Operations Research (OR) Teachout

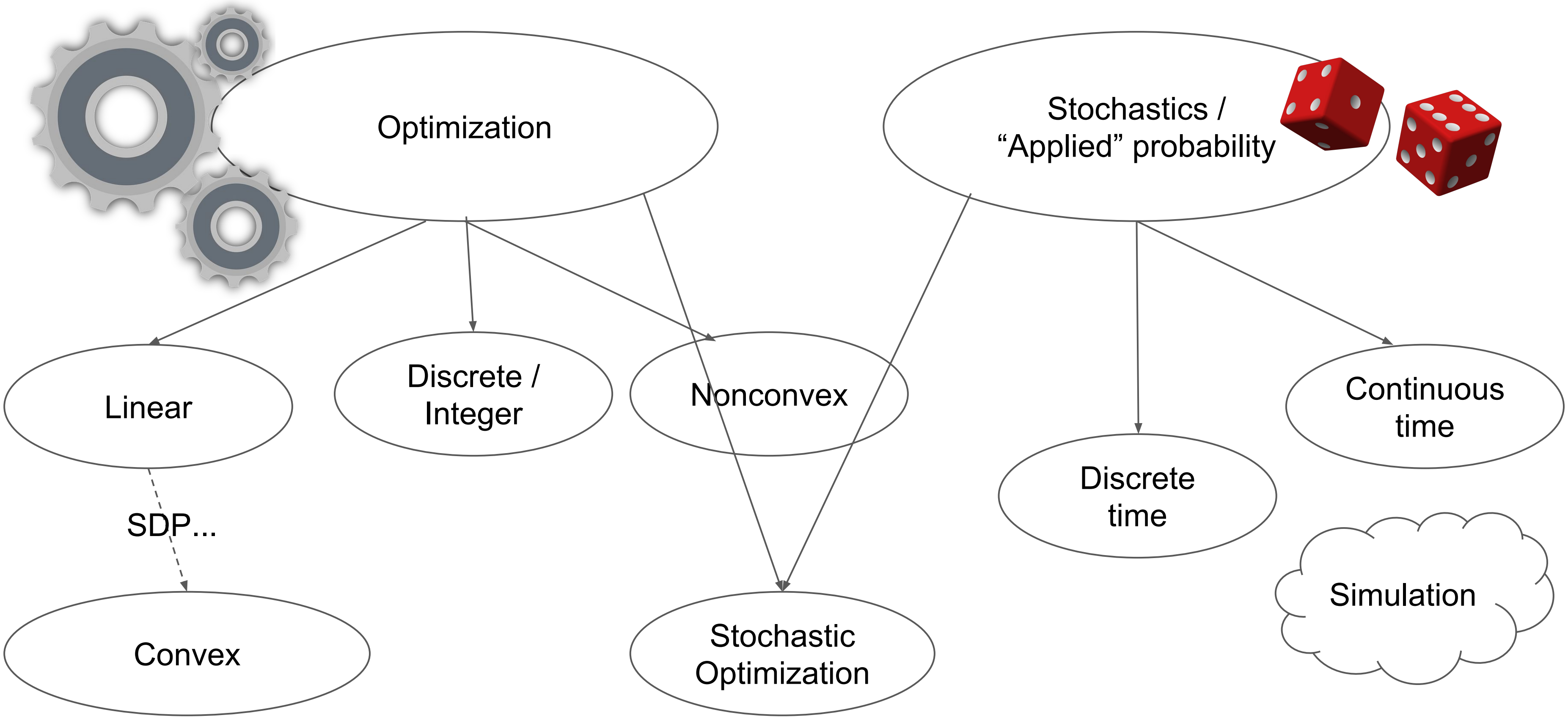
Agenda

1. Basic methods

2. Some widely-used OR models and applications
   - What can be solved?
   - Some formulations to taste...

3. Formulating OR models: Exercises

(Simple) Classification of OR models



Your (future) references for key methods

Linear Optimization

- *Bertsimas & Tsitsiklis: Introduction to Linear Optimization*

Continuous (mostly convex) Optimization

- *Ben-Tal & Nemirovski: Lectures Notes (Optimization III)*
- *Ben-Tal & Nemirovski: Lectures on Modern Convex Optimization*

Discrete Optimization

- *Nemhauser & Wolsey: Integer and Combinatorial Optimization*
- *Schrijver: Combinatorial Optimization*

Stochastic Optimization

- *Puterman: Markov Decision Processes*
- *Shapiro, Dentcheva, Ruszcynszki: Lectures on Stochastic Programming*
- *Powell: Approximate Dynamic Programming*

