JOURNAL OF APPLIED ECONOMETRICS
J. Appl. Econ. **18**: 23–46 (2003)
Published online 8 October 2002 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/jae.664

A SIMPLE FRAMEWORK FOR ANALYSING BULL AND BEAR MARKETS

ADRIAN R. PAGAN[a]* AND KIRILL A. SOSSOUNOV[b]

[a] *Economics Program, Research School of Social Sciences, Australian National University, Canberra, ACT 0200, Australia and Nuffield College, University of Oxford*
[b] *New Economic School, Moscow*

SUMMARY

Bull and bear markets are a common way of describing cycles in equity prices. To fully describe such cycles one would need to know the data generating process (DGP) for equity prices. We begin with a definition of bull and bear markets and use an algorithm based on it to sort a given time series of equity prices into periods that can be designated as bull and bear markets. The rule to do this is then studied analytically and it is shown that bull and bear market characteristics depend upon the DGP for capital gains. By simulation methods we examine a number of DGPs that are known to fit the data quite well—random walks, GARCH models, and models with duration dependence. We find that a pure random walk provides as good an explanation of bull and bear markets as the more complex statistical models. In the final section of the paper we look at some asset pricing models that appear in the literature from the viewpoint of their success in producing bull and bear markets which resemble those in the data.

1. INTRODUCTION

In the past two decades a great deal of attention has been paid to the documentation of many features of returns in equity markets. It has been shown that equity returns exhibit a range of features such as an equity premium, volatility clustering and fat-tailed densities e.g. see the surveys by Campbell, Lo and MacKinlay (1997) and Pagan (1996) *inter alia*. But much less attention has been paid to the feature that probably attracts more commentary than anything else, namely that there are extensive periods of time when equity prices rise and fall. Colloquially these periods of time are referred to as bull and bear markets respectively. Because it is less studied, the objective of this paper is to provide a framework for thinking about and analysing such *phases* of the market.

Since the movement from a bull market to a bear market phase (and conversely) involves a *turning point* in the market, Section 2 proposes a definition of such an event. The idea we use is motivated by similar research that has been carried out when detecting turning points in the business cycle—in particular the seminal work of Bry and Boschan (1971) and its application in King and Plosser (1994), Watson (1994) and Harding and Pagan (2002). We develop an algorithm that seems to be quite successful in locating periods in time that have been thought of as bull and bear markets in US equity prices. Once the turning points are established characteristics of the phases can be identified. This exercise is performed on monthly data for the equivalent of the S&P500 for the USA over the years 1835/1–1997/5.[1]

* Correspondence to: Adrian R. Pagan, Economics Program, Australian National University, Canberra, ACT 0200, Australia. E-mail: adrian.pagan@anu.edu.au

[1] Pagan (1998) does this exercise for two other countries, the UK and Australia. There are many similarities to the USA in the types of markets for these countries but, because most theoretical models have been calibrated to US data, we focus on it in the current paper.

Received 26 July 2000
Revised 2 October 2001

Section 3 gives a formal statement of the criterion selected to determine a turning point and Section 4 canvasses equivalent forms that are sometimes more useful for analysis. It emerges that the nature of bull and bear markets will depend upon the type of data generating process (DGP) for capital gains in the market. For example, if equity prices follow a random walk with normally distributed increments, all the characteristics of bull and bear markets will depend solely upon the mean and volatility of capital gains. In Section 5 we investigate the type of bull and bear markets generated by a number of statistical models that have been proposed as candidates for the DGP of the capital gains—a random walk, GARCH and EGARCH models and a hidden layer Markov chain model.

Section 6 looks at the type of markets generated by DGPs that come from economic rather than purely statistical models. We provide a general discussion about this and then focus in some detail upon models by Gordon and St–Armour (2000) and Campbell and Cochrane (2000). We also look at some work by Donaldson and Kamstra (1996) which explains the bull and bear markets of the 1920s and we consider some aspects of their explanation. Since economic models are generally of the calibrated variety they are capable of being simulated, whereupon the output from them may be passed through the dating algorithm to establish whether the bull and bear markets they imply match up with what has been observed. In general we find that all models have difficulty in producing realistic bull and bear markets and we use our framework to provide a simple explanation of why this is so. Finally, one by-product of the framework is that it facilitates the study of extreme events such as large increases in stock values during bull markets. We look at this use in the context of equity market behaviour in the 1920s. Experiments such as this may also be useful for Value at Risk work focusing on event risk.

2. DATING ALGORITHMS

In its earliest manifestation the definition of bull and bear (B&B) markets seems to correspond to that given in Chauvet and Potter (2000, p. 90, fn. 6): ‘In stock market terminology, bull (bear) market corresponds to periods of generally increasing (decreasing) market prices.’

Recent usage in the financial press, however, seems to have refined this to insist on the rise (fall) of the market being greater (less) than either 20% or 25% in order to qualify for these names. In many ways the more general definition offered in the quotation above would seem to be closer to that used to describe contractions and expansions in the business cycle literature while the second, by emphasizing extreme movements, would be analogous to ‘booms’ and ‘busts’ in the real economy. We will adopt the first definition, although it will become clear that the analysis could equally have been done with the second and we actually do measure this to some extent. Thus our definition implies that the stock market has gone from a bull to a bear state if prices have declined for a substantial period since their previous (local) peak. This definition does not rule out sequences of negative price movements in stock prices during a bull market or positive ones in bear markets, but we will have to provide some extra rules to restrict the extent of these movements.

Given this definition we need to be able to describe turning points in the series. In the business cycle literature an algorithm for doing this was developed Bry and Boschan (BB) (1971). It is important to recognize that the BB program is basically a pattern-recognition program and it seeks to isolate the patterns using a sequence of rules. Broadly these are of two types. First, a criterion is needed for deciding on the location of potential peaks and troughs. This is done by finding points which are higher or lower than a window of surrounding points. Second, durations between these points are measured and a set of censoring rules is then adopted which restricts the minimal lengths of any phase as well as those of complete cycles. Because one is simply seeking patterns in the data, the philosophy underlying the BB program is relevant to *any* series, but the nature of asset prices is sufficiently different from real quantities as to suggest that some modification may be needed in the precise way that pattern recognition is performed. In particular, while the determination of a set of initial peaks and troughs of the business cycle is done by using (monthly) data that are smoothed, and from which 'outliers' have been removed, this is not as attractive with monthly asset price data. In fact, the process of eliminating 'outliers' may actually be suppressing some of the most important movements in the series. Considerations such as these lead us to make a number of modifications to the BB procedures.

Our first deviation from BB is not to smooth any of the series, while the second relates to the size of window used in making the initial location of turning points. In the BB program this is six. It is not entirely clear how to choose this parameter when dealing with asset prices. But given the lack of smoothing it seemed sensible to make this slightly longer, and we eventually settled on eight months as the appropriate length for asset prices.

Our second deviation relates to some rule for deciding on the minimum time one can spend in any phase. In business cycle dating this is six months. To try to determine something that would be appropriate for stock prices we consider some of the earliest formal literature that emphasizes the terms 'bull and bear markets'. This literature, Dow Theory, was developed by Charles Dow at the turn of the century and popularized by W. P. Hamilton in editorials in the *Wall Street Journal*. Dow theory saw the stock market as composed of three distinct movements and distinguished between

> The daily fluctuation...a briefer movement typified by the reaction in a bull market or the sharp recovery in a bear market which has been oversold...and the main movement. which decides the trend over a period of many months' (Hamilton, 1919).

Hamilton also regarded the main or primary trend as 'The broad upward and downward movements known as bull and bear markets' while the secondary reaction was 'an important decline in a primary bull market or a rally in a primary bear market. These reactions usually last from three weeks to as many months.'

Since this paper shares with Dow theorists a fundamental interest in the primary movements, the quotations above point to a minimal length for a stock market phase of three months. We therefore set it at four months.[2]

Some minimum length to the complete cycle also needs to be prescribed. Dow Theory is somewhat vaguer about this. Dow defined a primary bull market as one with a broad upward movement, interrupted by secondary reactions, and averaging longer than two years. Furthermore, Hamilton says

> There are the broad market movements; upwards or downwards, which may continue for years and are seldom shorter than a year at the least (*Wall Street Journal*, 26 February 1909).

[2] It might be thought that the minimum phase length would be eight months since that would be an implication of the rule used to get the initial dates but later we allow a deviation from normal phase lengths based on quantitative movements in the stock price index.

Twenty-four months would therefore be one possibility but, with an eye on the hedging engaged in by Hamilton above, as well as the general recognition that bull and bear markets are unlikely to be equal in duration, the case for setting a complete cycle shorter than two years is strong. In business cycle dating the minimal cycle length is fifteen months, so we stay close to it at sixteen months. Moreover, this fits with the identification of original peaks and troughs as using a symmetric window of eight periods.

