H1 PSNR

$$f_{ij} = g_{ij} + n_{ij} \qquad PSNR(f,g) = 10\log_{10}\left(\frac{Max\_val}{MSE(f,g)}\right)$$

a) $$0 = 10\log_{10}\left(\frac{Max\_val}{MSE(f,g)}\right) \qquad /10$$

$$0 = \log_{10} Max\_val - \log_{10} MSE(f,g) \quad / + \log_{10} MSE(f,g)$$

$$\log_{10} MSE(f,g) = \log_{10} Max\_val \qquad / 10^{(\cdot)}$$

$$MSE(f,g) = Max\_val$$

$$\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{ij}-g_{ij})^2 = Max\_val \quad \wedge \quad n_{ij} = f_{ij} - g_{ij}$$

$$\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} n_{ij}^2 = Max\_val$$

$PSNR(f,g) = 0 \Leftrightarrow$ variance of the noise is equal to maximal signal value

b)

$$PSNR(u,g) = PSNR(f,g) + 20dB$$

$$10\log_{10}\left(\frac{MAX}{MSE(u,g)}\right) = 10\log_{10}\left(\frac{MAX}{MSE(f,g)}\right) + 20dB \quad , \; MAX = \text{max signal value}$$

$$10\log_{10} MAX - 10\log_{10} MSE(u,g) = 10\log_{10} MAX - 10\log_{10} MSE(f,g) + 20dB$$

$$10\log_{10}\left(\frac{MSE(f,g)}{MSE(u,g)}\right) = 20dB \qquad /10,\ 10^{(\cdot)}$$

$$\frac{MSE(f,g)}{MSE(u,g)} = 100$$

The filtering has reduced the MSE 100 times

c) $$n \to 0 \Leftrightarrow \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{ij}-g_{ij})^2 = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} n_{ij}^2 \to 0$$

$$\Leftrightarrow MSE(f,g) \to 0 \quad , \quad \frac{1}{MSE(f,g)} \to +\infty$$

$$\Leftrightarrow PSNR(f,g) = 10\log_{10}\left(\frac{MAX}{MSE(f,g)}\right) \to +\infty$$