches are affected. To nullify this affect the voltage source should be changed accordingly.

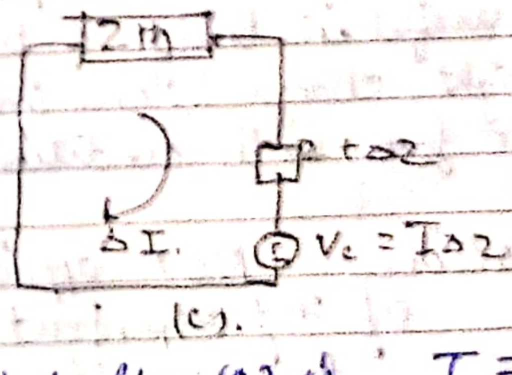
In a linear network N, having voltage source V (actually V_{oc}) with internal impedance Z_{th}, it delivers current I to the impedance Z. If the impedance changes from Z to $Z+\Delta Z$, then the change in current ΔI can be obtained by replacing voltage source with its internal impedance & placing a compensation voltage source of magnitude $V_c = I \cdot \Delta Z$ in series with impedance $(Z+\Delta Z)$ & its polarity is opposite to the flow of current I.



(a).



(b.)



(c).

The current in fig. (a) is $I = \dfrac{V}{Z_{th}+Z}$ — ①

$$= \frac{10j5 - \cancel{4/0} + \cancel{4/0} + 10j6 + 13 \cdot 44 - 65}{10 + 13j}$$

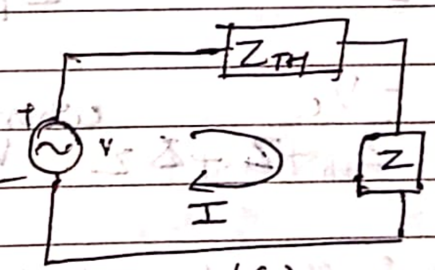
Z

DATE 29/09/22

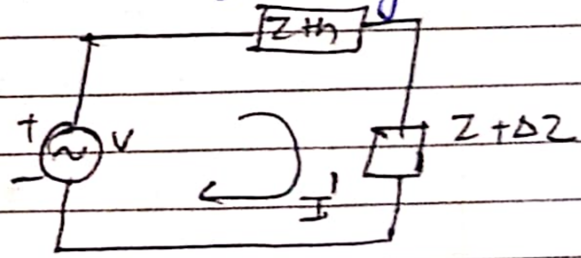
Compensation Theorem –

If a ~~circuit~~ circuit has certain distribution of voltages & currents across & through various branches and due to some reason if the impedance of one of the branch changes, the voltage & currents in other branches are affected. To nullify this affect the voltage source should be changed accordingly.

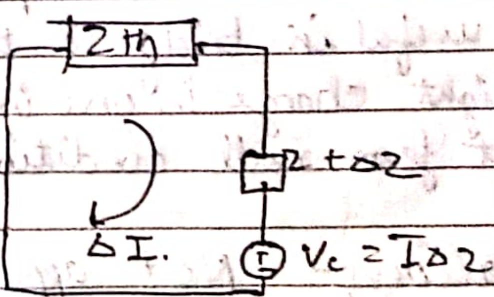
In a linear network N, having voltage source V (actually V_{oc}) with internal impedance Z_{th}, it delivers current I to the impedance Z. If the impedance changes from Z to $Z+\Delta Z$, then the change in current ΔI can be obtained by replacing voltage source with its internal impedance & placing a compensation voltage source of magnitude $V_c = I\Delta Z$ in series with impedance $(Z+\Delta Z)$ & its polarity is opposite to the flow of current I.



(a).



(b.)



(c).

The current in fig. (a) is $I = \dfrac{V}{Z_{th}+Z}$ — (1)

In Fig (ii), impedance is changed from Z to $Z+\Delta Z$

$\therefore$ Current $I' = \dfrac{V}{Z_{th} + (Z+\Delta Z)}$ —— ②

from Eqⁿ ① & ②

$$\Delta I = I' - I = \frac{V}{Z_{th}+(Z+\Delta Z)} - \frac{V}{Z_{th}+Z}$$

$$\Delta I = \frac{V(Z_{th}+Z) - V(Z_{th}+Z+\Delta Z)}{(Z_{th}+Z+\Delta Z)\cdot(Z_{th}+Z)}$$

$$= \frac{V[\cancel{Z_{th}}+\cancel{Z}-\cancel{Z_{th}}-\cancel{Z}-\Delta Z]}{(Z_{th}+Z+\Delta Z)\cdot(Z_{th}+Z)}$$

$$= \frac{-V\Delta Z}{(Z_{th}+Z+\Delta Z)\cdot(Z_{th}+Z)}$$

Since, $I = \dfrac{V}{Z_{th}+Z} \Rightarrow \Delta I = \dfrac{-I\Delta Z}{Z_{th}+Z+\Delta Z}$

$$\Delta I = \frac{-V_c}{Z_{th}+Z+\Delta Z}$$ where $V_c = I\Delta Z$.

This theorem is based on principle of superposition. It is useful in bridge & potentiometer circuit when a slight change in one impedance result, in a shift from null conditions.

Limitations- Not applicable to ckt, consisting of only dependent sources, not applicable to non-linear ckts.

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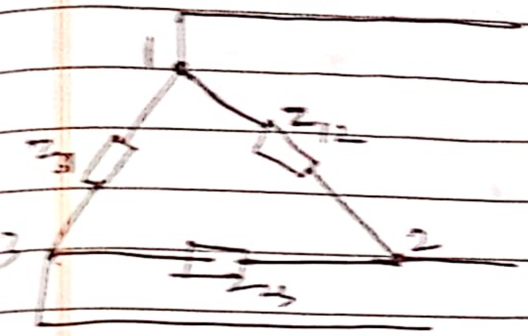
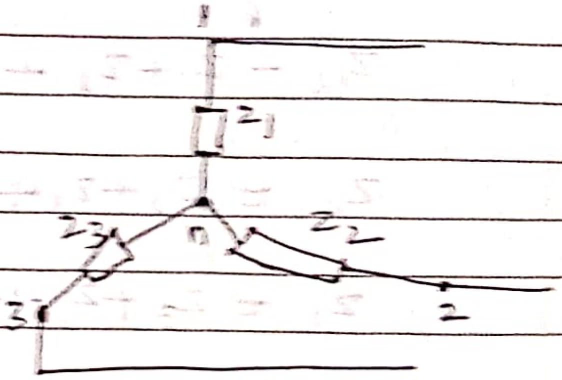
STAR DELTA TRANSFORMATION:-





For star connection: The impedance between terminal 1 & 2 is $Z_1 + Z_2$.

For delta connection: Two parallel paths,
∴ impedance between 1 & 2 is $Z_{12} \| Z_{23} + Z_{31}$

Impedance between terminal 1 & 2 is

$$Z_1 + Z_2 = \frac{Z_{12}(Z_{23} + Z_{31})}{Z_{12} + (Z_{23} + Z_{31})} \quad \text{—— (1)}$$

similarly impedance between 2 & 3

$$Z_2 + Z_3 = \frac{Z_{23} \cdot (Z_{12} + Z_{31})}{Z_{23} + (Z_{12} + Z_{31})} \quad \text{—— (2)}$$

similarly impedance between 3 & 1.

$$Z_3 + Z_1 = \frac{Z_{31}(Z_{12} + Z_{23})}{Z_{31} + (Z_{12} + Z_{23})} \quad \text{—— (3)}$$

Subtracting eq (2) from (1) & adding result to eq (3)

$$Z_1 = \frac{Z_{12} \cdot Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{23} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}, \quad Z_3 = \frac{Z_{31} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

similarly we can obtain,

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}.$$

$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1},$$

$$Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2},$$

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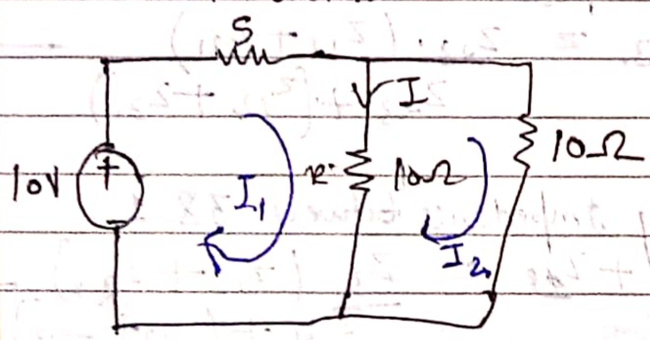
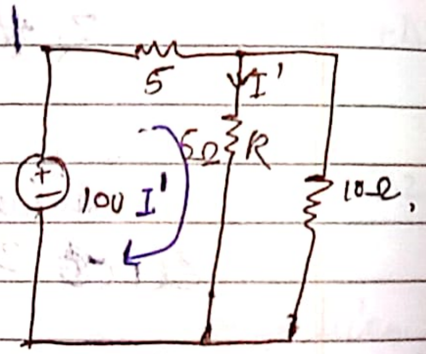
Ques. The resistance R is changed from 10 Ω to 5 Ω. Verify Compens-ation Theorem.





$\cancel{-5I_1 - 10I = 10}$ (1)

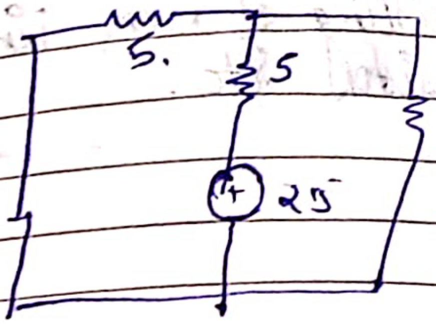
$$-5(I_1 - I_2) - 10(I_2 - I_1) = 10.$$

$\Delta I = I - I' = 0.3$

Using compensating theorem,

$V_c = I' \Delta Z = 0.5(-5)$

$= -2.5\,V$



$\Delta I = \frac{2.5}{5 + \frac{10}{3}}$

$= 0.30\,A.$

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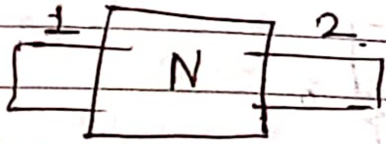
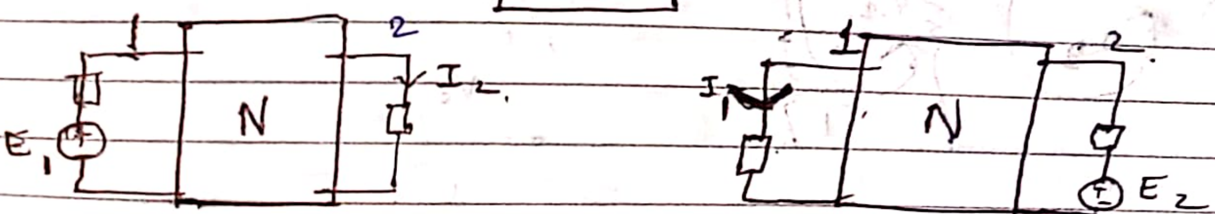
Reciprocity Theorem –

The ratio of excitation (→ Voltage) to response (→ current) remain invariant in a reciprocal network w.r.t interchange between the points of application of excitation and measurement of response.





E_1 in branch 1 produces current I_2 in branch 2.

E_2 is moved to branch 2 and response I_1 measured in

The reciprocity theorem asserts that

$$\frac{E_1}{I_2} = \frac{E_2}{I_1}.$$

Reciprocity theorem indicates that V + I are mutually interchangeable. V/I is called transfer resistance (impedance).

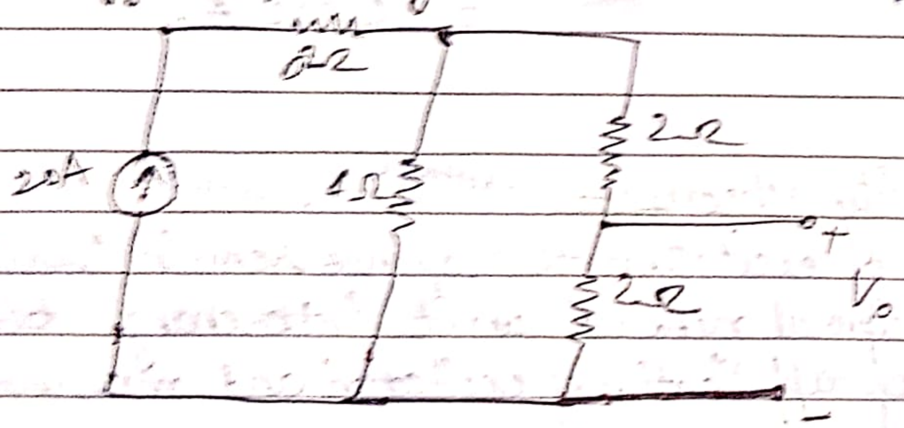
Limitations –

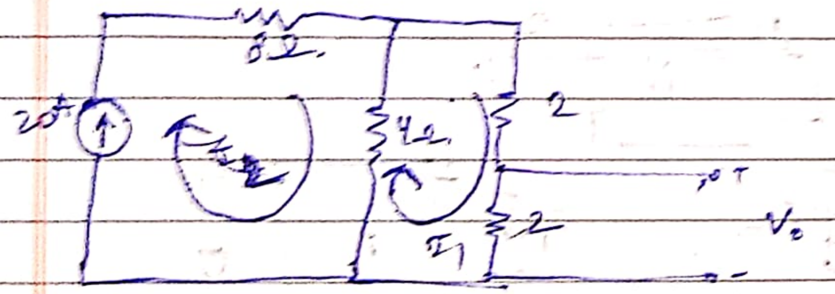
Not applicable to networks consisting of any dependent source. even if it is linear.

2. Not applicable to network consisting of any time varying element.

3. Not applicable to network consisting of non-linear elements like diode, transistor etc.

Ques Verify Reciprocating Theorem -





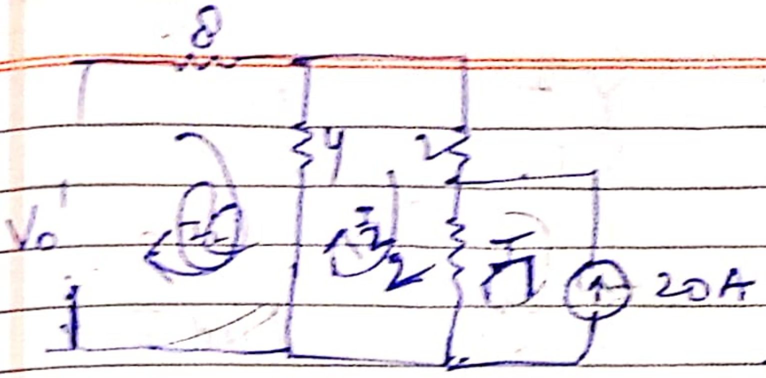
$I_2 = 20A$,

$\cancel{-2I_1 - 4I_1 - 2I_1 =}$

$$-2I_1 - 4(I_1 - I_2) - 2I_1 = 0$$
$$-2I_1 - 4(I_1 - 20) - 2I_1 = 0$$
$$-2I_1 - 4I_1 + 80 - 2I_1 = 0$$
$$-8I_1 = -80$$
$$\boxed{I_1 = 10}$$

$V_o = IR$

$= 20V$,



$I_1 = 20$

$\cancel{-4(I_2 - I_1) - 8 = 0}$

$-4I_2 + 80 - 8 = 0$

$\Rightarrow \cancel{-4I_2 + 72 = 0}$

$\cancel{-4I_2 = 72}$

$I_2 = \frac{72}{4}$

$I_2 = 18.$

$\cancel{-2I_2 - 2(I_2 - I_1) - 4I_2 = 0}$

$\cancel{-2I_2 - 2I_2 + 2I_1 - 4I_2 = 0}$

$\cancel{-2 \times 20 - 2I_2 + 2 \times 20 - 4I_2 = 0}$

$\cancel{-40 - 2I_2 + 40 - 4I_2 = 0}$

$-2I_2 - 2(I_2 - I_1) - 4I_2 = 0.$

$-2I_2 - 2I_2 + 40 - 4I_2 = 0$

$-8I_2 + 40 = 0$

$I_2 = \frac{40}{8}$

$\boxed{I_2 = 5}$

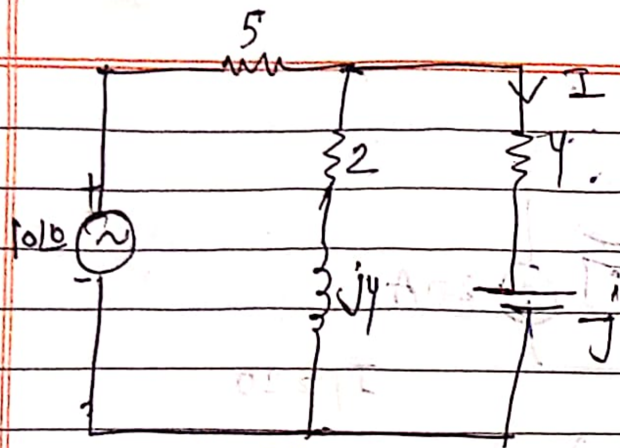
$V_0' = \cancel{4 \times 5}\ 4 \times 5$

$\boxed{V_0' = 20V}$

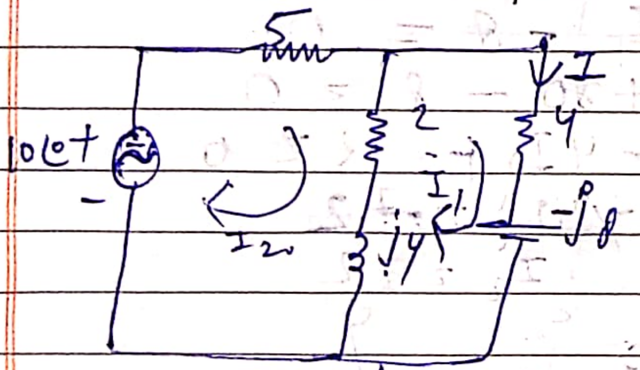
$\boxed{V_0 = V_0'}$

$\therefore$ Reciprocity theorem verified.

Ques.



Find I.



$-j_8 I$

$-j8I - 4I_1 - j4(I_1 - I_2) - 2(I_1 - I_2) = 0$

$-4I_1 - j_8 I_1 - j4 I_1 + j4 I_2 - 2I_1 + 2I_2 = 0$ — (i)

$-10\angle 0 + 5I_2 - 2(I_2 - I_1) - j4(I_2 - I_1) = 0$ — (ii)

$-10 + 0j - 5I_2 - 2I_2 + 2I_1 - j4I_2 + j4I_1 = 0$ — (ii)

from Eq^n (i) & (ii)

~~$-4I_1 - j8I$~~

$-6I_1 - 12jI_1 + j4I_2 + 2I_2 = 0$

$+2I_1 + j4I_1 - j4I_2 - 7I_2 = 10$

$-4I_1 - 8jI_1 - 5I_2 = -10$

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Current divided Rule –

$$Z_{eff} = \frac{(4-j8)(2+j4)}{(4-j8)+(2+j4)}$$

$$= \frac{(4-j8)(2+j4)}{(6-j4)}$$

$I_s = \frac{V}{Z_{eff}}$

$$\Rightarrow \frac{8.94\angle{-63.4} \times 4.47\angle{63.4}}{7.21\angle{-33.6}}$$

$$= \frac{39.96\angle{0}}{7.21\angle{-33.6}} = 5.54\angle{33.6}$$

$$I_s = \frac{V}{Z_{eff}}$$

$\Rightarrow$

TELLEGEN'S THEOREM -

The summation of instantaneous power or summation of complex power of sinusoidal source in a network is zero. Mathematically,

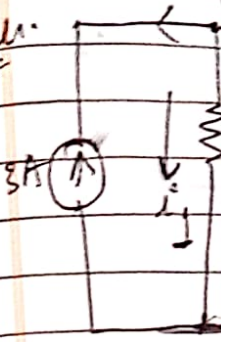
$$\sum_{k=1}^{b} v_k i_k = 0$$

where b is the no. of branches in a network
while for complex power,

$$\sum_{k=1}^{b} V_k I_k^* = \sum_{k=1}^{b} S_k = 0 \quad \text{where } S = V_k I_k^* = P + jQ.$$

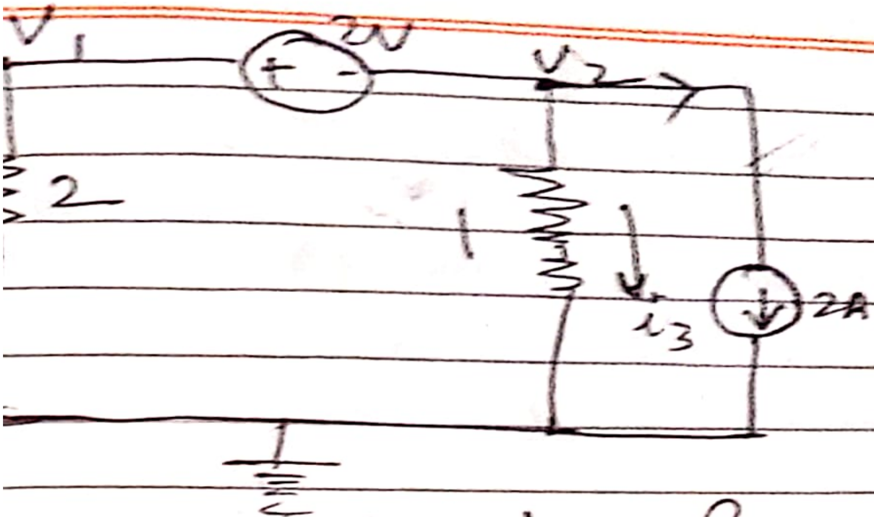
Tellegen's theorem is independent of circuit elements i.e. circuit having elements which are linear or non-linear, active or passive, time varying or time invariant. It is based on KVL & KCL.

Ques.



Ques.

6/10/22



$i_1, i_3 = ?$

node 1 & 2 are collectively called Super node.

$\frac{V_1}{2} - 3 + \frac{V_2}{1} + 2 = 0$ or

$V_1 + 2V_2 = 2$ —— (I)

$V_1 - V_2 = 2$ —— (II)

$3V_2 = 0$

$\boxed{V_2 = 0}$

Put V_2 in eq (1)

$V_1 + 2(0) = 2$

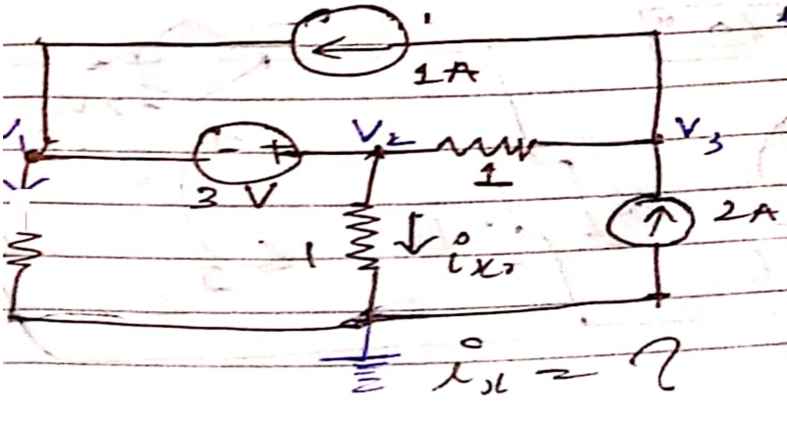
$\boxed{V_1 = 2}$

$i_1 = \frac{V_1}{2}$

$\boxed{i_1 = 1}$

$\boxed{i_3 = 0}$

$i_x = 2.5$



$i_x = ?$

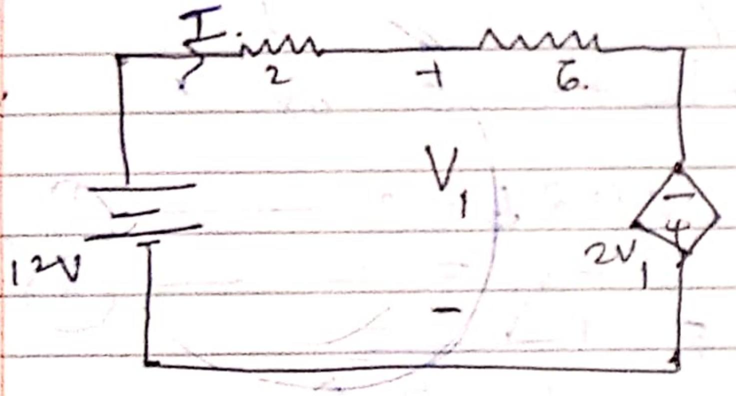
(Supernode)

$\frac{V_1}{1} - 1 + \frac{V_2}{1} + \frac{V_2 - V_3}{1} = 0$

$\frac{V_2 - V_3}{1} + 2 = 1$ → (node)

$V_2 - V_1 = 3$ — (II) ↳ (node 3)

Ans



~~12 − 2I~~

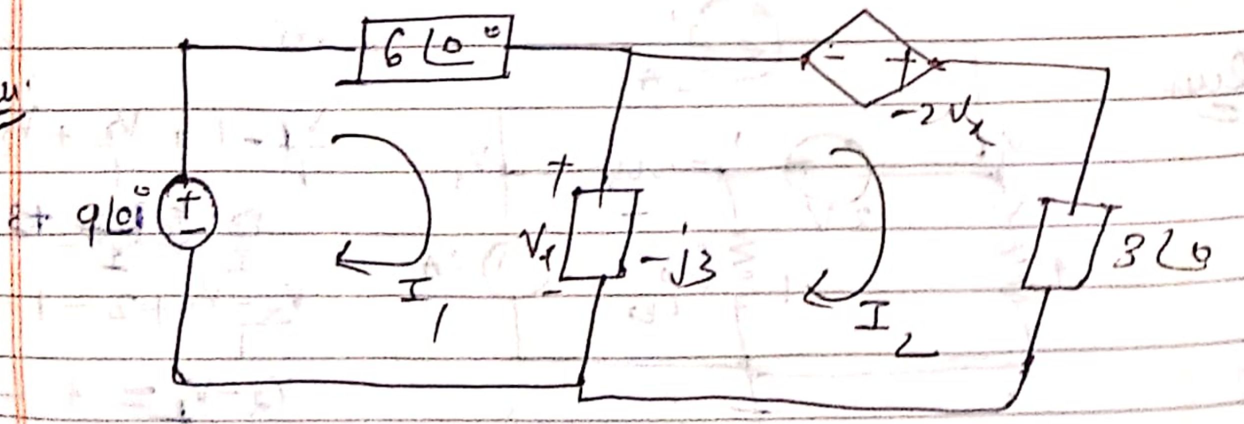
$-2I - V_1 + 12 = 0$ — (1)

~~2I~~ $12 - 8I + 2V_1 = 0$ — (ii)

$-8I + 2V_1 + 12 = 0$ — (ii)

~~I~~ $= -3$ $\boxed{I = 3A}$

Ans



$I_1 = 1.1\ 4.4$

$I_2 = 1.14 \angle -16$

$I_1 + I_2 = ?$

$$V_x = -j3(I_2 - I_1) \quad \text{--- (I)}$$

Mesh 1,

$$9\angle 0 - 6\angle 0 + j3(I_2 - I_1) = 0 \quad \text{--- (1)}$$

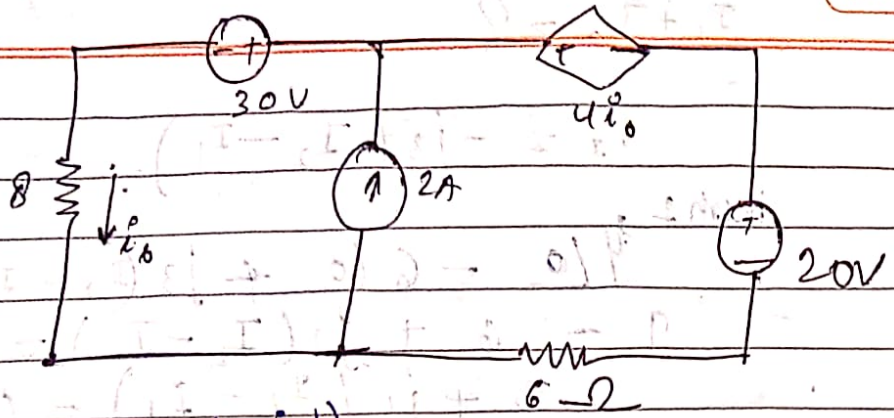
$$9 - 6 + j3(I_2 - I_1) = 0$$

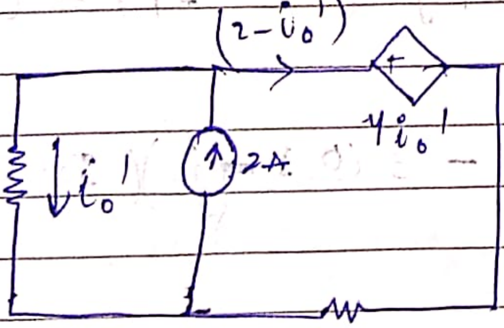
$$3 + j3(I_2 - I_1) = 0 \quad \text{--- (1)}$$

Mesh 2,

$$2(-j3)(I_2 - I_1) - 3\angle 0\ I_2 - (j3)(I_2 - I_1) = 0 \quad \text{--- (II)}$$

Que.