Finally, given the sharp movements that have been seen in stock prices it does seem as if some quantitative constraint needs to be appended to the rules above. Consider October 1987, for example. In terms of peaks and troughs the contraction lasted only for 3 months, after which a recovery occurred, so this would not be regarded as a bear market due to the duration of the price decline being too short. That seems unsatisfactory. Allowing bear markets to have less than a 3-month minimum duration would, however, almost certainly produce many spurious cycles. Hence an extra constraint that the minimal length of a phase (four months) can be disregarded if the stock price falls by 20% in a single month was appended to the rules. Appendix B sets out the rules used to establish the turning points of the series studied in this paper.

There are other algorithms that we might use. Instead of describing how a turning point occurs, we might instead define a binary random variable S_t taking the value unity when a bull market occurs at time t and zero for a bear market, and then describe how one goes from the state S_t to S_{t+1}. Such an algorithm was followed by Lunde and Timmermann (2000). Since the probability at time t of a peak would be $\Pr(S_t = 1, S_{t+1} = 0)$, and the method used by Lunde and Timmermann to date a switch from a bull to bear market focuses upon $\Pr(S_{t+1} = 0 | \ S_t = 1)$, it is clear that the differences reside in the need to specify $\Pr(S_t = 1)$ in the Lunde–Timmermann case, i.e. in practice one needs to know the initial state S_0 to perform their dating. Obviously a turning-point approach does not require knowledge of S_0. Both methods have their attractions. The fact that one does not need to guess at S_0 means that the turning-point method appeals. However, describing what would cause a transition between states rather than a turning point may make it easier to take account of the magnitude of price changes as a determinant of changes in states. To do so via the dating method one needs to incorporate the magnitude restriction as a censoring operation, and this will be done in the later numerical work. Of course we might use a combination of the two.

3. SOME FACTS ON BULL AND BEAR MARKETS

Figure 1 plots the natural log of the monthly stock price index $\ln P_t$ for the USA over the period 1835/1–1997/5. The series is equivalent to the S&P500 and the data sources are given in Appendix A. It is clear that there are many expansions and contractions in the series, particularly the enormous decline in stock values during the Great Depression.

To summarize this history we apply the algorithm that incorporates the dating methods discussed above to the series. Once we establish where the turning points occur it is possible to summarize various characteristics of the movements between each of these points; such expansions and contractions are termed phases. We consider five such measures of the nature of these phases.

(1) The average duration of each phase, D.
(2) The average amplitude of each phase, A. For convenience, we define 'amplitude' as the difference in the logs of the stock price from one turning point to another. This does not yield an exact measure of the actual percentage change in the equity price over a phase owing to the

Figure 1. Log US stock prices 1835/1–1997/5

fact that these movements are sometimes large and therefore the approximation $\ln(1+x) = \mathrm{x}$ breaks down.

(3) The average cumulated movements in $\ln P_t$ over each phase, C.

(4) The 'shape' of the phases as measured by their departure from being a triangle. The index used for this purpose is the 'excess' index in Harding and Pagan (2002), *EX*, which is the average over all phases of the value of $(C - 0.5A - 0.5(A \times D))/D$ for each phase.

(5) The fraction of expansions and contractions for which $A \geq 0.18$ and $A \leq -0.22$. These numbers translate into increases in the equity price of more than 20% and decreases of less than 20%. The motivation for considering such statistics is that some definitions of bull and bear markets require expansions and contractions that are of these magnitudes. We will refer to these as the B^+ and B^- proportions.

After defining S_t as a binary random variable taking the value unity if a bull market exists at time t and zero if it is a bear market, we can estimate the quantities above in the following way. First, the total time spent in an expansion is $\sum_{t=1}^{T} S_t$ and the number of peaks (hence expansions) is given by $NTP = \sum_{t=1}^{T-1}(1 - S_{t+1})S_t$.[3]

Therefore the average duration of an expansion will be [4]

$$\hat{D} = NTP^{-1}\sum_{t=1}^{T} S_t$$

[3] Notice that the algorithm makes turning points alternate so that the number of peaks and troughs must differ by one at most. In the asymptotic theory we treat the number as being the same.

[4] Some difficulties arise due to incomplete phases at both ends of the sample. Since we actually measure the features of completed phases in the tables of the text the summation should run from the beginning of the first completed phase until the end of the last one rather than over $1, \ldots, T$. For convenience we use the complete sum in these formulae.

The average amplitude of expansions will be

$$\hat{A} = NTP^{-1}\sum_{t=1}^{T} S_t \Delta \ln P_t$$

To obtain the cumulated change over any expansion we have to define $Z_t = S_t Z_{t-1} + S_t \Delta \ln P_t, Z_0 = 0$. Then Z_t contains the running sum of $\Delta \ln P_t$ provided $S_t = 1$, with the sum being automatically reset to zero whenever $S_t = 0$. Hence the total of cumulated changes over all expansions is

$$TC = \sum_{t=1}^{T} Z_t$$

with the average being

$$\hat{C} = NTP^{-1}\sum_{t=1}^{T} Z_t$$

All these quantities can be found up to a factor of proportionality from regressions, e.g. $\hat{C}$ comes from the regression of Z_t against unity. To get $\hat{C}$ one needs to adjust for an incorrect scaling factor, e.g. the regression coefficient in the regression just described would need to be multiplied by T/NTP. The estimated excess is also a multiple of $(1/NTP)$ where the multiple is the sum of $(\hat{C}_i - 0.5\hat{A}_i - 0.5\hat{A}_i\hat{D}_i)/\hat{D}_i$ over all the phases $i = 1, \ldots, NTP$. Finally, since the series $(1 - S_{t+1})S_t$ has unity at the peak of an expansion and zeros elsewhere, while Z_t contains the amplitude of each expansion at the point in time t, the amplitudes of expansions are the non-zero values of $(1 - S_{t+1})S_t Z_t$. Consequently,

$$B^{+} = NTP^{-1}\sum_{t=1}^{T-1} I[(1 - S_{t+1})S_t Z_t > 0.18]$$

where $I[a] = 1$ if a is true and zero otherwise. Bear market statistics are found in the same way by replacing S_t with $1 - S_t$.

To compute standard errors for the estimators we resort to some asymptotic theory. Consider $\hat{D} = NTP^{-1}\sum_{t=1}^{T} S_t$. We can write this as $\hat{D} = \hat{p}^{-1}T^{-1}\sum_{t=1}^{T} S_t$, where $\hat{p} = NTP/T$. Then, since

$$\frac{1}{T}\sum_{t=1}^{T} S_t$$

converges to $E(S_t)$, the delta method says that the distribution of $\sqrt{T}(\hat{D} - D)$ is asymptotically determined by that of $[p^{-2}\sqrt{T}(\hat{p} - p)]E(S_t)$. Now $\hat{p}$ is the regression coefficient of $(1 - S_{t+1})S_t$ against unity and so we can find its standard deviation using HAC standard errors. We use the formula in Hamilton (1994, p. 283, eq. (10.5.21)) for this, and account for 20th-order serial correlation. It is clear that all the statistics above are asymptotically multiples of $\hat{p}^{-1}$ and so the same theory applies.

As one might expect from the derivation, it is not clear that asymptotic theory will be very effective here as it hinges on getting a good approximation to the distribution of $\hat{p}^{-1}$. Indeed simulations of the $\hat{D}$ estimators from a model in which the underlying DGP for equity prices was a random walk with drift showed that the distribution was highly non-normal even when the sample size exceeds 1200. Thus the robust standard errors found by asymptotic analysis may be unreliable as guides to the dispersion of the estimators. Nevertheless, because we are generally interested in whether a particular model can generate the observable characteristics of bull and bear markets, it is possible to simulate sets of observations (of length equal to the sample size) from the DGP of that model, and thereby find p-values of tests for the hypotheses that the model and sample characteristics are the same.

Table I provides the statistics just described for the USA for three sample sizes as well as asymptotic standard errors. As we have noted above, the latter may be an unreliable indicator of the precision of the estimators so we provide (in the footnote to the table) the standard errors that one would get if the DGP had been a random walk with drift whose parameters are estimated over the period 1889/1–1997/5. There are some obvious differences but, by and large, one would get much the same story from using either set.

