

#2.2

$$f(x) = \begin{cases} 0, & (x \leq -3), \\ \frac{x^2 - 6x + 9}{16}, & (-3 < x \leq -1), \\ \frac{6 - 2x^2}{16}, & (-1 < x \leq 1), \\ \frac{x^2 - 6x + 9}{16}, & (1 < x \leq 3), \\ 0, & (x > 3). \end{cases}$$

$$h = \begin{cases} \frac{1}{2} & (-1 \leq x \leq 1) \\ 0 & \text{else} \end{cases}$$

$$f = h * h * h$$

$$F[f] = F[h * h * h] = F[h] \cdot F[h] \cdot F[h]$$

$$F[h](v) = \int_{-\infty}^{\infty} h(x) e^{-i2\pi v x} dx$$

$$= \int_{-1}^1 \frac{1}{2} e^{-i2\pi v x} dx$$

$$= \frac{1}{2} \int_{-1}^1 e^{-i2\pi v x} dx$$

$$= \frac{1}{2} \left[\frac{-1}{i2\pi v} e^{-i2\pi v} \right]_{-1}^1$$

$$= \frac{-1}{4i\pi v} \left(e^{-i2\pi v} - e^{i2\pi v} \right)$$

$$= \frac{1}{4i\pi v} \left(e^{i2\pi v} - e^{-i2\pi v} \right)$$

$$(e^{ix})' = ie^{ix}$$

$$\sin \phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$$

$$\phi = 2\pi v$$

$$= \frac{2i}{4i\pi v} \cdot \sin(2\pi v)$$

$$F[h](v) = \frac{\sin(2\pi v)}{2\pi v} = \text{sinc}(v)$$

$$F[f] = (F[h])^3 = \text{sinc}^3$$

$$F[f](v) = \text{sinc}^3(v)$$