Evolution of the Capital Asset Pricing Model (CAPM)

Anmaya Agarwal
Student
SVKM's Mukesh Patel School of Technology Management & Engineering (NMIMS University)
anmaya.agarwal02@nmims.edu.in

Pushpit Jain
Student
SVKM's Mukesh Patel School of Technology Management & Engineering (NMIMS University)
pushpit.jain26@nmims.edu.in

Prof. (Dr.) Abhay Kumar
Associate Professor
SVKM's Mukesh Patel School of Technology Management & Engineering (NMIMS University)
abhay.kumar@nmims.edu

Abstract

This chapter explores the evolution of the Capital Asset Pricing Model (CAPM), a foundational model in finance for assessing asset risk and return. It delves into the model's core assumptions, potential limitations garnered from criticism, and alternative approaches like the Fama-French 3-factor and 5-factor models. The chapter further presents original research investigating the effectiveness of CAPM in predicting stock and index prices. It utilises ex-ante beta and market return to calculate ex-ante CAPM returns, comparing them to actual market returns during the forecasting period. This analysis is juxtaposed with the traditional ex-post CAPM approach using historical data. By analysing both methods, the chapter offers valuable insights into the CAPM's efficacy for real-world investment decision-making. The results of the study showed that for 20 out of the 30 stocks sampled, the ex-ante CAPM gave a more accurate required rate of return compared to the ex-post CAPM. On average, the ex-ante CAPM was better by 5.78% compared to the ex-post CAPM for these 20 stocks. For the rest of the 10 stocks, the ex-post CAPM was better by 12.89% on average.

Keywords - Capital Asset Pricing Model, Fama-French 3-Factor Model, Fama-French 5-Factor Model, Ex-Post Analysis, Ex-Ante Analysis, Beta, Systematic Risk

1. Introduction

Savings and investments are transferred inside capital markets, which connect individuals in need of capital with suppliers. These providers, who are frequently represented by banks and investors, play an important role in lending money or making investments. Capital markets include both primary and secondary markets, with firms, governments, and individuals actively seeking capital. The stock market and bond market emerge as the primary components of these marketplaces.

Capital markets' principal goal is to improve transactional efficiency by facilitating the exchange of securities between suppliers and individuals or entities in need of financial resources. The risk and return associated with any given financial opportunity have a substantial impact on the efficiency of these transactions. The degrees of risk and reward involved have a direct impact on investment price.

It is worth noting that prior to the mid-twentieth century, the creation of a standardised asset pricing model received little attention. Sharpe [15] recognized that existing investing models were hesitant to evaluate the interaction between a security's risk-return relationship and its price behaviour.

2. What is the Capital Asset Pricing Model?

The CAPM was introduced by economists and researchers William Sharpe, John Lintner, Jan Mossin, and Jack Treynor during the early 1960s. Their work was built upon the theories put forth by Markowitz [14] and Tobin [17]. According to the original theories, the standard deviation of return may be used to measure return volatility, and the higher the standard deviation, higher the risk.

The CAPM is used to calculate the expected return on assets given when only systematic or market risk is present. The factors used to calculate this include the asset's beta (the volatility of an asset with respect to the broader market), the risk-free rate (typically the long term Government-Security bond rate), and the equity risk premium which is equivalent to the difference of the expected return on the

market or indice and the risk-free rate considered. The expected return from the CAPM is used as a discounting factor when performing evaluation of the asset using the Dividend Discount Model, Discounted Cash Flows Model or any other technique. [15]

The formula for calculating the expected return on an asset, given its risk (beta) is as follow -

$$\mathbf{R_E = R_F + \beta * (R_M - R_F)}$$

Where,
$\mathbf{R_E}$ = the expected return on the investment or the cost of equity
$\mathbf{R_F}$ = risk free rate (generally the long term government security bond rate)
$\boldsymbol{\beta}$ = systematic risk (calculated as risk of asset with respect to benchmark)
$\mathbf{R_M}$ = expected return by the market (generally the historical CAGR of the market over a period of time)

Investors demand compensation for two things for their investments, first is the time value of money as the value of money declines with time, and the other is compensation for the risk faced by investing in a risky asset.