The statistics of Table I are interesting. It is clear that bull markets tend to be longer than bear markets and the durations agree quite closely with those attributed to Hamilton in 1921 (Rhea, 1932, p. 37) who claimed that, over the preceding 25 years, bull markets had lasted an average of 25 months while bear markets had lasted 17 months. Over time it seems as if bear markets have become shorter and weaker while bull markets have become stronger. The US stock market also exhibits expansions and contractions that deviate quite a lot from a triangular shape and this tendency may have become more emphatic over time. The fact that there is a departure from a

Table I. Statistics on bull and bear markets in US stock price data[a,b]

	1835/1–1997/5	1889/1–1997/5	1945/1–1997/5
Bear duration	15	14	12
	(1.68)	(1.70)	(2.09)
Bull duration	25	25	27
	(2.70)	(3.17)	(4.74)
Bear amplitude	−0.31	−0.31	−0.23
	(0.03)	(0.04)	(0.04)
Bull amplitude	0.43	0.45	0.46
	(0.05)	(0.06)	(0.09)
Bear cumulated	−2.67	−2.59	−1.52
	(0.28)	(0.32)	(0.29)
Bull cumulated	7.29	7.71	7.70
	(0.79)	(0.96)	(1.40)
Bear excess	0.024	0.021	0.014
	(0.002)	(0.002)	(0.003)
Bull excess	0.019	0.026	0.030
	(0.004)	(0.004)	(0.006)
B^-	0.60	0.52	0.38
	(0.07)	(0.06)	(0.07)
B^+	0.83	0.88	0.93
	(0.09)	(0.10)	(0.16)

[a] Amplitudes are % changes, durations are in months. Asymptotic standard errors are in parentheses.
[b] Standard errors based on random walk simulation in Table IV are 1.93, 3.08, 0.03, 0.05, 0.68, 2.04, 0.005, 0.007, 0.09 and 0.07.

triangle in the evolution of the markets is also true for the US business cycle; see Sichel (1994) and Harding and Pagan (2002). Finally, it is clear that most bull markets rise more than 20% while a much smaller fraction of bear markets culminate in a fall of more than 20%.

As a check on the dating algorithm, Table II compares our results for the USA to some post-war stock market cycle dates quoted in Niemira and Klein (1994, Table 10.2, p. 431) that have been used by Chauvet and Potter (2000). Their results are in parentheses. The correspondence is quite good, except for an extra contraction from April 71 to November 71 (the Niermira/Klein dating stops before the last contraction identified). There was a 10% contraction over this period and it may be that the Niermira/Klein results incorporate some censoring based on the magnitude of movements in share prices.

Finally, as mentioned in the Introduction, most attention in the literature has been paid to the mean and volatility of $\Delta \ln P_t$. Moreover, Slutsky (1937) and Fisher (1925) both emphasized that what seem to be regular ups and downs in a series can simply arise from stochastic variation. Fisher termed this phenomenon the 'Monte Carlo cycle'. Malkiel (1973) noted that a random walk in stock prices would produce cycles. Hence, as there is clearly going to be some connection between B&B market phenomena and these two moments, we present values for the mean μ and standard deviation σ of $\Delta \ln P_t$ for each of the three samples in Table I. These quantities are used in the simulations of the next section.

Table III shows that the mean capital gain has been increasing over time and the standard deviation has been falling. However, the decline in the latter is really quite small and statistically insignificant.

Table II. Post-war US stock market cycles: two dating methods (Niermira/Klein in parentheses)

Peak	Trough
1946/5 (1946/4)	1948/2 (1948/2)
1948/6 (1948/6)	1949/6 (1949/6)
1952/12 (1953/1)	1953/8 (1953/9)
1956/7 (1956/7)	1957/12 (1957/12)
1959/7 (1959/7)	1960/10 (1960/10)
1961/12 (1961/12)	1962/6 (1962/6)
1966/1 (1966/1)	1966/9 (1966/10)
1968/11 (1968/12)	1970/6 (1970/6)
1971/4	1971/11
1972/12 (1973/1)	1974/9 (1974/12)
1976/12 (1976/9)	1978/2 (1978/3)
1980/11 (1980/11)	1982/7 (1982/7)
1983/6 (1983/10)	1984/5 (1984/7)
1987/8 (1987/9)	1987/11 (1987/12)
1990/5 (1990/6)	1990/10 (1990/10)
1994/1	1994/6

Table III. Mean and standard deviation of capital gains

	1835/1–1997/5	1889/1–1997/5	1945/1–1997/5
μ	0.0031	0.0039	0.0066
σ	0.044	0.0448	0.0404
μ/σ	0.07	0.09	0.16

4. THE ANALYTICS OF BULL AND BEAR MARKETS

To gain some appreciation of how the type of DGP determines B&B markets, return to how initial turning points in a series were selected. A peak was taken to have occurred at time t if the event

$$PK = [\ln P_{t-8}, \ldots, \ln P_{t-1} < \ln P_t > \ln P_{t+1}, \ldots, \ln P_{t+8}]$$

occurs, where P_t is the level of the stock price. Thus the probability of the event PK occurring will depend upon the joint distribution of $\{\Delta \ln P_{t+k}\}_{k=-8}^{8}$ and, to determine that probability, one requires a specification of the DGP for $\Delta \ln P_t$. For example, if $\Delta \ln P_t$ was $N(\mu, \sigma^2)$, then the $\Pr(PK)$ would be solely a function of μ/σ, since the turning points in $\ln P_t$ are identical to those in $\ln(P_t/\sigma)$. Consequently, it is likely that the probability will rise with μ (the mean capital gain) and decline with σ. Of course, there is more to the dating rules than that. After the initial turning points are found a set of censoring operations is applied that will change the probability of 'final' turning points. Unfortunately, it becomes very hard to assess the precise impact of those operations analytically and so we will be forced to resort to numerical simulation. Nevertheless, the insight obtained from looking at what determines the initial turning points is extremely valuable in analysing bull and bear markets. In particular, it is clear that, regardless of the model for $\Delta \ln P_t$, the ratio of μ to σ will be a key determinant of cycle characteristics. As an illustration of this point, note that the movements in this ratio in Table III are very suggestive about the actual changes over time in bull and bear market characteristics noted in Table I. Moreover, it is clear that any theoretical model which claims to provide an explanation of historical bull and bear markets will have to be capable of reproducing the historical values of μ and σ. Since μ is related to the equity premium one must therefore be able to replicate that as well as the volatility of capital gains. Whether this is sufficient is something that we investigate in the next section.

5. SOME STATISTICAL MODELS OF RETURNS

As the previous section showed, it is the DGP of $\Delta \ln P_t$ that is the key to understanding bull and bear markets. To this end we might categorize the potential DGPs into those for which the capital gain is a martingale and those for which it is not. The simplest martingale model would just be the basic random walk with drift.

$$\Delta \ln P_t = \mu + \sigma\varepsilon_t \tag{1}$$

where ε_t is n.i.d.(0,1).[5] Columns two and three of Table IV provide a summary of the bull and bear markets that would be seen in realizations of the DGP (1) when viewed through the dating filter described earlier.[6] In the simulations $\mu = 0.0039$ and $\sigma = 0.0448$ are taken from the US data for 1889/1–1997/5; see Table III. Ten thousand realizations of the DGP of the chosen statistical model are used to provide the sampling statistics. In parentheses under each quantity is the standard error of the estimator of that quantity using a sample of observations of the same length as

[5] Strictly speaking, this is not a martingale unless one removes the drift but we will keep the terminology.

[6] Our frame of reference for discriminating between models is the 'cycle information'. Of course equation (1), with values of μ and σ from Table III, can almost certainly be rejected as failing to replicate other features of the data. The simplest would just be the magnitude of $\Delta \ln P_t$ in any month. Since three standard deviations from the mean is an implausible outcome for $\Delta \ln P_t$, using US estimates of μ and σ over 1835/1–1997/5 would imply that a value of $\Delta \ln P_t$ less than -0.127 (a 12.7% contraction) should rarely come up, whereas it actually occurs about 1% of the time in the US data.

Table IV. US bull and bear markets generated by various statistical models[a,b]

	Data	RW $\mu = 0$	RW $\mu \neq 0$	GARCH $\mu \neq 0$	EGARCH $\mu \neq 0$	DDMS-DD
Dur bear	14	20** (2.36)	16 (1.93)	16 (1.92)	16 (1.95)	16 (2.00)
Dur bull	25	20** (2.35)	24 (3.08)	25 (3.26)	26 (3.41)	26 (3.48)
Amp bear	−0.31	−0.33 (0.034)	−0.27 (0.03)	−0.26 (0.04)	−0.30 (0.04)	−0.28 (0.04)
Amp bull	0.45	0.33** (0.034)	0.42 (0.05)	0.42 (0.06)	0.46 (0.05)	0.44 (0.06)
Cum bear	−2.59	−4.31* (1.14)	−2.70 (0.68)	−2.63 (0.78)	−2.87 (0.88)	−2.78 (0.85)
Cum bull	7.71	4.34** (1.09)	7.22 (2.04)	7.42 (2.16)	8.39 (2.41)	8.48 (2.61)
Ex bear	0.020	−0.00** (0.006)	−0.00** (0.005)	−0.000** (0.006)	0.006* (0.007)	0.005* (0.008)
Ex bull	0.026	0.00** (0.006)	0.00** (0.007)	0.000** (0.008)	0.011* (0.009)	0.007* (0.01)
B^-	0.52	0.69* (0.08)	0.56 (0.09)	0.50 (0.10)	0.56 (0.1)	0.50 (0.09)
B^+	0.88	0.80 (0.07)	0.88 (0.06)	0.85 (0.07)	0.89 (0.06)	0.84 (0.07)

[a] Amplitudes are logarithmic changes, durations are in months.
[b] Standard errors in parenthesis; 10 000 replications.
* p-value < 0.05; ** p-value < 0.01.

