

LATTICE DYNAMICS KIT

I. INTRODUCTION :

Lattice dynamics is an essential component of any postgraduate course in physics and material science. In particular it is essential for understanding the interaction of electro-magnetic waves and crystalline solids. In general, students find it difficult to understand involved concepts like ACOUSTICAL MODE, OPTICAL MODE, ENERGY GAP etc., which they cannot see for themselves in the laboratory. Such a difficulty can be overcome by introducing a laboratory exercise in which the student follows a carefully prescribed procedure which presents him with a simplified model of the system and allows him to verify well established theories. In the process he gains an insight into the concepts. The lattice dynamics kit provides such an experience in the study of dynamics of mono and di-atomic lattices.

The following experiments may be performed with the help of the kit.

- (i) Study of the dispersion relation for the mono-atomic lattice-Comparison with theory.
- (ii) Determination of the cut-off frequency of the mono-atomic lattice.
- (iii) Study of the dispersion relation for the di-atomic lattice – 'acoustical mode' and 'optical mode' energy gap. Comparison with theory.

II DETAILS OF KIT

The lattice dynamics kit has built-in audio oscillator that works on 220 V, 50 Hz and the only additional equipment needed is a general purpose C.R.O having XY mode. The Kit consists of the following parts:

- (i) Audio oscillator with amplitude control and facility to vary the frequency in the following ranges:

Switch position	Frequency range	Output Voltage
LO	0.8 – 8.9 KHz	4 – 6 V
HI	8.7 – 91 KHz	4 – 6 V

- (ii) The Lattice Dynamics Kit consists of an electrical transmission line which simulates *one dimensional mono and di-atomic lattices*. The toggle switches on the panel switches the system to the 'Mono-atomic mode' or the 'di-atomic mode'.

BACKGROUND INFORMATION :

Fig. 1 shows a mass and spring model for a one dimensional mono-atomic lattice. The particles having mass ' m ' connected by spring of force constant ' f '.

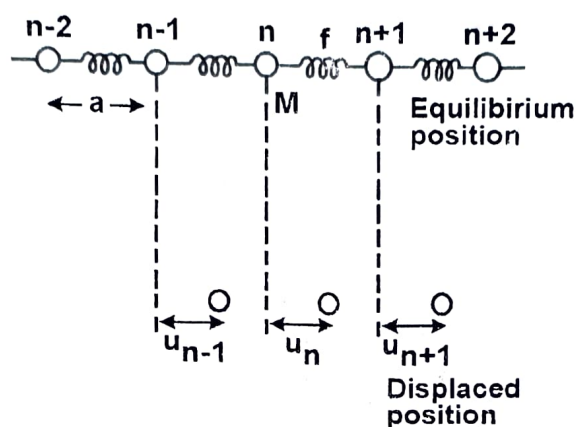


Fig. 1. One-dimensional linear mono-atomic lattice

$a \rightarrow$ lattice constant

$f \rightarrow$ Force constant

$M \rightarrow$ Mass of the atom

The equilibrium distance between the particles is ' a ' and the array is assumed to be infinitely long. Assuming only the nearest neighbour interaction, the equation of the motion of the n th atom is given by:

$$m\ddot{x}_n = f(U_{n+1} + U_{n-1}) - 2U_n \quad \dots(1)$$

which when solved gives the angular frequency

$$\omega^2 = \frac{4f}{m} \sin^2\left(\frac{ka}{2}\right) \quad \dots(2)$$

$$= \frac{2f}{m} (1 - \cos \theta) \quad \dots(3)$$

where k is the wave vector $\left(\frac{2\pi}{\lambda} \text{ or } \frac{\omega}{c}\right)$ and c is the velocity of propagation and $\theta = ka$ is the phase change per unit cell. The relation shows that the velocity of propagation is dependent on frequency i.e., dispersion is indicated. It also shows that there is a maximum frequency

$$v_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{1}{\pi} \sqrt{\frac{f}{m}} \quad \dots(4)$$

beyond which no transmission occurs. The array may thus be considered as a low-pass filter which transmits only in the range $0 - \nu_{\max}$.