- Within the CAPM formula, the risk-free rate plays a crucial role in accounting for the time value of money, offering a foundational return in the absence of risk. It establishes a baseline for calculating the anticipated return on an investment. Typically, the rate on long-term government security bonds is employed as the risk-free rate. This choice is rooted in the nature of government bonds as risk-free instruments with fixed payments, ensuring a known and predictable return.
- The other half of the formula determines the amount of compensation the investor requires for taking on more risk. The beta of a possible investment is the systematic risk of an asset assessed in relation to a market benchmark, such as the Nifty and Sensex. A beta exceeding one implies that a stock carries more risk compared to the market, whereas a beta below one suggests the stock is less risky than the market.

A study undertaken by Fama and MacBeth in 1973 [8] concluded that the statistics tend to generally support the CAPM. The study was performed using returns data for the New York Stock Exchange traded stocks for the years 1926 to 1968. More such studies were performed in support of the model like that of Black [2], who performed tests on CAPM under some assumptions, proving that a stock's expected return is linearly related with its beta and Black, Jensen and Scholes [11] who performed empirical tests.

2.1. Capital Asset Pricing Model Example

Ex 1 - An investor is contemplating an investment in a stock currently valued at Rs. 200 per share in the market, offering a 5% annual dividend. The stock exhibits a beta of 1.5 in relation to the market, indicating a higher level of volatility compared to the benchmark index, the BSE Sensex. Concurrently, the market's long-term government bond yields stand at 6%. The investor envisions a 12% annual growth in the market moving forward.

Compute the anticipated return of the stock using the Capital Asset Pricing Model (CAPM).

Ans. Given -
$\mathbf{R_E}$ = to find
$\mathbf{R_F}$ = 6%
$\boldsymbol{\beta}$ = 1.5
$\mathbf{R_M}$ = 12%

As per the CAPM model -
$R_E = R_F + \boldsymbol{\beta} * (R_M - R_F)$
$R_E = 0.06 + 1.5 * (0.12 - 0.06)$
$\mathbf{R_E = 0.15 \text{ or } 15\%}$

The expected return of the stock based on the CAPM formula is 15%.

2.2. Assumptions of the CAPM Model

The CAPM model operates based on the following assumptions [15, 11, 2]:

- ***Perfect capital markets:*** The CAPM assumes that capital markets are perfectly competitive, meaning there are no taxes, transaction costs, or restrictions on capital flows.Investors have the flexibility to trade securities at any time without incurring transaction costs.
- ***Rational investors:*** The model assumes that investors are rational actors who make decisions with the goal of maximizing their utility. It further assumes that investors have access to all pertinent information and possess the ability to analyze it efficiently, reflecting the notion that the markets are efficient.
- ***Homogeneous expectations:*** All investors in the market have the same expectations regarding future returns, standard deviations, and covariances of all assets. In other words, there is a consensus among investors about the future performance of securities.
- ***Single-period horizon:*** The CAPM is structured for a single-period analysis, operating under the assumption that investors are primarily focused on the expected returns and risks within a specific time frame. It does not take into account multi-period investment horizons.
- ***Risk measured by beta:*** The model operates under the assumption that the sole pertinent risk is systematic risk, quantified by beta (β). Beta gauges the responsiveness of an asset's returns to changes in the market portfolio's returns. The CAPM places emphasis on this systematic risk, positing it as the exclusive risk for which investors are compensated.

2.3. Criticisms of the CAPM Model

The CAPM model suffered a lot of criticism from the research community [1,10] for the unrealistic assumptions that it is based upon.

