# Simple Linear Regression

## Learning Activities

- learn to use scikit-learn to implement simple linear regression
- learn to create, train, and test a linear regression model on real data

## Import needed packages

For this lab, you will need to have the following packages:

- NumPy
- Matplotlib
- Pandas
- Scikit-learn

Execute the below to ensure you have the above to avoid importing issues.

- !pip install numpy==2.2.0
- !pip install pandas==2.2.3
- !pip install scikit-learn==1.6.0
- !pip install matplotlib==3.9.3

In [1]:
```
!pip install -q numpy pandas scikit-learn matplotlib
```

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
%matplotlib inline
```

## Load the data

Use the pandas library to load the dataset

In [3]:
```
df = pd.read_csv('my-fuel-consumption.csv')
```

In [4]:
```
df.sample(5)
```

Out[4]:

| | Model year | Make | Model | Vehicle class | Engine size (L) | Cylinders | Transmission | Fuel type | City (L/100 km) | Highway (L/100 km) | Combined (L/100 km) | Combined (mpg) | CO2 emissions (g/km) | CO2 rating | Smog rating |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **380** | 2026 | Toyota | Corolla (3-mode) | Compact | 2.0 | 4 | AV10 | X | 7.6 | 5.9 | 6.8 | 42 | 160 | 6 | 5 |
| **390** | 2026 | Toyota | Crown Signia AWD | Station wagon: Small | 2.5 | 4 | AV | X | 6.1 | 6.3 | 6.2 | 46 | 144 | 7 | 6 |
| **202** | 2026 | Hyundai | Elantra | Mid-size | 2.0 | 4 | AV1 | X | 7.8 | 5.9 | 6.9 | 41 | 162 | 6 | 6 |
| **192** | 2026 | GMC | Yukon | Sport utility vehicle: Standard | 5.3 | 8 | A10 | X | 15.7 | 12.0 | 14.0 | 20 | 329 | 3 | 6 |
| **4** | 2026 | Audi | Q8 55 TFSI quattro | Sport utility vehicle: Standard | 3.0 | 6 | AS8 | Z | 13.6 | 10.4 | 12.1 | 23 | 285 | 4 | 4 |

# Understand the data

**FuelConsumption.csv:**

We will use a fuel consumption dataset, `FuelConsumption.csv`, which contains model-specific ratings and estimated $CO_2$ emissions for new vehicles for retail sale in Canada.