Stochastics

- *Asmussen: Applied Probability and Queues*
- *Rogers & Williams: Diffusions, Markov Processes and Martingales (I, II)*
- *Karatzas & Shreve: Brownian Motion and Stochastic Calculus*

Simulation

- *Nelson: Foundations and Methods of Stochastic Simulation*

Software reference

Linear / Integer / (some convex)

- Gurobi, CPlex
- SCIP (open-source)

General Purpose optimization

- Standard libraries (e.g., optim in R, scipy.optimize in Python)
  - The BFGS method often good for smooth functions
  - The Nelder-Mead method works for anything but super slow
- Baron (nonlinear problems)

General purpose stochastics

- Libraries
- Code your own...

Simulation

- Arena, Simio
- Code your own...

Building blocks

Optimization

Make a decision to maximize/minimize a quantity given constraints
Quantity: cost, utility, time, happiness, love, whatever ...

A typical problem:

1. Formulate your decision criteria in math

$$\max_{x} \; f(x)$$
$$s.t. \quad g(x) \leq 0$$

2. Apply/design a solution method (exact / heuristic)

Linear programming

$$\begin{aligned} \max_{x} \ & c \cdot x \\ s.t. \ & A x \le b \\ & x \ge 0 \end{aligned}$$

Integer programming

$$\begin{aligned} \max_{x} \ & c \cdot x \\ s.t. \ & A x \leq b \\ & x \in Z \end{aligned}$$

Some Theory

Continuous optimization:

- LP / Convex: polynomial
- Discrete optimization -> IP (NP-complete, SAT reduces to IP)
  ...still want to solve them...
- Non-convex optimization: Hard (Baron)

IP solvable through LP ("a diamond in the rough")

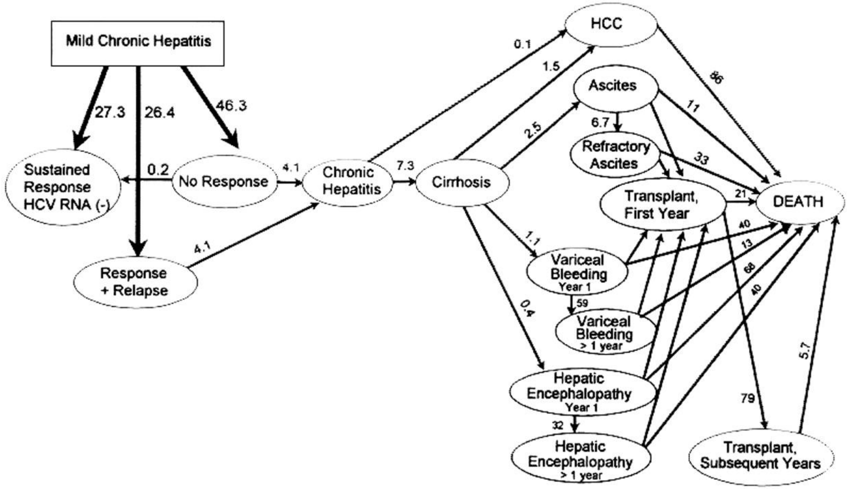
Sometimes good heuristics outperform IP, but IP is a good start and will work very well for "small" models.

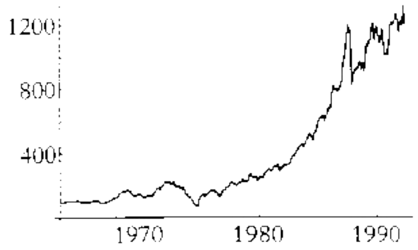
Many prototypical problems: Bin-packing, Knapsack, Set covering, TSP, ...

Stochastic processes

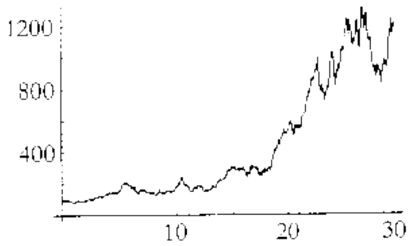
A typical problem:

1. Specify a process (e.g., a Markov Chain, Brownian Motion)
2. a) Compute/Simulate a functional of the process (policy-dependent?)
   b) Control?





