Distributivity

$$(f+g)*h = h*(f+g)$$

commutativity, allows us to consider only one case, as they are equivalent

$$\begin{aligned}
h*(f+g)(x) &= \int_{\mathbb{R}} h(x-x')\left(f(x')+g(x')\right)dx' \\
&= \int_{\mathbb{R}} \left[h(x-x')\cdot f(x') + h(x-x')g(x')\right]dx' \\
&= \int_{\mathbb{R}} h(x-x')\cdot f(x')\,dx' + \int_{\mathbb{R}} h(x-x')\cdot g(x')\,dx' \\
&= (h*f)(x) + (h*g)(x) = (f*h)(x) + (g*h)(x)
\end{aligned}$$

Shift invariance

$(T_b f) * g = T_b(f * g)$ for all translations T_b with $(T_b f)(x) := f(x-b)$.

$$\begin{aligned}
T_b(f*g)(x) &= (f*g)(x-b) \\
&= \int_{\mathbb{R}} f(x-b-x')g(x')\,dx' = L
\end{aligned}$$

$$\begin{aligned}
\left[T_b(f)*g\right]x &= \int_{\mathbb{R}} T_b(f)(x-x')g(x')\,dx' \\
&= \int_{\mathbb{R}} f(x-x'-b)g(x')\,dx' = P = L
\end{aligned}$$

$\square$