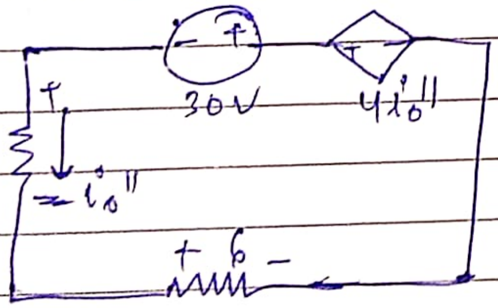
$$-4i_o' - 6(2-i_o') + 8i_o' = 0$$

$$\underbrace{-4i_o' - 12 + 6i_o'}_{} + 8i_o' = 0$$

$$-12 + 2i_o + 8i_o' = 0$$

$$10i_o = 12$$

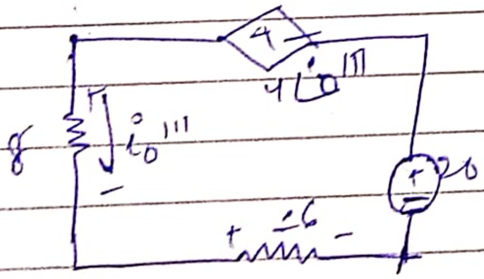
$$\boxed{i_o = 1.2A}$$



$$30 - 4i_o'' + 6i_o'' + 8i_o'' = 0$$

$$10i_o'' = -30$$

$$\boxed{i_o'' = -3A}$$



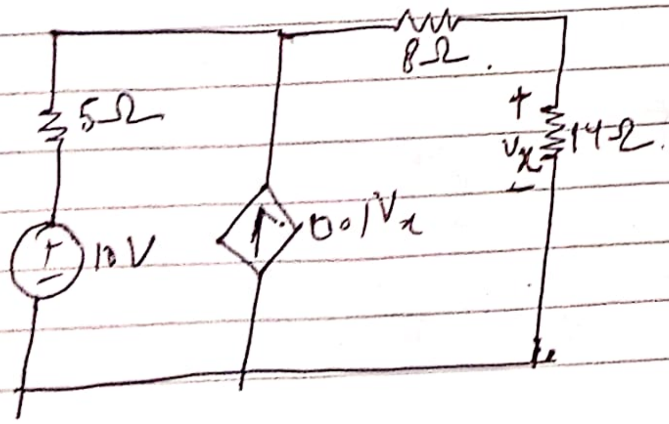
$$-4i_o''' - 20 + 6i_o''' + 8i_o''' = 0$$

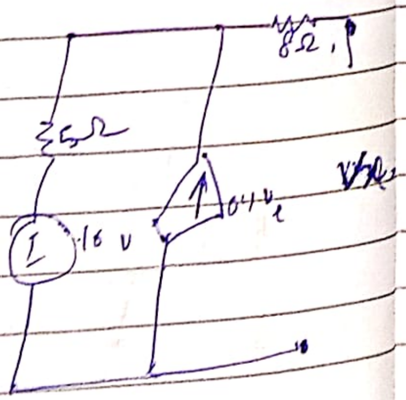
$$10i_o''' = 20$$

$$\boxed{i_o''' = 2A}$$

$$\boxed{I = 0.2}$$

Que 2





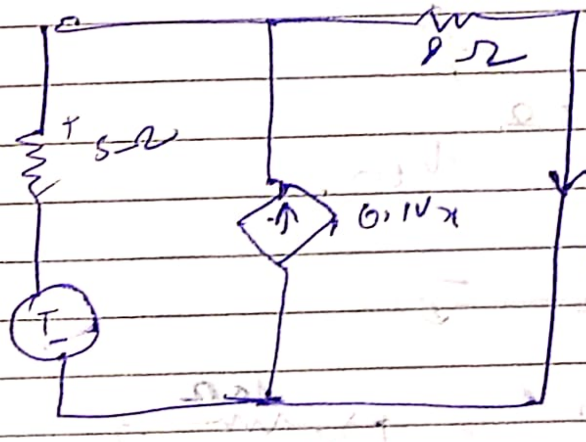
Que.

V_o

$$10 + 2.5V_x - V_x = 0$$
$$2.5V_x - V_x = -10.$$
$$\cancel{1.5V_x = -10}$$
$$V_x = \frac{-10}{1.5}$$

$$R_N = \frac{V_{TH}}{I_{SC}}.$$

$$10 + 5(0.1 V_x) - V_x = 0$$
$$10 + 0.5V_x - V_x = 0.$$
$$V_x = V_{th} = V_{oc}.$$
$$10 - 0.5V_x = 0$$
$$-0.5V_x = -10$$
$$V_x = \frac{100}{5}$$
$$\boxed{V_x = 20V}$$

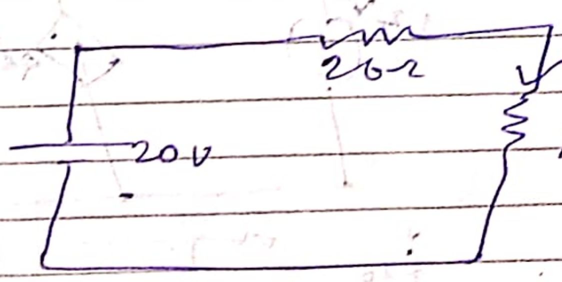


I_{sc}

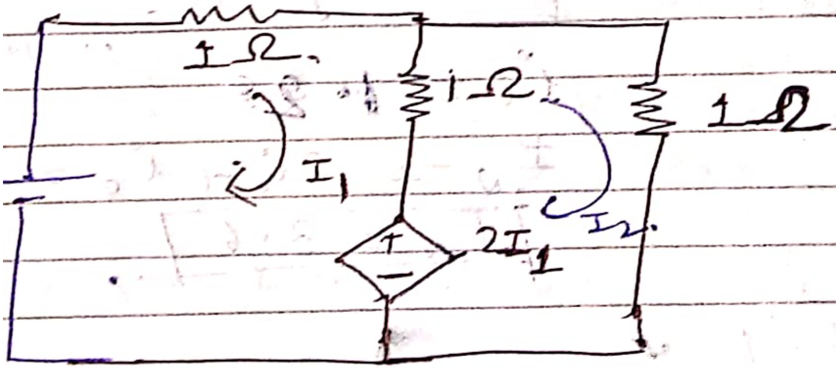
$V_x = 0$

$$I_{sc} = \frac{10}{5+8}$$
$$= \frac{10}{13}.$$

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{20}{10/13} = 26$$



$$I = \frac{20}{14+26}.$$
$$I = \frac{20}{40} \quad \frac{1}{2}$$
$$\boxed{I = 0.5A}$$



Thevenin eq. ckt.

when no independent source is there then
$$\boxed{V_{TH} = 0}$$

R_{TH}:

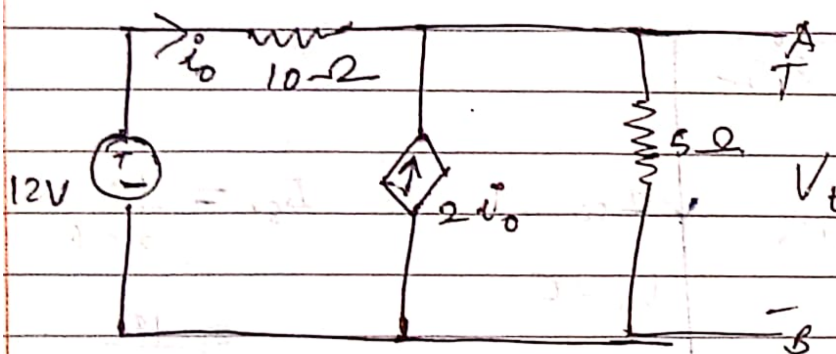
$V_0 - 1I_1 - 1(I_1 - I_2) - 2(I_1) = 0 \Rightarrow V_0 = 4I_1 - I_2$

$2I_1 - 1(I_2 - I_1) - 1I_2 = 0 \Rightarrow 3I_1 = 2I_2$

$\therefore V_0 = 4I_1 - \frac{3}{2}I_1$

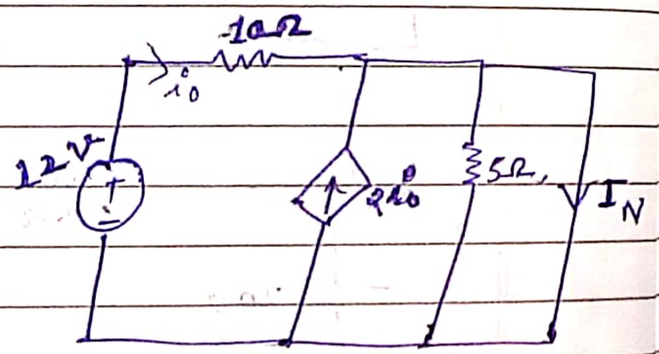
$V_0 = \frac{5}{2}I_1$

$$\boxed{\frac{V_0}{I_1} = \frac{5}{2} = R_{TH}}$$



Norton

V_{th}.

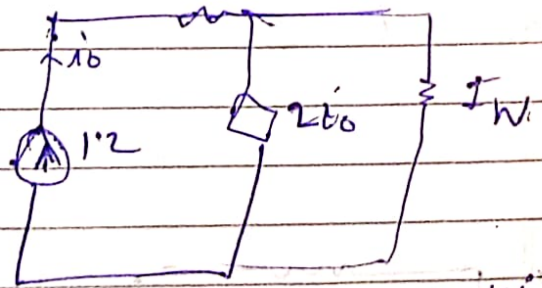


$I_N = ?$

$Z_{th} = \frac{V_{TH}}{I_N} = ?$

$2 - 10i_0 - 5i_0 =$

$i_0 = 0.48$.



$V_{th} = V_{5\Omega} = 5 \times 3i_0$

$= \cancel{8.86 \times 5}$

$= 15 \times 0.41$

$= 7.2$

$12 - 10i_0 - 5i_0$

$i_0 = 0.28$

$I_N = 2i_0 + i_0 = 3i_0$

$$\boxed{I_N = 3.6}$$

$Z_{TH} = \frac{V_{TH}}{I_N}$

$= \frac{7.2}{3.6}$

$$\boxed{Z_{TH} = 2}$$

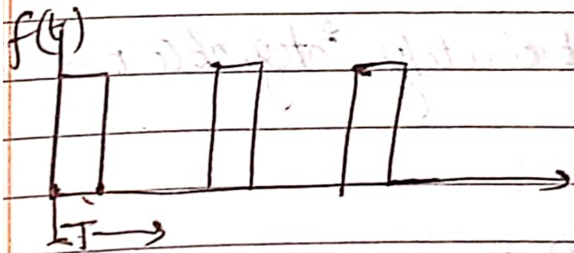
(1.)

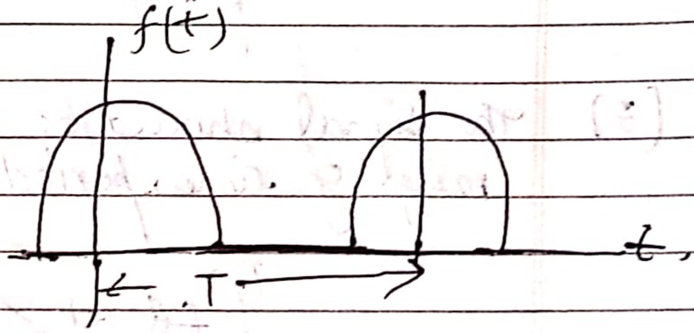
Unit-3

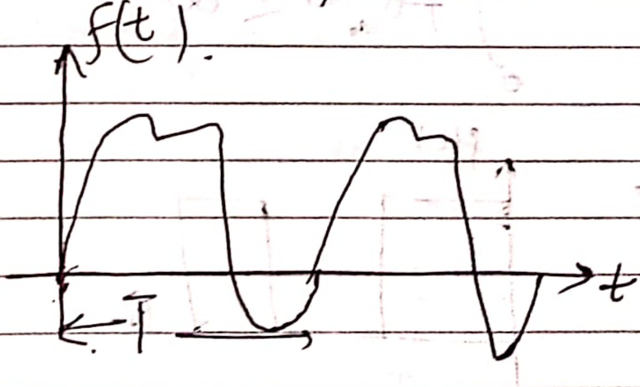
Fourier Series — Trigonometric / exponential

Any arbitary periodic function can be represented as by an infinite series of sinusoidal at harmonic related frequencies. Periodic fx^n can be represented as fourier series and a periodic wave form of fourier transform.

A signal $f(t)$ is said to be periodic of period t if $f(t) = f(t+T)$.

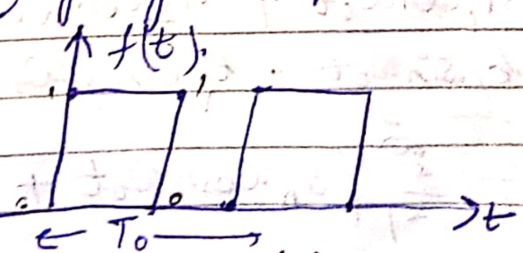




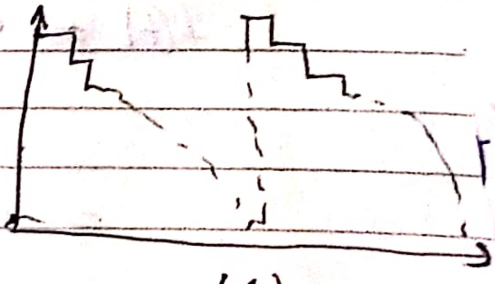


Condition for existence of fourier series → Dirichlet's Condition.

i) A signal should have finite no. of discontinuities over the Range of time period



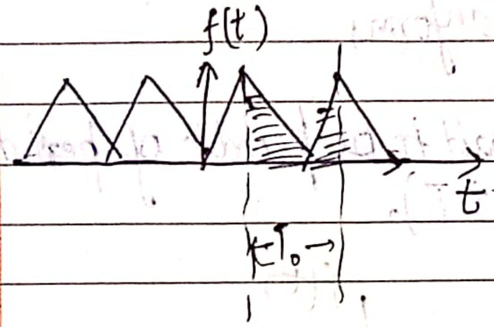
(A)

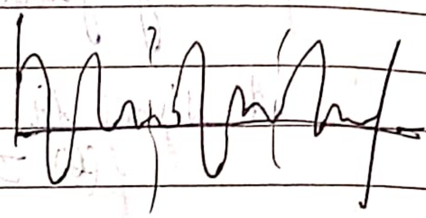


(B).

fig (A) can be determined by fourier serie as discontinuities are finite in it. But fig (B) cannot be determined as infinite discontinuities there.

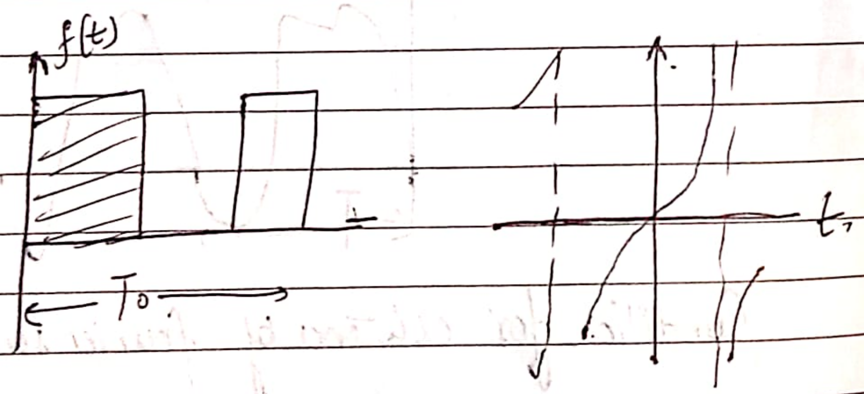
(2.) The signal should have finite no. of maxima and minima over the range of time period





(3) The signal should be absolutely integrable over the range of time period.

$$\int_0^{T} f(t)\,dt < \infty$$



Trigonometric Fourier Series –

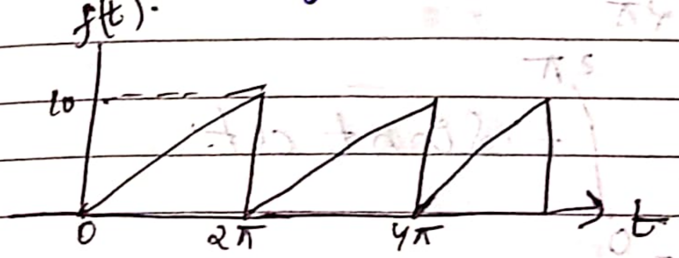
$$f(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \cdots a_n\cos n\omega_0 t + b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + \cdots + b_n\sin n\omega_0 t + \cdots$$

$$= a_0 + \sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t\right).$$

where, $a_0 = \frac{1}{T}\int_0^T f(t)\,dt.$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$\& \; b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt.$$



$\omega_0 T = 2\pi$

Time period $= 2\pi$

$$y = mx + c$$

$$f(t) = \frac{10}{2\pi}t + 0$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{10}{2\pi}t\,dt$$

$$= \frac{1}{2\pi}\times\frac{10}{2\pi}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= \frac{10}{\cancel{4\pi^2}}\cdot\frac{\cancel{4\pi^2}}{2} = 5.$$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi}\frac{10}{2\pi}t\cos nt\,dt$$

$$= \frac{\cancel{2}}{2\pi}\times\frac{\cancel{10}^{5}}{\cancel{2}\pi}\int_0^{2\pi}\underset{\text{I}}{t}\cdot\underset{\text{II}}{\cos nt}\,dt.$$

$$= \frac{5}{\pi^2}\left[t\cancel{\left(\frac{-\sin nt}{n}\right)}^{0} - 1\left(\frac{\cos nt}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{5}{\pi^2}\left[-\frac{\cos nt}{n^2}\right]_0^{2\pi}$$

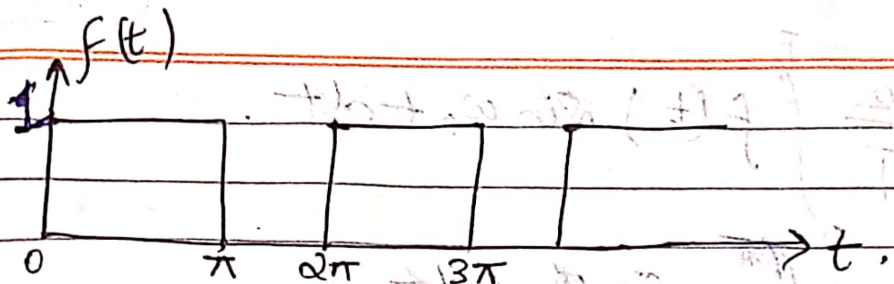
$$= \frac{5}{\pi^2}\left[\frac{-n}{n^2} + \frac{n}{n^2}\right]$$

$$= \frac{5}{\pi^2}[0] = 0,$$

$= \frac{-10}{n\pi}$ $$b_n = \frac{2}{2\pi}\int_0^{2\pi} \frac{10}{2\pi} t \sin nt \, dt$$

$$= \frac{1}{\pi} \cdot \frac{10}{2\pi}\int_0^{2\pi} t \cdot \sin nt \, dt$$

$$= \frac{5}{\pi^2}\int_0^{2\pi}$$



$f(t) = 1$ 0 to π.

$= 0$ π to 2π.

$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$.

$\omega_0 = 1$.

$a_0 = \frac{1}{2\pi}\int_0^{2\pi} 1\,dt$.

$= \frac{1}{2\pi}\int_0^{2\pi} 1\,dt$.

$= \frac{1}{2\pi}\left[t\right]_0^{2\pi} = \frac{1}{2\cancel{\pi}}\cancel{\pi}[\ \ ] = \frac{1}{2}$

$a_n = \frac{2}{2\pi}\int_0^{2\pi} 1 \cos n(1)t\,dt$.

$a_n = \frac{1}{\pi}\int_0^{2\pi} \cos nt\,dt$.

$a_n = \frac{1}{\pi}\left[-\frac{\sin nt}{n}\right]_0^{2\pi}$

$= \frac{1}{\pi}\left[-\frac{\sin 2\pi}{2\pi}\right]_0^{2\pi}$

$= \frac{1}{\pi}\left[0\right]$ $\boxed{a_n = 0}$

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$$b_n = \frac{2}{T}\int_0^T f(t)\sin \omega_0 t\,dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} 1 \sin \omega_0 t\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\cos \omega_0 t}{n}\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\cos nt}{n}\,dt$$

$$= \frac{1}{\pi}\left[\frac{\cos nt}{n}\right]_0^{\pi}$$

$$= \frac{-1}{n\pi}\left[\cos n\pi - \cos 0\right].$$

$=$

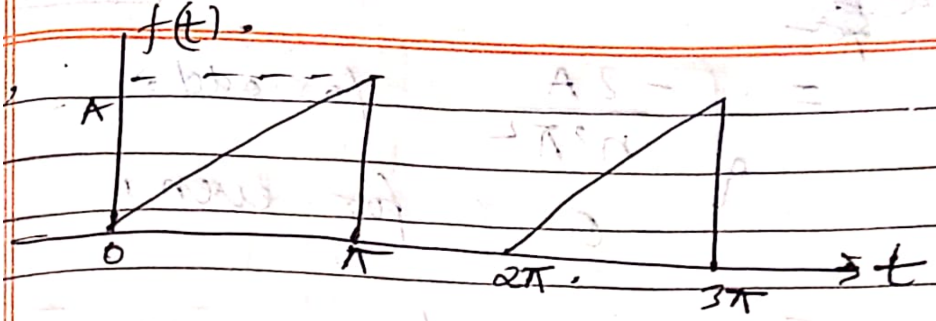
$b_n = 0$ when n is even

$= 1$

$$f(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t - - - a_n$$

$$= a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$= \frac{1}{2} +$$



$$f(t) = \frac{A}{\pi}t \quad \text{for } 0 \text{ to } \pi$$
$$ 0 \quad \text{for } \pi \text{ to } 2\pi.$$

$= 2\pi$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt.$$
$$= \frac{1}{2\pi}\int_0^{\pi} \frac{A}{\pi}t\,dt.$$
$$= \frac{1}{2\pi} \times \frac{A}{\pi}\left[\frac{t^2}{2}\right]_0^{\pi}.$$
$$= \frac{1}{2\pi} \times \frac{A}{\pi} \times \frac{\pi^2}{2}.$$
$$= \frac{A\cancel{\pi^2}}{4\cancel{\pi^2}} = \frac{A}{4}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt.$$
$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{\pi} \frac{A}{\pi}t\cos n(1)t\,dt.$$
$$= \frac{1}{\pi} \times \frac{A}{\pi}\int_0^{\pi} t \cdot \cos nt\,dt$$
$$= \frac{1}{\pi} \times \frac{A}{\pi}\left[t\left(\frac{-\sin nt}{n}\right) - 1\left(\frac{\cos nt}{n^2}\right)\right]_0^{\pi}$$
$$= \frac{A}{\pi^2}\left[\frac{\cos n\pi - \cos 0}{n^2}\right]$$
$$= \frac{A}{n^2\pi^2}\left[(-1)^n - 1\right]$$

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$$= \begin{cases} \dfrac{-2A}{n^2\pi^2} & \text{for odd,} \\ 0 & \text{for even,} \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt.$$

$$= \frac{2}{2\pi}\int_0^{\pi}\frac{A}{\pi}t\sin nt\,dt$$

$$= \frac{2}{2\pi}\cdot\frac{A}{\pi}\int_0^{\pi} t\cdot\sin nt\,dt$$

$$= \frac{1}{\pi}\cdot\frac{A}{\pi}\left[t\cdot\left(\frac{-\cos nt}{n}\right) - 1\left(\cancel{\frac{-\sin t}{n^2}}\right)\right]_0^{\pi}$$

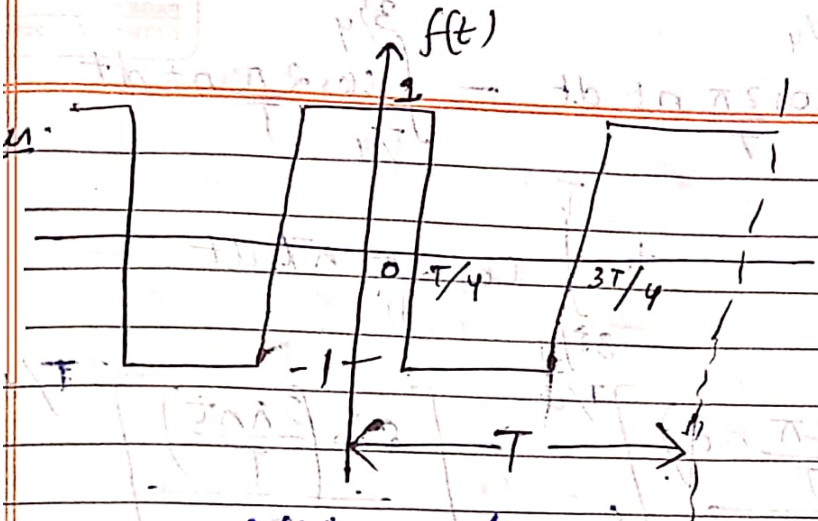
$$= \frac{A}{\cancel{\pi}^2}\left[\frac{-\cancel{\pi}(-1)^n}{n}\right]$$

$$= \frac{-A(-1)^n}{n\pi},$$

$$= \frac{-A}{n\pi} \cdot \text{for even}$$

$$\frac{A}{n\pi} \text{ for odd.}$$

a_0 - giving the average value



$$\omega_0 t = 2\pi$$

$$\omega_0 = \frac{2\pi}{T}$$

$f(t) = 1$ for 0 to $T/4$,
$f(t) = -1$ for $T/4$ to $3\pi/4$
$f(t) = 1$ for $3\pi/4$ to T,

$a_0 = 0$
↳ as the waveform is symmetric.

$$a_n = \frac{2}{T}\int_0^T f(t) \cos n\omega_0 t \, dt$$

$$a_n = \frac{2}{T}\left[\int_0^{T/4} 1 \cos n\frac{2\pi}{T} dt + \int_{T/4}^{3T/4} -1 \cos n\frac{2\pi}{T} dt + \int_{3T/4}^{T} 1 \cos n\frac{2\pi}{T} dt\right]$$

$$\cancel{a_n = \frac{2}{T}\left[\left[\frac{\sin n\frac{2\pi}{T}}{n\frac{2\pi}{T}}\right]_0^{T/4} + \left[-\left(\frac{\sin n 2\pi}{nT}\right)\right]_{T/4}^{3T/4} + \left[\sin n 2\pi + \left(\frac{\sin n 2\pi}{nT}\right)\right]_{3T/4}^{T}\right]}$$

$$a_n \Rightarrow \frac{2}{T}\left[\frac{\sin\left(n 2\frac{\pi}{4}\right)}{n\, T/4} + \left(-\sin\right.\right.$$

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$$a_n = \frac{2}{T}\left[\int_0^{T/4} \cos\frac{2\pi}{T} nt\, dt - \int_{T/4}^{3T/4} \cos\frac{2\pi}{T} nt\, dt + \int_{3T/4}^{T} \cos\frac{2\pi}{T} nt\, dt\right]$$

$$= \frac{2}{T}\left[\left[\frac{\sin\left(\frac{2\pi}{T}nt\right)}{\frac{2\pi}{T}n}\right]_0^{T/4} - \left[\frac{\sin\left(\frac{2\pi nt}{T}\right)}{\frac{2\pi}{T}n}\right]_{T/4}^{3T/4} + \left[\frac{\sin\left(\frac{2\pi nt}{T}\right)}{\frac{2\pi}{T}n}\right]_{3T/4}^{T}\right]$$

