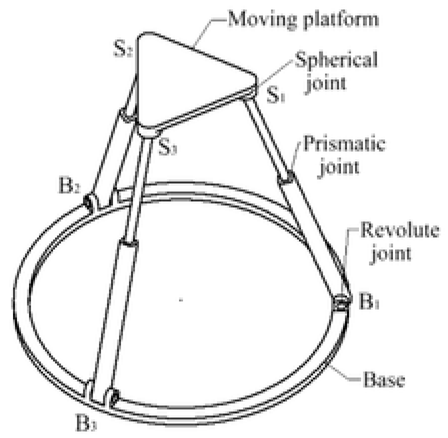
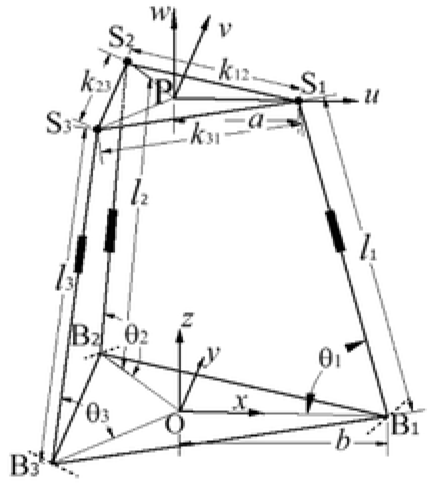
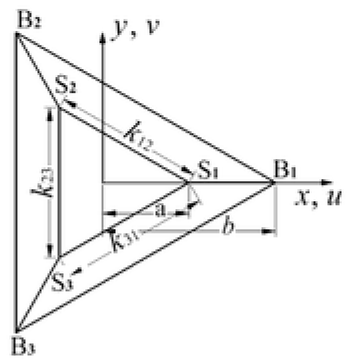


(a)



(b)



(c)

$$B_1 = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$$

$$B_2 = R_Z\left(\frac{2}{3}\pi\right) \cdot B_1 = \begin{bmatrix} \cos\left(\frac{2}{3}\pi\right) & -\sin\left(\frac{2}{3}\pi\right) & 0 \\ \sin\left(\frac{2}{3}\pi\right) & \cos\left(\frac{2}{3}\pi\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix}$$

$$B_3 = R_Z\left(\frac{4}{3}\pi\right) \cdot B_1 = \begin{bmatrix} \cos\left(\frac{4}{3}\pi\right) & -\sin\left(\frac{4}{3}\pi\right) & 0 \\ \sin\left(\frac{4}{3}\pi\right) & \cos\left(\frac{4}{3}\pi\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix}$$

$$S_1 = B_1 + \overrightarrow{L_1} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} + R_Y(-\theta_1) \begin{bmatrix} -l_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos(-\theta_1) & 0 & -\sin(-\theta_1) \\ 0 & 1 & 0 \\ \sin(-\theta_1) & 0 & \cos(-\theta_1) \end{bmatrix} \begin{bmatrix} -l_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -l_1\cos(-\theta_1) \\ 0 \\ -l_1\sin(-\theta_1) \end{bmatrix} = \begin{bmatrix} b - l_1\cos(-\theta_1) \\ 0 \\ -l_1\sin(-\theta_1) \end{bmatrix}$$

$$= \begin{bmatrix} b - l_1\cos(\theta_1) \\ 0 \\ l_1\sin(\theta_1) \end{bmatrix}$$

$$S_2 = B_2 + \overrightarrow{L_2} = \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + R_Z\left(\frac{2}{3}\pi\right) R_Y(-\theta_2) \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{2}{3}\pi\right) & -\sin\left(\frac{2}{3}\pi\right) & 0 \\ \sin\left(\frac{2}{3}\pi\right) & \cos\left(\frac{2}{3}\pi\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & 0 & -\sin(-\theta_2) \\ 0 & 1 & 0 \\ \sin(-\theta_2) & 0 & \cos(-\theta_2) \end{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_2\cos(-\theta_2) \\ 0 \\ -l_2\sin(-\theta_2) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_2\cos(-\theta_2) \\ 0 \\ -l_2\sin(-\theta_2) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ \frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}l_2\cos(-\theta_2) \\ -\frac{\sqrt{3}}{2}l_2\cos(-\theta_2) \\ -l_2\sin(-\theta_2) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_2\cos(\theta_2) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \end{bmatrix}$$