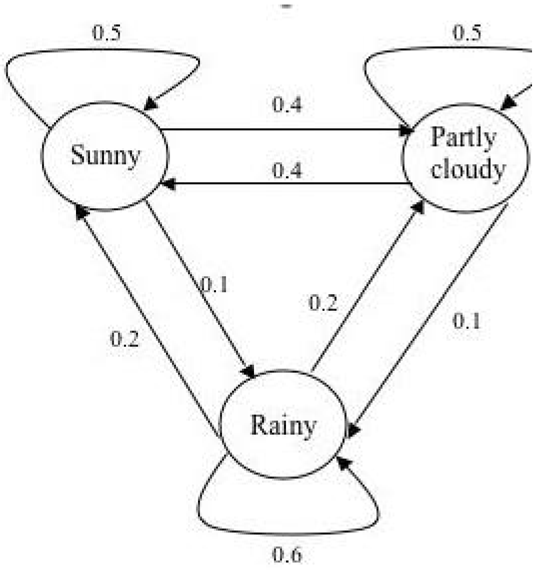
UK FTA index, 1963–92



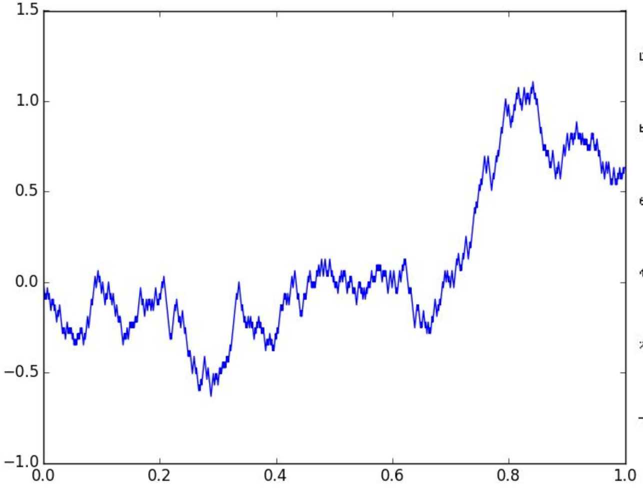
Exponential Brownian motion

Stochastic processes: Important instances

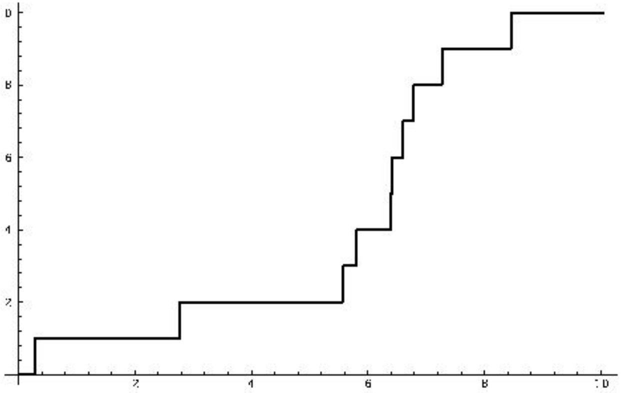
Markov chains + MDP



Brownian motion

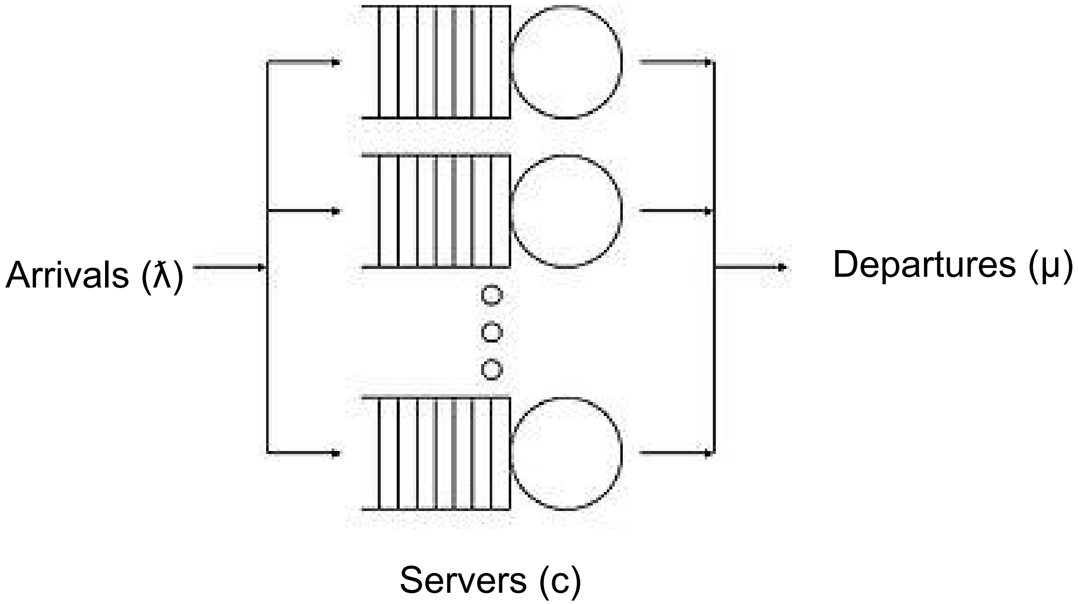


Poisson process