$\frac{2}{T}\times\pi\times\frac{T}{4}$

$\frac{2}{T}\times\pi n T/4$

$$= \frac{2}{T}\left[\frac{\sin\frac{\pi n}{2}}{\frac{2\pi n}{T}} - \left[\frac{\sin\left(\frac{3\pi n}{2}\right)}{\frac{2\pi n}{T}} - \frac{\sin\left(\frac{\pi n}{2}\right)}{\frac{2\pi n}{T}}\right] + \left[\frac{\sin 2\pi n}{\frac{2\pi}{T}n} - \frac{\sin\left(\frac{3\pi n}{2}\right)}{\frac{2\pi n}{T}}\right]\right]$$

$\frac{2}{T}\times\frac{T}{2\pi n}$

$$= T\,\frac{1}{\pi n}\left[\sin\left(\frac{\pi n}{2}\right) - \sin\left(\frac{3\pi n}{2}\right) + \sin\frac{\pi n}{2} + \sin 2\pi n + \sin\left(\frac{3\pi n}{2}\right)\right]$$

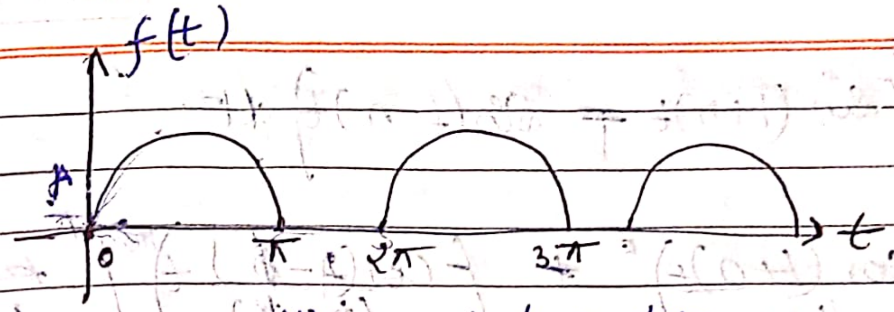
$$= \frac{1}{\pi n}\left[2\sin\frac{\pi n}{2} - 2\sin\frac{3\pi n}{2}\right]$$

$$a_n = \frac{2}{\pi n}\left[\sin\frac{\pi n}{2} - \sin\frac{3\pi n}{2}\right]$$

$n = 1 \quad a_1 = 4/\pi$ $\qquad n = 3, \quad a_3 = -4/3\pi$

$n = 2, \quad a_2 = 0$

Scanned with CamScanner



Time period $= 2\pi$

$$\omega_0 T = 2\pi$$

$$\omega_0 = 1$$

$$f(t) = A \sin t \quad \text{for } t = 0 \text{ to } \pi$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{\cancel{2}\pi} A \sin t\,dt$$

$$= \frac{1}{2\pi} A \left[-\cos t\right]_0^{\pi}$$

$$= -\frac{A}{2\pi}\left[(\cos(\pi) - \cos(0))\right]$$

$$= \frac{A}{2\pi}\left[\cancel{-}(-1) + 1\right]$$

$$= \frac{A}{2\pi} \times 2 = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos nt\,dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} A \sin t \cdot \cos nt\,dt$$

$$= \frac{A}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin(1+n)t + \sin(1-n)t\,dt\right]$$

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$$= \frac{A}{2\pi} \int_0^{\pi} \left[\sin(1+n)t + \sin(1-n)t \right] dt$$

$$= \frac{A}{2\pi} \left[\left(\frac{-\cos(1+n)t}{(1+n)} \right)_0^{\pi} + \left(\frac{-\cos(1-n)t}{(n-1)} \right)_0^{\pi} \right]$$

$$= \frac{A}{2\pi} \left[\frac{-\cos(n+1)t}{(n+1)} - \frac{\cos(n-1)t}{(n-1)} \right]_0^{\pi}$$

$$= \frac{A}{2\pi} \left[\frac{-(-1)^{n+1}}{(n+1)} + \frac{(-1)^{n-1}}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)} \right].$$

if $n = 2, 4, 6, \ldots$ (even).

$$a_n = \frac{A}{2\pi} \left[\frac{-(-1)}{n+1} + \frac{(-1)}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)} \right].$$

$$= \frac{A}{2\pi} \left[\frac{1}{(n+1)} - \frac{1}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)} \right]$$

$$= \frac{A}{2\pi} \left[\frac{2}{(n+1)} - \frac{2}{(n-1)} \right] = \frac{A}{2\pi} \times 2 \left[\frac{\cancel{n} - 1 - \cancel{n} - 1}{n^2 - 1} \right]$$

$$a_n = \frac{A}{\pi} \left[\frac{-2}{n^2 - 1} \right]$$

$$\boxed{a_n = -\frac{2A}{\pi(n^2-1)}}$$

if $n = 1, 3, 5, \ldots$ (odd).

$$= \frac{A}{2\pi}\left[-\frac{(1)}{(n+1)} + \frac{1}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)}\right]$$

$$\Rightarrow \boxed{a_n = 0}$$

$$a_n = \begin{cases} 0, & n \text{ is odd} \\ \dfrac{-2A}{\pi(n^2-1)} & n \text{ is even} \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T f(t) \sin \omega_0 t \, dt$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{\pi} A \sin t \sin nt \, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin nt \cdot \sin t \, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} \left(\cos(n-1)t - \cos(n+1)t\right) dt$$

$$= \frac{A}{2\pi}\left[\frac{\sin(n-1)t}{(n-1)} - \frac{\sin(n+1)t}{(n+1)}\right]_0^{\pi}$$

$$= \frac{A}{2\pi}\left[\frac{\sin(n-1)\pi}{(n-1)} - \frac{\sin(n+1)\pi}{(n+1)} - 0 + 0\right]$$

$\boxed{\sin(n \pm 1)\pi = 0}$

$$= \boxed{b_n = 0}$$

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$$\boxed{f(t) = \frac{A}{\pi} - \frac{2A}{\pi}\left[\frac{\cos 2t}{(2^2-1)} + \frac{\cos 4t}{(4^2-1)} + \frac{\cos 6t}{(6^2-1)} + \cdots\right]}$$

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Even Signal | Odd Signal

Even Signal

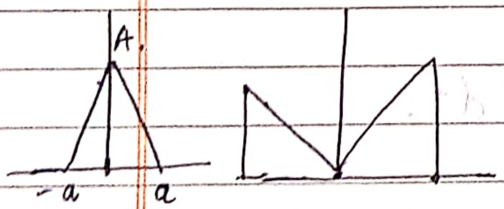
- Remain identical under folding operation / time reversal

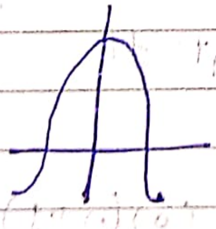
$x(t) \xrightarrow{T.R} x(-t) = x(t)$

E.g. $x(t) = \cos\omega t$.

$t \to -t$.

$x(-t) = \cos(-\omega t) = \cos\omega t$
$= x t$





- $b_n = 0$

$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt.$

$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt.$

Odd Signal

Doesnot remain identical under folding $x(-t) = -x(t)$

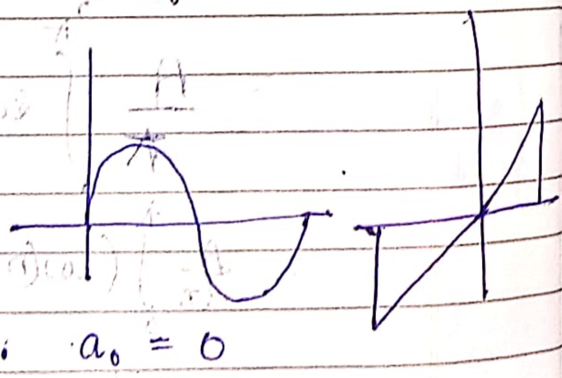
$x(t) = \sin\omega t$

$x(-t) = \sin(-\omega t) = -\sin\omega t$
$= -x(t)$.

Note-

1) Odd signal must be zero at $t = 0$.

2) Avg. or mean value of odd signal is zero. ($\to a_0$)



- $a_0 = 0$

$a_n = 0.$

$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\,dt.$

$\cos n\pi = (-1)^n$

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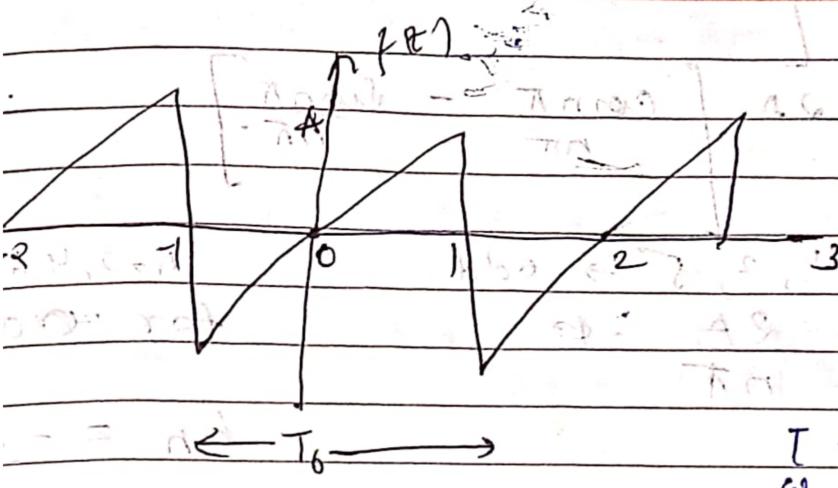


$T = 2$

$\omega_0 T = 2\pi$; $\omega_0 2 = 2\pi$

$f(t)\,dt = At$ $\boxed{\omega_0 = \pi}$

As it is odd signal,
then, the value of
a_0 & a_n is zero.

$$b_n = \frac{4}{2}\int_0^1 At \sin n\omega_0 t\,dt.$$

$$= 2A\int_0^1 \underset{I}{t} \cdot \underset{II}{\sin n\pi t}\,dt.$$

$$= 2A\left[t \cdot \frac{\cos n\pi\,t}{n\pi} - 1 \cdot \frac{\sin n\pi\,t}{n\pi}\right]_0^1.$$

$$= 2A\left[t \cdot \frac{\cos n\pi\,t}{n\pi} - \frac{\sin n\pi\,t}{n\pi}\right]_0^1$$

$$= 2A\left[\frac{\cos n\pi}{n\pi} - \frac{\sin n\pi}{n\pi} - 0 - 0\right]$$

$\sin n\pi = 0$ $\Rightarrow$

$$b_n = 2A\left[\frac{\cos n\pi}{n\pi} - \frac{\sin n\pi}{n\pi}\right]$$

$= b_n \Big[$

$$b_n = 2A\left[\frac{\cos n\pi}{n\pi} - \frac{\sin n\pi}{n\pi}\right]$$

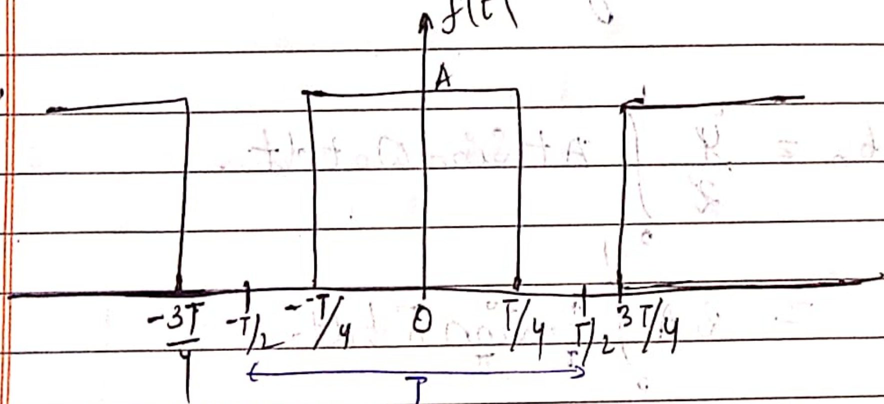
$n = 1, 3, 5 \rightarrow$ odd $\qquad n = 2, 4, 6 \rightarrow$ even

$b_n = \frac{2A}{n\pi}$ $\qquad$ for odd,

$$b_n = -\frac{2A}{n\pi}$$

$$f(t) = \frac{2A}{\pi}\left[\sin \pi t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3\pi t + ---\right]$$

Ans.



$f(t) = A. \qquad T = T, \qquad b_n = 0$ as this is an even function.

Here,

$$a_0 = \frac{2}{T}\int_0^{T/4} A\,dt.$$

$$= \frac{2A}{T}\Big[t\Big]_0^{T/4}$$

$$= \frac{2A}{T} \times \frac{T}{4} = \frac{A}{2}.$$

a_n.

$$\boxed{a_0 = \frac{A}{2}}$$

$\omega_0 T = 2\pi$ $\quad -\sin \omega t$

$\omega_0 = \frac{2\pi}{T}$

$$a_n = \frac{4}{T}\int_0^{T/4} A \cos n\omega_0 t \, dt$$

$$= \frac{4A}{T}\left[\cos n \right.$$

$$= \frac{4A}{T}\int_0^{T/4} \cos n\omega_0 t \, dt$$

$$\cancel{= \frac{4A}{T}\left[-\sin \right]_0^{T/4}}$$

$$= \frac{4A}{T}\int_0^{T/4} \cos n \frac{2\pi}{T} \, dt$$

$$= \frac{4A}{T}\left[\frac{\sin n 2\pi t}{nT} \right]_0^{T/4}$$

$$= \frac{4A}{T}\left[\frac{\sin 2\pi n t}{nT} \right]_0^{T/4}$$

$$= \frac{4A}{T}\left[\frac{\sin \frac{\pi n}{2}}{n.T} - \cancel{\sin \pi n} \right]$$

$$= \frac{4A}{T}\left[\frac{\sin \frac{\pi n}{2}}{nT} \right]$$

$$= \frac{4A}{T}\left[\frac{\sin \frac{\pi n}{2}}{n2\pi} \right]$$

$$= 2A \frac{\sin \frac{n\pi}{2}}{n\pi} \qquad = \frac{2A}{n\pi} \cdot \sin\frac{n\pi}{2}$$

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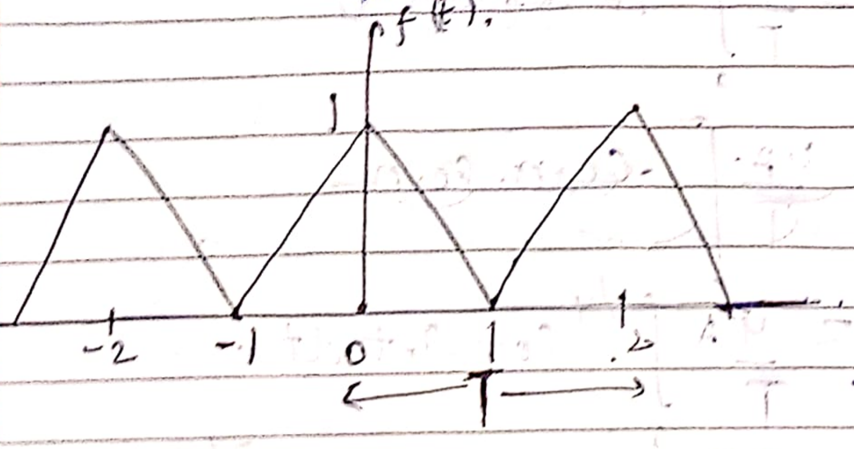
$a_1 = \frac{2A}{\pi}$, $a_2 = 0$, $a_3 = -\frac{2A}{3\pi}$,

Ques:



$T = 2$

$\omega_0 T = 2\pi$

$\omega_0 = \pi$

$$f(t) = 1 - |t|$$

as it is even signal, then the value of b_n is zero.

$$a_0 = \frac{2}{2}\int_0^1 (1-t)\,dt$$

$$a_0 = \int_0^1 (1-t)\,dt$$

$$a_0 = \left[t - \frac{t^2}{2}\right]_0^1$$

$$a_0 = \left[1 - \frac{1^2}{2} - 0\right]$$

$$a_0 = 1 - \frac{1}{2} \Rightarrow \boxed{a_0 = 1/2}$$

$$a_n = \frac{4}{2}\int_0^1 (1-t)\cos n\pi t\,dt$$

$$= 2\int_0^1 (1-t)\cos n\pi t\,dt$$

odd $= \frac{-4}{(n\pi)^2}$

$$= 2\int_0^1 \left(\frac{\sin n\pi t}{n\pi}\right)$$

$$= 2\int_0^1 \cos n\pi t - t\cos n\pi t\, dt$$

$$= 2\int_0^1 \cos n\pi t - t\cos n\pi t\, dt.$$

$$= 2\left[\frac{\sin n\pi t}{n\pi} - t\cdot\frac{\sin n\pi t}{n\pi} + \frac{1\cdot\cos n\pi t}{n\pi}\right]_0^1$$

$$= 2\left[\frac{\sin n\pi}{n\pi} - \frac{\sin n\pi}{n\pi} + \frac{\cos n\pi}{n\pi} - \frac{1}{n\pi^2}\right]$$

$$= 2\left[\frac{\cos n\pi - 1}{(n\pi)^2}\right]$$

for even,

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Half wave symmetry :- (only odd harmonic present)

$$f(t) = -f(t \pm T/2)$$

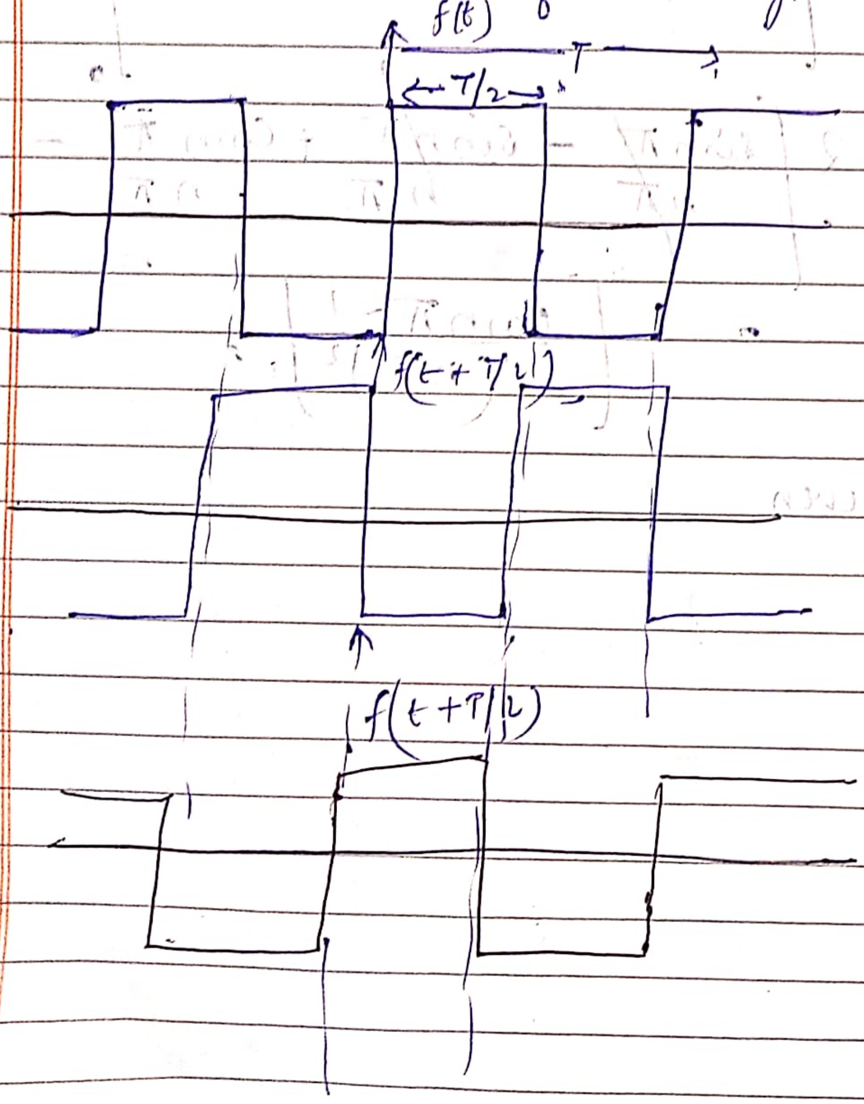
1) Perform time shifting.

$$f(t) \longrightarrow f(t + T/2)$$

2) Amplitude Reversal

$$f(t+T/2) \longrightarrow -f(t+T/2),$$

If $-f(t+T/2)$ is same as $f(t)$ it is half wave symmetry.



Odd & HWS

odd
$b_n \neq 0$
$a_n = 0$
$a_0 = 0$

only sine term.

HWS.
only odd harmonics.

Only sine terms with odd harmonics.

Even & HWS

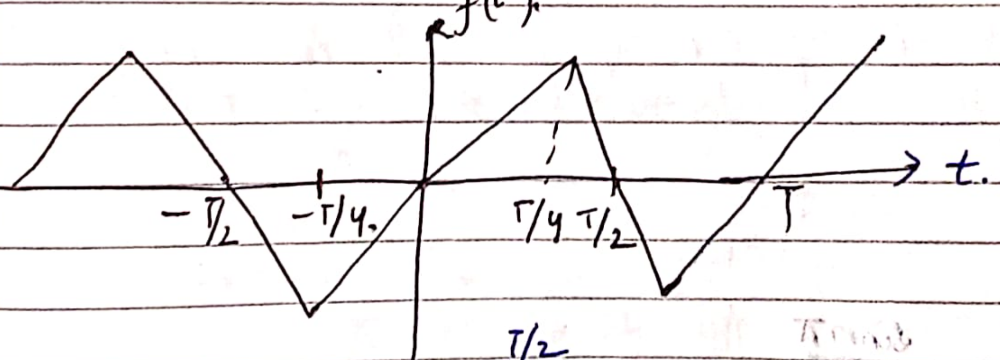
Even
$a_n \neq 0$
$b_n = 0$

only cosine terms,

HWS
only odd harmonics
$a_0 = 0$

Only Cosine terms with odd harmonics.

17/10/22



$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t,$$

$$\text{or } b_n = 2\times\frac{4}{T}\int_0^{T/4} f(t)\sin n\omega_0 t\,dt.$$

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$f(t) = \frac{4}{T} t$, $\omega_0 T = 2\pi$

$\omega_0 = \frac{2\pi}{T}$,

$b_n = \frac{4}{T}\int_0^{T/2} \frac{4}{T} t \sin n\left(\frac{2\pi}{T}\right) t \, dt$

$= \frac{4}{T}\times\frac{4}{T}\int_0^{T/2} t \cdot \sin n\omega_0 t \, dt$

$= \frac{4}{T^2}\left[t \cdot \left(-\frac{\cos n\omega_0 t}{n\omega_0}\right) - 1\cdot\left(-\frac{\sin n\omega_0 t}{n^2\omega_0^2}\right)\right]_0^{T/2}$

$\cancel{= \frac{4}{T^2}\left[-\frac{T}{2} \cos \frac{\frac{2\pi}{T} t}{n\frac{2\pi}{T}}\right.}$

$= \frac{4}{T^2}\left[-\frac{T}{2} \frac{\cos n \frac{2\pi}{T} t}{\frac{2\pi n}{T}} - \left(\frac{\sin n \frac{2\pi}{T}\frac{T}{2}}{n^2 \frac{4\pi^2}{T^2}}\right)\right]$

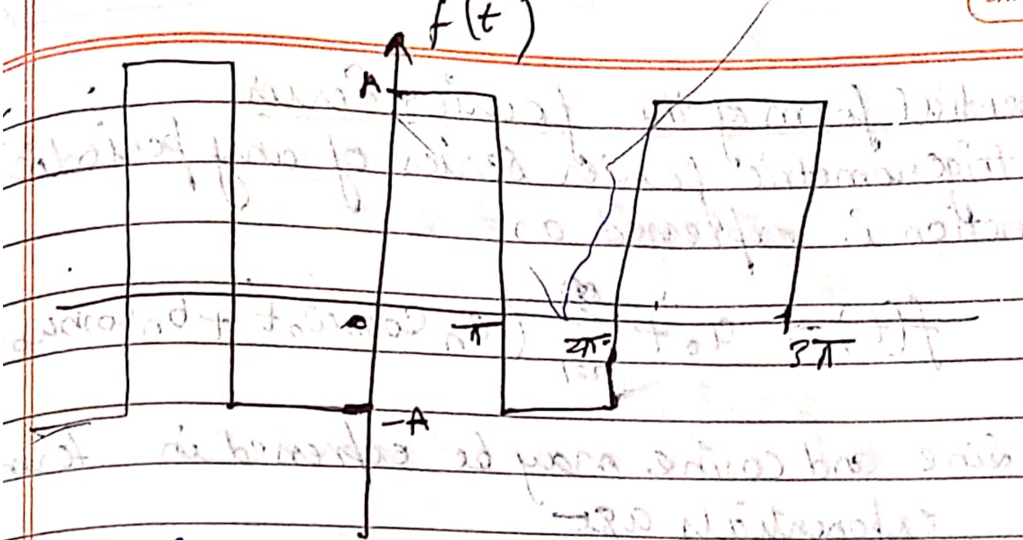
$=$

$b_n = \frac{8}{n^2\pi^2} \sin\frac{n\pi}{2}$,

$b_n = \frac{8}{n^2\pi^2}$ for $n = 1, 5, 9 \ldots$

$b_n = -\frac{8}{n^2\pi^2}$ for $n = 3, 7, 11 \ldots$

$f(t) = \frac{8}{\pi^2}\left[\sin\omega_0 t - \frac{1}{9}\sin 3\omega_0 t + \frac{1}{25}\sin 5\omega_0 t + \ldots\right]$



$$f(t) = A$$

$$b_n = \frac{4}{T}\int_0^{\pi/2} A \sin n\omega_0 t\, dt. \qquad \omega_0 = 2\pi.$$

$$b_n = A\left[\frac{-\cos n\omega_0 t}{\omega_0 n}\right]_0^{\pi/2}$$

$$= \frac{A}{\omega_0}\left[\frac{-\cos n\omega_0 t}{n}\right]_0^{\pi/2} \qquad T = \pi$$

$$= \frac{A}{2\pi}\left[\frac{-\cos n\, 2\pi\, t}{n}\right]_0^{\pi/2}$$

$$b_n = \frac{4}{T}\int_0^{\pi/2} A \sin n\omega_0 t\, dt$$

$$b_n = \frac{4A}{T}\int_0^{\pi/2} \sin n\omega_0 t\, dt.$$

$$b_n = \frac{4A}{\pi}\left[\frac{-\cos n\omega_0 t}{n\omega_0}\right]_0^{\pi/2}.$$

$$= \frac{4A}{\omega_0}\left[\frac{-\cos n\omega_0 t}{n}\right]_0^{\pi/2}$$

$$= -\frac{4A}{2\pi n}\left[-\cos 0\right] = \frac{4A}{2\pi n}.$$

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Exponential form of the fourier Series

The trigonometric fourier series of any periodic function is expressed as

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \cos n\omega_0 t) \quad -①$$

The sine and cosine may be expressed in terms of exponentials as

$$\cos n\omega_0 t = \frac{1}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right)$$

$$\sin n\omega_0 t = \frac{1}{2j}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right).$$

Substituting the above expressions in eq ①

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cdot \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} + b_n \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right)$$

In order to simplify this eq, like exponental terms are grouped, also $1/j = -j$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[\left(\frac{a_n - jb_n}{2}\right)e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2}\right)e^{-jn\omega_0 t}\right]$$

To simplify this expression, a new coeff is introduced to replace a & b.

$$\bar{c}_n = \frac{a_n - jb_n}{2}, \quad \bar{c}_n = \frac{a_n + jb_n}{2}, \quad \bar{c}_0 = a_0.$$

The new form of eq is,

$$f(t) = \bar{c}_0 + \sum_{n=1}^{\infty}\left(\bar{c}_n e^{jn\omega_0 t} + \bar{c}_n e^{-jn\omega_0 t}\right)$$

Letting n large through values from 1 to ∞ in this eq is equivalent to letting n range from $-\infty$ to ∞ (including zero) is compact eq^n.