- In a groundbreaking series of studies, Kahneman and Tversky (1979) [12] and Tversky and Kahneman (1992) [18] show that the average investor is not always a logical and "efficient machine." Many other behavioural economists and psychologists doubt the validity of investor behaviour in relation to the model assumptions. [13]
- The growth of discount brokers in the Indian capital market [19] has reinforced the assumption that transaction costs are non-existent. They do, however, charge a few tiny transaction fees. There may be other processing costs involved as well. It is, however, impossible to argue that there would be no taxes or inflation.
- The efficiency of CAPM is still debatable, owing mostly to the problematic issue of beta. A study conducted by Eugene Fama and Kenneth French [5] that examined share returns across major stock exchanges discovered that variations in betas failed to account for the variable performance of different stocks over time.
- Another CAPM assumption is that the risk-free rate remains constant across the discounting period. If, for example, interest rates on G-Sec Bonds rose to 7% or 8% during the previously specified 10-year holding term, the cost of capital for the investment would rise, potentially leading to the stock being expensive.
- CAPM's assumption that future cash flows can be reliably forecast for the discounting process is a source of criticism. The viability of CAPM is dependent on the capacity to accurately estimate future stock returns, without this, the CAPM model's necessity is called into question.

2.4. Practical Applications of the CAPM Model

In light of the criticisms surrounding the CAPM and the inherent assumptions shaping its application in portfolio construction, its overall effectiveness may be called into question. Nevertheless, there is potential utility in employing the CAPM for evaluating the reasonableness of future expected returns or to make comparisons.

For eg. consider an advisor proposing the addition of a stock to a portfolio, currently priced at Rs.350. The advisor justifies the price using the CAPM with a discount rate of 16%. In such a scenario, the investment manager can leverage this information to scrutinize the proposed stock's reasonableness by assessing its past performance and benchmarking it against peers. For instance, if the peer group has delivered slightly better returns at 12% over the last few years and the stock consistently

underperformed with 10% returns, the investment manager should consider seeking additional justification for the proposed higher expected return before accepting the advisor's recommendation.

3. Alternatives to the CAPM Model

Owing to criticisms directed at the Capital Asset Pricing Model (CAPM), several alternative models have been devised to enhance our comprehension of the risk-reward relationship in investments. Two of these alternative models will be examined in the following discussion.

3.1. Three Factor Model by Fama and French

Financial economists Eugene Fama and Kenneth French, in the year 1993, unveiled the Fama-French Three-Factor Model by adding two more factors to the CAPM. The model improved upon the CAPM by adding 2 more features that were able to explain the variation in stock returns. The factors included in the novel model are value, size, and market risk. [6]

- ***Market Risk (RMRF):*** This is similar to the beta or market risk factor in the CAPM, it represents the extra return given by the market index compared to the risk-free rate.
- ***Size (SMB - Small Minus Big):*** This factor embodies the idea that small-cap stocks typically exhibit better performance than large-cap stocks or the overall market index. It is employed to quantify the additional returns gained through investing in small-cap stocks compared to their larger counterparts.
- ***Value (HML - High Minus Low):*** This factor accounts for the inclination of value stocks to surpass the performance of growth stocks. It is computed as the disparity in returns between stocks characterized by high book-to-market ratios (value stocks) and those with low book-to-market ratios (growth stocks).

The model showed that in addition to the systematic or value, size and market risk are important for explaining the returns of diversified portfolios (those who have zero non-systematic risk). When you add these two factors. The model offers a more pragmatic explanation for the factors influencing the outperformance or underperformance of specific stocks or portfolios relative to the market. [8]

Given below is the formula for the Fama-French Three Factor Model

$$R_E = R_F + \beta_1 * (R_M - R_F) + \beta_2 * (SMB) + \beta_3 * (HML) + \varepsilon$$