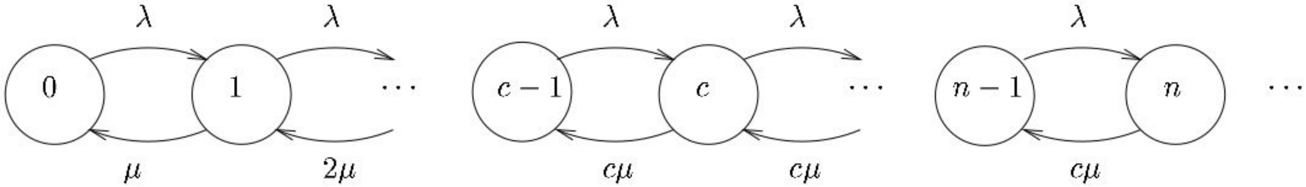
Models

Queueing



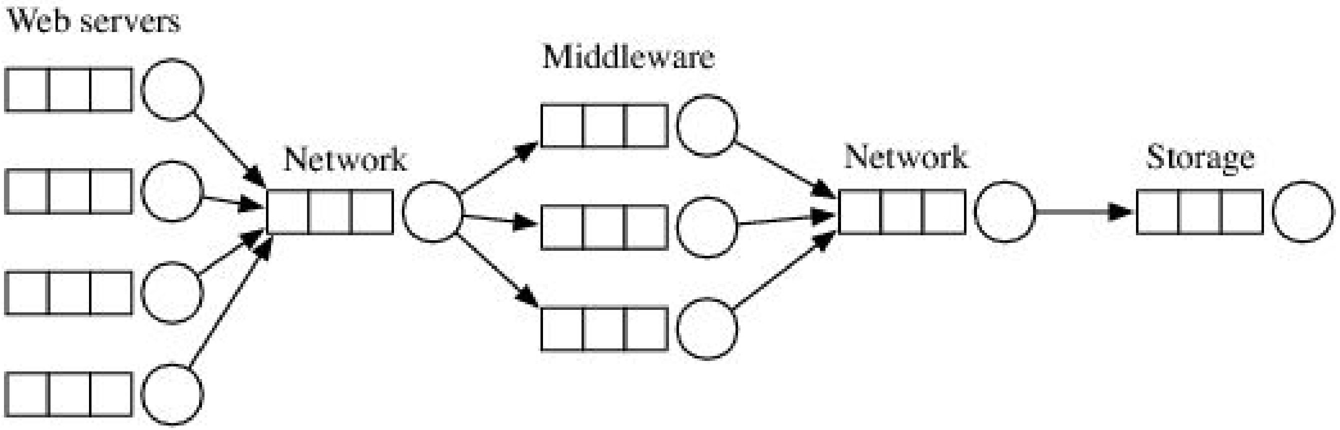
Examples:

- Hospital beds
- Airport check-in
- Supermarket lines
- Traffic lights
- Network traffic



Queueing Networks

An example:



Optimal Inspections and Maintenance

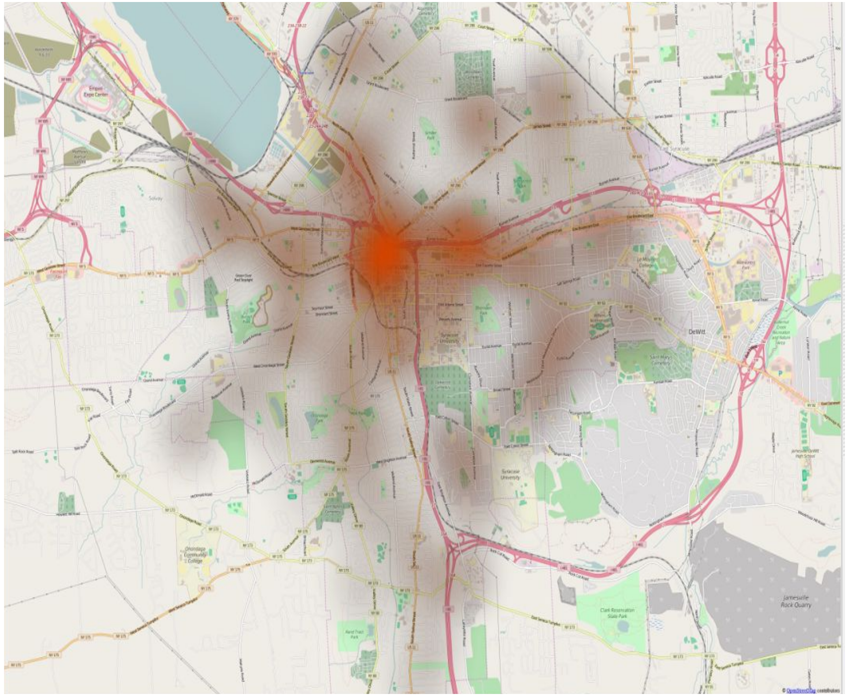
Optimal repair (MDP formulation)