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$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \bar{C}_n e^{jn\omega_0 t}}$$

The coeff $\bar{C}_n$ can be easily evaluated in terms of a_n & b_n. Substituting the expression of a_n & b_n in the expression of $\bar{C}_n$

$\bar{C}_n = \dfrac{a_n - jb_n}{2}$, sub, $a_n = \dfrac{2}{T}\int_0^T f(t)\cos n\omega_0 t$

+ $b_n = \dfrac{2}{T}\int_0^T f(t)\sin n\omega_0 t$

$$\bar{C}_n = \frac{1}{T}\int_0^T f(t)\cos n\omega_0 t\,dt - j\frac{1}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

$$= \frac{1}{T}\int_0^T f(t)(\cos n\omega_0 t - j\sin n\omega_0 t)\,dt$$

$$\boxed{\bar{C}_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt}$$

Exponential form has certain advantages over trigonometric form: only one integral is exponential, simpler integration

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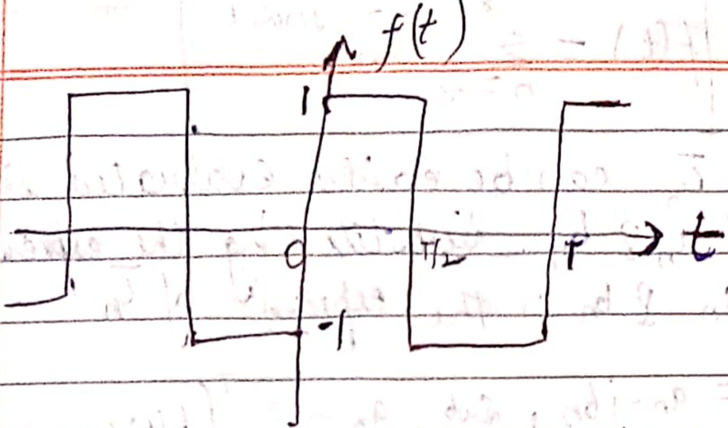
Ans-



$$f(t) = 1 \qquad \text{for } 0 < t < T/2$$

$$f(t) = -1 \qquad T/2 < t < T.$$

C_n will be imaginary

$$\boxed{C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt}$$

$$C_n = \frac{1}{T}\int_0^{T/2} 1 \cdot e^{-jn\omega_0 t}\, dt + \frac{1}{T}\int_{T/2}^{T} (-1) e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^{T/2} + \frac{1}{T}\left[\frac{(-1)e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{T/2}^{T}$$

$$= \frac{1}{-jn\omega_0}\left[e^{-jn\omega_0 T/2} - e^0\right] + \frac{1}{-jn\omega_0}\left[-e^{-jn\omega_0 T} - e^{-jn\omega_0 T/2}\right]$$

$\omega_0 T = 2\pi.$

$$e^{-jn\omega_0 T} = e^{-jn2\pi} = \cos 2n\pi - j\sin 2n\pi = 1$$

$$e^{-jn\omega_0 T/2} = e^{-jn\pi} = \cos n\pi - j\sin n\pi = (-1)^n.$$

Hence,

$$C_n = -\frac{1}{jn2\pi}\left[(-1)^n - 1\right] + \frac{1}{jn2\pi}\left[1 - (-1)^n\right]$$

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$$= \frac{1}{jn2\pi}\left[-(-1)^n + 1 + 1 - (-1)^n\right]$$

$$= \frac{1}{jn2\pi}\left[2 - 2(-1)^n\right] = \frac{2}{jn2\pi}\left[1-(-1)^n\right]$$

= Real + Img

$$\bar{C_n} = \frac{2}{jn\pi}, \text{ odd } n$$

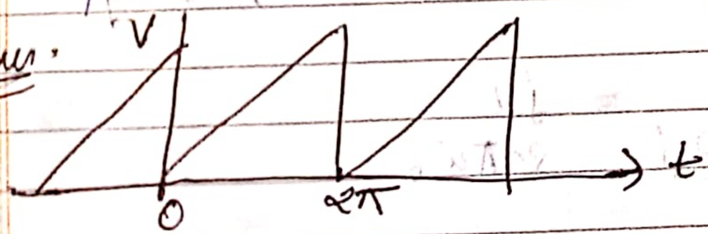
$$\bar{C_n} = 0, \text{ even } n.$$

$$\bar{C_0} = \frac{1}{T}\int_0^T f(t)\,dt = 0$$

exponential fourier series

$$\boxed{f(t) = \sum_{n=\infty}^{\infty} \bar{C_n} e^{jn\omega_0 t}}$$

$$= \frac{2}{j\pi}\sum_{n=-\infty}^{\infty} \frac{1}{n} e^{jn\omega_0 t}$$ for n, odd.



$$f(t) = \frac{V}{2\pi}t.$$

$$\bar{C_n} = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt.$$

$$= \frac{V}{2\pi}\int_0^{2\pi}\left(\frac{t}{2\pi}\right)e^{-jn\omega_0 t}\,dt.$$

$$\bar{C_0} = \left(\frac{V}{(2\pi)^2}\right)\int_0^{2\pi} t\,dt$$

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$$C_0 = \frac{V}{(2\pi)^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= \frac{V}{\cancel{4\pi^2}} \times \frac{\cancel{4\pi^2}}{2} - 0$$

$$= \frac{V}{2} = 0.5\,V$$

$$C_n = \frac{V}{(2\pi)^2}\left[\left[t \cdot \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^{2\pi} - \int_0^{2\pi} 1 \cdot \frac{e^{-jn\omega_0}}{jn\omega_0}\,dt\right]$$

$$= \frac{V}{(2\pi)^2}\Big[$$

$$= e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{-j2n\pi} = \cos 2n\pi - j\sin 2n\pi = 1 - 0 = 1$$

$$C_n = \frac{V}{(2\pi)^2}\left[\frac{2\pi}{-jn\omega_0} + \frac{1}{n^2\omega_0} - \frac{1}{n^2\omega_0^2}\right]$$

$$= \frac{-V}{2\pi n\omega_0 j} = \frac{jV}{2n\pi\omega_0}$$

$$\omega_0 T = 2\pi, \quad T = 2\pi$$
$$\omega_0 = 1$$

$$C_n = \frac{jV}{2n\pi}$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty}\left(\frac{jV}{2n\pi}\right)e^{jn\omega_0 t}$$

$$f(t) = \cdots - \underbrace{\frac{jV}{2\pi}e^{-j\omega_0 t}}_{n=-1} + \underbrace{\frac{V}{2}}_{n=0} + \underbrace{\frac{jV}{2\pi}e^{j\omega_0 t}}_{n=1}$$

$f(t) = \frac{V}{2} + \sum_{n=1} \left(\frac{-V}{n\pi}\right) \sin n\omega_0 t$

$f(t) = \frac{V}{2} - \frac{V}{\pi}\left(\sin t + \frac{\sin 2t}{2} + \frac{\sin 3t}{3} \ldots\right)$

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$C_0 = \frac{V}{2}$,

Trigonometric form from exponential form —

$$\boxed{\begin{aligned} a_n &= C_n + C_n^* \\ b_n &= j(C_n - C_n^*) \\ a_0 &= C_0 \end{aligned}}$$

$a_n = \frac{jV}{2n\pi} - \frac{jV}{2n\pi} = 0$

$b_n = j\left(\frac{jV}{2n\pi} - \left(\frac{-jV}{2n\pi}\right)\right) = j\left(\frac{2jV}{2n\pi}\right)$

$= \frac{\cancel{2}j^2 V}{\cancel{2}n\pi} = \frac{j^2 V}{n\pi}$

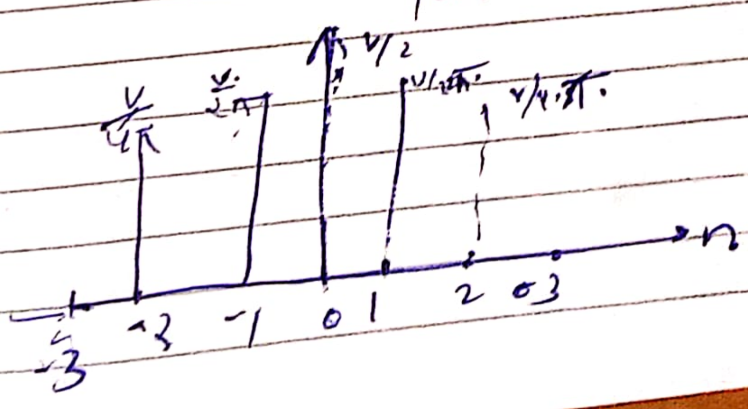
$= \frac{-V}{n\pi}$

a_0 -

Amplitude Spectrum =

$|C_n| = \sqrt{Re^2(C_n) + Im^2[C_n]}$

$= \sqrt{0^2 + \left(\frac{V}{2n\pi}\right)^2} = \frac{V}{2n\pi}$.



Phase spectrum -

$$\bar{C_n} = \frac{jV}{2n\pi} \quad \text{for } n \neq 0$$

$$\phi = \tan^{-1}\left[\frac{Im[\bar{C_n}]}{Re[\bar{C_n}]}\right]$$

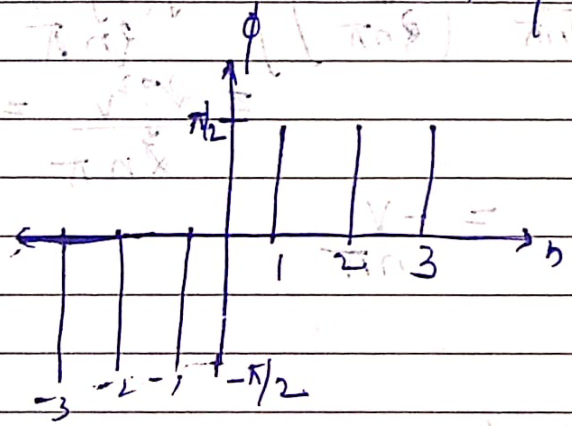
$n = 1, \; \bar{C_1} = \frac{jV}{2\pi}$

$n = -1 \; \bar{C_1} = -\frac{jV}{2\pi}$

$\phi = \tan^{-1}\left[\frac{jV/2\pi}{0}\right]$ and $\phi = \tan^{-1}\left[\frac{-jV/2\pi}{0}\right]$

$\phi = \tan^{-1}[\infty]$ $\quad \tan^{-1}[-\infty]$,

$\phi = \pi/2$ $\quad \phi = -\pi/2$.



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Fourier transform -

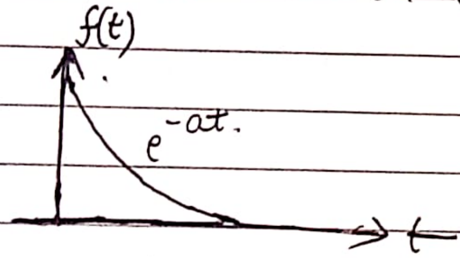
$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega.$$

$$\begin{cases} e^{-\infty} = 0 \\ e^{\infty} = \infty \\ e^{0} = 1 \end{cases}$$

Clues $f(t) = e^{-at} u t \quad a > 0.$

Single sided exponential



$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-aT} e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{-1}{a+j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty}.$$

$$= \frac{-1}{a+j\omega} \left[e^{\infty} - 0 \right] \qquad e^{\infty} = 0$$

$$\boxed{F(j\omega) = \frac{1}{a+j\omega}.}$$

Amplitude Spectrum -

$$|F(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

at $\omega = 0$, $|F(j\omega)| = \frac{1}{a}$,

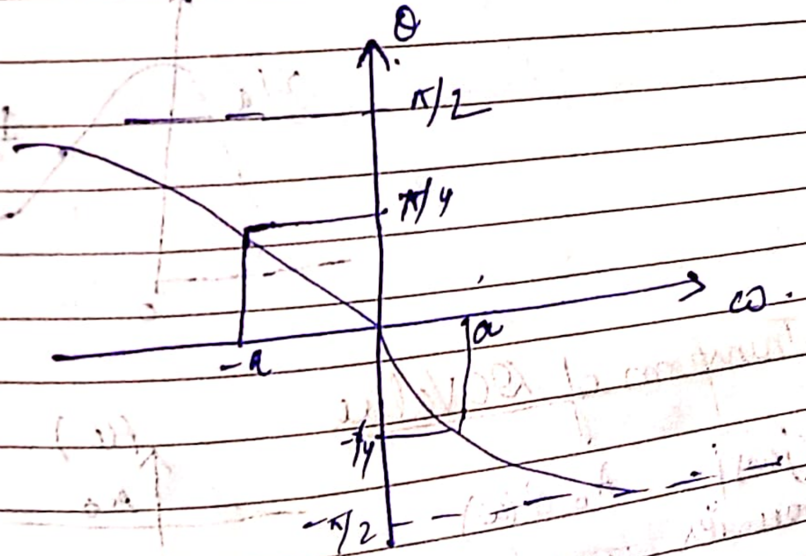
at $\omega = a$, $|F(j\omega)| = \frac{1}{\sqrt{2}\,a}$.

Phase spectrum

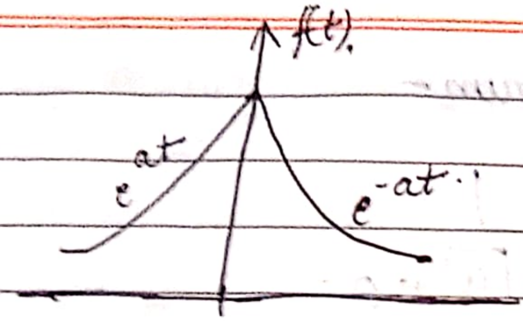
$$F(j\omega) = \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega} = \frac{a-j\omega}{a^2+\omega^2},$$

$$= \frac{a}{a^2+\omega^2} - \frac{j}{a^2+\omega^2},$$

$$\theta = \tan^{-1}\frac{b}{a}, \quad = -\tan^{-1}\frac{\frac{\omega}{a^2+\omega^2}}{a/a^2+\omega^2} = -\tan^{-1}\frac{\omega}{a}.$$



Clear.



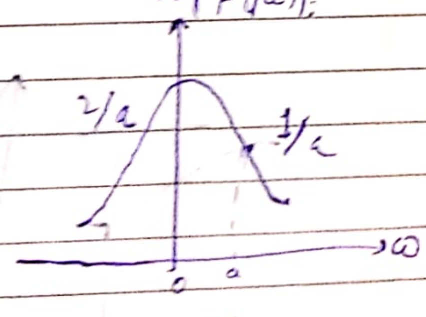
$$f(t) = e^{-a|t|} \text{ for all } t.$$

$$F(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

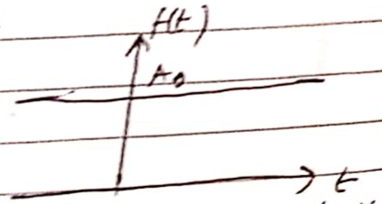
$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

$\omega = 0$, $|F(j\omega)| = \frac{2}{a}$

$\omega = a$, $|F(j\omega)| = \frac{1}{a}$



Fourier Transform of DC Value



Let $F(j\omega) = A_0 \delta(\omega)$ be the fourier transform of an unknown function $f(t)$.

$F(j\omega) = A_0 \delta(\omega)$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega.$$

Here, the area under the curve $= A_0 \times \infty$, which is not defined so, it is not integrable.

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$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A_0 \delta(\omega) e^{j\omega t}\, d\omega.$$

$$= \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \delta(\omega) e^{j\omega t}\, d\omega.$$

$$= \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \delta(\omega - 0) e^{j\omega t}\, d\omega$$

★ Sampling property –
Multiplying a signal $x(t)$ by a ~~unit~~ impulse, samples the value of the signal at the point at which the impulse is located.

$$\bigstar \cdot \int_{-\infty}^{\infty} \delta(t)\,dt = 1 \quad \text{impulse}$$

$$\bigstar,\ x(t)\delta(t) = x(t)\Big|_{t=0} \delta(t)$$

$$= x(0)\,\delta(t).$$

by victure of this property.

$$e^{j\omega t} = 1$$

in previous formulae,

Using multiplication property of impulse.

$$= \frac{A_0}{2\pi} \cdot \int \delta(\omega) \cdot d\omega,$$

$$f(t) = \frac{A_0}{2\pi} \cdot 1.$$

$\frac{A_0}{2\pi}$ is the signal having fourier transform as $A_0 \delta(\omega)$

$$2\pi \times \frac{A_0}{2\pi} = 2\pi\, A_0 \delta(\omega)$$

$$A_0 \rightleftharpoons 2\pi A_0 \delta(\omega).$$

$$\boxed{\begin{array}{l} F\cdot T \text{ of } A_0 = 2\pi A_0 \delta(\omega). \\ F\cdot T \text{ of } 1 = 2\pi\,\delta(\omega) \end{array}}$$

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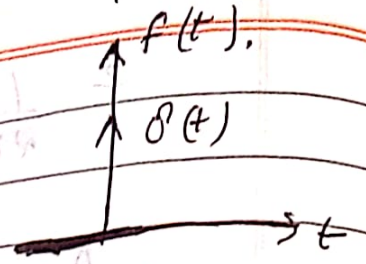


F.T of

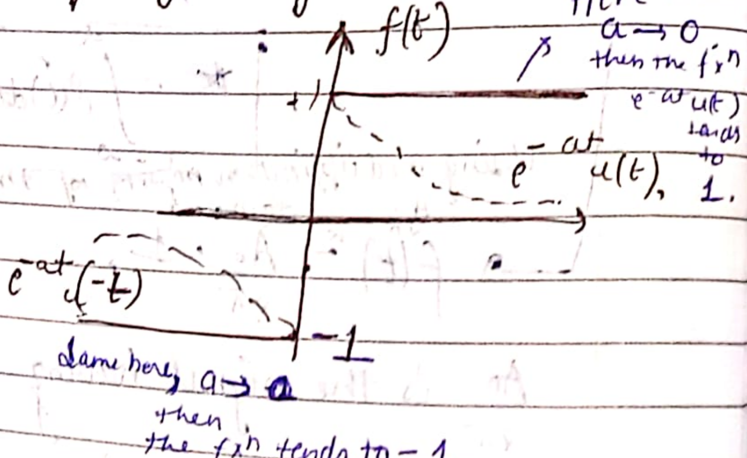
$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}$$

$$F(j\omega) = 1$$

According to sampling property, multiplying a signal x(t) by a unit impulse, samples the value of of the signal at the point at which δ is located.

Fourier transform of signum function -

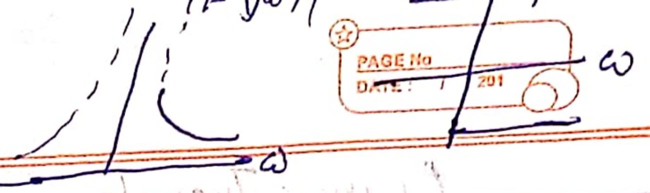


$$= \lim_{a\to 0}\left[e^{-at}u(t) - e^{-at}u(-t)\right].$$

$$= \lim_{a\to 0}\left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega}\right].$$

$$= \lim_{a\to 0}\left[\frac{-2j\omega}{a^2+\omega^2}\right]$$

$$= -\frac{2j\omega}{\omega^2} = \frac{2}{j\omega}.$$

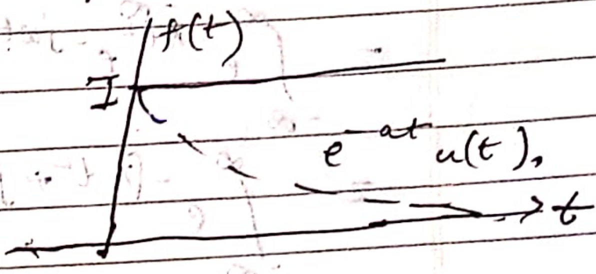


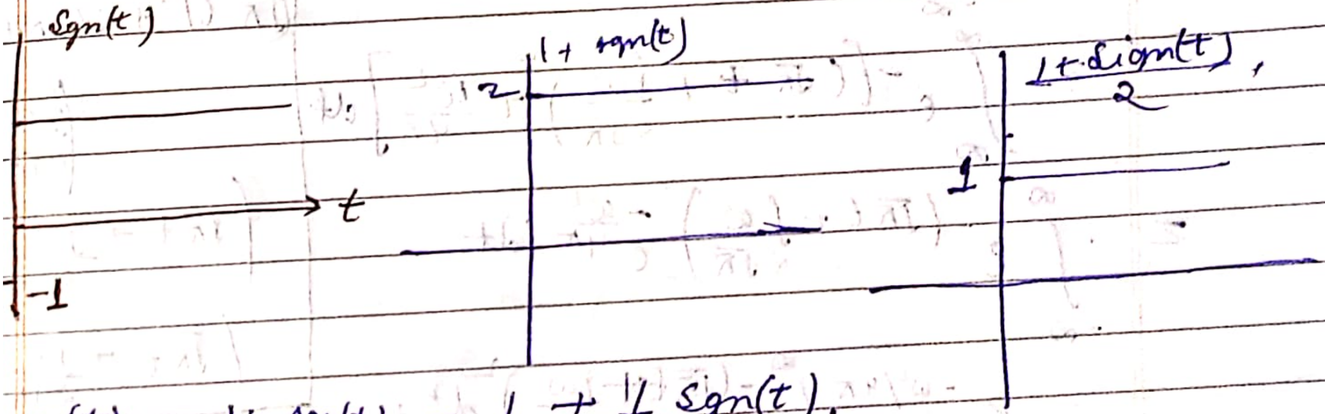
$$|F(j\omega)| = \left|\frac{2}{\omega}\right|$$

$$\angle F(j\omega) = -\tan^{-1}\left(\frac{b}{a}\right) = -\frac{\pi}{2} \quad \omega > 0$$

$$\frac{\pi}{2} \quad \omega < 0.$$

Fourier transform of Unit Step -





$$u(t) = \frac{1+\text{sgn}(t)}{2} = \frac{1}{2} + \frac{1}{2}\,\text{sgn}(t).$$

$$FT[u(t)] = FT\left[\frac{1}{2}\right] + \frac{1}{2}\,FT[\text{sgn}(t)]$$

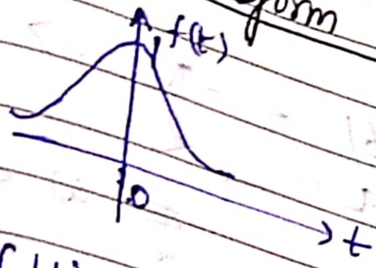
$$\boxed{F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}}$$

FT of $A_0 = 2\pi A_0 \delta(\omega)$

$A_0 = 1/2$

FT of $1/2 =$ $\cancel{2}\pi \times \frac{1}{\cancel{2}}\delta(\omega)$

Fourier Transform Gaussian Pulse



$$f(t) = e^{-\pi t^2}$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-j\omega t} dt.$$

$$= \int_{-\infty}^{\infty} e^{-(\pi t^2 + j\omega t)} dt$$

$$= \int_{-\infty}^{\infty} e^{-\left[\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2 + \frac{\omega^2}{4\pi}\right]} dt$$

$$= \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2} e^{-\frac{\omega^2}{4\pi}} dt$$

$$= e^{-\omega^2/4\pi} \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2} dt,$$

$\pi t^2 + j\omega t \rightarrow (a+b)^2$

$(\sqrt{\pi} t)^2 + 2(\sqrt{\pi} t)\left(\frac{1}{2\sqrt{\pi}} j\omega\right)$

$+ \left(\frac{j\omega}{2\sqrt{\pi}}\right)^2 - \left(\frac{j\omega}{2\sqrt{\pi}}\right)^2$

$\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2 - \frac{j^2\omega^2}{4\pi}$

$\left(\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}}\right)^2 + \frac{\omega^2}{4\pi}.$

Let $\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}} = u.$

$\sqrt{\pi}\, dt = du$

$dt = \frac{du}{\sqrt{\pi}}$

Also $u \to -\infty$ as $t \to -\infty$

$u \to \infty$ as $t \to \infty$

$\int_0^{\infty} e^{-a^2 t^2} dt = \frac{\sqrt{\pi}}{2a}$ $a > 0$

$$\therefore F(j\omega) = e^{-\omega^2/4\pi} \int_{-\infty}^{\infty} e^{-u^2} \frac{du}{\sqrt{\pi}}$$

even fn.

$$= \frac{e^{-\omega^2/4\pi}}{\sqrt{\pi}} 2 \int_0^{\infty} e^{-u^2} du$$

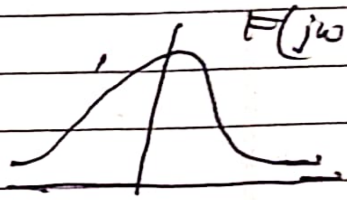
$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$

$$= \frac{e^{-\omega^2/4\pi}}{\sqrt{\pi}} \times 2 \times \frac{\sqrt{\pi}}{2} = e^{-\omega^2/4\pi}$$

Sub $\omega = 2\pi f$

$$= e^{-(2\pi f)^2/4\pi} = e^{-4\pi^2 f^2/4\pi} = e^{-\pi f^2}$$

Fourier transform of gaussian is gaussian.



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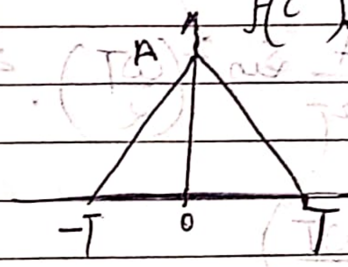


$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt.$$

$y = mx + c$

$$f(t) = -\frac{A}{T} t + A.$$

$$F(j\omega) = \int_{-T}^{0} A\left(1+\frac{t}{T}\right) e^{-j\omega t} dt + \int_{0}^{T} A\left(1-\frac{t}{T}\right) e^{-j\omega t} dt.$$

$$= 2A \int_0^T \left(1-\frac{t}{T}\right) \cos\omega t \cancel{dt}$$

$$= \int_0^T A\left(1-\frac{t}{T}\right) e^{j\omega t} dt + \int_0^T A\left(1-\frac{t}{T}\right) e^{-j\omega t} dt.$$

$$= 2A \int_0^T \left(1-\frac{t}{T}\right) \frac{e^{j\omega t} + e^{-j\omega t}}{2}.$$

$\frac{1.25\angle 0}{1\angle 90}$

$$F(j\omega) = 2A \int_0^T \left(1-\frac{t}{T}\right) \cos\omega t \, dt$$

$$= 2A\left[\left(1-\frac{t}{T}\right)\frac{\sin\omega t}{\omega}\right]_0^T - 2A\int\left(\frac{1}{T}\right)\frac{\sin\omega t}{\omega}\,dt$$

$$= 0 + \left[\frac{-2A}{T}\frac{\cos\omega t}{\omega^2}\right]_0^T$$

$$= \frac{-2A}{\omega^2 T}\left[\cos\omega T - 1\right]$$

$$= \frac{-2A}{\omega^2 T}\left[\cos\omega T - 1\right]$$

$$= \frac{2A}{\omega^2 T}\left[1 - \cos\omega T\right]$$

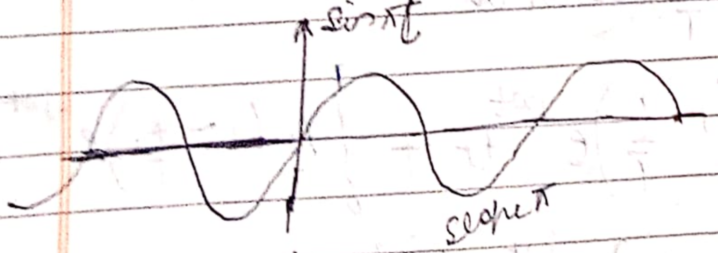
$$= \frac{2A}{\omega^2 T}2\sin^2\left(\frac{\omega T}{2}\right) = \frac{4A}{\omega^2 T}\sin^2\left(\frac{\omega T}{2}\right) = \frac{4AT}{\omega^2 T^2}\sin^2\left(\frac{\omega T}{2}\right)$$

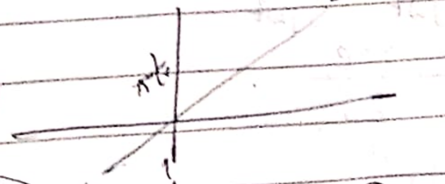
$$= \frac{AT}{\left(\frac{\omega T}{2}\right)^2}\sin^2\left(\frac{\omega T}{2}\right)$$

$$= AT\left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}\right]^2 = AT\,\mathrm{Sinc}^2\left(\frac{\omega T}{2}\right)$$

Sinc function =

$$\mathrm{Sinc}(t) = \frac{\sin\pi t}{\pi t}$$





Imp. Ques' { F.F. of Circular
Triangular
Rectangular

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$P = I^2 R = .25$

$V = I^2 R Z$

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Inverse fourier transform -

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega.$$

$$F(j\omega) = \delta(\omega)$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega$$

Using Sampling property of Impulse function

$$x(t)\delta(t) = x(t)\Big|_{t=0}\delta(t) = x(0)\delta(t).$$

Outer form

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

$$= \frac{1}{2\pi} e^{j\omega t}\Big|_{\omega=0}.$$

$$F^{-1}\left[\delta(\omega)\right] = \frac{1}{2\pi} \quad \text{or} \quad \delta(\omega) = F\left[\frac{1}{2\pi}\right]$$

$$\frac{1}{2\pi} \longleftrightarrow \delta(\omega).$$

now,

$$F(j\omega) = \delta(\omega-\omega_0).$$

$$F^{-1}\left[\delta(\omega-\omega_0)\right] = \frac{1}{2\pi} e^{j\omega_0 t}.$$

$$\delta(\omega-\omega_0) = F\left[\frac{1}{2\pi} e^{j\omega_0 t}\right].$$

$$\boxed{\therefore \frac{1}{2\pi} e^{j\omega_0 t} \longleftrightarrow \delta(\omega-\omega_0)}$$ → fourier transform pair.