**Features:**

- MODEL YEAR, MAKE, MODEL

- VEHICLE CLASS, ENGINE SIZE
- CYLINDERS, TRANSMISSION, FUEL TYPE
- FUEL CONSUMPTION CITY (L/100 km), FUEL CONSUMPTION HWY (L/100 km)
- FUEL CONSUMPTION COMBINED (L/100 km), FUEL CONSUMPTION COMBINED MPG (MPG)
- CO2 EMISSIONS (g/km)

**Activity:** Create a simple linear regression model from one of these features to predict C02 emissions of unobserved cars based on that feature.

# Basic data exploration

In [5]:
```
df.describe()
```

Out[5]:

| | Model year | Engine size (L) | Cylinders | City (L/100 km) | Highway (L/100 km) | Combined (L/100 km) | Combined (mpg) | CO2 emissions (g/km) | CO2 rating | Smog rating |
|---|---|---|---|---|---|---|---|---|---|---|
| **count** | 437.0 | 437.000000 | 437.000000 | 437.000000 | 437.000000 | 437.000000 | 437.000000 | 437.000000 | 437.000000 | 437.000000 |
| **mean** | 2026.0 | 3.051945 | 5.427918 | 12.097712 | 9.372311 | 10.873227 | 28.082380 | 255.219680 | 4.409611 | 4.956522 |
| **std** | 0.0 | 1.305678 | 1.876325 | 3.495363 | 2.308001 | 2.896746 | 8.522371 | 67.033008 | 1.340683 | 1.371544 |
| **min** | 2026.0 | 1.200000 | 3.000000 | 4.400000 | 4.400000 | 4.400000 | 15.000000 | 104.000000 | 1.000000 | 2.000000 |
| **25%** | 2026.0 | 2.000000 | 4.000000 | 9.600000 | 7.600000 | 8.800000 | 22.000000 | 204.000000 | 4.000000 | 4.000000 |
| **50%** | 2026.0 | 2.700000 | 4.000000 | 12.000000 | 9.300000 | 10.700000 | 26.000000 | 259.000000 | 4.000000 | 5.000000 |
| **75%** | 2026.0 | 3.500000 | 6.000000 | 14.600000 | 10.700000 | 12.700000 | 32.000000 | 298.000000 | 5.000000 | 6.000000 |
| **max** | 2026.0 | 6.700000 | 12.000000 | 23.500000 | 16.600000 | 19.200000 | 64.000000 | 451.000000 | 8.000000 | 7.000000 |

### Engine size (L)

- **Stats:** mean = 3.05, std = 1.31, min = 1.2, 25% = 2.0, median = 2.7, 75% = 3.5, max = 6.7
- **Meaning:**
  - Average engine size is about 3 liters.
  - Most engines range between 2 and 3.5 liters.
  - Most cars are “medium-sized” in engine power.

