STAT 8574

Assignment 1

1.1 $n = 129, \; y = 118$

$$P(y|\theta) \propto \theta^y (1-\theta)^{n-y}$$

$$P(\theta|y) \propto \theta^y (1-\theta)^{n-y} \times 1$$

$\propto$

Posterior Distribution $\propto$ Beta(119, 12)

$\propto$ Beta(y+1, n-y+1)

After simulating 1000 times,

mean(θ) = 0.908

MLE ($\hat{\theta}$) = $\frac{118}{129}$ = 0.914

Since, prior is not informative, mean of posterior is close to mean of data.

1.2) ~~$E(\theta) = \frac{a}{a+b} = 0.6$~~

The ~~lower~~ range is credible set posterior of The ~~range~~ of θ is btw (0.851 and 0.95) for (2.5% and 97.5%) respectively

1.2) $E(\theta) = 0.6$

$\frac{a}{a+b} = 0.6$

$a = 0.6a + 0.6b$

$0.4a = 0.6b$

$a = 1.5b$

Since we are not confident, let, $b = 2$, for a wider distribution, $\therefore a = 3$.

posterior = Beta(a^*, b^*)

$a^* = y + a = 118 + 3 = 121$

$b^* = n - y + b = 11 + 2 = 13$

Mean = 0.9034

Var = 0.00064

Credible Set

(2.5%, 97.5%) = (0.846, 0.948)

~~The Distribution mean~~

The posterior mean is close to the data set mean.

1.3) We are very confident.

So let,

$$b = 20$$

$$\therefore \quad a = 1.5 \times 20 = 30$$

$$\text{posterior}(\theta) = \text{Beta}(a^*, b^*)$$

$$a^* = 118 + 30 = 148$$

$$b^* = 11 + 20 = 31$$

Mean = 0.82

Var = 0.0008

Credible set

(2.5%, 97.5%) = (0.826, 0.0081)

The posterior mean is btw the prior mean of 0.6 and data mean of 0.91.

1.4) 0.715

The mean of posterior predictive distribution of observing btw 80 and 90 women, who report being happy is 0.715.

2)

$n = 100, \quad y = 58$

Q:

2.1) Let ~~prior ~ Unif(0,1)~~

$p_{prior} \sim Unif(0,1)$

$\sim Beta(1,1)$

We have a binomial likelihood function with,

$n = 100, \quad y = 58$

$p(\theta|y) \sim Beta(59, 43)$

~~lower Upper~~

~~Var 1 -0.035 0.724~~

HPD interval :

lower	Upper
-0.07329	0.7052619

2.2 Credible set

2.5%	97.5%
0.48855	0.6741274

2.3 After adding new data

HPD Interval

~~2.5%~~

lower	Upper
0.0233	0.5733

2.4) Since the HPD interval included 0.5, we can say that the population is evenly divided in terms of preferences for candidates A and B.

2.5) ~~0.247147~~ 0.3709391

3)

a) mean_joe → 0.195
Sd_joe → 0.116

mean_sam → 0.6
Sd_sam → 0.2

They have different priors.

3b

For joe - 1.89×10^{-16}

For sam - 0.2602016

3c

Credible set joe - 0.1 0.2

Credible set sam - 0.1 0.2