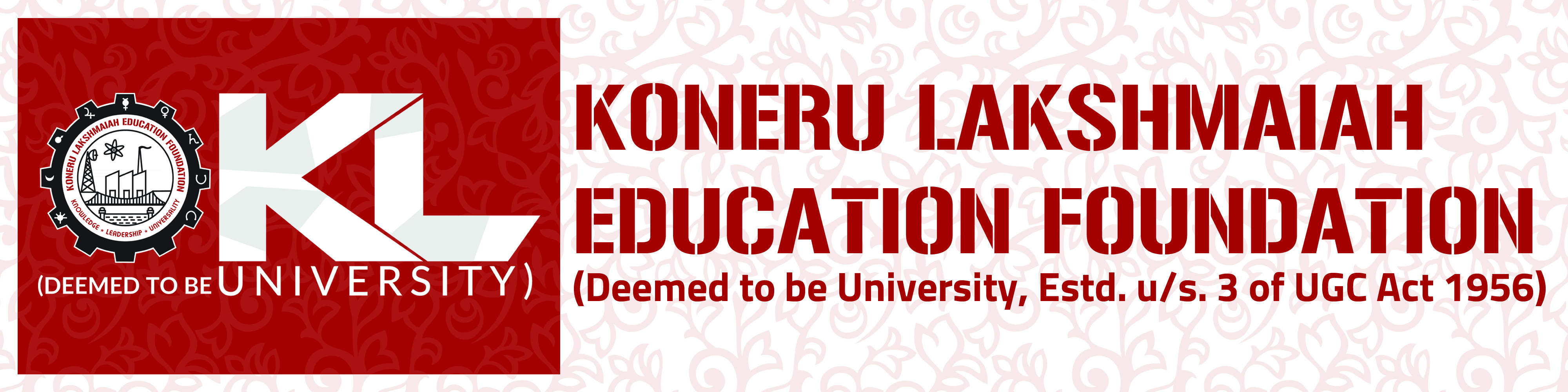
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**Department of Mathematics**

**DISCRETE STRUCTURES**

**I/IV-B.Tech-(I/II Sem), Academic Year: 2023-2024**

**Tutorial Problems-CO-2**

**Tutorial-4**

1. **Construct the truth tables for the following**
   1. [(pVq) Ʌ(~r)] ↔q
   2. (pVq) Ʌ( (~p) V (~r) )
2. **Prove the following are tautologies**
   * 1. *(p* ∧ *q)* → *p*
     2. *p* → *(p* ∨ *q)*
     3. ￢*p* → *(p* → *q)*
3. Show that ￢(￢p) and p are logically equivalent.
4. Use a truth table to verify the De Morgan law ￢(p ∧ q) ≡ ￢p ∨￢q.
5. Use resolution to show that the hypotheses “It is not raining, or Yvette has her umbrella,” “Yvette does not have her umbrella, or she does not get wet,” and “It is raining, or Yvette does not get wet” imply that “Yvette does not get wet.”
6. S.T. rs follows logically from the premises
   * 1. cVd, (cVd) → ￢h, ￢h →(aɅ ￢b), and (aɅ ￢b) → (rVs)

**Tutorial-5**

1. Let *P(x)*, *Q(x)*, and *R(x)* be the statements “*x* is a professor,” “*x* is ignorant,” and “*x* is vain,” respectively. Express each of these statements using quantifiers; logical connectives; and *P(x)*, *Q(x)*, and *R(x)*, where the domain consists of all people.
   1. No professors are ignorant.
   2. All ignorant people are vain.
   3. No professors are vain.
   4. Does (c) follow from (a) and (b)?
2. Let *P(x)*, *Q(x)*, and *R(x)* be the statements “*x* is a clear explanation,” “*x* is satisfactory,” and “*x* is an excuse,” respectively. Suppose that the domain for *x* consists of all English text. Express each of these statements using quantifiers,
3. logical connectives, and *P(x)*, *Q(x)*, and *R(x)*.
   1. All clear explanations are satisfactory.
   2. Some excuses are unsatisfactory.
   3. Some excuses are not clear explanations.
   4. Does (c) follow from (a) and (b)?
4. Let *P(x)*, *Q(x)*, *R(x)*, and *S(x)* be the statements “*x* is a baby,” “*x* is logical,” “*x* is able to manage a crocodile,” and “*x* is despised,” respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and *P(x)*, *Q(x)*, *R(x)*, and *S(x)*.
   1. Babies are illogical.
   2. Nobody is despised who can manage a crocodile.
   3. Illogical persons are despised.
   4. Babies cannot manage crocodiles.
   5. Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?
5. Let *S(x)* be the predicate “*x* is a student,” *F(x)* the predicate “*x* is a faculty member,” and *A(x, y)* the predicate “*x* has asked *y* a question,” where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.
   * + 1. Lois has asked Professor Michaels a question.
       2. Every student has asked Professor Gross a question.
       3. Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
       4. Some student has not asked any faculty member a question.
6. There is a faculty member who has never been asked a question by a student.
7. Some student has asked every faculty member a question.
8. There is a faculty member who has asked every other faculty member a question.
9. Some student has never been asked a question by a faculty member.

**Tutorial-6**

1. Prove that the sum of two odd integers is an even integer.
2. Prove that the product of two even integers is an even integer.
3. Prove that the sum of two rational numbers is a rational number.
4. Explain the process of an indirect proof by contradiction. Give an example to support your explanation.
5. Use an indirect proof by contradiction to prove that the square root of 2 is irrational.
6. Prove that there is no largest prime number using an indirect proof by contradiction.
7. Explain how you can use an indirect proof by contradiction to prove that there are infinitely many prime numbers.
8. Prove the statement "If n is a positive integer and n^2 is even, then n is even" using indirect proof by contrapositive.
9. Prove the statement "If p is a prime number and p ≠ 2, then p is odd" using indirect proof by contrapositive.
10. Prove the statement "If a + b = c, where a, b, and c are integers, then at least one of a, b, and c is even" using indirect proof by contrapositive.