### Cylinder

- **Stats:** mean = 5.43, min = 3, 25% = 4, median = 4, 75% = 6, max = 12
- **Meaning:**
  - Most cars have 4–6 cylinders.
  - Some small cars have 3 cylinders, and a few large engines have 12.
  - Think of cylinders as “engine muscles”—most cars have medium strength.

### City fuel consumption (L/100 km)

- **Stats:** mean = 12.1, min = 4.4, 25% = 9.6, median = 12, 75% = 14.6, max = 23.5
- **Meaning:**
  - On average, cars use 12 L of fuel per 100 km in the city.
  - Some are very efficient (4.4 L), while others are thirsty (23.5 L).
  - Most cars use fuel moderately; city driving can be expensive for less efficient vehicles.

### Combined fuel consumption (L/100 km)

- **Stats:** mean = 10.87, min = 4.4, 25% = 8.8, median = 10.7, 75% = 12.7, max = 19.2
- **Meaning:**
  - Average fuel consumption combining city and highway driving is ~10.9 L/100 km.
  - This gives a practical view of “typical” driving fuel use.

### Combined fuel efficiency (mpg)

- **Stats:** mean = 28.08, min = 15, 25% = 22, median = 26, 75% = 32, max = 64
- **Meaning:**
  - Cars get about 28 miles per gallon on average.
  - Lower mpg = more frequent trips to the gas station; higher mpg = fewer stops.

### $CO_2$ emissions (g/km)

- **Stats:** mean = 255, min = 104, 25% = 204, median = 259, 75% = 298, max = 451
- **Meaning:**
  - Average car emits 255 g/km of $CO_2$.

- Small cars can be very clean (104 g/km), while big engines pollute more (451 g/km).
- Most cars are moderately polluting; aiming below 204 g/km is greener.

In [6]:
```
df.isnull().sum()
```

Out[6]:
```
Model year              0
Make                    0
Model                   0
Vehicle class           0
Engine size (L)         0
Cylinders               0
Transmission            0
Fuel type               0
City (L/100 km)         0
Highway (L/100 km)      0
Combined (L/100 km)     0
Combined (mpg)          0
CO2 emissions (g/km)    0
CO2 rating              0
Smog rating             0
dtype: int64
```