Fourier Transform of $f(t) = \cos\omega_0 t$.

$$f(t) = \cos\omega_0 t = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right).$$

$$F(j\omega) = F\left[\frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right].$$

Since,

$$F\left[e^{j\omega_0 t}\right] = 2\pi\,\delta(\omega - \omega_0).$$

$$F\left[e^{-j\omega_0 t}\right] = 2\pi\,\delta(\omega + \omega_0)$$

$$F(j\omega) = \frac{1}{2} \times 2\pi\,\delta(\omega - \omega_0) + \frac{1}{2} \times 2\pi\,\delta(\omega + \omega_0).$$

$$\boxed{F(j\omega) = \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]}$$

\# Fourier transform of $f(t) = \sin\omega_0 t$.

$$f(t) = \sin\omega_0 t$$
$$= \frac{1}{2}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

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Ans. I.F.T.

$$F(j\omega) = \frac{j\omega+3}{(j\omega+1)^2} = \frac{j\omega+2+1}{(j\omega+1)^2},$$

$$= \frac{(j\omega+1)}{(j\omega+1)^2} + \frac{2}{(j\omega+1)^2}.$$

F.T. of

$t^0 e^{-at} u(t) = \frac{1}{(a+j\omega)}^{0+1}$

$t^n e^{-at} u(t) = \frac{1}{(a+j\omega)^{n+1}}$

$t e^{-at} u(t) = \frac{1}{(a+j\omega)^2}$

$$F(j\omega) = \frac{1}{(j\omega+1)} + \frac{2}{(j\omega+1)^2}$$

$$\cancel{F(j\omega) = \frac{1}{j\omega+1}}$$

$$F(j\omega) = e^{-1t}u(t) + 2te^{-t}u(t).$$

10/11/22.

Ans. IFT –

$$F(j\omega) = \frac{\omega^2+21}{\omega^2+9}.$$

$\omega^2+9 \,)\, \omega^2+21 \,(\, 1$
$\quad \cancel{\omega^2} + 9$
$\quad 12$

$$F(j\omega) = \cancel{\omega^2+9}\; 1 + \frac{12}{\omega^2+9}$$

$\omega^2+21 = (\omega^2+9)(1) + 12$

$\omega^2+21 = \omega^2+9+12$

ω^2+21

*. F.T of $\delta(t) \longrightarrow 1$

F.T of $e^{-a|t|} = \frac{2a}{\omega^2+a^2}$

$$= \frac{2(a)}{\omega^2+(3)^2}$$

$$= \frac{2(3)}{\omega^2+(3)^2}$$

~~FT~~

$$= 1 + 2 \cdot \frac{2(3)}{\omega^2+(3)^2}$$

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$F.T = \delta(t) + 2e^{-3(t)}$.

Class $F(j\omega) = \frac{j\omega+1}{(j\omega)^2+5j\omega+6}$

$= \frac{(j\omega+1)}{(j\omega+2)(j\omega+3)}$

$j^2\omega^2 + 5j\omega + 6 = 0$

$j^2\omega^2 + 5j\omega + 6 = 0$

$j^2\omega + (3+2)j\omega + 6 = 0$

$j\omega^2 + 3j\omega + 2j\omega + 6 = 0$

$j\omega(j\omega+2) 3(j\omega+2) = 0$

$(j\omega+3)(j\omega+2) = 0$

$\cancel{j\omega^2 + 5j\omega + 6 = 0}$

$F(j\omega) = \frac{K_1}{j\omega+2} + \frac{K_2}{j\omega+3}$.

$\cancel{F(j\omega) = \frac{K_1}{j\omega+2}}$

$K_1 = \left.\frac{j\omega+1}{j\omega+3}\right|_{j\omega=-2} = -1$

$K_2 = \left.\frac{j\omega+1}{j\omega+2}\right|_{j\omega=-3} = 2$.

$F(j\omega) = \frac{-1}{(j\omega+2)} + \frac{2}{(j\omega+3)}$.

$= \frac{-(j\omega+3)+2(j\omega+2)}{(j\omega+2)(j\omega+3)}$

$= \frac{-j\omega-3+2j\omega+4}{(j\omega+2)(j\omega+3)}$

$= \frac{1+j\omega}{(j\omega+2)(j\omega+3)}$

$F(j\omega) = -e^{-2t}u(t) + 2e^{-t}u(t)$

Qn. $F(j\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$

$(j\omega)^2 + 3j\omega + 2 = 0$

$j^2\omega^2 + (2+1)j\omega + 2 = 0$

$j^2\omega^2 + 2j\omega + j\omega + 2 = 0$

$j\omega(j\omega + 1) + 2(j\omega + 1)$

$(j\omega + 2)(j\omega + 1)$

$F j(\omega) = \dfrac{-j\omega}{(j\omega+2)(j\omega+1)} =$

$F(j\omega) = \dfrac{A}{(j\omega+2)} + \dfrac{B}{(j\omega+1)}$

$\Rightarrow A(j\omega+1) + B(j\omega+2)$

$-j\omega = Aj\omega + A + Bj\omega + 2B$

$A + B = -1$

$A + 2B = 0$

$A = -2$
$B = 1$

$= \dfrac{-2}{(j\omega+2)} + \dfrac{1}{(j\omega+1)}$

$= -2e^{-2(t)}u(t) + e^{-t}u(t)$

Steady state Response of network to periodic Signal -

In practice, many circuits are driven by non-sinusoidal periodic function. To obtain the steady state response of the circuit to a non-sinusoidal periodic function requires the application of fourier series, ac phasor analysis & superposition principle. The procedure involves following steps -

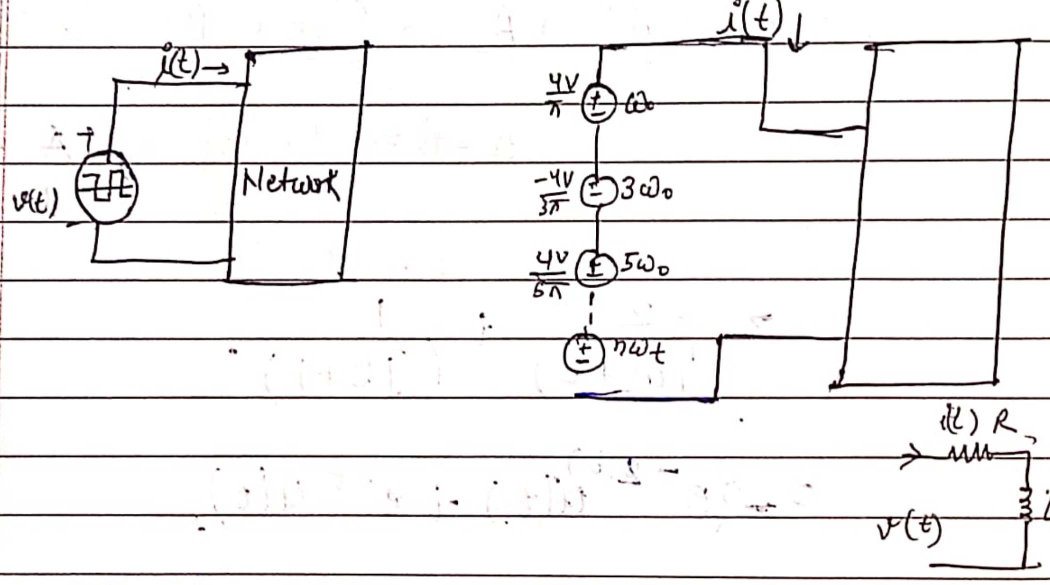
1. Express the excitation as fourier series.
2. Transform the circuit elements from time domain to frequency domain

$R \rightarrow R$, $L \rightarrow j\omega nL$, $C \rightarrow \frac{1}{j\omega nC}$ for n^{th} harmonic

3. Find the response of dc & ac component in fourier series

4. Add the individual dc & ac response using the superposition principle.

Ques: Steady state response when a square wave source excites series RL circuit.



The fourier series is

$$v(t) = \frac{4V}{\pi}\left(\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t + \cdots\right)$$

If $V = \frac{\pi}{4}$ & for simplicity $\omega_0 = 1$ rad/sec.

then

$|A|\cos(\omega t + \theta) \longleftrightarrow |A|\angle\theta$

$\longleftrightarrow Ae^{j\theta}$.

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$V(t) = \cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t + - - - -$

The phasors corresponding to first three terms are.

$V_1 = 1e^{j0}$; $V_3 = \frac{1}{3}e^{-j180}$, $V_5 = \frac{1}{5}e^{j0}$

Let $R = 1\Omega$ & $L = 1H$

$\therefore Z(j\omega n) = R + j\omega nL$ or $Y(j\omega n) = \frac{1}{R + j\omega nL}$.

$Y(jn) = \frac{1}{R + jnL} = \frac{1}{1 + jn \cdot 1} = \frac{1}{1 + jn}$.

$n = 1$; $Y(j1) = \frac{1}{1 + j \cdot 1} = \frac{1}{1+j} \times \frac{1-j}{1-j} = \frac{1 - j1}{1 - j^2} = \frac{1 - j1}{2}$

$= 0.5 - j0.5$

$= 0.707 \angle -45$

$\cancel{n = 2 ; Y(j2) = \frac{1}{1+j} = \frac{1}{1+2j}}$

$n = 3$; $Y(j3) = \frac{1}{1+3j} = \frac{1}{1+3j} \times \frac{1-3j}{1-3j} = \frac{1-3j}{1-3j^2} = \frac{1-3j}{10}$

$= 0.1 - 0.3j$

$= 0.316 \angle -71.56°$

$n = 5$; $Y(j5) = \frac{1}{1+j5} = \frac{1}{1+j5} \times \frac{1-j5}{1-j5}$

$= \frac{1-j5}{1-(j5)^2} = \frac{1-j5}{26} = 0.038 - j0.192$

$= 0.195 \angle -78.80$

Since $I = YV = I_1 + I_3 + I_5 + - - -$

& $I_1 = Y_1V_1 = 0.707 \angle -45 \times 1e^{j0} = 0.707 e^{-j45} \times 1e^{j0}$

$= 0.707 e^{-j45}$.

$I_3 = Y_3V_3 = 0.316 e^{-j71.56} \times \frac{1}{3}e^{-j180} = 0.105e^{-j251.56}$

$I_5 = Y_5V_5 = 0.195 e^{-78.80} \times \frac{1}{5}e^{j0} = 0.039 e^{-j78.80}$

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Then the total current has following first three terms.

$$i(t) = 0.707\cos(t-45) + 0.105\cos(3t-251.6) + 0.039\cos(5t-78.8^\circ) + \cdots$$

EFFECTIVE VALUE / RMS VALUE

- Let the periodic, non-sinusoidal waveform be represented by

$$f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \cdots + b_1\sin\omega t + b_2\sin 2\omega t + \cdots$$

or $$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n\sin n\omega t)$$

$$F_{rms} = \left\{\sqrt{\frac{1}{T}\int_0^T\left[a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n\sin n\omega t)\right]^2 dt}\right\}$$

$$= \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 \cdots}$$

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Unit - IV

Laplace Transform

$$F(s) = \int_0^{\infty} f(t) e^{-\sigma t} e^{-j\omega t} dt.$$

Let $s = \sigma + j\omega$.

$$\boxed{\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st} dt}$$

$$\boxed{\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s) e^{st} ds}$$

Basic Theorem

1. The laplace transform of sum of two functions equal to the sum of laplace transform of the individual f^{xn}.

$$\mathcal{L}[f_1(t) + f_2(t)] = \int_0^{\infty} [f_1(t) + f_2(t)] e^{-st} dt$$

$$= \int_0^{\infty} f_1(t) e^{-st} dt + \int_0^{\infty} f_2(t) e^{-st} dt$$

$$= F_1(s) + F_2(s)$$

2. The laplace transform of a constt. times a function is equal to the constt. times the laplace transform

of the function.

$$\mathcal{L}[K f(t)] = \int_0^\infty K f(t) e^{-st} dt = K F(s).$$

3. Differentiation Theorem -

If a fx^n $f(t)$ & its derivative are both Laplace transformable then the Laplace transform of the first derivative of a time fx^n $f(t)$ is 's' times the Laplace transform of $f(t)$ minus the limit of $f(t)$ as $t \longrightarrow 0_+$

$$\mathcal{L}\left[\frac{d f(t)}{dt}\right] = sF(s) - f(0_+)$$

$$\mathcal{L}\left[\frac{d^2 f(t)}{dt}\right] = \mathcal{L}\left[\frac{d}{dt}\left[\frac{d f(t)}{dt}\right]\right] = s\mathcal{L}\left[\frac{d f(t)}{dt}\right] - \frac{d f(t)}{dt}\Big|_{t=0^+}$$

$$= s\left[sF(s) - f(0^+)\right] - f'(0^+) = s^2F(s) - sf(0^+) - f'(0^+).$$

4. Differentiation by s. -

Differentiation by s in complex frequency domain corresponds to multiplication by t in time domain. i.e.

$$\mathcal{L}[t f(t)] = -\frac{dF(s)}{ds}.$$

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5. Integration Theorem-
The Laplace transform of the first integral of a fxn $f(t)$ w.r.t time is the laplace transform of $f(t)$ divided by s,
i.e.

$$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

6. Integration by s-

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_0^\infty F(s)\,ds,$$

7. Initial Value Theorem-
If the Laplace transform of $f(t)$ is $f(s)$ & the first derivative of $f(t)$ is Laplace transformable then the initial value of $f(t)$ is

$$f(0^+) = \underset{t\to 0^+}{Lt}\ f(t) = \underset{s\to\infty}{Lt}\left[s\,F(s)\right]$$

If time limit exists.

8. Final Value Theorem-
If the Laplace transform of $f(t)$ is $f(s)$ & if the ~~[illegible]~~ s times $f(s)$ is analytic on the imaginary axis & is right half of s-plane then, limit $t\to\infty$.

$$\underset{t\to\infty}{Lt}\ f(t) = \underset{s\to 0}{Lt}\left[s\,F(s)\right].$$

Ques. Find Laplace transform of

(1) $f(t) = e^{at}$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{at} e^{-st}\, dt$$

$$= \int_0^\infty e^{at-st}$$

$$= \int_0^\infty e^{-(s-a)t}\, dt.$$

$$= \frac{1}{s-a}$$

Ques (2). $f(t) = u(t) = 1$ for $t > 0$,
0 for $t < 0$.

$$f(s) = \int_0^\infty 1\, e^{-st}\, dt.$$

$$= \left[\frac{e^{-st}}{s}\right]_0^\infty dt$$

$$= \left(\frac{e^{-\infty}}{s} - \frac{e^{0}}{s}\right).$$

$$= \frac{1}{s}$$

(3) Step function

$$f(t) = K\, u(t)$$

$$f(s) = \int_0^\infty K\, ut\; e^{-st}\, dt. \quad = \frac{K}{s},$$

$\frac{e^{i\theta}+e^{-i\theta}}{2i}$

Table

$f(t)$	$f(s)$
e^{at}	$\frac{1}{s-a}$
$u(t)$	$\frac{1}{s}$
$K\,u(t)$	$\frac{K}{s}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s+\omega^2}$

(iv) $f(t) = \sin\omega t$

$$f(t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$f(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j}\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) \cdot e^{-st}\, dt$$

$$= \int_{0}^{\infty} \frac{e^{j\omega t - st}}{2j} - \frac{e^{-j\omega t} \cdot e^{-st}}{2j}\, dt$$

$$= \frac{1}{2j} \int_{0}^{\infty} e^{j\omega t - st} - e^{-j\omega t - st}$$

$$\cancel{= \frac{1}{2j}\left[e^{j\omega -st} - e^{-\infty - st}\right]}$$

$$\cancel{= \frac{1}{2j}\left[e^{j\omega t}\right.}$$

$$= \frac{1}{2j}\left[e^{j\omega t - st}\Big|_{0}^{\infty} - e^{-j\omega t - st}\Big|_{0}^{\infty} \right]$$

$$= \frac{1}{2j}\left[\frac{e^{(j\omega - s)t}}{j\omega - s}\right]_0^{\infty} - \left[\frac{e^{-(j\omega + s)t}}{j\omega + s}\right]_0^{\infty}$$

$$= \frac{1}{2j}\left[\frac{1}{j\omega - s}\right] - \left[\frac{-1}{j\omega + s}\right]$$

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$$= \frac{1}{2j}\left[\frac{1}{j\omega - s} + \frac{1}{j\omega + s}\right]$$

$$= \frac{1}{2j}\left[\frac{(j\omega + s) + (j\omega - s)}{(j\omega - s)(j\omega + s)}\right]$$

$$= \frac{1}{\cancel{2}\cancel{j}}\left[\frac{\cancel{2}\cancel{j}\omega}{(j\omega)^2 - s^2}\right] =$$

$$= \frac{\omega}{-\omega^2 - s^2}$$

~~Wae~~ f(t)

$\mathcal{L}[u(t)]$	$\frac{1}{s}$.

Table

$f(t)$	$f(s)$
e^{at}	$1/s-a$.
$u(t)$	$1/s$.
$K\,u(t)$	K/s.
$\sin\omega t$	$\omega/s^2+\omega^2$
$\cos\omega t$	$s/s^2+\omega^2$.
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$.
$\mathcal{L}[t^n]$	$\frac{n!}{s^{n+1}}$
$\mathcal{L}[e^{-t}.t^2]$	$\frac{2}{(s+1)^3}$
$\mathcal{L}[e^{-at}.t^n]$	$\frac{n!}{(s+a)^{n+1}}$
$\mathcal{L}[\delta(t)] = 1$.	

Ques- $f(t) = \cos \omega t$

Ans. Find Laplace transform of Hyperbolic sine,

$$f(t) = \sinh at = \frac{1}{2}\left(e^{at} - e^{-at}\right)$$

$$f(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$$

$$= \int_0^\infty \frac{1}{2}\left(e^{at} - e^{-at}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty \left(e^{at} - e^{-at}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty \left(e^{at} \cdot e^{-st} - e^{-at} \cdot e^{-st}\right) dt$$

$$= \frac{1}{2}\int_0^\infty \left(e^{at-st} - e^{-at-st}\right) dt.$$

$$= \frac{1}{2}\int_0^\infty \left(e^{(a-s)t} - e^{(a-s)t}\right. dt.$$

$$= \frac{1}{2}\left[\frac{e^{(a-s)t}}{a-s}\right]_0^\infty - \left[\frac{e^{(a-s)t}}{-a-s}\right]_0^\infty$$

$$= \frac{1}{2}\left[e. \right.$$

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Ans: Hyperbolic Cosine

$$f(t) = \frac{1}{2}\left(e^{at} + e^{-at}\right).$$

—— X —— X —— X —— X —— X ——

Topics:

Formula of L.T.
advantages.
Basic Theorems – differentiation
– Integration
– initial value
– final Value
– Standard function
Property : e^{-as} times the function
L.T. $[t^n]$
Numericals
Singularity function : – Unit Ramp ; Parabolic fx^n,
Unit Step, Unit Impulse
Shifting & Scaling
Shifting theorem $\mathcal{L}\left[f(t-a)\,U(t-a)\right] = e^{-as} F(s)$

Waveform Synthesis
L.T. of waveform.
L.T. of gate fx^n, Sawtooth wave, Exp – fx^n etc.
L.T. of periodic fx^n –
Transform ckt, application of L.T. to electrical ckt.

Qn: Damped sine $f(t) = e^{-at} \sin\omega t$,

$$= \mathcal{L}\left[\frac{1}{2j}\left[e^{-at}\left(e^{j\omega t} - e^{-j\omega t}\right)\right]\right].$$

$$= \frac{1}{2j}\mathcal{L}\left[e^{-(a-j\omega)t} - e^{-(a+j\omega)t}\right].$$

$$= \frac{1}{2j}\left[\frac{1}{s+a-j\omega} - \frac{1}{s+a+j\omega}\right]$$

$$= \frac{\omega}{(s+a)^2+\omega^2}$$

Property - e^{-at} times a function

$$\mathcal{L}\left[e^{-at}f(t)\right] = \int_0^\infty e^{-at}f(t)e^{-st}\,dt,$$

$$= \int_0^\infty f(t)e^{-(s+a)t}\,dt$$

$$\boxed{\mathcal{L}\left[e^{-at}f(t)\right] = F(s+a)}.$$

e.g. $\mathcal{L}\left[e^{-at}\sin\omega t\right] = \frac{\omega}{(s+a)^2+\omega^2}.$

$\cos^2 t = \frac{1+\cos 2\theta}{\cancel{\sin 2\theta}}$ $\quad \frac{1+\cos 2\theta}{2}$

Ans. $f(t) = \cos^2 t.$

$\frac{2s^2+4}{2s(s^2+4)}$

$f(t) = \frac{1+\cos 2t}{2}$

$f(s) = \int_0^\infty f(t)\, e^{-st}\, dt$

$f(s) = \int_0^\infty \frac{1+\cos 2t}{2}\, e^{-st}\, dt.$

$f(s) = \int_0^\infty \frac{1}{2} L[1] + \frac{1}{2} L[\cos 2t]$ ar

$= \frac{1}{2} L[1] + \frac{1}{2} L[\cos 2t].$

$= \frac{1}{2} \quad + \frac{1}{2}\frac{s}{s}$

$f(t) = t^n$

$$\mathcal{L}[t^n] = \int_0^\infty t^n e^{-st}\,dt$$

$$= t^n\left[\frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty n t^{n-1}\frac{e^{-st}}{-s}\,dt$$

$$= -\frac{1}{s}\left[t^n e^{-st}\right]_0^\infty + \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt$$

$$= -\frac{1}{s}\left[\infty^n e^{-\infty} - 0^n e^{-0}\right]$$

$$= -\frac{1}{s}\left[\infty - 0 - 0\right]$$

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Ques. $F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$

$s^2+6s+25$

$\boxed{x_1 -3+4i, x_2 -3-4i}$

$= \cancel{\frac{A}{s+6} + \frac{B}{s-3+4i}} + \frac{C}{s-3-4i}$

$\Rightarrow A = \cancel{(s+6)(s-3+4i)}$

$(-6+6)(-6$

$s+6 = -6+4i$

$s = -6$

$= \frac{A}{s+6} + \frac{B}{-3+4i} + \frac{C}{-3-4i}$

$A = (-3+4i)(-3-4i)$

$x = -2 + 2i$

$x + 2 - 2i =$

Que. $F(s) = \dfrac{1}{s^2+4s+8}$

s^2+4s+8

$x_1 = -2 \pm 2i$.

$x_1 = -2+2i$

$x_2 = -2-2i$

$= \dfrac{A}{s-2+2i} + \dfrac{B}{s-2-2i}$

$= \dfrac{1}{(s+2-2i)(s+2+2i)}$

$A = \dfrac{1}{(-2+2i+2+2i)}$ $A = -0.25i$

$B = \dfrac{1}{-2-2i-2+2i}$ $B = -0.25$

$\Rightarrow f(s) = \dfrac{-0.25i}{(s-2+2i)} + \dfrac{(-0.25)}{s-2-2i}$

$=$

$\dfrac{-\frac{j}{4}}{(s-2+2i)} + \dfrac{-\frac{1}{4}}{(s-2-2i)}$

$-j/4\left(e^{-2+2i} - e^{-2-2i}\right)$

$= -j/4\,($

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$$\frac{1}{2!}\frac{d^2}{ds^2}$$

$$\frac{1}{j!}\frac{d^j}{ds^j}$$

Qus: $F(s) = \dfrac{2s^2+5s+2}{(s+1)^3}$

$$F(s) = \frac{A_0}{(s+1)^3} + \frac{A_1}{(s+1)^2} + \frac{A_2}{(s+1)}$$

$$A_0 = (s+1)^3 F(s)\Big|_{s=1} = \frac{d}{ds}\left(2s^2+5s+2\right)\Big|_{s=-1} = -1$$

$$A_1 = (s+1)^2 \cdot F(s) \; ; \; \frac{d}{ds}\left(2s^2+5s+2\right) = 1$$

~~$A_2 = (s+1)\ F(s)$~~

$$A_2 = \frac{1}{2!}\frac{d}{ds}\left[(s+1)^3 F(s)\right]\Big|_{s=-1} = 2$$

$\mathcal{L}\left[e^{-at} f(t)\right] = F(s+a)$

$$\mathcal{L}\left[e^{-t}\cdot t\right] = \frac{1}{(s+1)^2}$$

$$\mathcal{L}\, t = \frac{1}{s^2}$$

Qus: $F(s) = \dfrac{2s^3-9s^2+4s+10}{s^2-3s-4}$

$$s^3-3s-4 \,\big)\, 2s^3-9s^2+4s+10 \,\big(\, 2$$
$$2s^3 \quad -6s-8$$
$$-9s^2+10s+18$$

$$(s^3-3s-4)(2) + (-9s^2+10s+18) = (2s^3-9s^2+4s+10)$$

$2-5, 2$
$2-3$

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$$s^2-3s-4 \overline{) 2s^3-9s^2+4s+10} \; (2s-3$$
$$\underline{\cancel{2s^3} \mp 6s^2 \mp 8s}$$
$$-\cancel{3s^2}+12s+10$$
$$\underline{\mp 3s^2 \pm 9s \pm 12}$$
$$3s-2$$

$$= (s^2-3s-4)(2s-3)+(3s-2)$$

$$F(s) = 2[s] - 3.1 + \frac{1}{s+1} + \frac{2}{s-4}.$$

$$f(t) = 2 \cdot \frac{d}{dt}\delta(t) - 3.\delta(t) - e^{-t} + 2e^{4t},$$

Que $F(s) = \frac{s^2+s-3}{s^2+3s+2}$

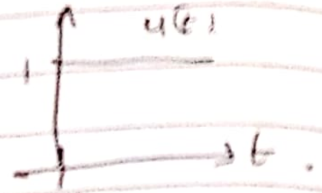
(first long division & then partial fraction constt.)..