$$p(x_{t+1} = j | x_t = i, a_t);$$
$$1 \leqslant i, j \leqslant n; \quad t = 0, 1, \ldots T-1 \quad (1)$$

where:

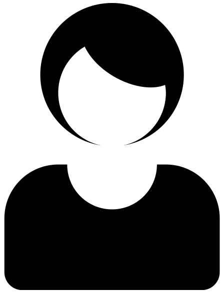
x_t = condition state of the facility at the beginning of year t,
i, j = indices of elements in the set of discrete conditions,
a_t = M & R activity performed during year t,
n = number of possible states the facility can be in,
T = number of years in the planning horizon.

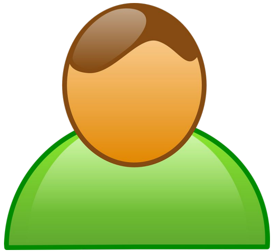
Extend to inspections...

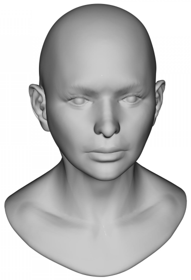


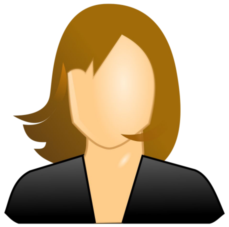
Water pipes???

Marriage problem (optimal stopping)





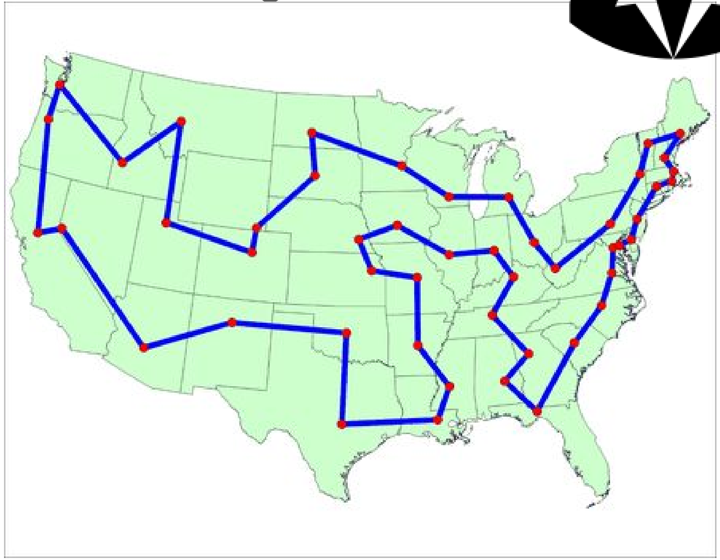


Logistics: Vehicle routing

Basis:

- Shortest path
- Traveling salesman



Full-fledged

$$Min \quad \sum_{r=1}^{R} c_r y_r$$

$$s.t. \quad \sum_{r=1}^{R} \alpha_{ir} y_r \geq 1, \quad \forall i = 1, 2, \ldots, n$$

$$y_r \in \{0, 1\}, \quad \forall r = 1, 2, \ldots, R.$$

Ambulance location/dispatch?

Maximal covering location problem

(MCLP)

$$\text{Maximize} \sum_{i \in V} d_i y_i$$

subject to

$$\sum_{j \in W_i} x_j \geqslant y_i \quad (i \in V),$$

$$\sum_{j \in W} x_j = p,$$

$$x_j \in \{0, 1\} \quad (j \in W),$$

$$y_i \in \{0, 1\} \quad (i \in V).$$

Logistics: Inventory management

How to reorder?