**Missing values check**

- All columns have 437 valid entries
- This means there are no missing numbers in any of these features

# Feature Selection

Select features that might be indicative of CO2 emission

In [7]:
```
df.rename(columns={
    "Model year":"ModelYear", "Make":"Make", "Model":"Model", "Vehicle class":"VehicleClass",
    "Engine size (L)":"EngineSize", "Cylinders":"Cylinders", "Transmission":"Transmission",
    "Fuel type":"FuelType", "City (L/100 km)":"Fuel_CityL100Km", "Fuel_Highway (L/100 km)":"HighwayL100Km",
    "Combined (L/100 km)":"FuelConsumption_Comb", "Combined (mpg)":"FuelConsumption_Mpg",
    "CO2 emissions (g/km)":"CO2Emissions", "CO2 rating":"CO2Rating", "Smog rating":"SmogRating"
}, inplace=True)
```

In [8]:
```
cdf = df[['EngineSize', 'Cylinders', 'FuelConsumption_Comb', 'CO2Emissions']]
cdf.sample(4)
```

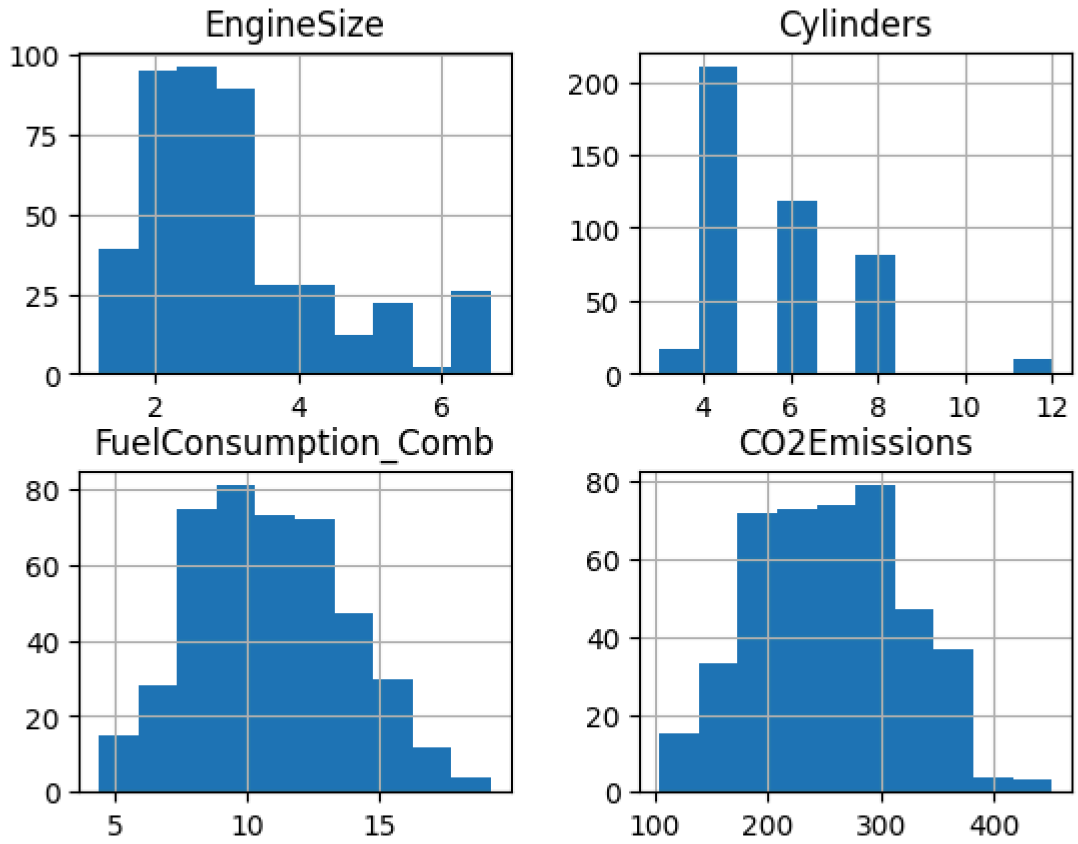
Out[8]:

| | EngineSize | Cylinders | FuelConsumption_Comb | CO2Emissions |
|---|---|---|---|---|
| **214** | 2.5 | 4 | 8.1 | 189 |
| **317** | 2.0 | 4 | 7.4 | 171 |
| **391** | 1.6 | 3 | 10.5 | 245 |
| **285** | 2.5 | 4 | 6.0 | 141 |

# Feature Visualization

Histogram, scatter plot

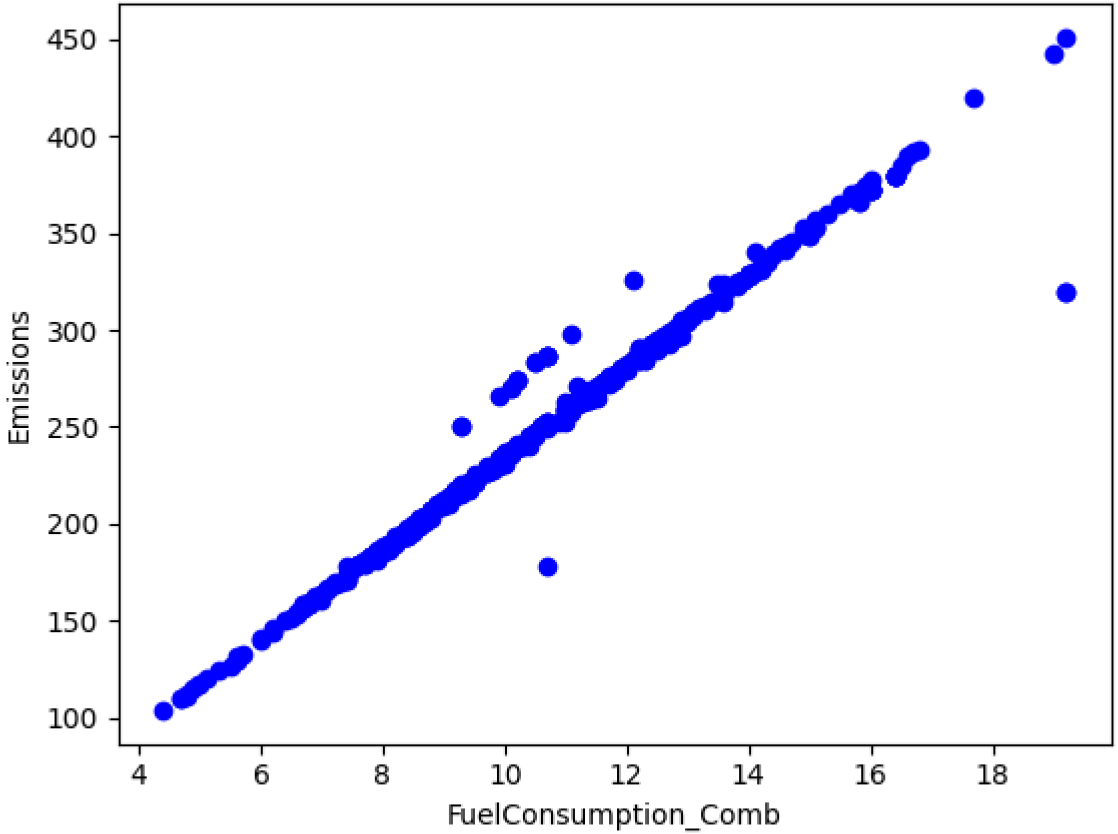
In [9]:
```
viz = cdf[['EngineSize','Cylinders', 'FuelConsumption_Comb', 'CO2Emissions']]
viz.hist()
plt.show()
```




**Insights**

- Most cars have 4,6,8 cylinders
- Engine sizes are between 2 and 4

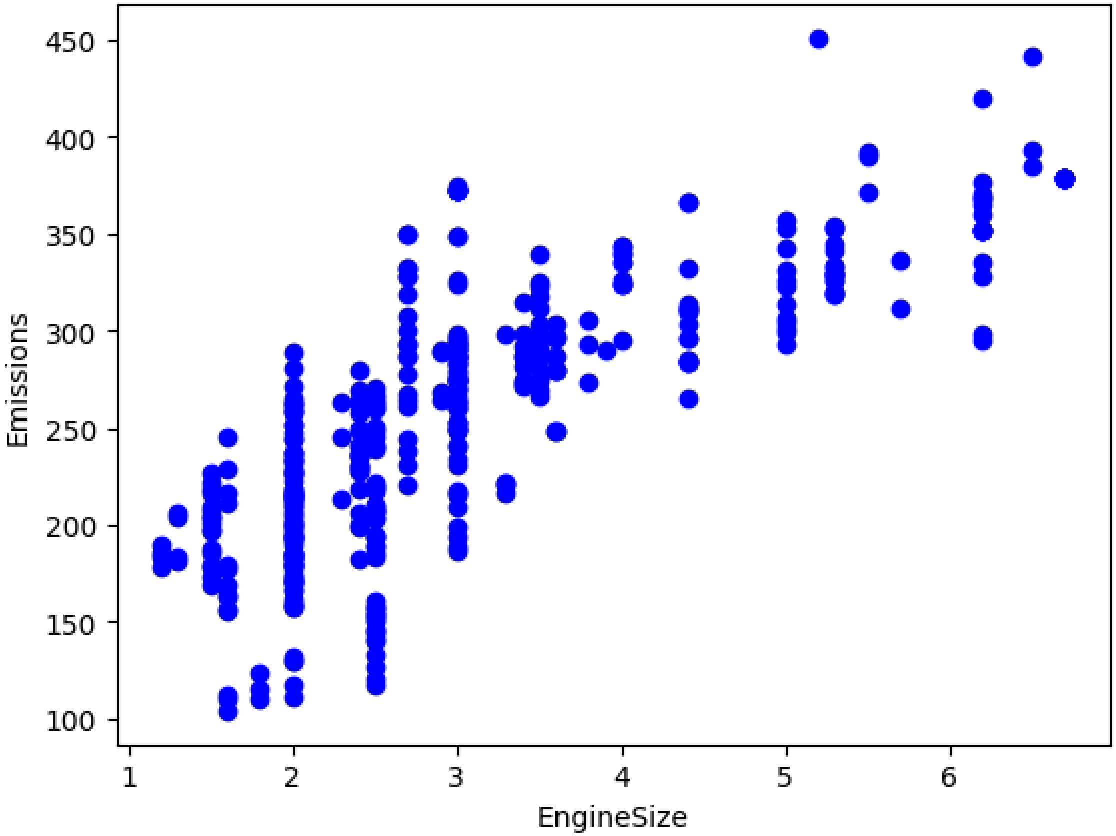
In [10]:
```
plt.scatter(cdf.FuelConsumption_Comb, cdf.CO2Emissions,  color='blue')
plt.xlabel("FuelConsumption_Comb")
plt.ylabel("Emissions")
plt.show()
```




**Insights**

- One car group has a strong linear relationship between it's combined fuel consumption and CO2 emissions.
- Another group has a linear relation but it is weak.

In [11]:
```
plt.scatter(cdf.EngineSize, cdf.CO2Emissions,  color='blue')
plt.xlabel("EngineSize")
plt.ylabel("Emissions")
plt.show()
```




