Multi-Agent Systems

Homework Assignment 1

MSc AI, VU

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Deadline: Wednesday Nov 16, 2021 (23h59)

1 Game Theory: Optimality Concepts and Nash Equilibrium

1.1 Iterated Best response dynamics for a simple matrix game

Consider the following simultaneous two player game: Player 1 (row player) can play U, M or D, while player 2 (column player) can pick L, C or R. They pay-off is given below:

	L	C	R
U	$4,3$	$2,0$	$8,2$
M	$8,2$	$7,6$	$-1,1$
D	$6,-3$	$0,0$	$1,-1$

- Are there any dominated strategies?
- The concept of *Nash equilibrium* plays a central role in game theory and will be discussed at length in the next assignment(s). But some basic intuition can be gleaned from the following approach (known as: best response dynamics). One way to visualise Nash equilibria is to interpret the simultaneous game as a repeated sequential one: One player is allowed to make a move, then the other player makes a move, after which the first player can make another move and so on. What happens when you do this for this game?
- Can you think of simple games where the outcome of this dynamics would be qualitatively different?

1.2 Travelers' dilemma: Discrete version

An airline severely damages identical antiques purchased by two different travelers. The management is willing to compensate them for the loss of the antiques, but they have no idea about the actual value, After some research, the management is convinced that the value is either 1, 2 or 3 (in some appropriate unit), and they ask the two travelers to separately choose the appropriate number. To incentivise the travelers to come up with the correct number they introduce the following rules for the procedure:

1. If both travelers pick the same number, they will be reimbursed that amount.
2. If they pick different numbers, management will assume that the lowest number is the correct one, and pay both of them the lower figure, but he person with the lower number will get a bonus of size a for honesty, while the one who picked the higher number will get a penalty of size a.

Questions

1. Write down the pay-off matrix for this game.
2. Determine the pure Nash equilibriua (PNE, there might be none, one or multiple ones).
3. Are there mixed Nash equilibria?
4. Does this knowledge about the NE help the travelers in their decision?
5. Write down the regret matrix. This matrix is similar to the pay-off matrix, but now specifies the regret (rather than the pay-off) for each action profile.
6. What are the regret minimisation strategies?

1.3 Cournot's Duopoly (continuous version)

Cournot's duopoly is a game that models economic competition with strategic substitutes. Strategies are called *strategic substitutes* if an increase in your strategy will cause the competitor to decrease the use of his strategy (*"if you increase your part, I will reduce mine"*). This contrasts with *strategic complements* when an increase in the strategy of one player causes the other player to follow suit (*"if you increase your part, I will do the same."*.)

Two companies make an interchangeable product (e.g. bottled water). Both need to determine (simultaneously!) the quantity they will produce (say for next week). Call these quantities q_1 and q_2, respectively. The unit price p (price of each unit, e.g. one bottle) of the product in the market depends on the total produced quantity $q_1 + q_2$. Specifically

$$p(q_1, q_2) = \alpha - \beta(q_1 + q_2) \qquad (\alpha, \beta > 0).$$

Firm 1 can produce each unit at a unit-cost c_1, whereas the unit-cost for firm 2 equals c_2.

- What is the best response for each company given the quantity the other company will produce?
- Suppose the companies need **not** decide on their quantity at the same time, but can react to one another (an unlimited number of times). What will be the outcome? (Provide a diagram.)

1.4 Ice cream time!

Three competing ice-cream vendors (Alice, Bob and Charlize) are trying to sell their refreshments to tourists on the beach. We are making the following assumptions:

- the beach has total length of 1, while its width is uniform and much smaller than its length. So the beach can be represented as a line segment of length 1, and each position on the beach can be represented by the position parameter $0 \leq x \leq 1$.
- Tourists are uniformly distributed along the total length of the beach and will buy their ice-cream at the stall that is closest to their location;

Questions:

1. On a beautiful summer morning Charlize makes her way to the beach and upon arrival finds that her two competitors have already set up their stalls: Alice at location $a = 0.1$ and Bob at location $b = 0.8$. Discuss what Charlize's best response is: i.e. what location should she choose, given $a = 0.1$ and $b = 0.8$?
2. Same question as above, but now assume that all we know is that $a = 0.1$ and $a < b \leq 1$.
3. Earlier that morning, Bob arrived and discovered that Alice had already set up her stall at $a = 0.1$, while Charlize hadn't shown up yet. But Bob knows that Charlize will arrive before too long, and that she will try to position herself in such a way as to maximize her revenue. What location should Bob pick in order to maximize his own expected revenue?
4. At sunrise that morning, Alice arrived before both Bob and Charlize, and set up her stall at location $a = 0.1$. However she knows for sure that the other two vendors will show up soon. Where **should she have set up** her stall in order to maximise her expected revenue?