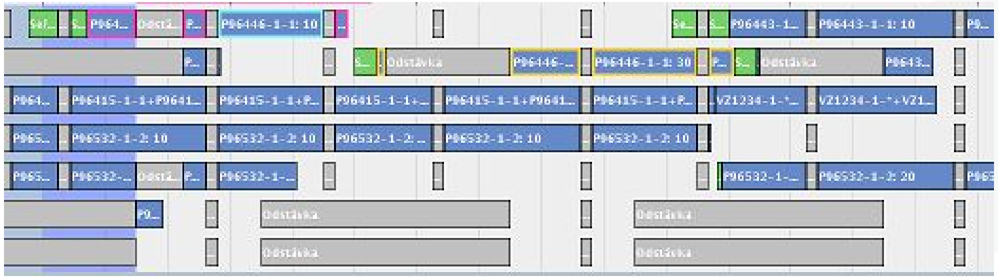
- Item availability

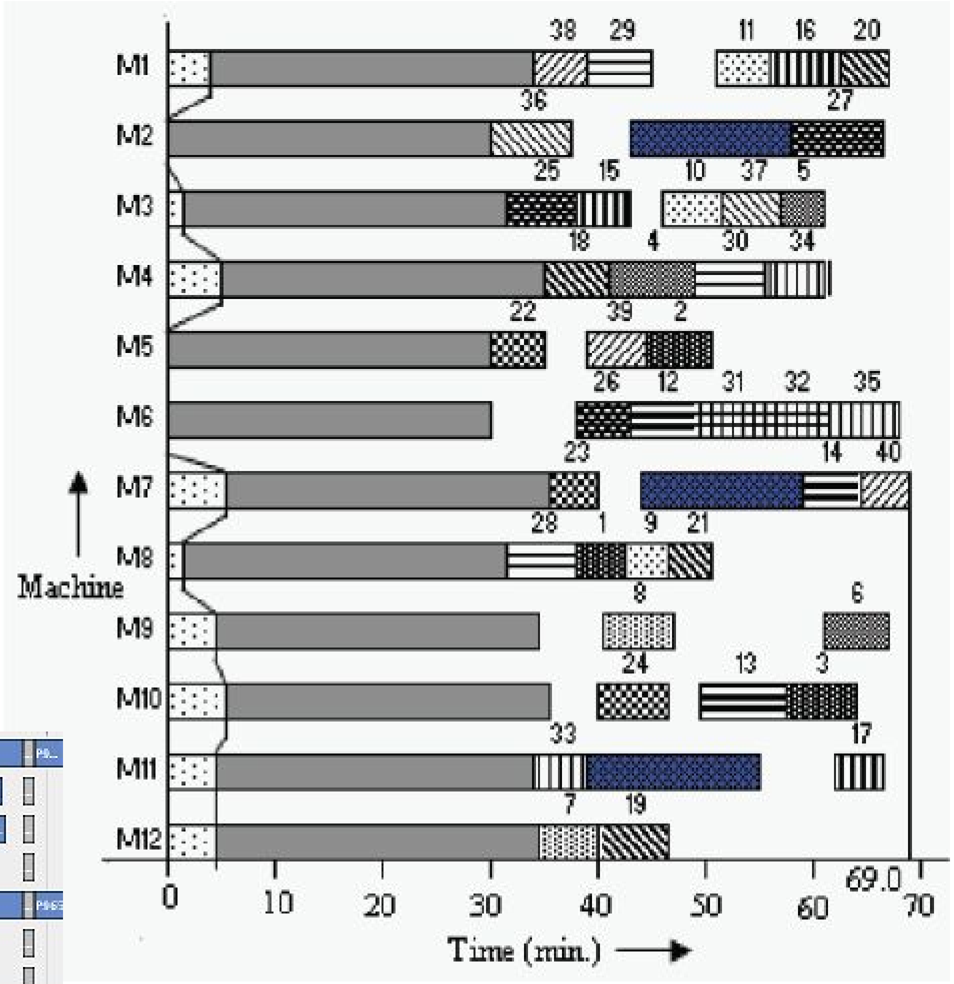
  vs

- Holding costs

Scheduling

- Machine scheduling
- Allocating resources
  to complex projects (job shop)