Ans: $\delta(t) - 3e^{-t} + e^{-2t},$

Singularity Function

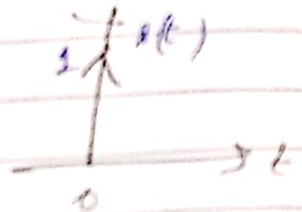
1. Unit Step

$u(t) = 1, \quad t \geq 0$
$\quad\quad\; 0 \quad t < 0$



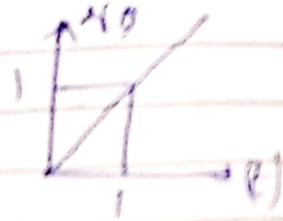
2. Unit Impulse (Dirac Delta f^n)

$\delta(t) = 0 \quad t \neq 0$
$\quad\quad\; 1 \quad t = 0.$



3. Unit ramp.

$r(t) = t\,u(t) = t \quad t \geq 0$
$\quad\quad\quad\quad\quad\; 0 \quad t < 0$



Laplace Transform -

$\mathcal{L}[u(t)] = \int 1 \cdot e^{-st}\,dt = \frac{1}{s}.$

$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t) e^{-st}\,dt = e^{-st}\Big|_{t=0} = e^0 = 1$

Ramp function → $\mathcal{L}[t\,u(t)] = \int_0^\infty t\,e^{-st}\,dt = \frac{1}{s^2}.$

Parabolic f^n. $\mathcal{L}[t^2 u(t)] = \frac{2}{s^3}.$

Scaling

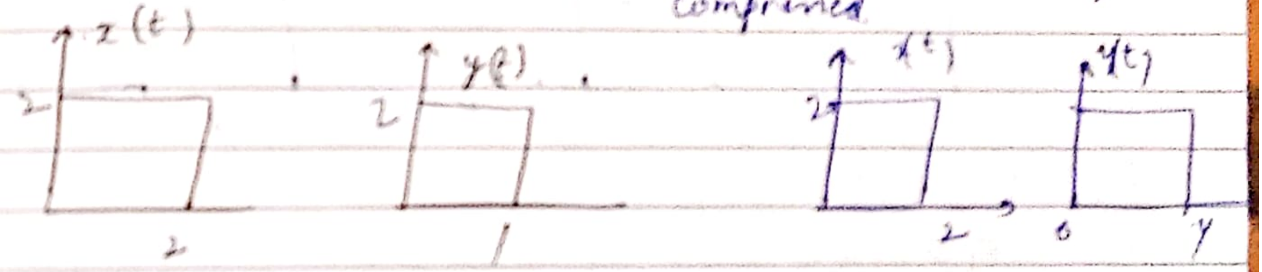
Time   Amplitude

Time scaling ; Expansion or Compression

$x(t) \xrightarrow{T.S.} y(t) = x(\alpha t) \quad \alpha \neq 0$

$|\alpha| > 1$ eg $\alpha = 2$. $|\alpha| < 1$ e.g. $= 0.5$

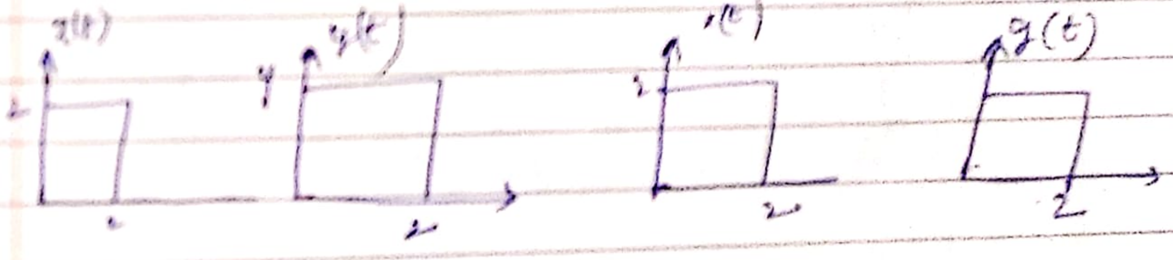
compressed



Amplitude scaling

$x(t) \xrightarrow{A.S.} y(t) = \beta(x(t))$

$|\beta| > 1$; $\beta = 2$ & $|\beta| < 1$; $\beta = 0.5$



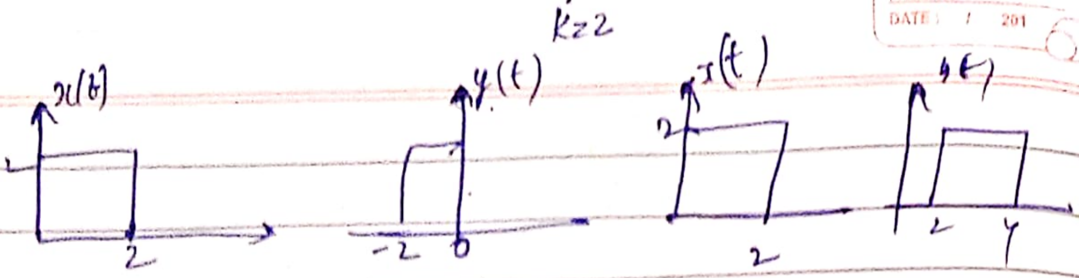
Shifting

Time   Amplitude

Time shifting

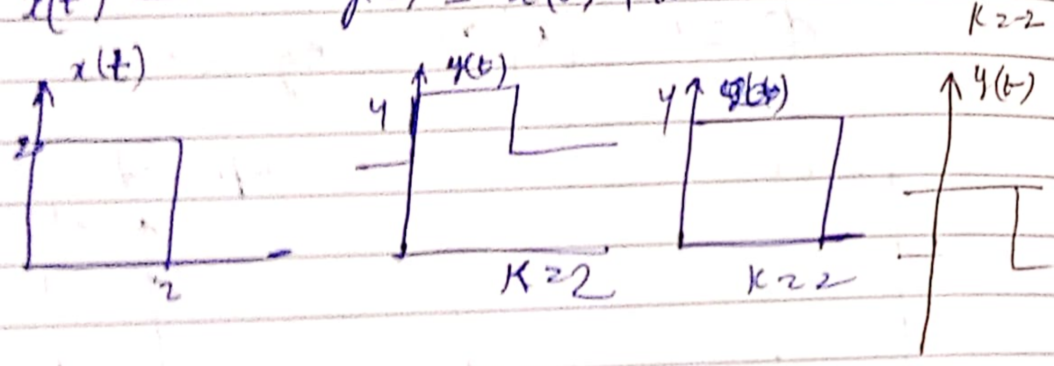
$x(t) \xrightarrow{T.S.} y(t) = x(t+k)$

$K > 0$ (+ve, advances) $K < 0$ (-ve, delays)



Amplitude Shifting -

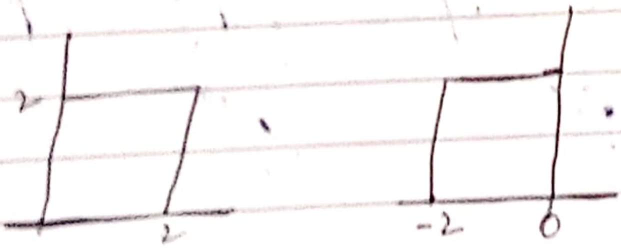
$x(t) \xrightarrow{A.S.} y(t) = x(t) + K$



Reversal

Time Reversal (sp. case of time scaling with α=-1)

$x(t) \xrightarrow{T.R.} y(t) = x(-t)$



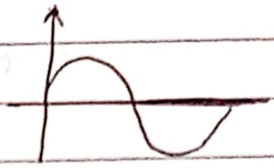
Shifting Theorem -

The Laplace transform of any function shifted or delayed by a time interval of 'a' is e^{-as} times the transform of the function i.e.

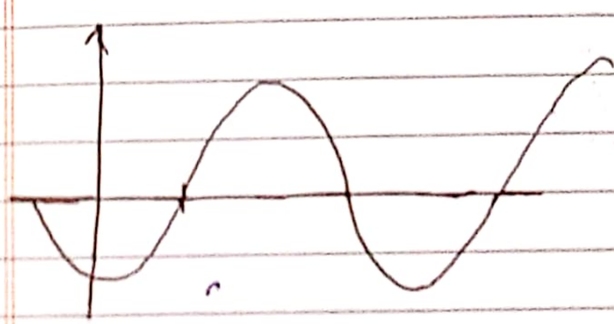
$$\mathcal{L}[f(t-a)\,U(t-a)] = e^{-as}F(s).$$

$f(t) = \sin\omega t$

$F(s) = \dfrac{\omega}{s^2+\omega^2}$

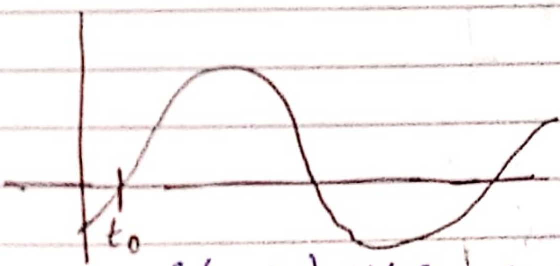


1.)



$$f(t-t_0) = \sin\omega(t-t_0)$$
$$= \mathcal{L}[\sin\omega t\cos\omega t_0 - \cos\omega t\sin\omega t_0]$$
$$= \frac{\omega\cos\omega t_0 - s\sin\omega t_0}{s^2+\omega^2}$$

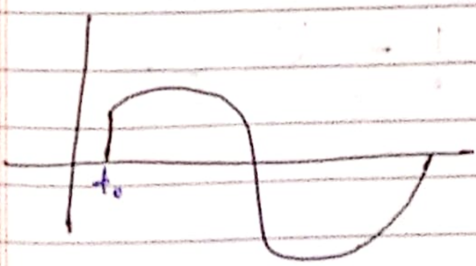
2.)



$$f(t-t_0)\,U(t) = \sin\omega(t-t_0)\,U(t).$$

$$F(s) = \frac{\omega\cos\omega t - s\sin\omega t_0}{s^2+\omega^2}, \quad (\because \text{L.T. is for } 0<t<\infty)$$

3)

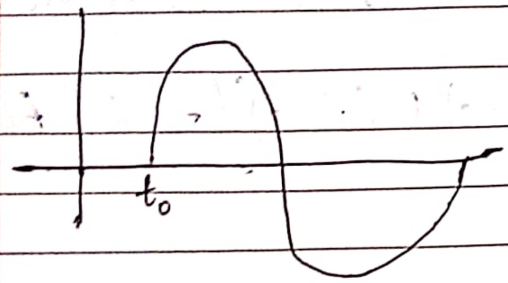


$$f(t)\,U(t-t_0) = \sin\omega t\,U(t-t_0).$$

$$f(t)\,U(t-t_0) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}\,U(t-t_0)$$

$$f(s) = e^{-t_0 s}\frac{[\omega \cos \omega t_0 + s \sin \omega t_0]}{s^2+\omega^2}$$

(4.)



$f(t-t_0)\,U(t-t_0)$

$= \sin\omega(t-t_0)\,U(t-t_0)$

$F(s) = e^{-t_0 s}\,\mathcal{L}[\sin \omega t]$

$= e^{-t_0 s} \cdot \frac{\omega}{s^2+\omega^2}$

\# Singularity function

$\mathcal{L}[u(t)] = \frac{1}{s}$

$\mathcal{L}[r(t)] = \frac{1}{s^2}$

$\mathcal{L}[\delta(t)] = 1$

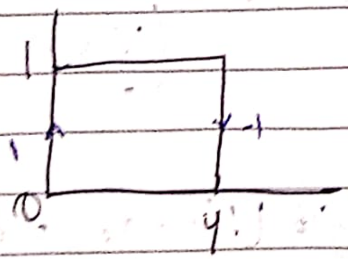
Shifting theorem.

$\mathcal{L}[u(t-a)] = e^{-as}\frac{1}{s}$

$\mathcal{L}[r(t-a)] = e^{-as}\frac{1}{s^2}$

$\mathcal{L}[\delta(t-a)] = e^{-as} \cdot 1$

Ques

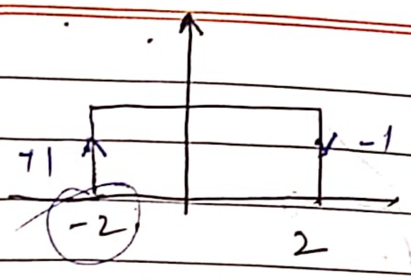


$1 \cdot u(t) - 1 \cdot u(t-4)$

$u(t) - u(t-4)$

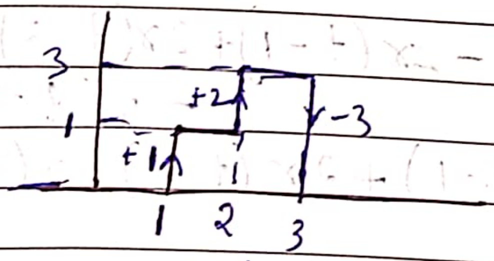
$= \frac{1}{s} - e^{-as}\frac{1}{s}$

Ans:



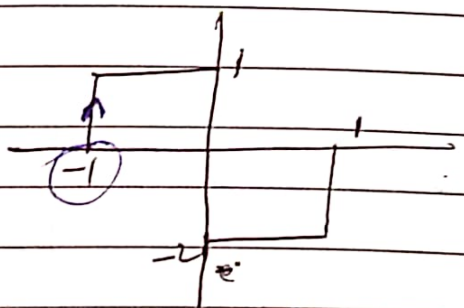
$u(t+2) - u(t-2)$

(3)



$u(t-1) + 2u(t-2) + 3u(t-3)$

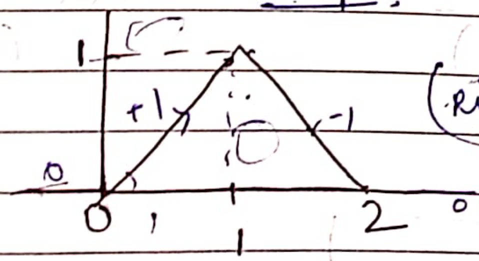
(4)



$u(t+1) - 3u(t) + 2u(t-1)$

Ramp:

(5)



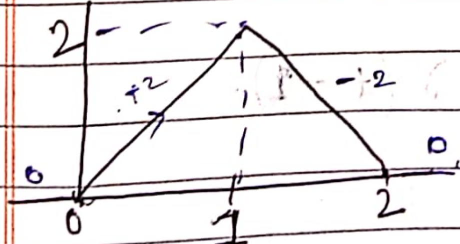
(Rising slope. Falling slope).

(Present − Previous slope)

$$(1-0)\,r(t) + (-1-(+1))\,r(t-1) + (0-(-1))\,r(t-2)$$

$$r(t) - 2r(t-1) + 1\,r(t-2)$$

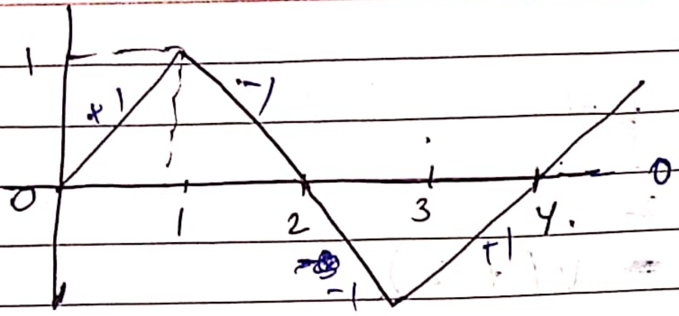
(6)



$$(2-0)\,r(t-0) + (-2-(+2))\,r(t-2) + (0-(-2))\,r(t-2)$$

$$= 2r(t) - 4r(t-2)\; r(t-2) + 2(r(t-2)$$

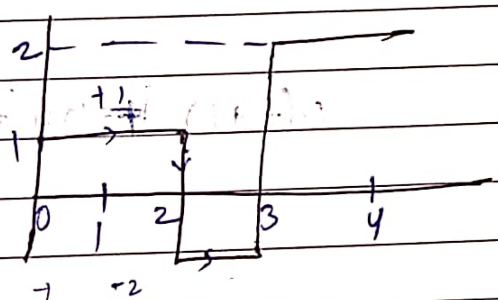
Ques.



$$(1-0)\, r(t) - 2r(t-1) + 2r(t-3) - r(t-4)$$

$$= r(t) - 2r(t-1) + 2r(t-3) - r(t-4)$$

Ques.

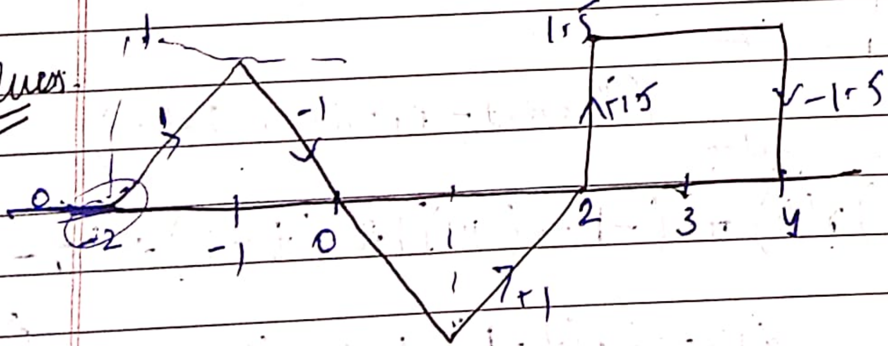


$\cancel{u(t) - 2u(t-1)}$

$\cancel{= u(t) - 2u(t-2)}$

$$= u(t) - 3u(t-2) + 4u(t-3)$$

Ques.

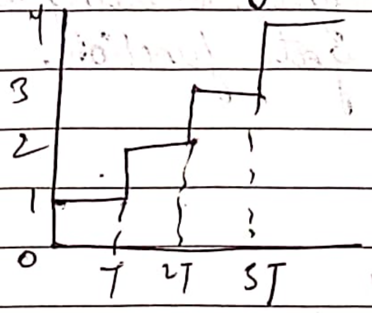


$\cancel{(1-0)\, r(t)}$

$$(1-0)\, r(t+2) - r(t-2)$$

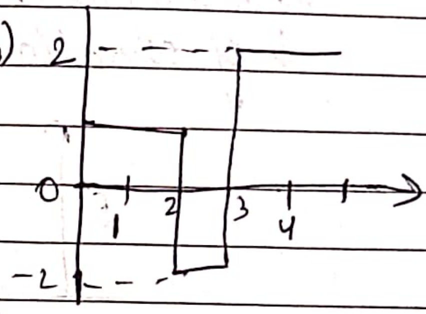
Ques. Find L.T of

a)

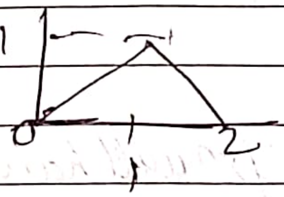


$\frac{1}{s(1-e^{-st})}$

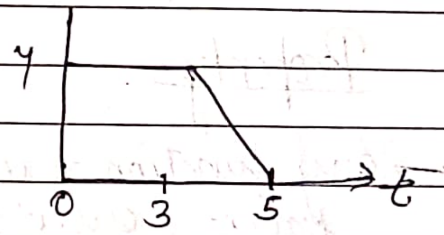
(b)



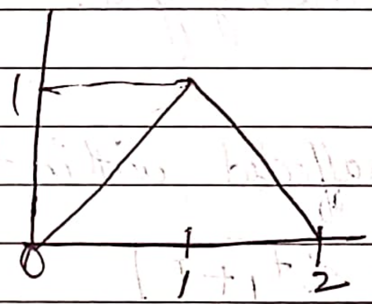
(c)



(d)



(e)



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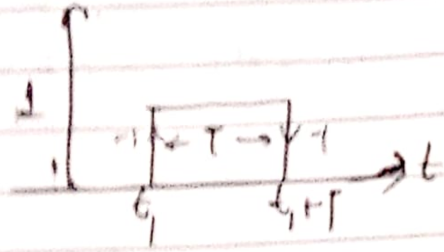
Gate function

A Rectangular Pulse of unit height starting at $t = t_1$ & of duration T is called gate function.



Property

→ Any function multiplied by gate $f x^n$ will have zero value, outside the duration of gate.

$$\left[(t+T) < t < t_1\right]$$

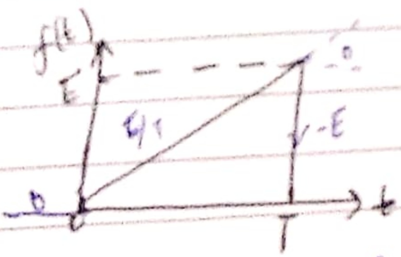
& value of the function will be unaffected within the duration of gate i.e. $\left[t_1 < t < t_1 + T\right]$

$\star$ $G_{t_1}(T) = u(t-t_1) - u(t-t_1-T)$

→ $G_0(T) = u(t) - u(t-T)$

$$\mathcal{L}[G_0(T)] = \frac{1}{s} - \frac{1}{s}e^{-sT}.$$

Ques.



Find L.T. of saw-tooth wave.

Method I - $f(t) = \left(\frac{E}{T} - 0\right) r(t) + \left(0 - \frac{E}{T}\right) r(t-T) - E\,u(t-$

$$= \frac{E}{T} r(t) - \frac{E}{T} r(t-T) - E U(t-T).$$

$$= \frac{E}{T}\cdot\frac{1}{s^2} - \frac{E}{T}\frac{1}{s^2} e^{-Ts} - \frac{E}{s} e^{-Ts}.$$

Method II - Using gate function -

$$f(t) = \frac{E}{T} t \times G_0(T) = \frac{E}{T} t\left[U(t) - U(t-T)\right].$$

$$= L\left(\left(\frac{E}{T} t\right) U(t)\right) - L\left[\left(\frac{E}{T} t\right) U(t-T)\right].$$

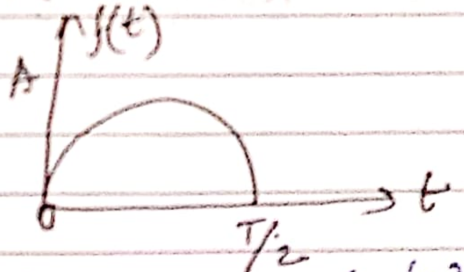
$$= \frac{E}{T}\frac{1}{s^2} - L\left[\frac{E}{T}(t-T+T)\right] U(T-T)$$

$$= \frac{E}{Ts^2} - L\left[\frac{E}{T}\left((t-T)+T\right)\right] U(t-T)$$

$$= \frac{E}{Ts^2} - L\frac{E}{T}(t-T)U(t-T) - L\frac{E}{T}\cdot T U(t-T).$$

$$= \frac{E}{Ts^2} - \frac{E}{Ts^2} e^{-Ts} - \frac{E}{s} e^{-Ts}.$$

Que



$\sin\omega t = \frac{\omega}{s^2+\omega^2}$

$U\frac{T}{2}$

$$f(t) = A\sin\omega t \cdot G_0\left(\frac{T}{2}\right).$$

$$F(t) = A\sin\left(\frac{2\pi}{T}\right) t\left[U(t) - U\left(t - \frac{T}{2}\right)\right]$$

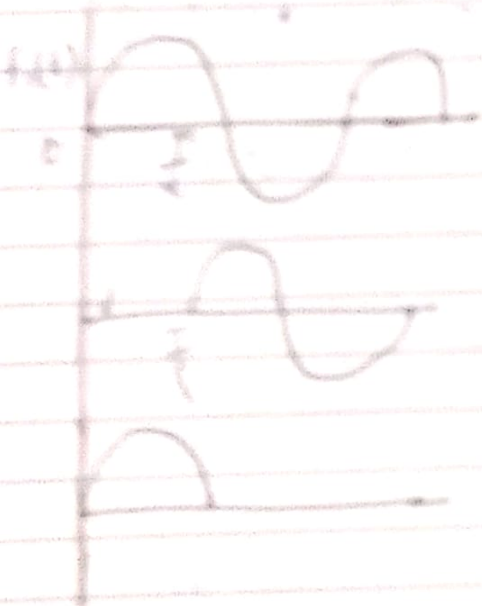
$$= A\, L\left[\sin\left(\frac{2\pi}{T}\right) t\cdot U(t)\right] - A\, L\left[\sin\left(\frac{2\pi}{T}\right) t\cdot U\left(\frac{t-T}{2}\right)\right]$$

$$= A \cdot \frac{\left(\frac{2\pi}{T}\right)}{s^2 + \left(\frac{2\pi}{T}\right)^2} - A \, \mathcal{L}\, \sin \frac{2\pi}{T}\left[t - \frac{T}{2} + \frac{T}{2}\right] \cdot U\left(t - \frac{T}{2}\right)$$

$$= -A \, \mathcal{L}\left[\sin \frac{2\pi}{T}\left(t - \frac{T}{2}\right) + \pi\right] U\left(t - \frac{T}{2}\right)$$

$$+ A \, \mathcal{L}\left[\sin \frac{2\pi}{T}\left(t - \frac{T}{2}\right) \cdot U\left(t - \frac{T}{2}\right)\right]$$

$$\frac{A\left(\frac{2\pi}{T}\right)}{s^2 + \left(\frac{2\pi}{T}\right)^2} + A \cdot \frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2} e^{-Ts/2}$$



$f(t) = f_1(t) + f_2(t)$

$f_1(t) = A \sin \omega t = A \sin \frac{2\pi}{T} t \cdot u(t)$

$f_2(t) = A \sin \frac{2\pi}{T}\left(t - \frac{T}{2}\right) u\left(t - \frac{T}{2}\right)$

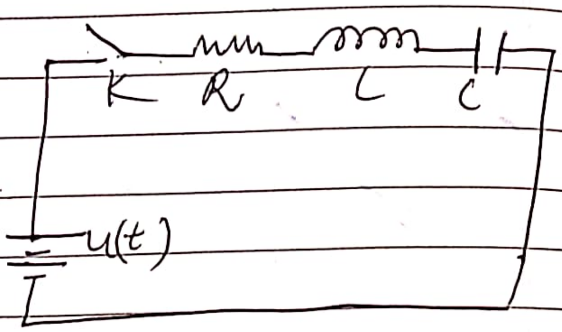
L.T. of Periodic function :-

The Laplace transform of a periodic function with period 'T'

Ques A series RLC circuit

$R = 3\,\Omega,\ L = 1H,\ C = 0.5F.$

Obtain expression for current.



$$u(t) = L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt + R\,i(t).$$

$$\frac{1}{s} = L\left[sI(s) - i(0^-)\right] + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{i(0^-)}{s}\right] + RI(s).$$

$$\frac{1}{s} = LsI(s) + \frac{1}{C}\frac{I(s)}{s} + RI(s).$$

$$\Rightarrow I(s)\left(Ls + \frac{1}{Cs} + R\right) = \frac{1}{s}$$

$$\Rightarrow I(s) = \frac{1/s}{\left(Ls + \frac{1}{Cs} + R\right)}$$

$$= \frac{\frac{1}{s}}{\left(\frac{1s}{1} + \frac{1}{0.5s} + \frac{3}{1}\right)}$$

I = R
U = I R

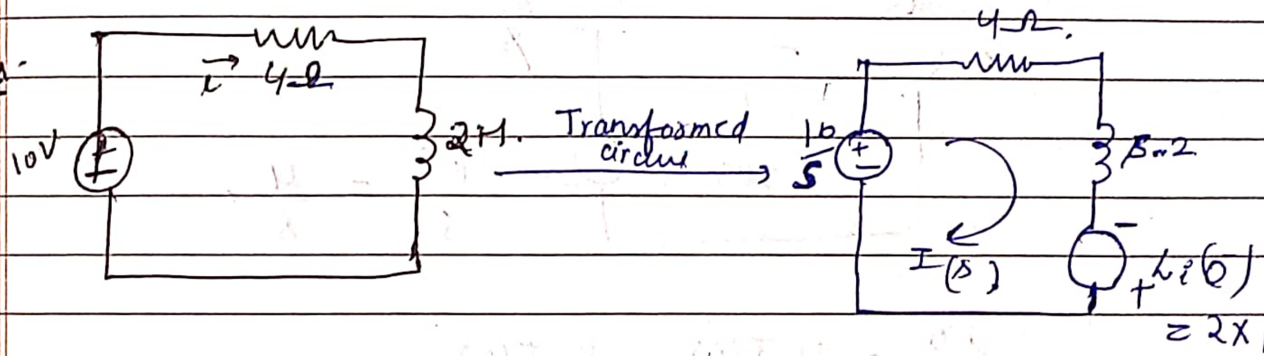
$$= \frac{\frac{1}{s}}{\left(\frac{0.5s^2 + 1 + 1.5s}{0.5s}\right)}$$

$$= \frac{0.5s}{(0.5s^2 + 1 + 1.5s)s}$$

$$= \frac{0.5s}{\cancel{(0.5s^2 + 1 + 1.5s)}}$$

$$I(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

Ckt.



$$\frac{10}{s} - 4I(s) - 2s\,I(s) + 2 = 0$$

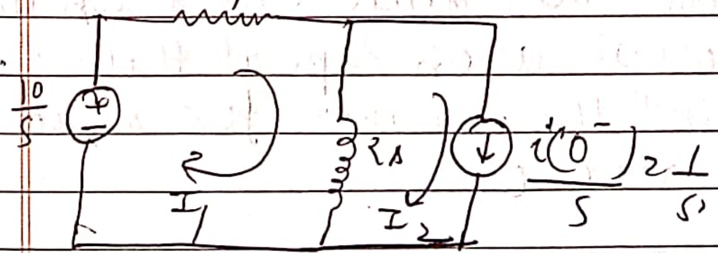
$$I(s) = \frac{5+s}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$= \frac{5}{2s} - \frac{3}{2(s+2)}$$

$$I(s) = \frac{5}{2} - \frac{3}{2}e^{-2t}$$

Method-2

Ckt.