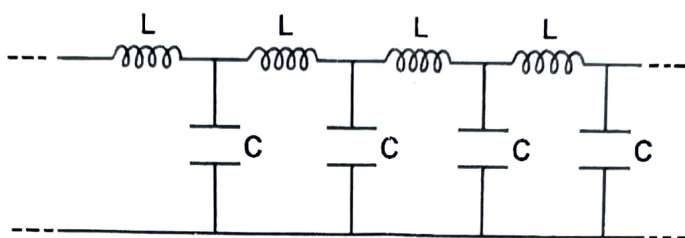


Fig. 2. Electrical analogue of linear mono-atomic lattice

The electrical analogue of the mono-atomic lattice is shown in Fig. 2. The dispersion relation for this circuit is

$$\omega^2 = \frac{2}{LC} (1 - \cos \theta) \quad \dots(5)$$

where θ is the phase change introduced by each section (unit cell) of the filter.

Thus, one has a precise analogy with the one dimensional mono-atomic lattice, with

$C \leftrightarrow m$ and $\left(\frac{1}{L}\right) \leftrightarrow f$. By measuring the phase difference between the input and

output voltages of the circuit shown in Fig. 2 as a function of frequency, the dispersion relation may be verified.

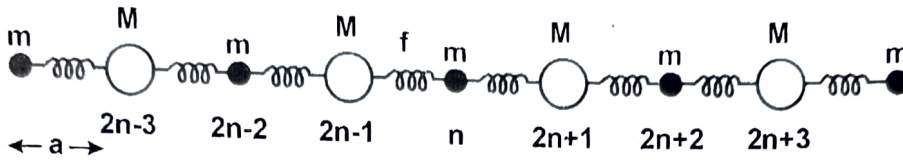


Fig. 3. Linear diatomic lattice of lattice parameter 'a' mass 'm' and 'M' and force constant 'f'

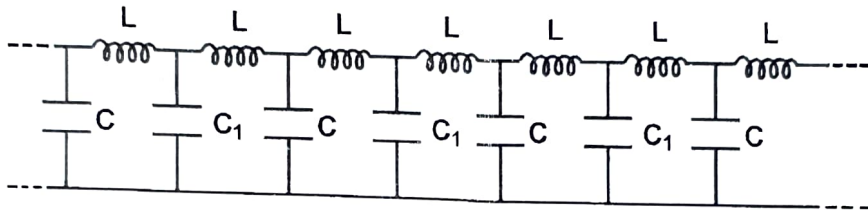


Fig. 4. Linear diatomic lattice -- electrical analogue

The di-atomic lattice with alternative masses 'm' and 'M' shown in Fig. 3 can be simulated by the transmission line with alternative capacitors **C** and **C₁** shown in Fig.

4. The dispersion relations for the mechanical and electrical analogues are given below:

$$\omega^2 = f \left(\frac{1}{m} + \frac{1}{M} \right) + f \left[\left(\frac{1}{m} + \frac{1}{M} \right)^2 - \frac{4 \sin^2 \theta}{mM} \right]^{1/2} \quad \dots(6)$$

(mechanical)

$$\omega^2 = \frac{1}{L} \left(\frac{1}{C} + \frac{1}{C_1} \right) + \frac{1}{L} \left[\left(\frac{1}{C} + \frac{1}{C_1} \right)^2 - \frac{4 \sin^2 \theta}{CC_1} \right]^{1/2} \quad \dots(7)$$

(electrical)

In contrast to the mono-atomic lattice, there are now two frequencies ω_+ and ω_- corresponding to a particular value of the wave vector \mathbf{k} . In a plot of ν versus θ (Fig. 5), this leads to two branches; the one corresponding to ν_- is called the acoustical branch and the one corresponding to ν_+ is called the optical branch. The frequency gap between the two branches depends on (M/m) .