R_E = expected return on investment or cost of equity
R_F = risk free rate (generally the long term government security bond rate)
β = Factor's coefficient (sensitivity)
R_M = expected return by the market (generally the historical CAGR of the market over a period of time)
SMB (Small minus Big) = denotes the historical excess returns observed in small-cap companies compared to their large-cap counterparts.
HML (High minus Low) = signifies the historical excess returns of value stocks (those with a high book-to-price ratio) relative to growth stocks (those with a low book-to-price ratio).
ε **=** Risk

This factor captures the historical outperformance of small-cap stocks over their large-cap counterparts, signifying the return differential between a small-cap stock portfolio and a large-cap stock portfolio. Following the determination of SMB, the calculation of its beta coefficient (β) is achievable through linear regression. It's worth noting that a beta coefficient may assume either positive or negative values.

Similarly, this factor delineates the variance in returns between portfolios comprising stocks with high book-to-market ratios (value stocks) and those with low book-to-market ratios (growth stocks). The computation of the beta coefficient for the HML factor, akin to the SMB factor, involves linear regression. The HML beta coefficient, much like its counterpart SMB, can exhibit either a positive or negative value.

3.2. Five Factor Model by Fama and French

Recently, financials researchers have worked further upon the Three-Factor model incorporating some more elements that are considered important like "quality," "momentum," and "low volatility". The Five-Factor Model, introduced in 2015 [7], builds upon the original three factors and adds two more factors to offer a more inclusive structure for elucidating the dynamics of stock returns. The two new factors in this model are:

- ***Profitability (RMW - Robust Minus Weak) [7]:*** This factor was introduced to accommodate the historical trend where stocks of highly profitable companies consistently outperformed those of less profitable ones.
- ***Investment (CMA - Conservative Minus Aggressive) [7]:*** This factor encapsulates the past tendency for stocks of companies adopting conservative investment strategies to outperform those of companies with more aggressive investment policies.

4. Can we apply Machine Learning to improve the CAPM Model ?

One of the major criticisms of the CAPM model is that the historical beta is not consistent in the future and the historical market returns might not be indicative of the future returns given by the market. So, what if we predicted the stock prices and index prices of the future and calculated the beta based on future prices, will the cost of equity calculated by this method be more indicative of the real returns expected by investors? Obviously, discrepancies are to be expected as we will be predicting the stock market using basic tools and if we were able to accurately predict the market then the pricing models will be obsolete.
The purpose of this study is to compare the Ex-Post beta (beta based on historical prices) and the Ex-Ante beta (beta based on predicted prices) in the CAPM model. Is the expected return of the Ex-Ante beta more indicative of the real return given by an asset (stocks in this case)?

4.1. Methodology

- All the 30 stocks in the BSE Sensex index as on 2 Dec 2023 have been picked for this experiment and the benchmark index which will be used to calculate the market return and beta of each stock is the BSE Sensex index.
    - We have picked large-cap stocks as predicting them is easier since they are generally more stable compared to mid-cap or small-cap stocks. [1]
    - Also, since the benchmark used is the Sensex, it makes sense to include stocks in the index.
- Historical data of stocks and indices from 1 Jan 2013 to 31 Dec 2022 (10 years of data) is gathered from the BSE website. [3]
- For the risk free rate, yield on India 10 Years Government Bonds [20] have been used, but due to the volatility in the yield, the risk free rate has been assumed as 7%.
- The data was split into training and testing datasets.
    - Training data contained data from 1 Jan 2013 to 31 Dec 2021 - this data was used to train the prediction model.
    - Testing data contained data from 1 Jan 2022 to 31 Dec 2022 - the model was used to make predictions for the stock prices for this period.
    - Facebook Prophet [4,9,16] model was utilised for predicting the stock prices.
- The FB Prophet model was trained on the training data and predictions were generated for the dates in the testing data i.e. 1 Jan 2022 to 31 Dec 2022.
- The forecasted values were used to calculate the Ex-Ante Beta and Ex-Ante Market return.
    - The model returned three types of forecasted values - upper bound, normal, and lower bound.
    - The values or bound are chosen as per the view on the stock, here, since we have the data for the testing period, the bound with the least Mean Absolute Percent Error with respect to the testing data is chosen.
- Ex-Post Beta and Ex-Post Market Return for the training data time period were calculated
- Using the above results, Ex-Ante Beta and Ex-Ante Market Return were calculated for the testing data time period using the forecasted values.
- Cost of capital for the Ex-Ante period and Ex-Post period using the CAPM formula were calculated.