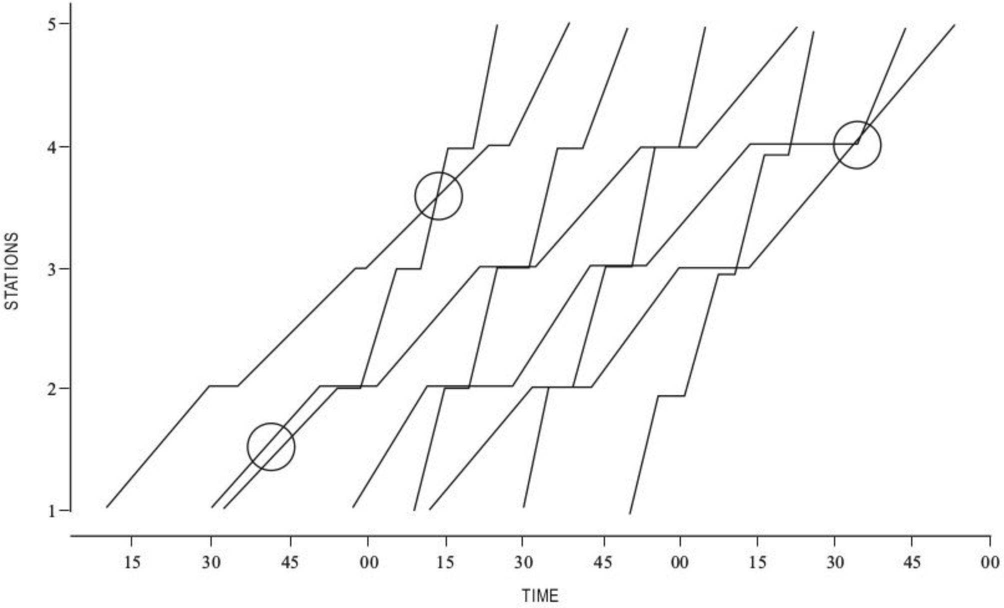




More scheduling!

Crew scheduling (airlines, nurses…)

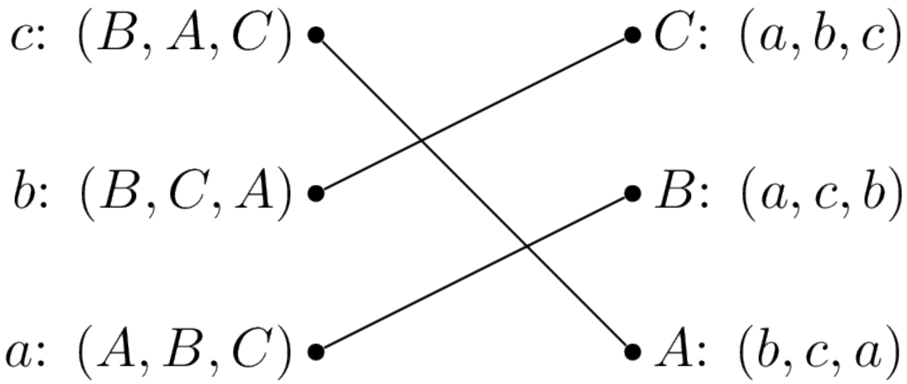
Train timetables



Unit commitment

Mechanism design (econ/OR)

Stable matching



Gale-Shapley algorithm

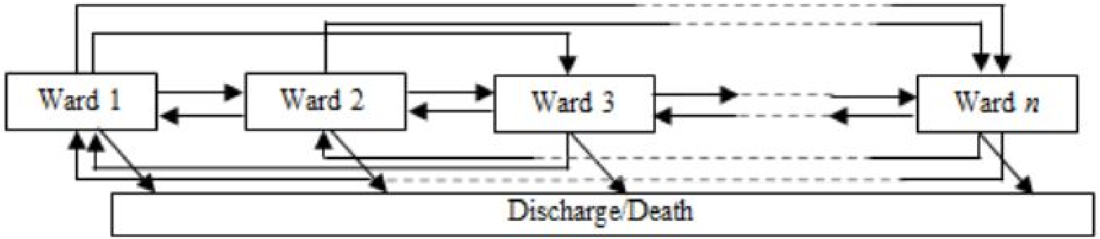
Applications:

- Econ job market
- Residency matching (extension)

Semi-Markov processes: a “demo”

- Like Markov chains, but heterogeneous transition times

Application:
Patient flows between hospital wards (semi-Markov cluster)