1889/1–1997/5. However, since the estimators are non-normal, p-values were also computed. If there is a single asterisk then the p-value is less than 0.05, while a double asterisk means it is less than 0.01. If there is no asterisk the p-value is greater than 0.05.

It is clear that the random walk with drift does quite well at replicating the bull and bear markets actually observed, but the pure random walk ($\mu = 0$) fails quite badly. In fact, with the latter we would expect symmetry in the characteristics of the phases, since the probabilities of an initial peak and a trough occurring would be identical. The only exception to this comes with respect to the B^+ and B^- statistics. There the different censoring thresholds (-0.22 and 0.18) utilized to ensure that B&B market movements produce a 20% movement in P_t are the sources of the asymmetries in these proportions. Given this result on the importance of μ it is clear that an explanation of its magnitude will be a key element in getting the nature of bull and bear markets right. A notable deficiency with the random walk model is its implication that phases should, on average, look like triangles, whereas this is clearly not so.

A possible extension to the basic random walk model is motivated by the fact that (1) implies that capital gains are normally distributed, while the sample excess kurtosis for the 1899–1997 period is 9.2. Consequently, one wants to adopt a DGP that produces realizations for $\Delta \ln P_t$ from a non-normal density. One response would be to change the density for ε_t to others with fatter tails, e.g. Student's t, but it is more interesting to generate the excess kurtosis 'endogenously'. Some standard ways of doing that are to treat $\Delta \ln P_t$ as being either a GARCH(1,1) or EGARCH(1,1) process, i.e. σ in (1) is replaced by σ_t which varies with the past history of returns. Accordingly, these models were fitted to US capital gains over 1889/1–1997/5 yielding:

$$\sigma_t^2 = 0.000087 + 0.13e_{t-1}^2 + 0.83\sigma_{t-1}^2$$

$$\ln \sigma_t^2 = -0.314 - 0.06\varepsilon_{t-1} + 0.26\left(|\varepsilon_{t-1}| - \sqrt{\frac{2}{\pi}}\right) + 0.95 \ln \sigma_{t-1}^2$$

In order to perform a valid comparison with the random walk model we also make the means of the GARCH and EGARCH capital gains identical to those in the data, i.e. $\mu = 0.0039$.

Columns four and five of Table IV then process the simulated output from the GARCH and EGARCH processes. Given the symmetry of the GARCH process it is not surprising that it has little effect upon mean durations, but in general it seems that the GARCH model does not add much to the explanation of B&B markets over that provided by the random walk. Given that the GARCH specification produces fatter tails in the distribution of $\Delta \ln P_t$, it is a somewhat surprising outcome that bull and bear markets are slightly less extreme under it. Of course it has to be remembered that it is *cumulated* shocks that are important for bull and bear markets and the GARCH model is just as likely to produce a large positive shock as a negative one and these operate to offset one another. The EGARCH model tends to provide a better match to most of the phase characteristics than the GARCH model does. In particular, it is the only model that has the ability to produce shapes of phases that resemble the data, although even then the p values suggest that there is still some discrepancy between them.

The above models have $\Delta \ln P_t$ being a martingale difference. It has long been observed that there is little linear dependence in $\Delta \ln P_t$, motivating the search for some non-linear structure. One might fit some general non-linear models, such as neural networks or threshold autoregressions, but in some ways it would be nicer to be able to produce the non-linearity in a framework that preserves the flavor of the topic being examined. Because of the emphasis being laid upon the two types of markets, it is useful to try to obtain the requisite non-linearity by utilizing the literature on hidden layer Markov chains. Hamilton's (1989) work is the best-known example of this in econometrics, although in other fields there have been other versions.

In its simplest form Hamilton's model replaces (1) with

$$\Delta \ln P_t = (\mu_0 + \sigma_0 e_t)(1 - z_t) + (\mu_1 + \sigma_1 e_t)z_t \tag{2}$$

where z_t is a random variable taking the values of zero and unity whose evolution is governed by a Markov chain with transition probabilities $p_{00} = \Pr(z_t = 0|z_{t-1} = 0)$ and $p_{11} = \Pr(z_t = 1|z_{t-1} = 1)$. Which state corresponds to which type of market is essentially arbitrary. For the purpose of presentation and comparison of results we identify the bull state as that which has a higher mean capital gain and label it the state corresponding to $z_t = 1$, although we stress again this is quite arbitrary. Many applications of Hamilton's model and its extensions have been made in the econometric literature. Pagan and Schwert (1990) applied the basic model to US stock returns from 1835 until 1925. Recently attempts have been made to generalize this model to allow for the transition probabilities to depend upon the length of time spent in a particular state, i.e. to produce duration dependence. Maheu and McCurdy (2000) is a good example. They fitted a model in which the transition probabilities had the same format as in Durland and McCurdy (1994), namely

$$\Pr(z_t = j | z_{t-1} = j, d_t) = \frac{\exp(\psi_{1j} + \psi_{2j} d_t)}{1 + \exp(\psi_{1j} + \psi_{2j} d_t)} \tag{3}$$

where d_t, the duration of time spent in the jth state in the current phase at time t, is constrained to not exceed 16. The model they preferred was called DDMS-DD and we took the parameters from their Table V to simulate it. One problem is that they fit the model to monthly US returns rather than to capital gains. Whilst the volatility of the returns series is much the same as capital gains, since the dividend yield shows relatively small monthly variation, the mean of returns is higher than that of capital gains. Hence we adjusted the μ_0 and μ_1 parameter estimates in their model by a scaling factor of 1.7 so that the overall mean of the simulated data agreed with that in Table IV. By keeping the mean and variance of the simulated returns equal to that of the data we are therefore solely studying the effect of introducing duration dependence into the model. Data is simulated from this model in column 6 of Table IV. It provides no improvement on the results from the EGARCH model.[7]

In summary, one can conclude that the broad characteristics of B&B markets are a consequence of the random walk nature of the data but that the 'shapes' of the markets are not well accounted for by any of the statistical models studied here. It does seem however that whatever feature is

Table V. US bull and bear markets generated by the Gordon–St Armour model, equivalent random walk and data (1960/1–1992/6)[a,b]

	Data	GSA	RW
Bear duration	11	10	15
	(2.67)	(4.92)	(3.5)
Bull duration	25	52*	27*
	(5.48)	(21.30)	(7.2)
Bear amplitude	−0.27	−0.12*	−0.20
	(0.059)	(0.070)	(0.04)
Bull amplitude	0.44	0.41	0.39
	(0.099)	(0.162)	(0.09)
Bear cumulated	−1.49	−0.3	−1.76
	(0.323)	(1.88)	(0.95)
Bull cumulated	6.81	16.7	7.70
	(1.52)	(16.3)	(4.69)
Bear excess	0.029	0.000	0.00**
	(0.004)	(0.024)	(0.007)
Bull excess	0.025	0.003	0.00**
	(0.008)	(0.026)	(0.013)
B^-	0.5	0.22	0.33
	(0.11)	(0.16)	(0.16)
B^+	1.0	0.71	0.83
	(0.22)	(0.21)	(0.14)

[a] Amplitudes are logarithmic changes, durations are in months.
[b] Standard errors in parenthesis; 1000 replications.
* p-value < 0.05; ** p-value < 0.01.

[7] Although it should be noted that Maheu and McCurdy estimated the model with data from 1835 and not 1899.

needed to account for the shapes, it is likely that it will involve some asymmetry in the conditional density of the capital gains process.