- Also, the real return for the testing data was calculated.
- Finally, we compared the real returns and checked which cost of capital (Ex-Ante or Ex-Post) is closer to the real return of the testing period.

4.2. Observations

Table 1 contains three columns as follows -

- **Real Returns (Real Testing Data) -** This column shows the real return of each of the stocks for the testing period i.e. 1 Jan 2022 to 31 Dec 2022.
- **Ex-Post CAPM (Real Training Data)** - This column presents the required rate of return for the stock as per the traditional CAPM. The essential inputs, beta and market return, are derived from historical data (training period) spanning from January 1, 2013, to December 31, 2021. This computation is referred to as the Ex-Ante CAPM.
- **Ex-Ante CAPM (Forecasted Training Data)** - This column presents the required rate of return for the stock as per the CAPM. The essential inputs, beta and market return, are derived from the forecasted data (testing period) spanning from January 1, 2022, to December 31, 2022. This computation is referred to as the Ex-Post CAPM.

Stocks	Real Returns (Real Testing Data)	Ex-Post CAPM (Real Training Data)	Ex-Ante CAPM (Forecasted Training Data)
Asian_Paints	-9.65%	11.76%	13.51%
Axis_Bank	34.12%	15.60%	4.31%
Bajaj_Fin	-8.97%	14.31%	6.51%
Bajaj_Finserv	-90.88%	13.47%	11.50%
Bharti_Airtel	16.71%	12.04%	38.49%
HCL_Tech	-21.66%	10.69%	8.04%
HDFC_Bank	7.08%	13.16%	4.46%
HUL	8.35%	10.43%	9.03%
ICICI_Bank	16.50%	15.64%	18.63%
Indusind	33.88%	15.60%	-7.18%
Infosys	-20.48%	11.18%	5.40%
ITC	51.33%	11.45%	52.19%
JSW_Steel	15.11%	14.20%	-27.61%
Kotak	0.14%	13.34%	9.57%
L&T	8.54%	13.54%	4.86%
M&M	50.57%	13.00%	11.21%
Maruti	11.74%	13.12%	39.69%
Nestle	-0.37%	9.79%	7.20%
NTPC	32.02%	11.21%	47.55%
Power_Grid	4.37%	10.52%	7.72%
Reliance	5.98%	13.45%	10.48%
SBI	30.23%	14.81%	33.62%
Sun_Pharma	17.97%	10.95%	7.86%
Tata_Motors	-21.98%	15.01%	10.35%
Tata_Steel	-90.15%	14.55%	9.68%
TCS	-14.64%	10.39%	8.29%

Tech_Mahindra	-43.06%	10.97%	7.95%
Titan	2.94%	12.23%	9.95%
Ultratech_Cement	-9.88%	12.74%	7.01%
Wipro	-45.34%	10.24%	11.68%

Table 1. Real Return of testing period Vs Expected Return as per Ex-Post CAPM Vs Expected Return as per Ex-Ante CAPM