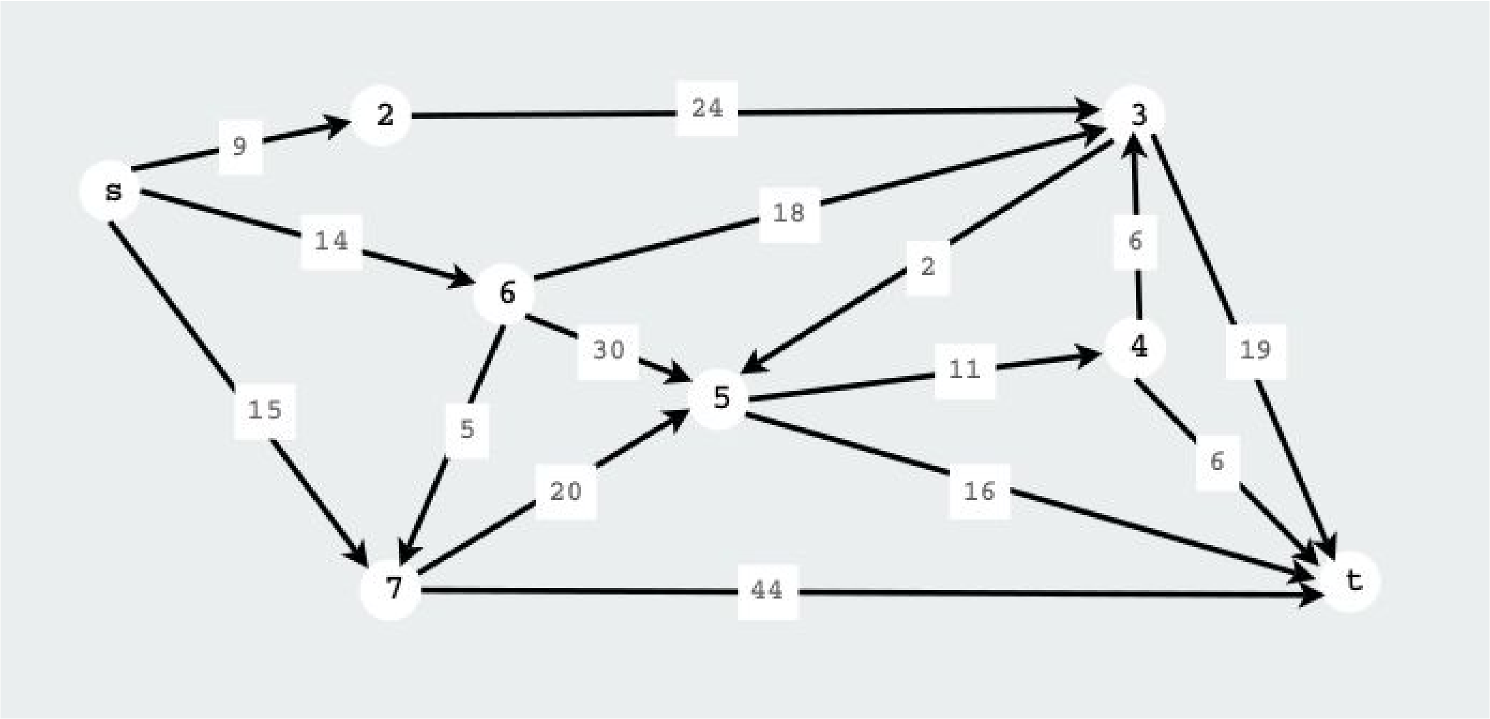
Yields:

- Patient clusters, transition probs, sojourn time distributions
- [Petitions?]

Formulations

Find the Shortest path with LP

From **s** to **t**



The Wagner-Whitin Inventory Model

- Schedule orders over T time periods (one item)
- Demand d_t, must be met
- Per-unit order cost c, fixed order cost K
- Holding cost h

Shift scheduling

You are a supermarket manager who needs to assign workers to shifts.

pattern	*Hours of Work*	*Total Hours*	*Cost*
1	10 a.m. to 6 p.m.	8	$ 50.00
2	1 p.m. to 9 p.m.	8	$ 60.00
3	12 p.m. to 6 p.m.	6	$ 30.00
4	10 a.m. to 1 p.m.	3	$ 15.00
5	6 p.m. to 9 p.m.	3	$ 16.00

Hour	*Staffing Requirement*
10 a.m. to 11 a.m.	3
11 a.m. to 12 a.m.	4
12 a.m. to 1 p.m.	6
1 p.m. to 2 p.m.	4
2 p.m. to 3 p.m.	7
3 p.m. to 4 p.m.	8
4 p.m. to 5 p.m.	7
5 p.m. to 6 p.m.	6
6 p.m. to 7 p.m.	4
7 p.m. to 8 p.m.	7
8 p.m. to 9 p.m.	8

Design liver allocation districts!

- 58 donor service areas: Divide them into districts within which livers are transported
- Minimize disparity between liver supply and demand [supply varies!]
- Don’t transport livers too far (not more than 5h)
- Districts contiguous, “convex” (for efficiency) -> cluster districts around DSA
- Not too many districts
- At least 6 transplant centers within a district (to have a competition)

To remember

Optimization

- Linear programming
- Integer programming
- Convex optimization: easy

Stochastics

- Markov decision processes
- Brownian motion
- Poisson process