6. SOME ECONOMIC MODELS

6.1. General Analysis

Most economic models of stock prices can be expressed as

$$P_t^r = E_t\left[\sum_{j=1}^{\infty} IMRS_{t+j,t} D_{t+j}^r\right]$$

where $P_t^r = P_t/P_{ct}$ is the real stock price at the end of period t, $D_t^r = D_t/P_{ct}$ are real dividends paid in time t, $IMRS_{t+j,t}$ is the inter-temporal marginal rate of substitution (or pricing kernel) between time t and $t+j$ and P_{ct} is a consumption price deflator. It is useful to write $D_{t+j}^r = D_t^r(1+g_{t+j,t})$ so that the expression for P_t has the form

$$P_t^r = D_t^r E_t\left[\sum_{j=1}^{\infty} IMRS_{t+j,t}(1+g_{t+j,t})\right] \tag{4}$$

$$P_t^r = D_t^r K_t \tag{5}$$

reflecting the conditioning of the expectation upon information at time t. Then, provided $\Delta \ln K_t$ is a second-order stationary process so that $\Delta E(\ln K_t) = 0$, we may write the log of (5) in real terms as

$$\Delta \ln P_t^r = \Delta \ln D_t^r + \Delta(\ln K_t - E(\ln K_t)) \tag{6}$$

while, in nominal terms, it becomes

$$\Delta \ln P_t = \Delta \ln D_t + \Delta(\ln K_t - E(\ln K_t)) \tag{7}$$

$$= \Delta \ln D_t + \Delta \xi_t \tag{8}$$

Equation (6) was written to show that B&B market characteristics depend only upon the deviation of $\ln K_t$ from its expected value and the latter is irrelevant to their nature.

All economic models which have been advanced to explain B&B markets can be interpreted as different ways of producing a DGP for $\Delta \ln P_t$. However, not all can be decomposed as in (7). Examples would be Campbell and Kyle (1993) and Lux and Marchesi (1999), in which the DGP of $\Delta \ln P_t$ comes from the interaction of decisions by heterogeneous agents, and equity prices are not the sum of discounted dividends, since only a fraction of agents adhere to fundamentals in making decisions. However, the majority of models do adhere to this framework and can be classified according to how they propose to model one (or more) of three of the elements in (7). These three categories are:

(a) The nature of the expectation operator, E_t.
(b) The specification of $IMRS_{t+j,t}$
(c) The nature of the dividend process as given by D_t and $g_{t+j,t}$.

Which of the above are we likely to need as elements in any successful explanation of B&B markets.? To get some feel for the answer to this question, take a benchmark case where $\Delta \ln D_t \sim N(\mu_D, \sigma_D^2)$, $\Delta \ln D_t^r \sim N(\mu_D^r, \sigma_D^{2r})$ and there is a constant discount rate. Then $IMRS_{t+j,t} = (1/(1+r))^j$ and

$$K_t = \sum_{j=1}^{\infty} \left(\frac{1+\mu_D^r}{1+r} \right)^j$$

implying that $\Delta \ln K_t = 0$ and the log of capital gains will be a random walk with mean μ_D and volatility σ_D^2. Thus in this benchmark case the capital gains process has the same moments as the dividend growth process. Moreover it is clear from (7) that the mean of the capital gains process will always be the same as that for dividend growth. But the volatility of $\Delta \ln P_t$ will depend upon the relative strength of the volatilities of $\Delta \ln D_t$ and $\Delta \xi_t$.

Is there a difference between the moments of the observed capital gains and the nominal dividend growth processes?. To gain some appreciation of the likelihood of any such difference it can be noted that, over the period 1875/1–1997/5, the ratios μ/μ_D and σ/σ_D were 1.3 and 1.03 respectively, while, over the post-World War II era, they become 1.12 and 2.97 respectively. Hence, to explain post-World War Two markets one needs a substantial contribution from $\Delta \xi_t$, i.e. $\Delta \ln K_t$ to the volatility of capital gains. Plots of the series show this decline in volatility in a striking way—see Figure 2.

So the stylized facts suggest that it will be necessary for models to produce substantial volatility in $\Delta \ln K_t$ and we look at a few proposals along these lines. A first candidate might be Cecchetti *et al.* (2000) who propose a model in which the endowment process (which we will describe as dividends) follows a Markov switching process but in which there are distorted beliefs, in the sense that the expectation operator uses a different measure from that which describes the endowment process. Thus the agents have misperceptions about the transition probabilities of the MS process

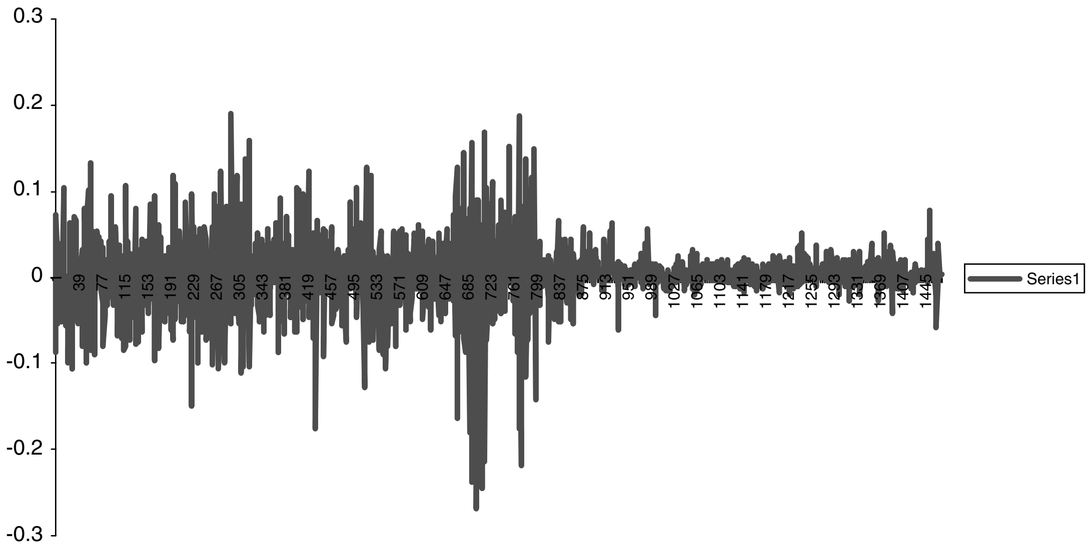

Figure 2. Dividend growth 1875/1–1995/12

for dividends. Beliefs about the true transition probabilities are modelled as a two-state process and it is this that enables $\Delta \ln K_t$ to be volatile. However, although this is an intriguing model, it is hard to assess it here because it is calibrated to yearly data. Instead we concentrate upon two other models that are also designed to produce the requisite volatility but are calibrated for monthly data and so we are able to simulate monthly observations on equity prices so as to study the implied B&B markets.

6.2. The Gordon–St Armour Model

As noted above we will be concerned here with models in which volatility in $\Delta \ln K_t$ is produced by inducing $IMRS_{t+j,t}$ to be a stochastic process. Gordon and St Armour (GSA) (2000) make it a two-state process, representing states of optimism and pessimism. In GSA the utility function has the form

$$\Theta \frac{(\Theta^{-1} C_t)^{1-\gamma_{S(t)}}}{1-\gamma_{S(t)}}$$

where $\gamma_{S(t)}$ can assume one of two values, γ_0 and γ_1, depending on the outcome of a binary random variable $S(t)$. The random variable $S(t)$ is driven by an exogenous Markov switching process. Thus the aggregate $IMRS_{t+j,t}$ has the form

$$IMRS_{t+j,t} = \psi_{S(t)0,j} IMRS_{t,t+j}^{S(t),0} + \psi_{S(t)1,j} IMRS_{t,t+j}^{S(t),1}$$

where $\psi_{kl,j}$ are probability weights determined by the transition probabilities from state k to l $(k, l = 0, 1)$ and

$$IMRS_{t,t+j}^{k,l} = \beta^j (\Theta^{-1} C_{t+j})^{-\gamma_l} / (\Theta^{-1} C_t)^{-\gamma_k}$$

for $k, l = 0, 1$.

Given a joint process for real dividend and consumption growth we can simulate Gordon and St Armour's model. This produces a value for K_t. Then, as nominal dividends follow a random walk, we can calibrate the process with values of μ_D and σ_D used in GSA ($\mu_D = 0.0048$, $\sigma_D = 0.012$), and subsequently generate simulated values for P_t. The latter are then passed through our dating algorithm to find the average characteristics of bull and bear markets implied by the GSA model. These are given in Table V along with the actual characteristics for their estimation period. Clearly, some of the model statistics are very unlike those for actual bull and bear markets. But the variance of these statistics, if the true model was GSA's, is very high and so it is hard to reject it as a description of the data. Nevertheless, since the standard errors of the simulated statistics are often so different from those found from asymptotic theory—which are designed to be robust to the true DGP—one feels that there is a serious question mark about the ability of this model to generate realistic B&B markets. Examining the simulated data it emerges that $\Delta \ln P_t$ has a mean and standard deviation of 0.0047 and 0.0364 respectively, while the actual values over their data period are 0.0051 and 0.0432, so it does not appear that the failure to match historical B&B markets is due to a failure to get these two moments right. To check this conclusion the last column of Table V examines simulated data from a process $\Delta \ln P_t \sim N(0.0047, 0.0364)$. Comparing this column with the preceding one shows that the strong bull and bear markets coming from the GSA model have to be due to some difficulties with the higher-order moments of $\Delta \ln P_t$. Indeed, inspection of simulated data shows that the problem seems to be that K_t jumps whenever γ changes value and this has a profound effect upon the probabilities of getting a turning point. It is interesting to note that, whilst $\mu = 0.0047$ and $\mu_D = 0.0048$ are virtually the same (as expected), the GSA model amplifies the standard deviation significantly—from $\sigma_D = 0.012$ to $\sigma = 0.036$—and this is a very desirable outcome.

