$$y = w_0 + w_1 e^{-x_1} + w_2 x_1 + w_3 x_1 x_2$$

Assuming squared loss;

loss function will be $(y - \hat{y})^2$

WKT:-

$$w_{new} = w_{old} - \eta \frac{\partial Loss}{\partial w_{old}}$$

$\eta \rightarrow$ learning rate

$$Loss = (y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2)^2$$

→ Partial derivative w.r.t w_0

$$\frac{\partial Loss}{\partial w_0} = -2(y - w_0 - w_1 e^{-x_1} - w_2 x_1 + w_3 x_1 x_2)$$

→ Partial derivative wrt w_1

$$\frac{\partial Loss}{\partial w_1} = 2(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2)(-1 e^{-x_1} + (+ w_1) e^{-x_1}$$

$$\frac{\partial Loss}{\partial w_1} = 2(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2)(w_1 e^{-x_1} - e^{-x_1})$$

$$\frac{\partial Loss}{\partial w_1} = 2(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2) e^{-x_1}(w_1 - 1)$$

$$\frac{\partial Loss}{\partial w_1} = 2e^{-x_1}\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)(w_1 - 1)$$

Partial derivative wrt w_2 :-

$$\frac{\partial Loss}{\partial w_2} = 2\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)(-x_1)$$

$$\frac{\partial Loss}{\partial w_2} = -2x_1\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)$$

Partial derivative wrt w_3 :-

$$\frac{\partial Loss}{\partial w_3} = 2\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)(-x_1 x_2)$$

$$\frac{\partial Loss}{\partial w_3} = -2x_1 x_2\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)$$

Therefore the updated weights are :-

$$w_{0_{new}} = w_0 + \eta \cdot 2\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 + w_3 x_1 x_2\right)$$

$$w_{1\,new} = w_1 - 2\eta(w_1 - 1)e^{-x_1}\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)$$

$$w_{2new} = w_2 + \eta * 2x_1\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)$$

$$w_{3new} = w_3 + \eta * 2x_1 x_2\left(y - w_0 - w_1 e^{-x_1} - w_2 x_1 - w_3 x_1 x_2\right)$$