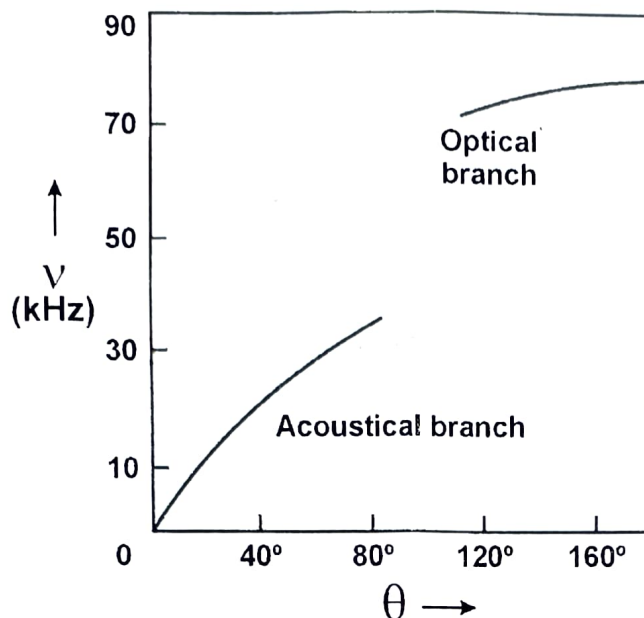
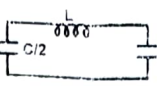


Fig. 5. Dispersion relation for the diatomic lattice

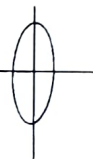
EXPERIMENTAL PROCEDURE :

Mono-atomic lattice

- (i) Connect the Kit to the mains. Keep **Amplitude**, **R₁** & **R₂** knob at maximum. Flipp the toggle switch Lo.
- (ii) Feed the output of the Lattice Dynamics Kit to a general purpose CRO. Connect "H" to the horizontal input & "V" to the verticle input of C.R.O.. Operate the CRO Horizontal input on the EXTERNAL MODE (or switch C.R.O. to XY mode).

(iii) Flip both the toggle switches towards the 'Monatomic' side. The transmission line (lattice) now consists of ten sections (unit cells) and it is excited by a constant source (audio oscillator in series with R_1). Each unit of the line may be thought of as consisting of  To eliminate the reflected wave and thus to simulate an infinite line, the line should be terminated by the resistance equal in value to the characteristic impedance $\sqrt{L/C}$. Vary R_2 until the Lissajous figure approaches nearly circle figure may be slightly distorted

(iv) Flip toggle switch to LO and start with the lowest frequency available & Vary the frequency of the & audio oscillator with the help of TTP. Determine the frequencies at which the phase differences between the input and output voltages of the simulated lattice is $\frac{n\pi}{2}$ where $n = 1, 2, \dots$ etc. At low frequencies, the phase difference will be practically zero and the C.R.O. pattern shows a



As the frequency is increased, at a particular frequency the pattern shows (or approach to) a circle (if you are not getting circle adjust R_2). The phase difference is now $\pi / 2$. At this frequency, the total phase difference is 90° and the phase difference per cell unit is $90^\circ / 10 = 9^\circ$. Note down this reading. Vary the frequency still further, determine and tabulate the phase difference as a function of θ . Calculated frequency is evaluated using equation (5) for different values of θ .

TABLE FOR MONO ATOMIC LATTICE(SI. No.1516052)

S.No.	C.R.O. pattern	N ₀	e	Observed Freq (KHz)	Calculated freq (KHz)
1.	/	0	0	---	----
2.	O	90	9	3.739	3.644
3.	\	180	18	7.487	7.266
4.	O	270	27	11.290	10.844
5.	/	360	36	15.083	14.355
6.	O	450	45	18.941	17.777
7.	\	540	54	22.532	21.089
8.	O	630	63	26.023	24.272
9	/	720	72	29.320	27.304
10	O	810	81	32.550	30.169
11.	\	900	90	35.315	32.847
12.	O	990	99	37.837	35.323
13.	/	1080	108	40.465	37.581
14	O	1170	117	42.401	39.608
15	\	1260	126	43.929	41.390
16	O	1350	135	45.255	42.917
17	/	1440	144	---	44.180