Stocks	Deviation from Real Return of Ex-Post CAPM	Deviation from Real Return of Ex-Ante CAPM
Asian_Paints	21.40%	23.16%
Axis_Bank	18.51%	29.80%
Bajaj_Fin	23.29%	15.48%
Bajaj_Finserv	104.35%	102.38%
Bharti_Airtel	4.67%	21.78%
HCL_Tech	32.35%	29.70%
HDFC_Bank	6.08%	2.62%
HUL	2.08%	0.69%
ICICI_Bank	0.87%	2.12%
Indusind	18.28%	41.06%
Infosys	31.66%	25.88%
ITC	39.88%	0.86%
JSW_Steel	0.91%	42.73%
Kotak	13.21%	9.44%
L&T	5.00%	3.69%
M&M	37.57%	39.36%
Maruti	1.38%	27.94%
Nestle	10.16%	7.57%
NTPC	20.81%	15.53%
Power_Grid	6.15%	3.35%
Reliance	7.46%	4.50%
SBI	15.42%	3.39%
Sun_Pharma	7.02%	10.12%
Tata_Motors	36.99%	32.33%
Tata_Steel	104.70%	99.83%
TCS	25.02%	22.92%
Tech_Mahindra	54.02%	51.00%
Titan	9.29%	7.01%
Ultratech_Cement	22.62%	16.89%
Wipro	55.58%	57.02%

Table 2. Deviation from Real Return of Ex-Ante CAPM Vs Deviation from Real Return of Ex-Post CAPM

Table 2. Contains the following columns -

- **Deviation from Real Return of Ex-Post CAPM -** This column calculates the deviation between the real returns and the required rate of return given by the Ex-Post CAPM of Table 1.
- **Deviation from Real Return of Ex-Ante CAPM -** This column calculates the deviation between the real returns and the required rate of return given by the Ex-Ante CAPM of Table 1.

4.3. Results

The code and the data used can be found on this link -
https://github.com/Anmaya1856/Ex-Ante_CAPM_Vs_Ex-Post_CAPM.git

The cells highlighted in green in Table 2 show the stocks whose deviation between the real returns and required rate of return given by the Ex-Ante CAPM is less than the deviation of the Ex-Post CAPM.
There are 20 stocks out of the 30 we have sampled whose Ex-Ante CAPM gives a more accurate required rate of return as compared to the Ex-Post CAPM.

- On an average the Ex-Ante CAPM is better by 5.78% compared to the Ex-Post CAPM for the 20 stocks.
- For the rest of the 10 stocks the Ex-Post CAPM is better by 12.89% on average.
- A few notable results are of the following stocks -
    - Hindustan Unilever Ltd. (HUL) whose Ex-Ante CAPM has a deviation of just 0.69% as compared to the real return of HUL for the testing period.
    - ITC Ltd. whose Ex-Ante CAPM has a deviation of just 0.86% as compared to the real return of ITC for the testing period.
    - HDFC Bank whose Ex-Ante CAPM has a deviation of just 2.62% as compared to the real return of HDFC Bank for the testing period.
    - Larsen & Toubro Ltd., Reliance Industries Ltd., Power Grid Corporation of India Ltd., and State Bank of India whose Ex-Ante CAPM has a deviation of in the range of 3.35% to 4.5% as compared to the real returns for the testing period.

5. Conclusion

In conclusion, the experiment focused on comparing the performance of Ex-Ante Capital Asset Pricing Model (CAPM) with that of the traditional Ex-Post CAPM in predicting stock prices and index prices for a sample of 30 stocks. The highlighted results in Table 2 reveal that, for 20 out of the 30 sampled stocks, the Ex-Ante CAPM demonstrated greater accuracy in estimating the required rate of return compared to the Ex-Post CAPM.

On average, the Ex-Ante CAPM outperformed the Ex-Post CAPM by 5.78% for these 20 stocks, suggesting that predicting future prices and using them in the calculation of beta and market premium led to more precise predictions of returns. Notable instances include Hindustan Unilever Ltd. (HUL), ITC Ltd., HDFC Bank, Larsen & Toubro Ltd., Reliance Industries Ltd., Power Grid Corporation of India Ltd., and State Bank of India, where the Ex-Ante CAPM exhibited particularly impressive results, with deviations ranging from 0.69% to 4.5% compared to the actual returns during the testing period.