$$S_3 = B_3 + \overrightarrow{L_3} = \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + R_Z\left(\frac{4}{3}\pi\right) R_Y(-\theta_3) \begin{bmatrix} -l_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{4}{3}\pi\right) & -\sin\left(\frac{4}{3}\pi\right) & 0 \\ \sin\left(\frac{4}{3}\pi\right) & \cos\left(\frac{4}{3}\pi\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_3) & 0 & -\sin(-\theta_3) \\ 0 & 1 & 0 \\ \sin(-\theta_3) & 0 & \cos(-\theta_3) \end{bmatrix} \begin{bmatrix} -l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_3\cos(-\theta_3) \\ 0 \\ -l_3\sin(-\theta_3) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_3\cos(-\theta_3) \\ 0 \\ -l_3\sin(-\theta_3) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} \\ -\frac{\sqrt{3}}{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}l_3\cos(-\theta_3) \\ \frac{\sqrt{3}}{2}l_3\cos(-\theta_3) \\ l_3\sin(\theta_3) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_3\cos(\theta_3) \\ -\frac{\sqrt{3}}{2}b + \frac{\sqrt{3}}{2}l_3\cos(\theta_3) \\ l_3\sin(\theta_3) \end{bmatrix}$$

$$\overrightarrow{K_{S1,S2}} = S_2 - S_1 = \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_2\cos(\theta_2) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \end{bmatrix} - \begin{bmatrix} b - l_1\cos(\theta_1) \\ 0 \\ l_1\sin(\theta_1) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_2\cos(\theta_2) - b + l_1\cos(\theta_1) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_2\sin(\theta_2) - l_1\sin(\theta_1) \end{bmatrix} = \begin{bmatrix} -\frac{3b}{2} + \frac{1}{2}l_2\cos(\theta_2) + l_1\cos(\theta_1) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_2\sin(\theta_2) - l_1\sin(\theta_1) \end{bmatrix}$$

$$\overrightarrow{K_{S2,S3}} = S_3 - S_2 = \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_3\cos(\theta_3) \\ -\frac{\sqrt{3}}{2}b + \frac{\sqrt{3}}{2}l_3\cos(\theta_3) \\ l_3\sin(\theta_3) \end{bmatrix} - \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_2\cos(\theta_2) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \end{bmatrix} = \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_3\cos(\theta_3) + \frac{b}{2} - \frac{1}{2}l_2\cos(\theta_2) \\ -\frac{\sqrt{3}}{2}b + \frac{\sqrt{3}}{2}l_3\cos(\theta_3) - \frac{\sqrt{3}}{2}b + \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_3\sin(\theta_3) - l_2\sin(\theta_2) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}l_3\cos(\theta_3) - \frac{1}{2}l_2\cos(\theta_2) \\ -\sqrt{3}b + \frac{\sqrt{3}}{2}l_3\cos(\theta_3) + \frac{\sqrt{3}}{2}l_2\cos(\theta_2) \\ l_3\sin(\theta_3) - l_2\sin(\theta_2) \end{bmatrix}$$

$$\overrightarrow{K_{S3,S1}} = S_1 - S_3 = \begin{bmatrix} b - l_1\cos(\theta_1) \\ 0 \\ l_1\sin(\theta_1) \end{bmatrix} - \begin{bmatrix} -\frac{b}{2} + \frac{1}{2}l_3\cos(\theta_3) \\ -\frac{\sqrt{3}}{2}b + \frac{\sqrt{3}}{2}l_3\cos(\theta_3) \\ l_3\sin(\theta_3) \end{bmatrix} = \begin{bmatrix} b - l_1\cos(\theta_1) + \frac{b}{2} - \frac{1}{2}l_3\cos(\theta_3) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_3\cos(\theta_3) \\ l_1\sin(\theta_1) - l_3\sin(\theta_3) \end{bmatrix} = \begin{bmatrix} \frac{3b}{2} - l_1\cos(\theta_1) - \frac{1}{2}l_3\cos(\theta_3) \\ \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_3\cos(\theta_3) \\ l_1\sin(\theta_1) - l_3\sin(\theta_3) \end{bmatrix}$$