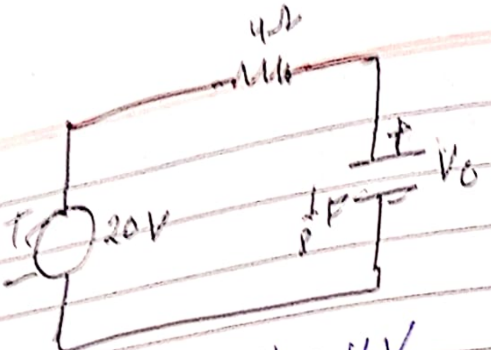


$$I_2(s) = \frac{1}{s}$$

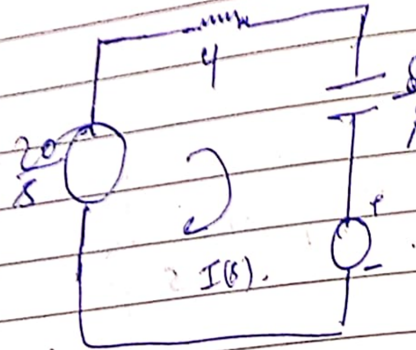
$$\frac{10}{s} = 4I_1(s) - 2s\left(I_1(s) - I_2(s)\right) = 0$$

$$I_1(s) = \frac{s+5}{s(s+2)}$$

Ques



$V_c(0^-) = 4V,$

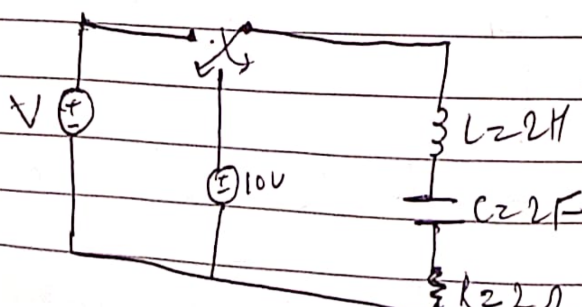


$\frac{V(0^-)}{s} = \frac{4}{s},$

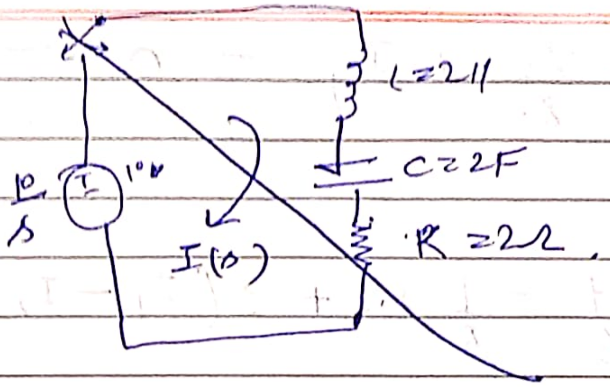
$\frac{20}{s} - 4\,I(s) - \frac{8}{s}\,I(s).$

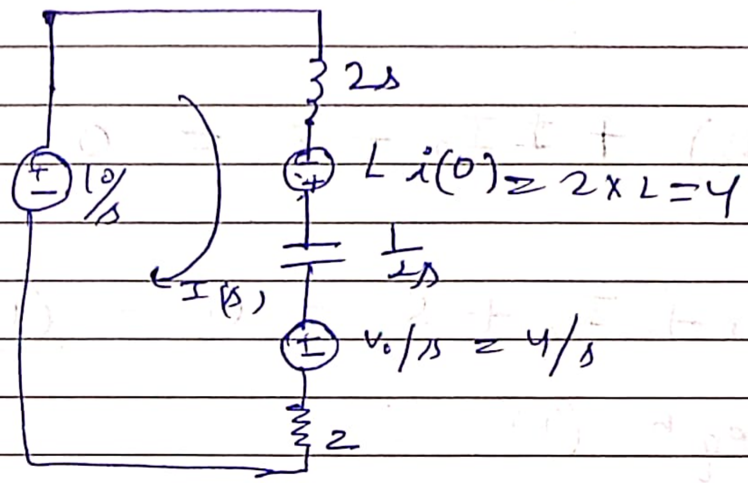
Ques

In series, RLC ckt, the switch is moved from position 1 to 2 at $t=0$ initially it remain at position for along time; the initial current in inductor is 2A & voltage across cap. at that instant is 4V. Find the expression for inductor current at $t=0$,



Ques

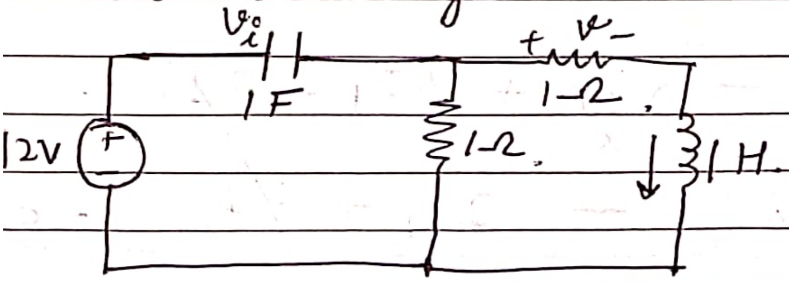


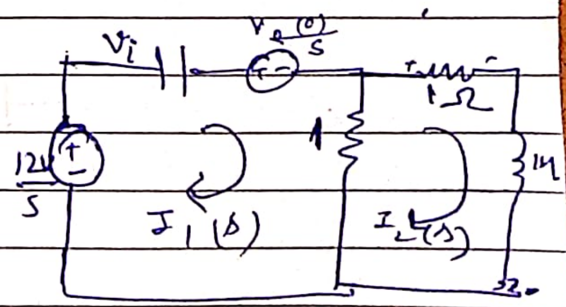


$$\Rightarrow \frac{10}{s} - 2s\,I(s) + 4 - \frac{1}{2s}\,I(s) - \frac{4}{s} - 2\,I(s) = 0$$

$$I(s) = \frac{12+8s}{4s^2+4s+1} = \frac{2(s+3)}{\left(s+\frac{1}{2}\right)^2} = \frac{2\left[\left(s+\frac{1}{2}\right)+1\right]}{\left(s+\frac{1}{2}\right)^2}$$

Determine voltage 'v'





$v_i(0) = 4V$

$i(0) = 2\,A.$

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$\Rightarrow \cancel{\frac{12}{8} - \frac{12}{8} + \frac{V_2}{8} + I_1 = 0} \quad \text{— (I)}$

$$-\frac{12}{8} + \frac{4}{8} - \frac{1}{8} I_1 + 1(I_1 - I_2) = 0 \quad \text{— (I)}$$

In mesh 2,

$$1(I_2 - I_1) + 1 I_2 + 8 = 0 \quad \text{— (II)}$$

$$\Rightarrow I_2 - I_1 + I_2 + 8 = 0 \quad \text{— (II)}$$

Taking eq^n (I)

$$-\frac{12}{8} + \frac{4}{8} - \left(\frac{1}{8} I_1\right) + I_1 - I_2 = 0$$

$$\Rightarrow \frac{-12 + 4}{8} - \left(\frac{1}{8} I_1\right) + I_1 - I_2 = 0$$

$$\times 2 \quad \frac{-8}{8} - \frac{I_1}{8} + I_1 - I_2 = 0 \quad \text{— (I)}$$

$$8 \quad - I_1 + 2 I_2 = 0$$

$$\frac{-16}{8} - \frac{2I_1}{8} + 2I_1 - 2I_2 = 0$$

$$8 \quad - I_1 + 2 I_2 = 0$$

$$\frac{-16}{8} - \frac{2I_1}{8} - 8 - I_1 = 0$$

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$$-\frac{16}{s} - \frac{2I_1}{s} - s - I_1 = 0$$

$$-16 - 2I_1 - \cancel{s} - I_1 = 0$$

$$\cancel{-16 - 2I_1 - s}$$

$$-16 - s - 3I_1 = 0$$

$$-3I_1 = 16 + s$$

$$I_1 = -\frac{16+s}{3}$$

$$\frac{12}{s} - \frac{1}{s} I_1(s) - \frac{4}{s} - 1\left(I_1(s) - I_2(s)\right) = 0$$

$$\left(\frac{1}{s} + 1\right) I_1(s) - I_2(s) = \frac{8}{s} \quad — (1)$$

$$-1\left(I_2(s) - I_1(s)\right) - 1\, I_2(s) - sI_2(s) + 2 = 0$$

$$I_1(s) - I_2(s)(2+s) = -2 \quad — (2)$$

$$\Delta = \begin{vmatrix} \frac{1}{s}+1 & -1 \\ 1 & -(2+s) \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} \frac{1}{s}+1 & \frac{8}{s} \\ 1 & -2 \end{vmatrix}$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{2s+10}{s^2+2s+2}$$

$$v = 1 \times I_2(s) = \frac{2s+10}{s^2+2s+2} = 2\left[\frac{s+1}{(s+1)^2+1}\right] + 8\left[\frac{1}{s^2+2s+2}\right]$$

$$v(t) = 2e^{-t}\cos t + 8e^{-t}\sin t$$

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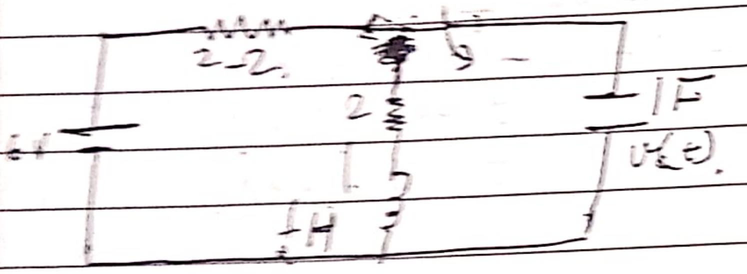
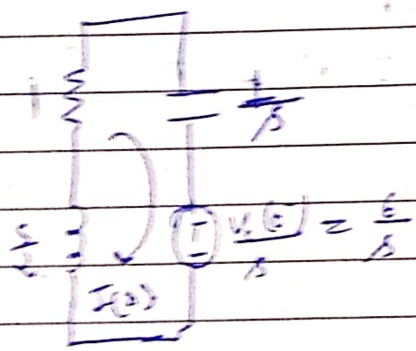
Ques: In the circuit shown switch S is in position 1 for a long time & moved to position 2 at $t=0$. find the voltage across the capacitor for $t \geq 0$.





$$-\frac{1}{2}s\,I(s) - 1\,I(s) - \frac{6}{s} - \frac{1}{s}I(s) = 0$$

$$\cancel{-\frac{1}{2}s\,I(s)} - \qquad - \cancel{\frac{6}{s}}$$

$$-I(s) - \left(\frac{1}{2}s + \frac{6}{s} + \frac{1}{s}\right) I(s) = 0$$

$$\cancel{-I(s)\;\frac{s^3+}{2s^2}}$$

$$-I(s) - \left(\frac{s^2+12+2}{2s}\right) I(s) = 0$$

$$-I(s) - \frac{s^2-12-2}{2s} I(s) = 0$$

$$-I(s) - \frac{s^2-12-2\,I(s)}{2s} = 0$$

$$-I(s) - s^2 I(s) - 14\,I(s) = 0$$

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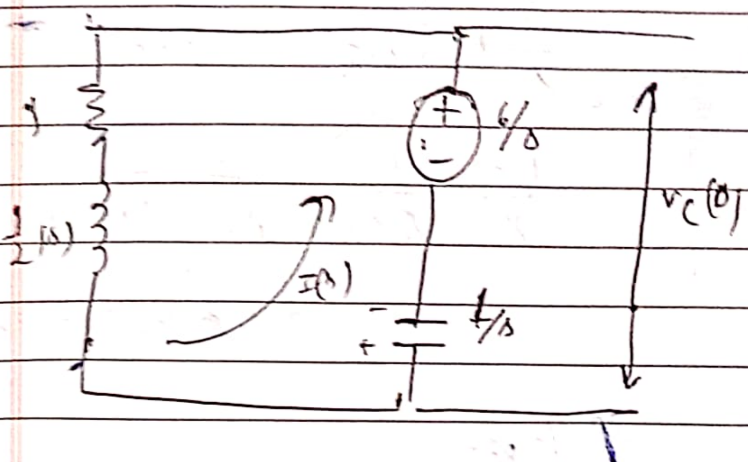
$$I(s) = s^2 - 14$$

$$\boxed{I(s) = \frac{-12}{s^2+2s+2}}$$



$$-\frac{1}{s} I(s) + \frac{6}{s} - V_c(s) = 0.$$

or

$$-\frac{1}{s} I(s) + \frac{6}{s} = V_c(s).$$

$$V_c(s) = \frac{6(s^2+2s+12) - 12s}{s(s^2+2s+12)}$$

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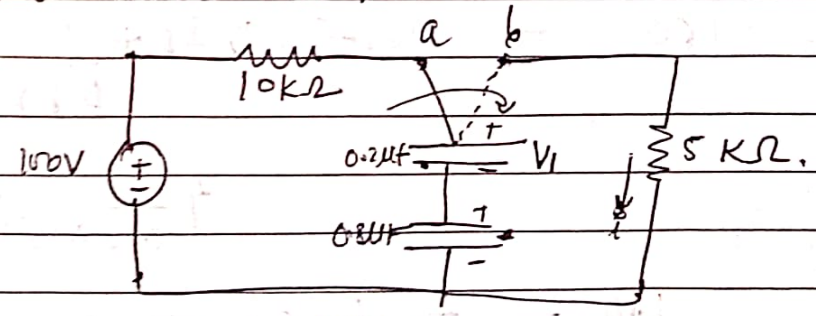
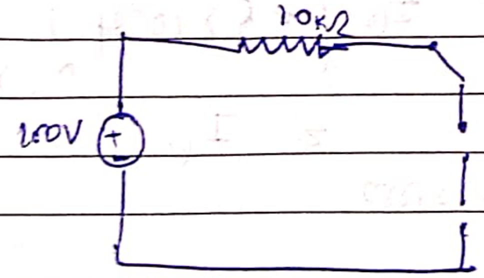
Que. In circuit shown, has been in position A for a long time at $t=0$, the switch is thrown to position B. Find current I.





Initial vol across both capacitor is 100 V.

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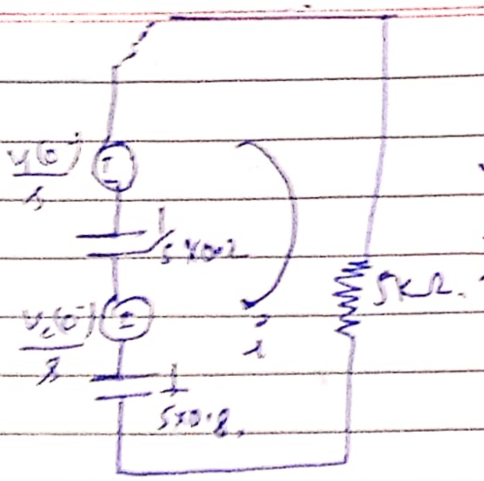


$$\frac{100}{s} + \frac{1}{s \times 0.8} i + \frac{1}{s \times 0.2} i + 5i = 0$$

$$\frac{100}{s} + \frac{1}{s \times 0.8 \times 10^{-6}} i + \frac{1}{s \times 0.2 \times 10^{-6}} i$$

$$+ 5000\, i = 0$$

$$\frac{V_1(0^-)}{s} - 5 \times 10^3 I(s) - \frac{1}{s \times 0.8 \times 10^{-6}} I(s) + \frac{V_2(0^-)}{s} - \frac{1}{s \times 0.2 \times 10^{-6}} I(s)$$

$$\frac{1}{s}\left[V_1(0^-) + V_2(0^-) - 5 \times 10^3 I(s) - \frac{1}{0.8 \times 10^{-6}} I(s) - \frac{1}{0.2 \times 10^{-6}} I(s) \right] = 0$$

$$\frac{1}{s}\left[100 - 5 \times 10^3 I(s) - \frac{1}{0.8 \times 10^{-6}} I(s) - \frac{1}{0.2 \times 10^{-6}} I(s) \right] = 0$$

$$100 = -5 \times 10^3 I_s + \frac{1}{0.8 \times 10^{-6}} I(s) + \frac{1}{0.2 \times 10^{-6}} I(s).$$

$$100 = I(s)\left(-5 \times 10^3 s + \frac{1}{0.8 \times 10^{-6}} + \frac{1}{0.2 \times 10^{-6}} \right)$$

$$\frac{100}{(-5 \times 10^3)s + 6250000.} = I(s)$$

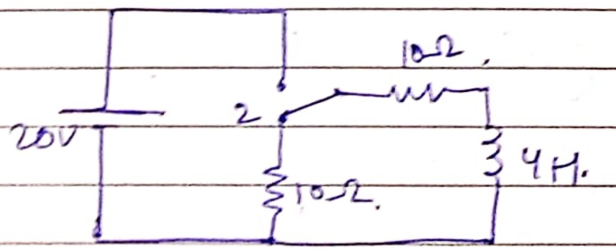
$I(s) = \frac{-0.02}{s+1.25}$

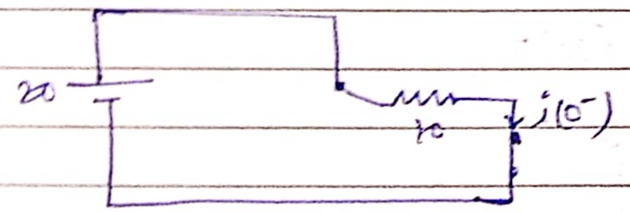
~~0.022~~

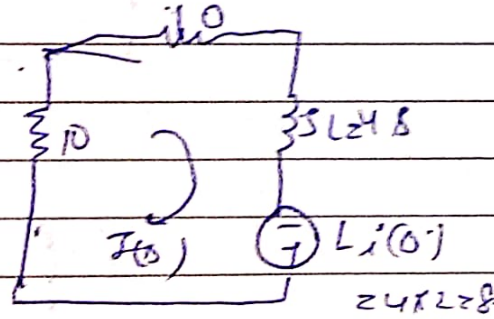
$I(s) =$

Que The ckt shown in fig. is initially under steady state condition the switch is moved from position ① to position ② at $t=0$ find the current after switching.

$\Rightarrow$

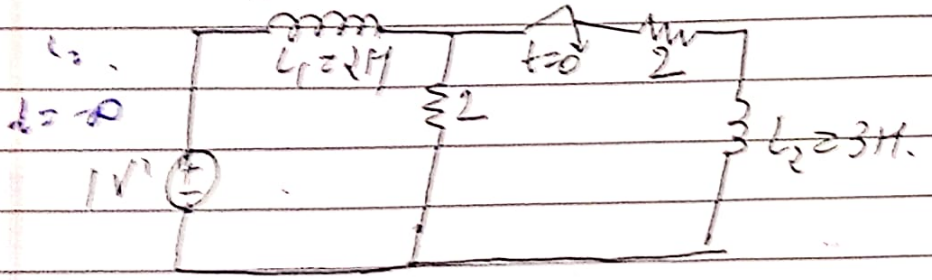


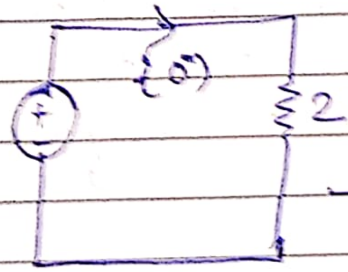




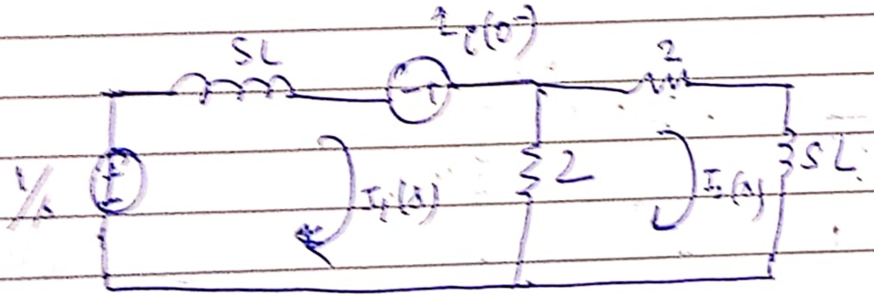
2)

Que Determine the current in inductor L_2 in shown in fig after the switch is closed at $t=0$. Assume the voltage is applied at $t=-\infty$.





$\boxed{i(0^-) = 1/2}$



$I_2(s) = ?$

$\Rightarrow$ $\frac{1}{s}$

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Ques- Define initial value & final value Theorem.

Ans- Initial value theorem states that if the laplace transform of f(t) is f(s) & the first derivative of f(t) is laplace transformable then the initial value of f(t) is

$$f(0^+) = \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} [s\,F(s)].$$

if time limit exists.

Final Value theorem states that if the L.T. of f(t) is f(s) & if the s times the f(s) is analytic on the imaginary axis & in right half of s-plane then,

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} [s\,F(s)].$$

Ques- Verify initial value theorem.

(a) $f(t) = 5e^{-4t}$

$$\boxed{\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\,F(s)}$$

~~f(t) = 5 e L~~

$F(s) = \frac{5}{s+4}$

$s\,F(s) = \frac{5s}{s+4}$

$5e^{-4 \cdot 0} = \lim_{s \to \infty} \frac{5}{1+\frac{4}{s}}$.

$5 = 5$

H.P.

$f(t)e^{-at}$ e^{-t}

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(b) $f(t) = 2 - e^{5t}$

$$f(s) = \frac{2}{s} - \frac{1}{s+5}$$

$$f(s) = \frac{2(s+5) - s}{(s)(s+5)}$$

$$= \frac{2s+10-s}{(s)(s+5)} = \frac{s+10}{s(s+5)}$$

$$s\,f(s) = \frac{\cancel{s}(s-10)}{\cancel{s}(s-5)} = \frac{(s-10)}{(s-5)}$$

$$\lim_{s\to\infty} \frac{\left(1 - \frac{10}{s}\right)}{(s-5)} = 0$$

Qu. Verify final value theorem.

$f(t) = 2 + e^{-3t}\cos 2t$

$$f(s) = \frac{2}{s} + \frac{s+3}{(s+3)^2+4}$$

$$s\,f(s) = 2 + \frac{s^2+3s}{(s+3)^2+4}$$

$e^{\infty} = 0$

$$\lim_{t\to\infty}\left[2+e^{-3t}\cos 2t\right] = 2$$

$$\lim_{t\to 0}\left[\frac{2}{1} + \frac{s(s+3)^2}{(s+3)^2+4}\right] = \frac{2(s+3)^2+4+s+3+4}{(s+3)^2+4}$$

$= 2.$

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Ques: Verify final value theorem

$f(t) = 6(1-e^{-t})$

$f(t) = 6 - 6e^{-t}$

$\cancel{= \frac{6}{s} - \frac{6}{s+6}}$

$= \frac{6}{s} - \frac{6}{s+1}$

$= \frac{6}{s} - \frac{6}{s+1}$

$\lim_{t\to\infty} f(t) = 6(1-e^{-\infty})$

$= 6$

$\lim_{s\to 0} f(s) = \left(\frac{6\cancel{s}}{\cancel{s}} - \frac{6s}{s+1}\right) = \left(6 - \frac{6s}{s+1}\right)$

$= \left(\frac{6(s+1)}{s+1}\right) = 6$

$= 6$

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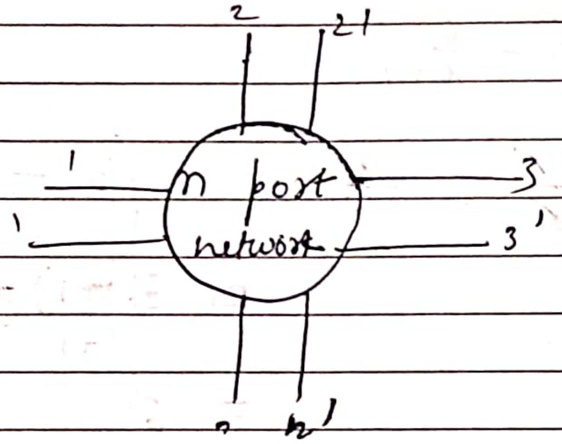
Unit-5

Terminal pair or port.



Network functions-

Transform Impedance

The tranform impedance at a port has been defined as the ratio of voltage transform to current transform for a network in zero state (no initial condition).

$$Z(s) = \frac{V(s)}{I(s)}$$

Transform Admittance,

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

The voltage transform & current transform that define transfer impedance & admittance must relate to the same port

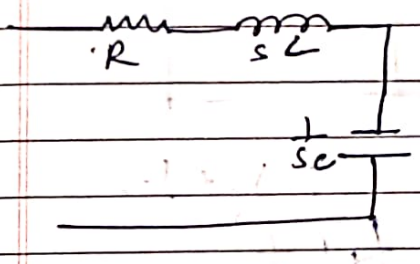
$R + sL + \frac{1}{sC}$

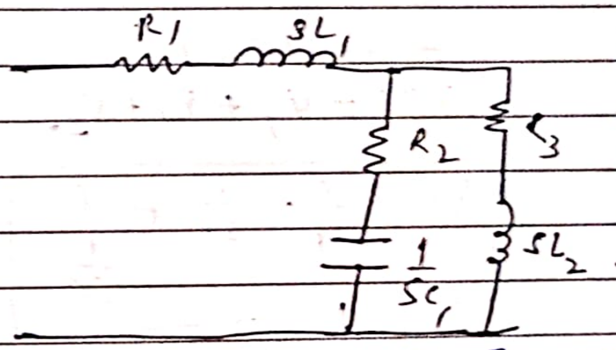
for e.g. $11'$, $22'$

The impedance or admittance found at a given port is called driving point impedance or admittance,

→ Immitance is a common name which can be used for both impedance and admittance.

Ques. Obtain the driving point impedance or transform impedance for the network shown.

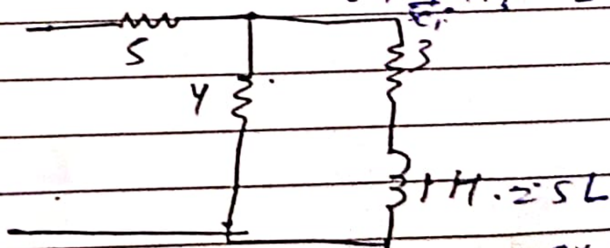




$$Z(s) = R + sL + \frac{1}{sC}$$

$$Z(s) = (R + sL_1) + \left[R_2 + \frac{1}{sC_1}\right] \,\|\, \left(R_3 + sL_2\right)$$

$$= (R + sL_1) + \frac{\left(R_2 + \frac{1}{sC_1}\right) \cdot \left(R_3 + sL_2\right)}{R_2 + \frac{1}{sC_1} + R_3 + sL_2}$$



$= sL$

$= s \times 1 = s$

$$\frac{1}{4} + \frac{1}{3} + \frac{1}{1}$$

$$Z(s) = 5 + (4 \,\|\, 3 + s)$$

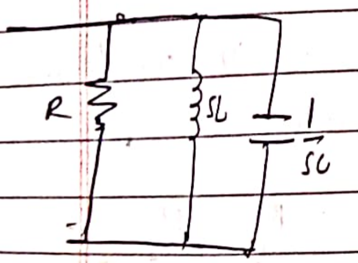
$$\left(5 + \frac{4 \times (3+s)}{4+3+s}\right)$$

$\frac{1}{Y(s)}$

$Z(s) = \frac{s+47/9}{s+7}$

Ques. Obtain $Y(s)$.

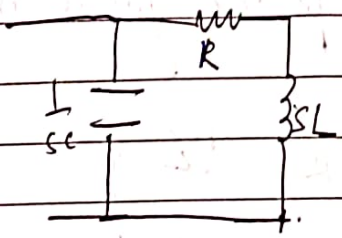
In parallel
$Y = Y_1 + Y_2$



$Y(s) = Y_1 + Y_2 + Y_3.$

$$\boxed{Y(s) = \frac{1}{R_L} + \frac{1}{S_C} + \frac{1}{\frac{1}{SC}}}$$

$$Y(s) = \frac{SL + R + s^2RLC}{SRL}$$



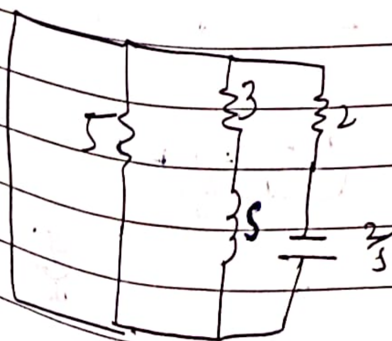
$Y(s) = Y_1 + Y_2 + Y_3.$

$= \frac{1}{SC} + \frac{1}{(R+SL)}$

$= \frac{\cancel{RSC + SC}}{\cancel{SC(R+SL)}}$

$= \frac{RSC + (s^2LC) + 1}{R + SL}$

Ques.



