Q1.

(a) Class conditional distribution for $C1$:

$$p(x|C1) = \begin{cases} \frac{1}{1-(-1)} ; & x \in [-1,1] \\ 0 ; & \text{otherwise} \end{cases}$$

i.e $$p(x|C1) = \begin{cases} \frac{1}{2} ; & x \in [-1,1] \\ 0 ; & \text{otherwise} \end{cases}$$

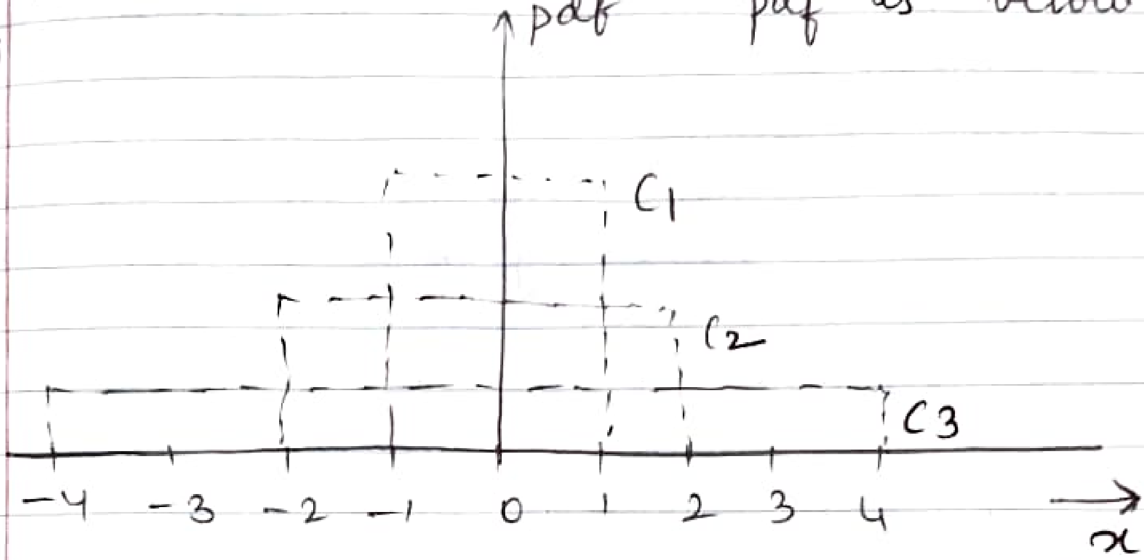
for $C2$:-

$$p(x|C_2) = \begin{cases} \frac{1}{4} ; & x \in [-2,2] \\ 0 ; & \text{otherwise} \end{cases}$$

For $C3$:-

$$p(x|C_3) = \begin{cases} \frac{1}{8} ; & x \in [-4,4] \\ 0 ; & \text{otherwise} \end{cases}$$

(b) At optimal decision boundary, the posterior probabilities are equal. Here we have pdf as below.



Basis the graph, the boundary between C1 and C_2 is at $x = \pm 1$.

Boundary between C_2 and C_3 is at $x = \pm 2$

Boundary between C_1 and C_3 also lies at $x = \pm 1$

Using all three we can say:-

For:- C1 $\longrightarrow$ $x \in [-1, 1]$
For C2 $\longrightarrow$ $x \in [-2, -1] \cup [1, 2]$
For C3 $\longrightarrow$ $x \in [-4, -2] \cup [2, 4]$

(c) Bayes Error Rate

Between $[-4, -2]$ only C_3 exists, so no error in that range

Between $[-2, -1]$ both C_3 and C_2 are present.

Now we know that

$$P(C_2|x) \propto P(C_2) \cdot P(x|C_2)$$

i.e $P(C_2|x) \propto \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

Similarly $P(C_3|x) \propto \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$

$P(C_2|x)$ is greater and hence in this region points classified as C_3 should contribute to error.

$$\therefore \text{Error}_1 = \int_{-2}^{-1} P(C_3|x)\,dx = \frac{1}{48}(1) = \frac{1}{48}$$

Between $[-1, 1]$, all three classes exist.

$P(C_1|x) \propto \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Based on these posterior probabilities we see that it is data points classified as C_2 and C_3 which will contribute to error.

$$\therefore \text{Error}_2 = \int_{-1}^{1} P(C_2|x)\,dx + \int_{-1}^{1} P(C_3|x)\,dx$$

$$= \frac{1}{12}(2) + \frac{1}{48}(2)$$

$$= \frac{5 \times 2}{48} = \frac{5}{24}$$

For region [1,2], C_2 and C_3 overlap.

and $P(C_2|x) > P(C_3|x)$

$$\therefore \text{Error}_3 = \int^{2} P(C_3|x)\,dx = \frac{1}{48}(1) = \frac{1}{48}$$

For [2,4], only C_3 exists

$\therefore$ No error in this range

Total Bayes Error =

$\text{Error}_1 + \text{Error}_2 + \text{Error}_3$

$$= \frac{1}{48} + \frac{5}{24} + \frac{1}{48} = \frac{5}{24} + \frac{1}{24} = \frac{6}{24} = \frac{1}{4} = 0.25 \text{ Ans}$$