$$\left|\overrightarrow{K_{S1,S2}}\right|^2 = k_{12}^2 = \overrightarrow{K_{S1,S2}}^T \overrightarrow{K_{S1,S2}}$$

$$= \begin{bmatrix} -\frac{3b}{2} + \frac{1}{2} l_2 \cos(\theta_2) + l_1 \cos(\theta_1) & \frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} l_2 \cos(\theta_2) & l_2 \sin(\theta_2) - l_1 \sin(\theta_1) \end{bmatrix} \begin{bmatrix} -\frac{3b}{2} + \frac{1}{2} l_2 \cos(\theta_2) + l_1 \cos(\theta_1) \\ \frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} l_2 \cos(\theta_2) \\ l_2 \sin(\theta_2) - l_1 \sin(\theta_1) \end{bmatrix}$$

$$= \left(-\frac{3b}{2} + \frac{1}{2} l_2 \cos(\theta_2) + l_1 \cos(\theta_1)\right)^2 + \left(\frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} l_2 \cos(\theta_2)\right)^2 + (l_2 \sin(\theta_2) - l_1 \sin(\theta_1))^2$$

$$= \left(\frac{3b}{2}\right)^2 - 2\frac{3b}{2}\left(\frac{1}{2} l_2 \cos(\theta_2)\right) - 2\frac{3b}{2} l_1 \cos(\theta_1) + \left(\frac{1}{2} l_2 \cos(\theta_2)\right)^2 + 2\frac{1}{2} l_2 \cos(\theta_2)\, l_1 \cos(\theta_1) + (l_1 \cos(\theta_1))^2 + \left(\frac{\sqrt{3}}{2} b\right)^2$$

$$- 2\left(\frac{\sqrt{3}}{2} b\right)\left(\frac{\sqrt{3}}{2} l_2 \cos(\theta_2)\right) + \left(\frac{\sqrt{3}}{2} l_2 \cos(\theta_2)\right)^2 + (l_2 \sin(\theta_2))^2 - 2(l_2 \sin(\theta_2))(l_1 \sin(\theta_1)) + (l_1 \sin(\theta_1))^2$$

$$= \frac{9b^2}{4} - \frac{3b}{2}(l_2 \cos(\theta_2)) - 3bl_1 \cos(\theta_1) + \frac{1}{4} {l_2}^2 \cos(\theta_2)^2 + l_2 \cos(\theta_2)\, l_1 \cos(\theta_1) + {l_1}^2 \cos(\theta_1)^2 + \frac{3}{4} b^2 - \left(\frac{3}{2} b\right)(l_2 \cos(\theta_2))$$

$$+ \frac{3}{4} {l_2}^2 \cos(\theta_2)^2 + {l_2}^2 \sin(\theta_2)^2 - 2(l_2 \sin(\theta_2))(l_1 \sin(\theta_1)) + {l_1}^2 \sin(\theta_1)^2 =$$

$$= 3b^2 - \frac{3b}{2}(l_2 \cos(\theta_2)) - 3bl_1 \cos(\theta_1) + {l_2}^2 \left(\frac{1}{4} \cos(\theta_2)^2 + \sin(\theta_2)^2 + \frac{3}{4} \cos(\theta_2)^2\right) + l_2 \cos(\theta_2)\, l_1 \cos(\theta_1)$$

$$+ {l_1}^2 (\cos(\theta_1)^2 + \sin(\theta_1)^2) - \left(\frac{3}{2} b\right)(l_2 \cos(\theta_2)) - 2(l_2 \sin(\theta_2))(l_1 \sin(\theta_1))$$

$$= 3b^2 - 3bl_2 \cos(\theta_2) - 3bl_1 \cos(\theta_1) + {l_2}^2 + l_1 l_2 \cos(\theta_2) \cos(\theta_1) + {l_1}^2 - 2l_1 l_2 \sin(\theta_2) \sin(\theta_1)$$