$= \frac{C}{3}$

$= \frac{1}{5} + \frac{1}{3+S} + \frac{1}{\left(\frac{2}{1}+\frac{2}{S}\right)}$

$= \frac{1}{5} + \frac{1}{3+S} + \frac{1}{\left(\frac{4+S}{S}\right)}$

$\frac{2}{1}+\frac{2}{s}$

$\frac{2s+2}{s}$.

~~$= \frac{1}{5} + \frac{1}{3+s} + \frac{2}{(4+s)}$,~~

~~$= \frac{s}{5} \cdot \frac{}{5(3+s)(4+s)}$.~~

~~$= \frac{1}{5} + \frac{1}{3+s} + \frac{1}{\left(2+\frac{2}{s}\right)}$.~~

~~$= \frac{}{(5)(3+s)\left(\frac{2}{s}\right)}$.~~

$= \frac{1.6+3.3s+0.7s^2}{3+4s+s^2}$.

07/12/22

Network function

The Ratio of Output to Input Quantity ⇒ It may be either Voltage or Current or Impedance or Admittance.

Voltage transfer Ratio-

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}.$$

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)}.$$

Current transfer

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}.$$

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}.$$

3. <u>Transfer Impedance</u> -

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

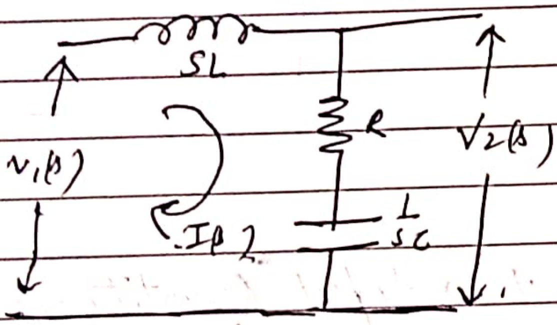
4. <u>Transfer Admittance</u> -

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

Que. Find voltage transfer function.

→

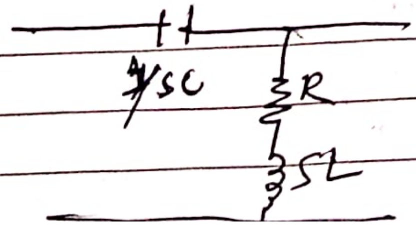


$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\cancel{I(s)}\left(R + \frac{1}{SC}\right)}{\cancel{I(s)}\left(R + SL + \frac{1}{SC}\right)}$$

$$= \frac{\frac{SRC + 1}{\cancel{SC}}}{\frac{RSC + S^2LC + 1}{\cancel{SC}}}$$

$$= \frac{SRC + 1}{RSC + S^2LC}$$

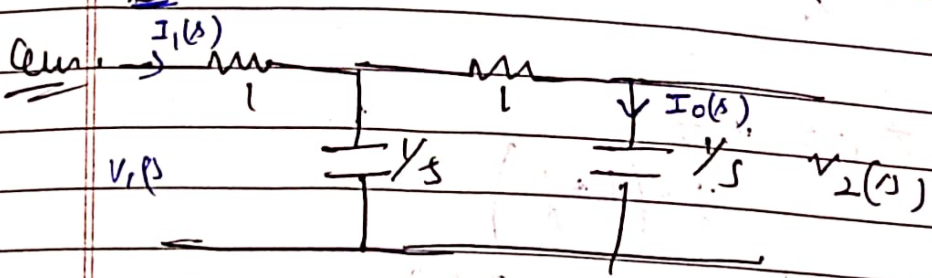
Que.



$$= \frac{\frac{1}{SC} \parallel R}{\frac{1}{SC} + R + SL}$$

$$= \frac{1 + RSC + S^2LC}{SC}$$

Ans.



$$G_{21} = \frac{V_2(s)}{V_1(s)} =$$

$$Z(s) = \left(\frac{1}{1} + \frac{1}{s}\right) // \left(\frac{1}{1} + \frac{1}{s}\right) \Rightarrow \cancel{\left(\frac{s+1}{s}\right) // \left(\frac{s+1}{s}\right)}$$

$$\cancel{V_1(s) = I_1(s) \cdot Z(s)} \quad 1 + \frac{1}{s} // \left(\frac{1}{1} + \frac{1}{s}\right)$$

$$= \left(\frac{1}{1} + \frac{1}{s}\right) // \left(\frac{s+1}{s}\right)$$

$$= \frac{\frac{1}{1} + \frac{1}{s} \times \frac{s+1}{s}}{\left(\frac{1}{1} + \frac{1}{s}\right) + \left(\frac{s+1}{s}\right)}$$

$$= \frac{\frac{s+1}{s} * \frac{s+1}{s}}{\frac{s+1}{s} + \frac{s+1}{s}} = \frac{\frac{(s+1)^2}{\cancel{s^2}}}{\frac{s(s+1) + s(s+1)}{\cancel{s^2}}}$$

$$= \frac{(s+1)^{\cancel{2}}}{\cancel{(s+1)}(s+s)}$$

$$= \frac{s+1}{2s}$$

$$= \frac{s^2+3s+1}{s(s+2)}$$

$$V(s) = I(s) \cdot Z(s)$$

$$\boxed{V_1(s) = I(s) \cdot \frac{s+1}{2s}, \quad \frac{s^2+3s+1}{s(s+2)}}$$

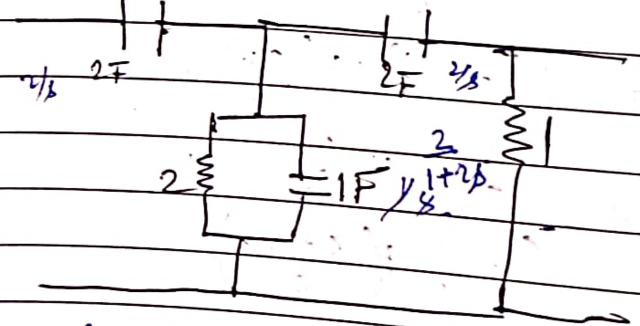
$V_2(s) = I_0(s) \cdot \times \frac{1}{s} = \frac{I_0 s}{s+2} \times \frac{1}{s}$

$I_0(s) = \frac{I_1(s) \times \frac{1}{s}}{\frac{1}{s} + \left(1 + \frac{1}{s}\right)}$

$= \frac{I_1 s^{-1}}{\left(\frac{1}{s} + \frac{s+1}{s}\right)} = \frac{I_1(s)}{\frac{1+s+1}{s}} = \frac{I_1 s^{1}}{\frac{s+2}{s}}$

$g_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{s^2+3s+1}$

Que:



$G_{21}(s) = ?$

$\frac{2+\frac{1}{1}}{2+1} = \frac{3}{1}$

$\frac{1}{2}$

$Z(s) = \frac{2}{s} \parallel \left(\frac{2}{1} + \frac{1}{s}\right) \parallel \left(\frac{2}{s} + \frac{1}{1}\right)$

$= \frac{2}{s} \parallel \left(\frac{2s+1}{s}\right) \parallel \left(\frac{2+s}{s}\right)$

$= \frac{\frac{2}{s} \times \frac{2s+1}{s} \times \frac{(2+s)}{s}}{s+s+s}$

$= \frac{\frac{2(2s+1)(2+s)}{3s^3}}{3s}$

$$z(s) = \frac{12s^2+12s+1}{2s(4s^2+8s+1)}.$$

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$$= \frac{(4s+2)(2+s)}{3s(s^3)} = \frac{8s+4s^2+4+2s}{3s \cdot s^3}$$

$$= \frac{10s+4s^2+4}{3s \cdot s^3}$$

$$z(s) = \frac{\cancel{2}}{2s} + \frac{2}{1+2s} \parallel \left(\frac{\cancel{2}}{2s} + \frac{1}{1}\right).$$

$$= \frac{\cancel{2}}{2s} + \frac{2}{1+2s} \parallel \left(\frac{\cancel{2}+2s}{2s}\right).$$

$$= \frac{\cancel{2}}{2s} + \left(\frac{\frac{2}{1+2s} \times \frac{\cancel{2}+2s}{2s}}{\frac{2}{1+2s} + \frac{\cancel{2}+2s}{2s}}\right)$$

$$\cancel{= \frac{\cancel{2}}{2s} + \left(\frac{\frac{4+2s}{s+2s^2}}{\frac{2s+(2+s)(1+2s)}{(1+2s)s}}\right).}$$

$$= \frac{\cancel{2}}{2s} + \left(\frac{4+2s}{2s+2+4s+s+2s^2}\right)$$

$$= \frac{\cancel{2}}{2s} + \left(\frac{4+2s}{2s^2+2+7s}\right)$$

$$\cancel{= \frac{\cancel{2}}{2s} + \frac{4+2s}{2s^2+2+7s}}$$

$$= \frac{1}{2s} + \left(\frac{\frac{2}{1+2s} \times \frac{1+2s}{2s}}{\frac{2}{1+2s} + \frac{1+2s}{2s}}\right).$$

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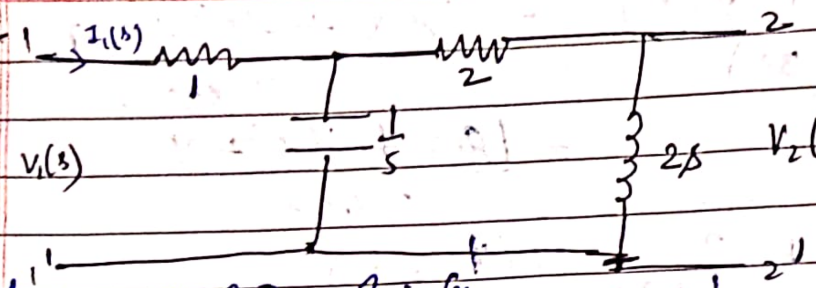
sXL $\frac{1}{sC}$

Q.



find Z_{11}? Z_{21} = ? G_{12}.

$$I_L(s) = \frac{V_2(s)}{2s}$$

Vol. across capacitor,

~~$V_C(s) = V_2(s) + V_2(s) + 2I_2(s)$~~

$$V_C(s) = V_2(s) + V_{2s} = V_2(s) + 2I_L(s) = V_2(s) + 2\frac{V_2(s)}{2s}$$

$$= V_2(s)\left[\frac{s+1}{s}\right]$$

$$I_C(s) = \frac{V_C(s)}{\frac{1}{s}} = V_2(s) \cdot \frac{(s+1)}{s} \cdot s = (s+1)V_2(s)$$

$$I_1(s) = I_C(s) + I_L(s) = (s+1)V_2(s) + \frac{V_2(s)}{2s} = V_2(s)\left(\frac{s+1+1}{2s}\right)$$

$$= \left(\frac{2s^2+2s+1}{2s}\right)V_2(s)$$

$$V_1(s) = V_C(s) + 1 \times I_1(s)$$

$$V_1(s) = V_2(s)\left(\frac{s+1}{s}\right) + 1\left[\frac{2s^2+2s+1}{2s}\right]V_2(s) = V_2(s)\left[\frac{2s^2+4s+}{2s}\right]$$

$$= V_2(s) + \frac{2V_2(s)}{2s} = V_2(s)\left[\frac{s+1}{s}\right]$$

$$Z_{11} = \frac{V_1(s)}{I_1(s)}$$

$$Z_{21}(s) = V_2(s)$$

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

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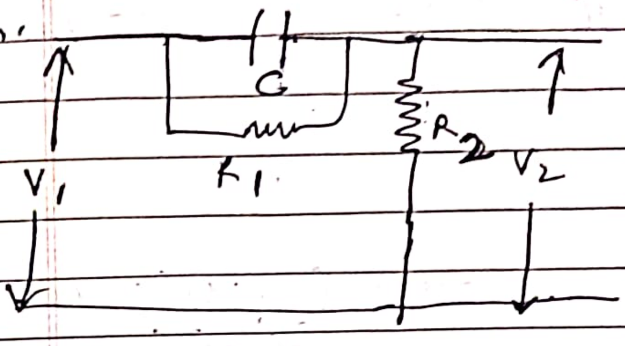
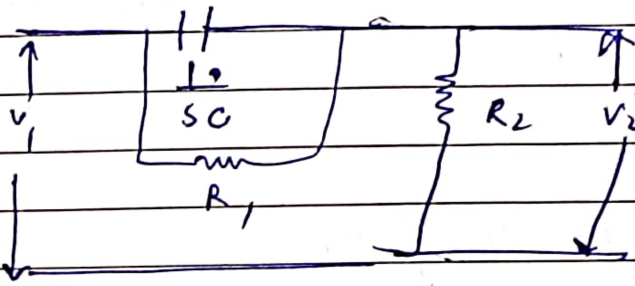
Ques.



find $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

$= \frac{I \cdot R_2}{I \cdot Z}$



$V_2 = V_1(s) R$

$$Z = \left(\frac{1}{SC} + \frac{1}{R_1}\right)$$

$$Z = \left(\frac{R_1 + SC}{SC\ R_1}\right)$$

$$\boxed{G_{21}(s) = \frac{s + \frac{1}{CR_1}}{s + \frac{1}{CR_1} + \frac{1}{CR_2}}}$$

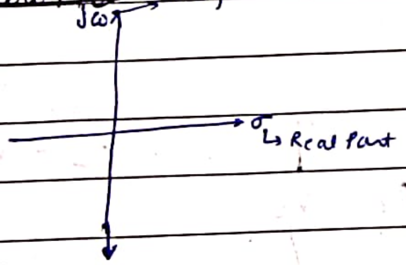
$2s+4=0$
$2s=-4$
$s=-2.$

Poles & Zeroes of Network function:-

$$T(s) = \frac{K(s-z_1)(s-z_2)\cdots\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots\cdots(s-p_n)}$$

$Z \rightarrow$ Zeroes o
$P \rightarrow$ Poles, x

Pole-Zero Plot:- imag.



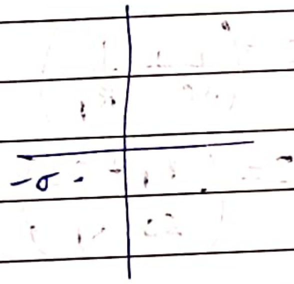
$$T(s) = \frac{(2s+4)(s+4)}{s(s+1)(s+3)}$$

Ques. Obtain pole & zero location fxⁿ.

(i) $f(t) = e^{-\sigma t}.$

$$F(s) = \frac{1}{s+\sigma}$$

$x = 0$
$z = -\sigma.$

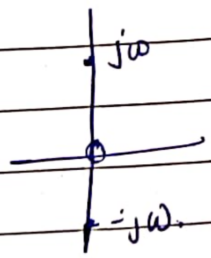


(ii) $f(t) = \cos\omega t$

$$F(s) = \frac{s}{s^2+\omega^2};$$

$s = 0$
$z = \pm j\omega$

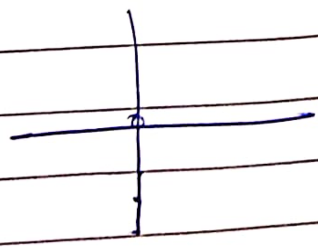
z



$\frac{s}{s^2+\omega^2}$

(iii) $f(t) = e^{-\sigma t}\cos\omega t$

$$F(s) = \frac{s+\sigma}{\omega^2+(s+\sigma)^2}$$



$z = (-\sigma \pm j\omega)$

Necessary Conditions for Transfer Functions (with common factors in N(s) & D(s) cancelled).

The coefficients in the polynomials N(s) & D(s) of T = N/D must be real & those for D(s) must be positive.

Poles & zeroes must be conjugate if imaginary or complex.

The Real Part of poles must be negative or zero. If the real part is zero, then that pole must be simple. This includes the origin.

The polynomial D(s) may not have any missing term between that of highest & lowest degrees, unless all even or all odd terms are missing.

The polynomial N(s) may have terms missing, & some of the coefficients may be negative.

The degree of N(s) may be as small as zero independent of the degree of D(s).

for G & α: The max. degree of N(s) is equal to the degree of D(s).

) for Z and Y: The max. degree of N(s) is equal to the degree of D(s) plus one.

- Check whether given functions are suitable in representing the transfer functions or not.

$G_{21}(s) = \frac{3s+2}{5s^3+4s^2+1}$

(ii) $\alpha_{21}(s) = \frac{2s^2+5s+1}{s+7}$

$Z_{21}(s) = \frac{1}{s^2+2s}$

(iv) $G_{21}(s) = \frac{2s^2+5}{3s^2+9s+1}$

. No; coefficient is missing in polynomial D(s).

). No; the degree of N(s) is greater than D(s)

(iii) Yes, all cond. are satisfied.
(iv) Yes " " "

Necessary conditions for Driving point Immittance Functions (with common factors in $N(s)$ & $D(s)$ cancelled):

1. The coefficients in the polynomials $N(s)$ & $D(s)$ must be real & positive.
2. Poles & Zeroes must be conjugate if imaginary or complex.
3. The Real part of all poles & zeroes must be -ve or zero, if the real part is zero, then that pole or zero must be simple. i.e. all the roots of $N(s)=0$ & $D(s)=0$ lie on the left half of s-plane & simple roots may lie on the imaginary or $j\omega$-axis.
4. The polynomials $N(s)$ & $D(s)$ may not have missing terms between those of highest or lowest degree unless all even or all odd terms are missing.
5. The highest degree of $N(s)$ & $D(s)$ may differ by zero or one only.
6. The lowest degree of $N(s)$ & $D(s)$ may differ by either zero or one only.

Example

(i) $Z(s) = \dfrac{4s^4+s^2-3s+1}{s^3+2s^2+2s-40}$

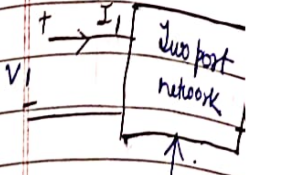
(ii) $Y(s) = \dfrac{5(s^3+2s^2+3s+4)}{s^4+2s^3+6s^2}$

(iii) $Z(s) = \dfrac{s^2+s+2}{2s^2+s+1}$

Sol:- (i) No,
(ii) No,
(iii) Yes.

Ch-2

Two port network

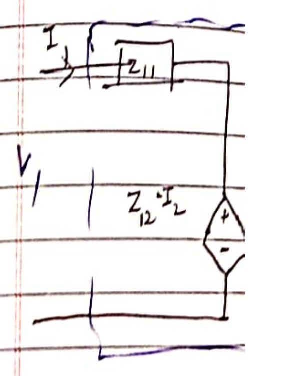


Open circuit para

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z \\ Z \end{bmatrix}$

$V_1 = Z_{11} I_1 +$

$V_2 = Z_{21} I_1$



Case I - $I_2 = 0$

i/p driving point impedance. $Z_{11} = \frac{V_1}{I_1}$

$Z_{21} = \frac{V_2}{I_1}$

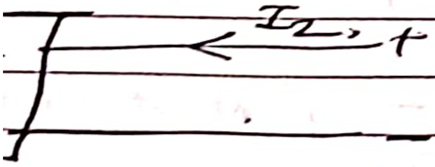
forward transfer impedance

Case II - $I_1 = 0$

$Z_{12} = \frac{V}{I}$

Scanned with CamScanner

k =

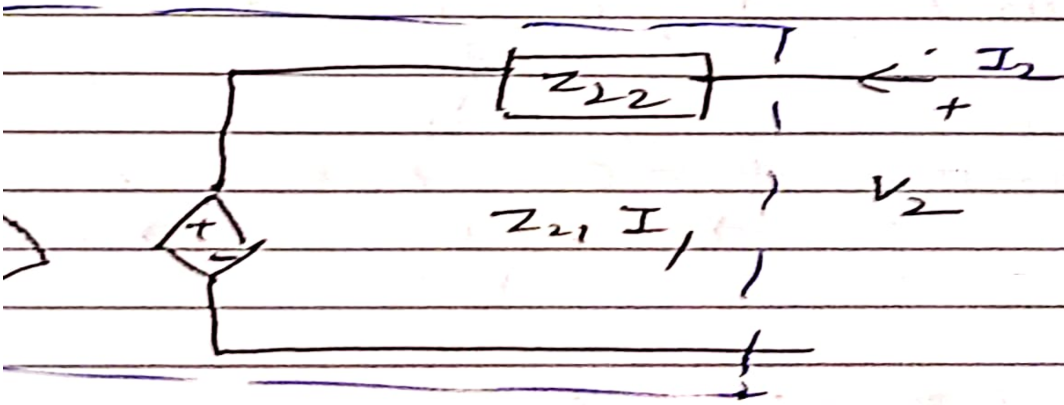


...ameter .. (z parameter).

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$+ z_{12} I_2$ — 1

$_1 + I_{22} I_2$ — 2



0 (output port open circuit).

$\Big|_{I_2 = 0}$

$\Big|_{I_2 = 0}$

0 (i/p port is open ckt).

$\frac{V_1}{I_2}\Big|_{I_1 = 0}$ Reverse transfer impedance.

output driving point impedance.

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = 0}$$

15|12|22

Y Parameter [Y] – (short ckt parameter)

$$I = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2.$$

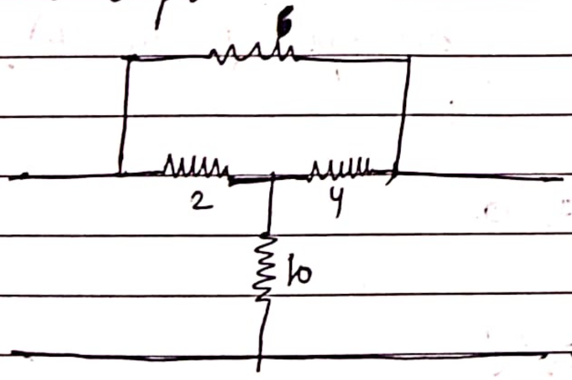
Case I –; $V_2 = 0$ Port 2 is shorted.

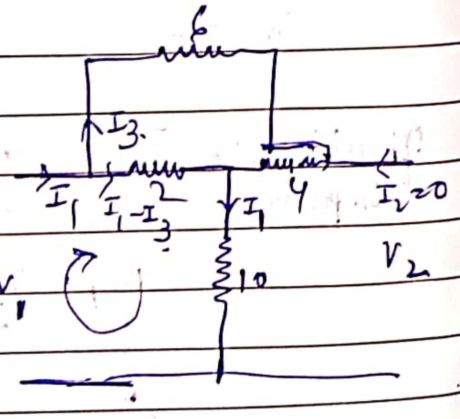
$$Y_{11} = \frac{I_1}{V_1}, \quad Y_{21} = \frac{I_2}{V_1}$$

Case II – $V_1 = 0$, Port 1 is shorted.

$$Y_{11} = \frac{I_1}{V_2} \quad Y_{22} = \frac{I_2}{V_2}$$

Ques Find Z parameter –





$$-V_1 - 2(I_1 - I_3) - 10 I_1 = 0 \; — ①$$

$$-V_1 - 2I_1 + 2I_3 - 10 I_1 = 0$$

$$-V_1 - 12I_1 + 2I_3 = 0$$

$$V_1 = 12I_1 - 2I_3 \; — ①$$

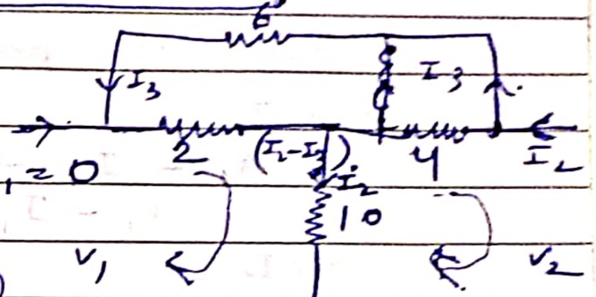
$$V_2 = 4I_3 + 10 I_1 \; — ②$$

$$I_3 = I_1 \times \frac{2}{2+(6+4)} = \frac{I_1}{6}$$

$$V_1 = \frac{35}{3} I_1$$

$$V_2 = \frac{32}{3} I_1$$

$$\boxed{\because Z_{11} = \frac{35}{3} \quad Z_{21} = \frac{32}{3}}$$



II $I_1 = 0$

~~$V_1 - 2[I_1 - I_3] =$~~

$$V_1 - 2I_3 - 10I_2 = 0 \quad \text{--- (I)}$$

$$-V_2 - 4(I_2 - I_3) - 10 I_2 = 0 \quad \text{--- (II)}$$

$$I_3 = I_2 \times \frac{4}{2+(6+4)}$$

$$= I_2 \times \frac{4}{12}$$

$$I_3 = \frac{I_2}{3}$$

$$V_1 = 2I_3 + 10 I_2$$

$$2 \times \frac{I_2}{3} + 10 I_2$$

$$= \frac{2I_2}{3} + \frac{10 I_2}{1}$$

$$\frac{2I_2 + 30 I_2}{3}$$

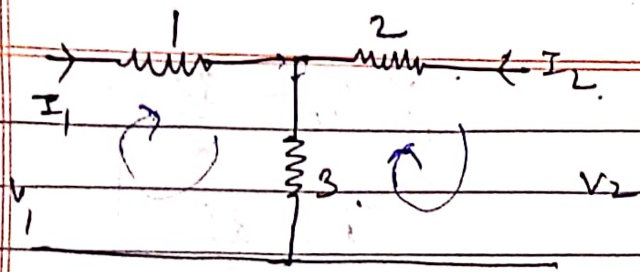
$$= \frac{32 I_2}{3}$$

V_2

$V = I Z$

$Z = \frac{V}{I}$

Ques.



Find z parameter.

case I –

$$V_1 - I_1 - 3(I_1 - I_2) = 0 \quad -(1)$$

$$V_1 - I_1 - 3I_1 + 3I_2 = 0 \Rightarrow V_1 - 4I_1 + 3I_2$$

case II –

$$V_1 = 4I_1 + 3I_2 \quad -(1)$$

$$-3(I_2 - I_1) \cancel{+} 2I_2 + V_2 = 0 \quad -(\text{II})$$

$$-3I_2 + 3I_1 \cancel{+} 2I_2 + V_2 = 0$$

$$-5I_2 + 3I_1 + V_2 = 0$$

$$V_2 = 5I_2 + 3I_1 \quad -(2)$$

Take I_1 as 0

$$V_1 = 3I_2$$

$$I_2 = \frac{V_1}{3} \qquad Z = V_1$$

$z_{11} = 4$, $z_{12} = 3$, $z_{22} = 5$

Take I_2 as 0

$$V_1 = 4I_1$$

$$\frac{V_1}{4} = I_1$$

from eq^n (2)

$$I_2 = \frac{V_2 - 3I_1}{5}$$

Sub. I_2 in eq^n (1)

$$V_1 = 4I_1 + 3I_2$$

$$= 4I_1 + 3\left(\frac{V_2 - 3I_1}{5}\right)$$

$$V_1 = \frac{4I_1}{1} + \frac{3V_2 - 9I_1}{5}$$

$$= \frac{20I_1 + 3V_2 - 9I_1}{5}$$

$$V_1 = \frac{11I_1 + 3V_2}{5}$$

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$B_1 \quad \cancel{I_1} \quad 5V_1 = 11I_1 + 3V_2$

$$I_1 = \frac{5}{11} V_1 - \frac{3}{11} V_2$$

$$\quad\; y_{11} \qquad\qquad\qquad y_{12}$$

write eqⁿ ① in terms of I_2

Put I_1 in eqⁿ ②,

$$V_2 = 5I_2 + 3\left(\frac{5}{11} V_1 - \frac{3}{11} V_2\right)$$

$$\cancel{V_2 = 5I_2 + \left(\frac{15}{11} V_1 - \frac{9}{11} V_2\right)}$$

$$V_2 = 5I_2 + 3\left(V_2 - \frac{8I_2}{3}\right)$$

$\frac{11}{4}$

$I_2 - 3I_1$

$\frac{9V_2}{3}$