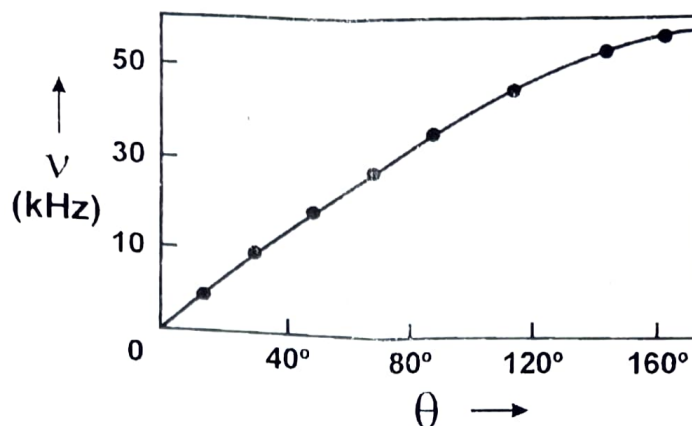


Fig. 6. Dispersion curve for the mono-atomic lattice
 $L = 1 \text{ mH}$, $C = 0.04 \text{ } \mu\text{F}$
 • \rightarrow experimental — \rightarrow theoretical

- (v) Tabulate and plot the observed and calculated values of ν as a function of θ (as per sample readings attached). Comment on the agreement between the theoretical and experimental values. Typical results are shown in Fig. 6.
- (iv) Find out the maximum frequency of transmission and compare with the theoretical value of $\frac{1}{\pi} \sqrt{\frac{L}{C}}$

NOTE: To get proper circle, you will have to adjust the potentiometer R_2 .

Di-atomic lattice

- (i) Flip both the toggle switches on the front panel of the lattice dynamics kit towards the '**Diatomic**'. The alternate capacitors are now changed to $C_1 = 0.147 \mu\text{F}$.
- (ii) Repeat the procedure outlined for the mono-atomic lattice and tabulate the frequency vs phase as given below and plot the same on graph. Note the existence of the energy gap.