6.3. The Campbell and Cochrane Model

Campbell and Cochrane (CC) (1999) also produce a model that allows the $IMRS_{t+j,t}$ to change over time. However, unlike the GSA solution, it now changes continuously. Specifically they make

$$IMRS_{t+j,t} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

where C_t is consumption and the state $\ln S_t$ evolves as a heteroscedastic autoregressive process. To simulate data from their model they need to make some assumptions about the parameters—δ (the risk-free rate) and γ—as well as the nature of the processes generating consumption and dividends. The logs of these latter variables are taken to be random walks with drift. Correlation between the innovations of the two series is allowed. The mean of the monthly real dividend growth rate was set to that of consumption, which they quantified from statistics on annual consumption over 1899–1992. For this reason we will utilize data on bull and bear markets over 1899/1–1997/5 as the benchmark to assess this model. For later reference CC's calibration assumes that $\mu_D = 0.00398$ and $\sigma_D = 0.0388$.[8] CC assess their model by its ability to replicate the equity premium, $\exp(E(\ln(P_t/D_t))$ and the volatility of $\ln(P_t/D_t)$. Thus our experiment provides an assessment from an alternative perspective. As we noted earlier, the term $E(K_t) = E(\ln(P_t/D_t)$ used in their assessment methods is irrelevant to the nature of B&B markets.

Table VI gives the results. It is apparent that the CC model replicates the data quite well and is superior to the random walk model in showing some ability to match the phenomenon captured by the 'excess' statistic. As the CC model is known to produce a leverage effect in volatility (something the EGARCH model was designed to capture) this result would be expected, given the outcomes in Table IV for the EGARCH model. In many ways its performance is reminiscent of what one gets with the EGARCH model. Thus it seems a promising candidate for further analysis of the nature of B&B markets.

6.4. Model Emphasizing Dividend Behaviour

Another approach is to focus upon the dividend process as the cause of B&B markets. The issue then becomes what type of process one should specify for dividend growth. Assuming that real dividends follow a random walk of the form

$$\Delta \ln D_t^r = \ln(1 + g_t) = \mu_D^r + \varepsilon_t \tag{9}$$

with $IMRS_{t+1,t} = (1 + r)^{-1}$, produces $K_t = [1 - r \times \exp(\mu_D^r + \frac{1}{2}\sigma_\varepsilon^2)]^{-1}$ provided $r \times \exp(\mu_D^r + \frac{1}{2}\sigma_\varepsilon^2) < 1$. Thus the price–dividend ratio becomes a constant which depends on the volatility of

[8] We used a program available on John Cochrane's web page to generate the data. The value CC assigned to μ_D^r comes from per capita real consumption data and to get μ_D this needs to be inflated with the rate of price inflation. Using the PPI yearly inflation rate over 1899–1997 as a deflator, the latter adds 0.0024 to their $\mu_D^r = 0.00158$.

Table VI. US bull and bear markets generated by the Campbell–Cochrane model, equivalent random walk and data (1899/1–1997/5)[a,b]

	Data	CC
Bear duration	14	16 (1.83)
Bull duration	25	23 (2.80)
Bear amplitude	−0.31	−0.32 (0.04)
Bull amplitude	0.45	0.47 (0.05)
Bear cumulated	−2.59	−3.10 (0.76)
Bull cumulated	7.71	7.63 (1.85)
Bear excess	0.020	0.006* (0.006)
Bull excess	0.026	0.009* (0.008)
B^-	0.52	0.67 (0.10)
B^+	0.88	0.92 (0.05)

[a] Amplitudes are logarithmic changes, durations are in months.
[b] Standard errors in parentheses; 1000 replications.
* p-value < 0.05; ** p-value < 0.01.

dividend growth as well as its mean. To make K_t stochastic therefore requires a more complicated process for real dividend growth than (9). One that has appeared in various guises in the literature allows for some serial correlation in $\ln(1+g_t)$, i.e. the model for dividend growth becomes

$$\Delta \ln D_t^r = \ln(1+g_t) = \mu_D^r + \phi(\ln(1+g_{t-1}) - \mu_D^r) + \varepsilon_t$$

Then one might attempt to evaluate (4) with this specification. This is quite difficult since

$$1 + g_{t+j,j} = \prod_{k=1}^{j}(1+g_{t+k})$$

and so

$$K_t = \sum_{k=1}^{\infty}(1+r)^{-k}E_t\prod_{i=1}^{k}(1+g_{t+i})$$

where $(1+r)^{-1} = IMRS_{t+1,t}$. If ε_t is $N(0,\sigma_\varepsilon^2)$ then

$$1 + g_{t+k,k} = \prod_{j=1}^{k}(1+g_{t+j})$$

is log-normal with $\ln(1 + g_{t+k,k})$ having moments

$$\mu_k = \mu_D^r k + \frac{\phi\ (1-\phi)^k}{1-\phi}(\ln(1+g_t) - \mu_D^r)$$

$$\sigma_k^2 = \frac{\sigma_\varepsilon^2}{(1-\phi)^2}\left(k - 2\phi\frac{1-\phi^k}{1-\phi} + \phi^2\frac{1-\phi^{2k}}{1-\phi^2}\right)$$

Therefore

$$E_t\prod_{i=1}^{k}(1+g_{t+i}) = \exp\left[\mu_D^r k + \frac{\phi\ (1-\phi^k)}{1-\phi}(\log(1+g_t) - \mu_D^r) + \frac{1}{2}\sigma_\upsilon^2\right]$$

and

$$K_t = \left(\frac{1+g_t}{\exp\mu_D^r}\right)^{\frac{\phi}{1-\phi}} A\sum_{k=1}^{\infty}\left[\left((1+r)^{-1}\exp\left(\mu_D^r + \frac{\sigma_\varepsilon^2}{2(1-\phi)^2}\right)\right)^k \right.$$
$$\left. \times\left(\exp\left(\frac{-\phi^2}{1-\phi^2}\frac{\sigma_\varepsilon^2}{2(1-\phi)^2}\right)\right)^{(\phi^{2k})} B^{(\phi^k)}\right]$$

where

$$A = \exp\left(\frac{\sigma_\varepsilon^2}{2(1-\phi)^2}\right)\left(\frac{\phi^2}{2(1-\phi)^2} - \frac{\phi}{1-\phi)}\right)$$

$$B = \exp\left(\frac{\phi}{1-\phi}\frac{\sigma_\varepsilon^2}{(1-\phi)^2}\right)\left(\frac{\exp\mu_D^r}{1+g_t}\right)^{\frac{\phi}{1-\phi}}$$

K_t is an infinite sum of different powers of a single lognormal variable. However, the term $\exp(\mu_D^r/(1+g_t))^{\phi/(1-\phi)}$ in B makes it hard to find the exact value of K_t. If, however, the sum is approximated by setting $B = 1$ then

$$K_t = C(1+g_t)^{\frac{\phi}{1-\phi}}$$

and so the price–dividend ratio is stochastic and depends upon the growth rate of dividends at the point that the expectation is being taken.[9] Moreover, as ϕ rises, i.e. the degree of persistence of shocks into the dividend growth rate increases, K_t will rise.

Given the formula above for K_t, and using the relation between real equity prices and real dividends in (6), it follows that

$$\Delta\ln P_t^r = \mu_D^r + (1-\phi)^{-1}\varepsilon_t$$

Now, we might expect that volatility in nominal and real equity prices are much the same due to stickiness in consumer prices, so that $\Delta\ln P_t$ will be well approximated by $N(\mu_D, (\sigma/(1-\phi))^2)$.

[9] Because μ_D and g_t are monthly values it is likely that B will be very close to unity even for quite high values of ϕ. For example, if $\mu_D^r = 0.004$, $g_t = 0.006$, $\phi = 0.8$ then $B = 0.992$.