However, it is important to note that for the remaining 10 stocks, the Ex-Post CAPM outperformed the Ex-Ante CAPM by an average of 12.89%. This suggests that, in some cases, historical data-based predictions may still be more reliable than real-time data-driven forecasts.

This suggests that the approach should be decided based on the risk appetite of the investor. As seen above, Ex-Ante CAPM is better than the Ex-Post CAPM but the magnitude is much smaller compared to the times when the Ex-Post CAPM is better than the Ex-Ante CAPM. There are a few sectors where the Ex-Ante CAPM almost consistently outperforms the Ex-Post CAPM but there is no sector where this is absolute.

6. Limitations & Future work

- We have used Auto-correlation based forecasting here, which has its limitations due to only one variable being present.
    - Adding more variables i.e. doing multivariate time series could make the stock and index predictions more accurate hence, making the Ex-Ante Beta and Market Premium more accurate.
- We have only tested the CAPM here but the same techniques can be applied to the Fama French models which are left for readers to try and test.
- CAPM only takes into consideration the unsystematic risk and market risk premium which even if we know accurately might not give zero deviation between the real return and the required rate of return calculated by the CAPM.
- The prediction of small-cap and mid-cap stocks still needs advances in Machine Learning as they are very volatile, especially in a developing economy. This might lead to large errors in the beta calculation which can lead to errors in the rate of return calculated by the Ex-Ante CAPM.
- The forecasted stock prices given by the facebook prophet model has three types of prices - bull (optimistic i.e. price rise), neutral (price more or less remains the same), bear (pessimistic i.e. price decline). The investor must choose which type to choose given his view on the stock, industry and overall market condition. Any error here would lead to error in the Beta calculation which in turn will lead to error in the required rate of return given by CAPM.

References

[1] Banz, R. W. (1981). The relationship between return and market value of common stocks. Journal of financial economics, 9(1), 3-18.
[2] Black, F. (1972). Capital market equilibrium with restricted borrowing. The Journal of business, 45(3), 444-455.
[3] BSE Website. https://www.bseindia.com/markets/equity/EQReports/StockPrcHistori.html
[4] Facebook Prophet model https://facebook.github.io/prophet/
[5] Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. the Journal of Finance, 47(2), 427-465.
[6] Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of financial economics, 33(1), 3-56.
[7] Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. Journal of financial economics, 116(1), 1-22.
[8] Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. Journal of political economy, 81(3), 607-636.
[9] Garlapati, A., Krishna, D. R., Garlapati, K., Rahul, U., & Narayanan, G. (2021, April). Stock price prediction using Facebook Prophet and Arima models. In 2021 6th International Conference for Convergence in Technology (I2CT) (pp. 1-7). IEEE.
[10] Jagannathan, R., & McGrattan, E. R. (1995). The CAPM debate. Federal Reserve Bank of Minneapolis Quarterly Review, 19(4), 2-17.
[11] Jensen, M. C., Black, F., & Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests.
[12] Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. Econometrica, 47(2), 263-292.
[13] Levy, H. (2010). The CAPM is alive and well: A review and synthesis. European Financial Management, 16(1), 43-71. https://doi.org/10.1111/j.1468-036X.2009.00530.x
[14] Markowitz, H. (1952). Portfolio selection, Journal of Finance, 7, 77-91
[15] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), 425-442.
https://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.1964.tb02865.x
[16] Taylor, S. J., & Letham, B. (2018). Forecasting at scale. The American Statistician, 72(1), 37-45.
[17] Tobin, J. (1958). Liquidity preference as behavior towards risk. The review of economic studies, 25(2), 65-86.
[18] Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and uncertainty, 5, 297-323.
[19] Vyas, A. (2023). STUDY ON PERCEPTION OF INVESTORS TOWARDS DISCOUNT BROKERS. International Journal of Management, Public Policy and Research, 2(4), 75-79.

[20] Yield on long term risk free securities in India
https://www.worldgovernmentbonds.com/bond-historical-data/india/10-years/#google_vignette