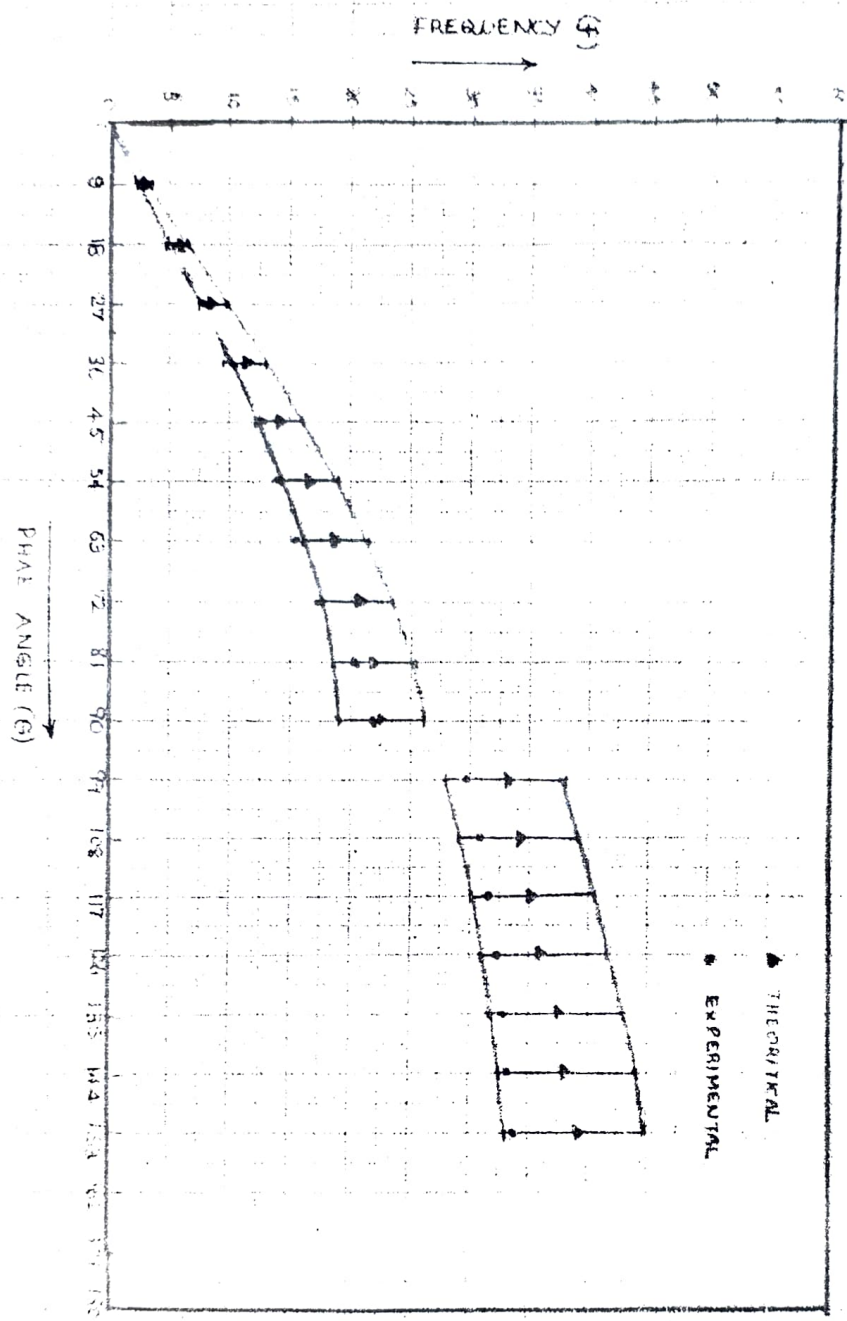
TABLE FOR DI-ATOMIC LATTICE(SI. No. 1516052)

S.No.	C.R.O. pattern	Ne	e	Observed Freq (KHz)	Calculated freq (KHz)
1.	---	0	0	—	---
2.	O	90	9	2.654	2.9
3.	\	180	18	5.134	5.8
4.	O	270	27	8.097	8.6
5.	/	360	36	10.372	11.4
6.	O	450	45	13.028	14.0
7.	\	540	54	15.090	16.5
8.	O	630	63	16.951	18.0
9.	/	720	72	18.314	20.5
10.	O	810	81	23.558	21.5
....	BAND GAP	
....		
11.	O	990	99	34.495	33.0
12.	/	1080	108	35.128	34.0
13.	O	1170	117	35.836	35.0
14.	\	1260	126	36.828	35.8
15.	O	1350	135	—	37.5

MANUFACTURER'S NOTES :

1. This instrument is based on admittance analogy. In this case **C** corresponds to m and $1/L$ to f . You may kindly refer to Dynamical Analogies by OLSON (pp: 232 – 256) published by Van Nostrands.
2. Observations printed in manual varies instrument to instrument. We are attaching herewith actual readings (dial vs frequency) taken on this particular instrument.
3. Lissajous figure may be slightly distorted due to CRO used or tolerance in the components of the kit. It may be minimized controlling X and Y gain in CRO and **R₁**, **R₂** controls in the kit. It is important to show the phase change with respect to frequency change inclined Lissajous figure distortion free.
4. In this region of Energy band gap the Lattice does not transmit any energy in spite of constant input. On CRO we will observe only the horizontal voltages. At the end of energy gap the optical Branch appears in the form of Lissajous figure.
5. Frequency may be directly observed on digital frequency counter. One BNC connector near the left corner of the top is provided for the purpose. IF required, we may supply a low cost frequency counter for the same.

DIATOMIC LATTICE



Frequency