Thus the introduction of serial correlation into the dividend growth process amplifies the volatility of equity prices and $\mathrm{var}(\Delta \ln P_t)/\sigma_D^2$ will be $1 + \phi/1 - \phi$. This latter effect produces a dilemma for an investigator. By increasing ϕ it is possible to realize much larger values of $\ln K_t$ and thereby increase the magnitude of realizations of $\ln P_t$. But the rise in volatility in $\Delta \ln P_t^r$ also means that the average bull market becomes shorter and of smaller amplitude. Thus the explanation of a particular episode may compromise the ability to explain average market outcomes. Some amplification of volatility is desirable, however, since there is not enough volatility in dividend growth itself to explain the volatility in stock prices. As we observed earlier, the ratio of σ to σ_D for post-war data is around 3, so that a value of $\phi = 0.8$ would be needed to produce the correct volatility in capital gains, given the observed dividend volatility. This is quite a high degree of persistence and, although it is smaller than that used by Barsky and de Long (1993), it is subject to the same criticism that has been made of their argument.

6.5. Studying Extreme Events

The history of asset price movements is replete with instances of extreme behaviour, e.g. the movement in equity prices in the USA during the 1920s. Extreme events in asset markets are interesting for a number of reasons. One is that they provide a very demanding test of any postulated model. Another is that they are frequently used to shed light on the importance of bubbles within these markets. The latter motivation has produced several papers which have enquired into the nature of the bull and bear markets of the 1920s. Donaldson and Kamstra (1996) is probably the best known of these. They argued that the high price–dividend ratios of the 1920s were a consequence of the dividend processes at that time. To establish this fact they fit models to $y_t = 1 + g_t/1 + r_t$, where r_t is an interest rate on low-risk commercial paper plus a constant equity premium and

$$K_t = E_t\left[\sum_{j=1}^{\infty}\prod_{k=1}^{j} y_{t+k}\right]$$

The models have the form

$$(1 - \rho_1 L - \rho_2 L^2)(1 - \beta L)y_t = a + (1 - \rho_1 L - \rho_2 L^2)\psi_{t-1} + u_t \tag{10}$$

where ψ_{t-1} is a non-linear function of past y_t and u_t has a GARCH structure. They recursively estimate the parameters of this process.

Since the objective is to find a large value of K_t during the 1920s bull market, we might begin by examining the degree of persistence in the y_t process. Ignoring the non-linearity in (10), persistence could be measured by the roots of the equation $(1 - \rho_1 L - \rho_2 L^2)(1 - \beta L) = 0$ and, from Table III in Donaldson and Kamstra (1996), these roots are very close to unity (the dominant root is 1.02 using estimates over samples from 1899/1 up to 1919/12 and 1925/12). Consequently, although the analysis of the previous section had $\ln(1 + g_t)$ rather than y_t evolving as an AR(1) structure, we might expect that the result found there would still hold, i.e. the strong persistence has the potential for producing a large value of K. One of their simulations (reported in their Figure 4) effectively sets $\phi = 0.97$ and such an outcome can indeed be observed. Of course, this was Barsky and de Long's point and the question which arose in comment on their paper was what the evidence was for such persistence in dividend growth. Yearly real dividend growth does not show any. Monthly dividends are harder to analyse owing to their seasonality and occasional very large spikes. Donaldson and Kamstra removed these effects by performing seasonal adjustment with the X-11 program followed by smoothing operations to eliminate spikes in the series to generate the values for y_t. Thus there has to be a question mark over the origin of the persistence they find. It should also be noted that Donaldson and Kamstra maintain that the non-linearity and GARCH effects in (10) are very important to getting a large enough K_t. We find it hard to evaluate this argument since they compute K_t by simulation methods and so it is necessary to assume that all moments of this random variable exist, and this is problematic with GARCH structures.

Instead, we turn to another way of looking at the issue of whether the 1920s can be explained by fundamentals, asking whether a particular model is capable of generating outcomes such as occurred at that period in time, i.e. rather than focusing upon the average B&B market characteristics as in earlier sections we look at the likelihood of extreme movements being generated by a particular model. Let us concentrate first on bull markets. Because we have a method for demarcating the periods of a bull market it is possible for us to simulate from a given process for equity prices and then determine whether any observed movement in a bull market is likely to have come from the assumed process. The computation is like that used in value at risk analysis.

A number of questions might then be asked. First, was the amplitude and the duration of the 1920s bull market unusual? Second, was the particular conjunction of those two characteristics observed at that time unusual? Essentially this analysis asks whether it was possible for there to be a sequence of dividend outcomes and a realization of K_t which supports a bull market defined with a specific amplitude and duration. It differs from Donaldson and Kamstra's work in that it does not directly address whether the actual dividends and likely value of K_t during the 1920s were such as to produce the necessary outcome for K_t.

To examine the first of these questions we select a model to simulate from and then summarize the history of amplitudes and durations of bull and bear markets from a long simulation. We have produced 3000 bull and bear markets in this way by simulating a random walk model with the μ and σ values from 1835/1 to 1997/5. Figure 3 cross-plots the amplitudes against the durations of the bull markets; also marked on this graph is the 1920s configuration.

It is clear that the 1920s bull market is not an improbable event when equity prices evolve as a random walk with the historical μ and σ. Another way to compare model output and data is to fit a linear relation between the amplitudes and the durations from the simulated data and to then place that relation on a similar graph to Figure 3, along with historical bull market characteristics. Figure 4 does this and it shows that the random walk is quite a satisfactory model for bull markets. Its main deficiency is the inability to explain some long bull markets that failed to produce a large rise in stock prices. Figure 5 is the equivalent of Figure 4 for bear markets and it is equally clear that the bear market at the end of the 1920s would never have been predicted by the random walk model. Thus, even though this model is capable of producing quite large declines in share prices, they only come with long-duration bear markets, and this one was quite short.[10]

7. CONCLUSION

We have tried to present a framework that might be used for studying bull and bear markets in asset prices. To do this we first defined the idea of local peaks and troughs in asset prices and

[10] It is worth remembering that we use local turning points to demarcate the phases. Although there was a strong recovery from the trough in June 1932 to a peak in February 1934, the level had not returned to the peak of the 1920s. The market had essentially recovered only to its level of 1924.

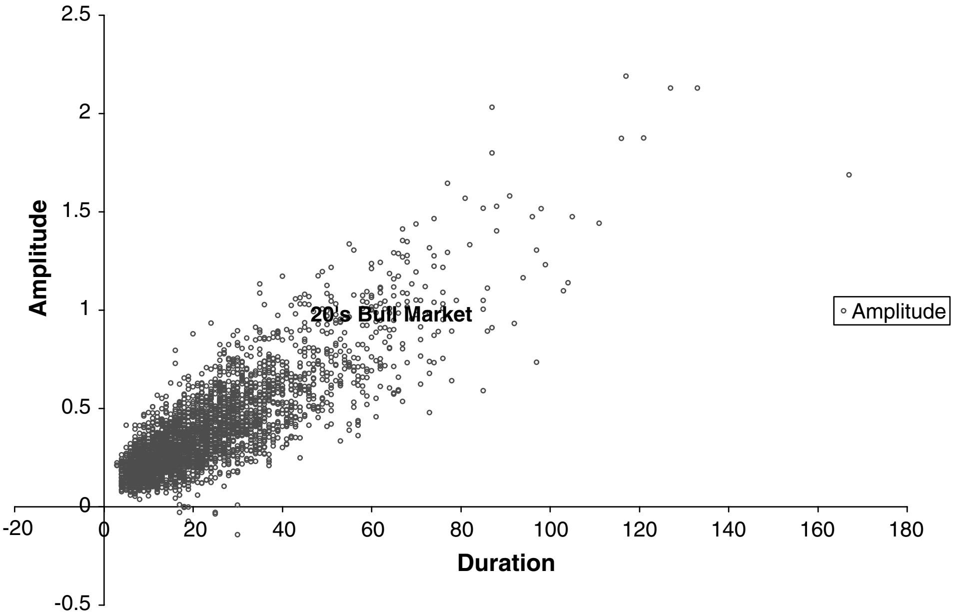

Figure 3. Amplitudes versus durations from simulated random walk model

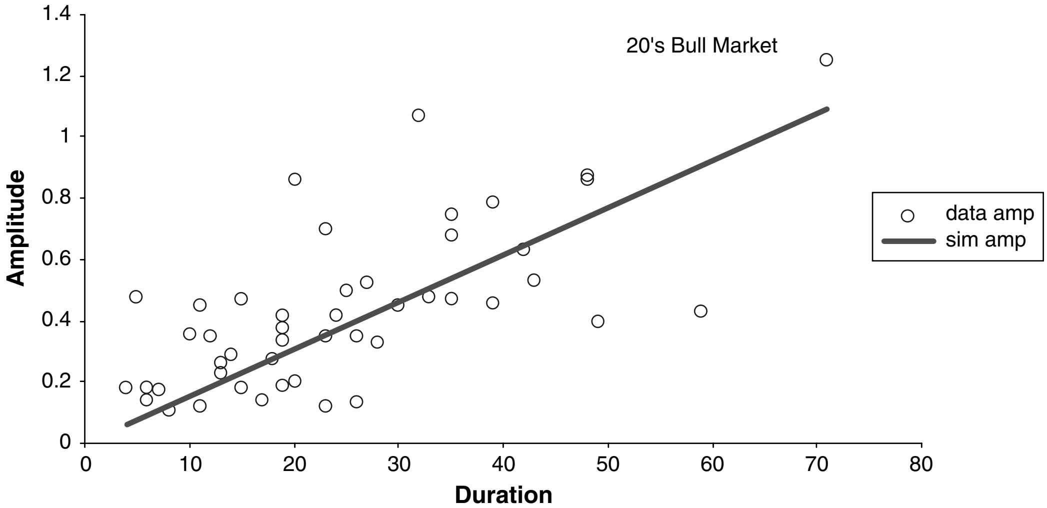

Figure 4. Amplitudes versus durations of actual US bull markets and from simulated random walk model

then observed that the proposed definitions meant that the characteristics of such markets come from the stochastic process driving capital gains. A number of statistical and economic models were then used to evaluate whether they were capable of producing bull and bear markets like those seen over a long period of time in the USA. However, many other models might be used

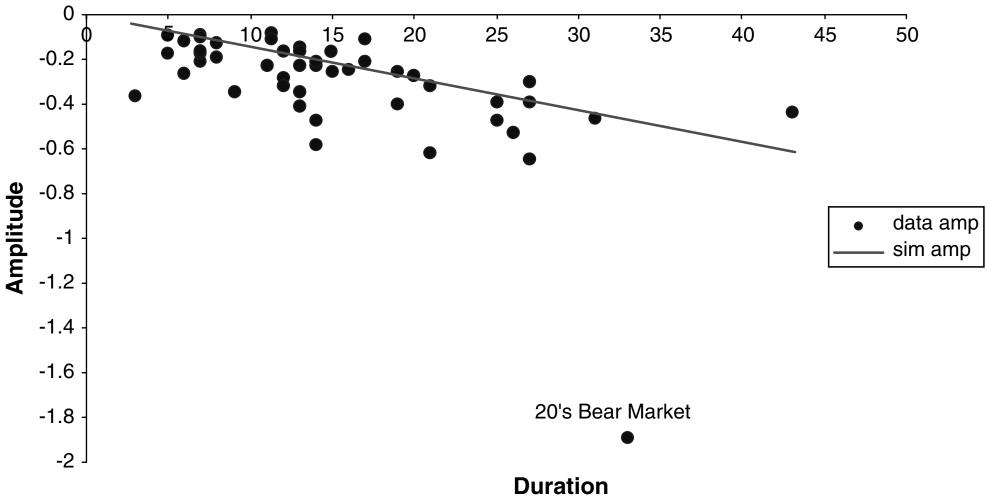

Figure 5. Amplitudes versus durations of actual US bear markets and from simulated random walk

to generate a process for capital gains, realizations of which may be fed into our dating process, thereby enabling us to directly study the factors which give rise to bull and bear markets. For example, VAR models have sometimes been proposed that have asset prices as one of the variables within the system so that it would be possible to study which of the shocks that drive the VAR are responsible for bull and bear markets.

APPENDIX A: STOCK MARKET DATA

The US data is over 1835/1–1997/5 and consists of combining series from Schwert (1990) from 1835/1 to 1870/12 and the S&P index thereafter. From 1871/1 to 1956/12 this data was taken from series 11011 in the NBER macroeconomic database. Missing observations in 1914 due to the closure of the NYSE at the outbreak of World War I were linearly interpolated. Dividends are those derived from a comparison of the S&P index with and without dividends and were obtained from Allan Timmerman.

APPENDIX B: PROCEDURE FOR PROGRAMMED DETERMINATION OF TURNING POINTS

1. Determination of initial turning points in raw data.
   (a) Determination of initial turning points in raw data by choosing local peaks (troughs) as occurring when they are the highest (lowest) values in a window eight months on either side of the date.
   (b) Enforcement of alternation of turns by selecting highest of multiple peaks (or lowest of multiple troughs).
2. Censoring operations (ensure alternation after each).
   (a) Elimination of turns within 6 months of beginning and end of series.
   (b) Elimination of peaks (or troughs) at both ends of series which are lower or higher).

(c) Elimination of cycles whose duration is less than 16 months.
(d) Elimination of phases whose duration is less than 4 months (unless fall/rise exceeds 20%).
3. Statement of final turning points

ACKNOWLEDGEMENTS

We thank many people who commented on earlier versions of this paper, particularly Allan Timmermann, Mardi Dungey and two anonymous referees.

REFERENCES

Barsky R, de Long B. 1993. Why does the Stock Market Fluctuate? *Quarterly Journal of Economics* **108**: 291–311.

Bry G, Boschan C. 1971. *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, NBER: New York.

Campbell JY, Cochrane JH. 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* **107**: 205–251.

Campbell JY, Cochrane JH. 2000. Explaining the poor performance of consumption-based asset pricing models. *Journal of Finance* **55**: 2863–2878.

Campbell JY, Kyle AS. 1993. Smart money, noise trading and stock price behaviour. *Review of Economic Studies* **60**: 1–34.

Campbell JY, Lo AW, MacKinlay AC. 1997. *The Econometrics of Financial Markets*. Princeton University Press: Princeton, NJ.

Cecchetti P-SLam, Mark NC. 2000. Asset pricing with distorted beliefs: are equity returns too good to be true. *American Economic Review* **90**: 787–805.

Chauvet M, Potter S. 2000. Coincident and leading indicators of the stock market. *Journal of Empirical Finance* **7**: 87–111.

Donaldson RG, Kamstra M. 1996. A new dividend forecasting procedure that rejects bubbles in asset prices: the case of the 1929's stock market crash. *Review of Financial Studies* **9**: 333–383.

Durland JM, McCurdy TH. 1994. Duration dependent transitions in a Markov model of U.S. GNP growth. *Journal of Business and Economic Statistics* **12**: 279–288.

Fisher I. 1925. Our unstable dollar and the so-called business cycle. *Journal of the American Statistical Association* **23**: 179–202.

Gordon S, St-Armour P. 2000. A preference regime model of bull and bear market. *American Economic Review* **90**(4): 1019–1033.

Hamilton JD. 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* **57**: 357–384.

Hamilton JD. 1994. *Time Series Analysis*. Princeton University Press: Princeton, NJ.

Hamilton WP. 1919. Stock market analysis. *Wall Street Journal* 9 August: 1919. Reprinted in Rhea R. 1932. *The Dow Theory*. Barron's: New York; 181–182.

Harding D, Pagan AR. 2002. Dissecting the cycle: a methodological investigation. *Journal of Monetary Economics* **49**: 365–381.

King RG, Plosser CI. 1994. Real business cycles and the test of the Adelmans. *Journal of Monetary Economics* **33**: 405–438.

Lunde A, Timmermann A. 2000. Duration dependence in stock prices: an analysis of bull and bear markets. Mimeo, University of California, San Diego.

Lux T, Marchesi M. 1999. Volatility clustering in financial markets: a micro-simulation of interacting agents. Mimeo, University of Bonn.

Maheu J, McCurdy TH. 2000. Identifying bull and bear markets in stock returns. *Journal of Business and Economic Statistics* **18**: 100–112.

Malkiel BG. 1973. *A Random Walk Down Wall Street*. Norton: New York.

Niermira MP, Klein PA. 1994. *Forecasting Financial and Economic Cycles*. Wiley: New York.

Pagan AR. 1996. The econometrics of financial markets. *Journal of Empirical Finance* **3**: 15–102.
Pagan AR. 1998. Bulls and bears: a tale of two states. Walras-Bowley Lecture: Montreal.
Pagan AR, Schwert GW. 1990. Alternative models for conditional stock volatility. *Journal of Econometrics* **45**: 267–90.
Rhea R. 1932. *The Dow Theory*. Barron's: New York.
Schwert GW. 1990. Indexes of United States stock prices from 1802 to 1987. *Journal of Business* **63**: 399–426.
Sichel DE. 1994. Inventories and the three phases of the business cycle. *Journal of Business and Economic Statistics* **12**: 269–277.
Slutsky EE. 1937. The summation of random causes as the source of cyclic processes. *Econometrica* **5**: 105–146.
Watson M. 1994. Business cycle durations and postwar stabilization of the U.S. economy. *American Economic Review* **84**: 24–46.