**Insights**

**Positive Correlation**

- As engine size increases, $CO_2$ emissions also increase.
- Upward trend from left (small engines) to right (large engines).

**Smaller Engines - Lower Emissions**

- Vehicles with 1.5L–2.5L engines mostly emit between 100–250 g/km.
- These are usually compact cars or efficient models.

**Data Spread Shows Some Variation**

- Even within the same engine size:
- Emissions vary slightly depending on vehicle design, fuel type

# Extract the input features and labels from the dataset

- Let us pick engine size
- Extract the input feature and targer / output variables.

In [15]:
```
#For EngineSize
X = cdf.EngineSize.to_numpy()
y = cdf.CO2Emissions.to_numpy()
```

# Create, Train and Test Datasets

**Steps**

- Split the dataset into two parts: training set and testing set.
- Train a SLM using the training set.
- Use the trained model to make predictions on the unseen testing set.
- The test data already has the real answers.
- After training the model, use the test data to check its predictions.
- Compare the predicted values with the actual values.
- This tells how well the model performs on new, unseen data.
- This measurement is called **out-of-sample accuracy.**

In [16]:
```
from sklearn.model_selection import train_test_split

X_train, X_test, y_train, y_test = train_test_split(X,y,test_size=0.2, random_state=42)
```

In [17]:
```
type(X_train), np.shape(X_train), np.shape(X_train)
```

Out[17]:
```
(numpy.ndarray, (349,), (349,))
```

# Build a Simple Linear Regression Model

In [19]:
```
#Using scikit-learn
from sklearn import linear_model

#create a model
regress = linear_model.LinearRegression()

#train the model on training data
regress.fit(X_train.reshape(-1,1), y_train)

#print the coefficients
print('Coefficients:', regress.coef_[0])
print('Intercept:', regress.intercept_)
```

```
Coefficients: 39.886630889809524
Intercept: 134.94676409410977
```

**Coefficient and Intercept:** They define the slope and best fit line to the training data

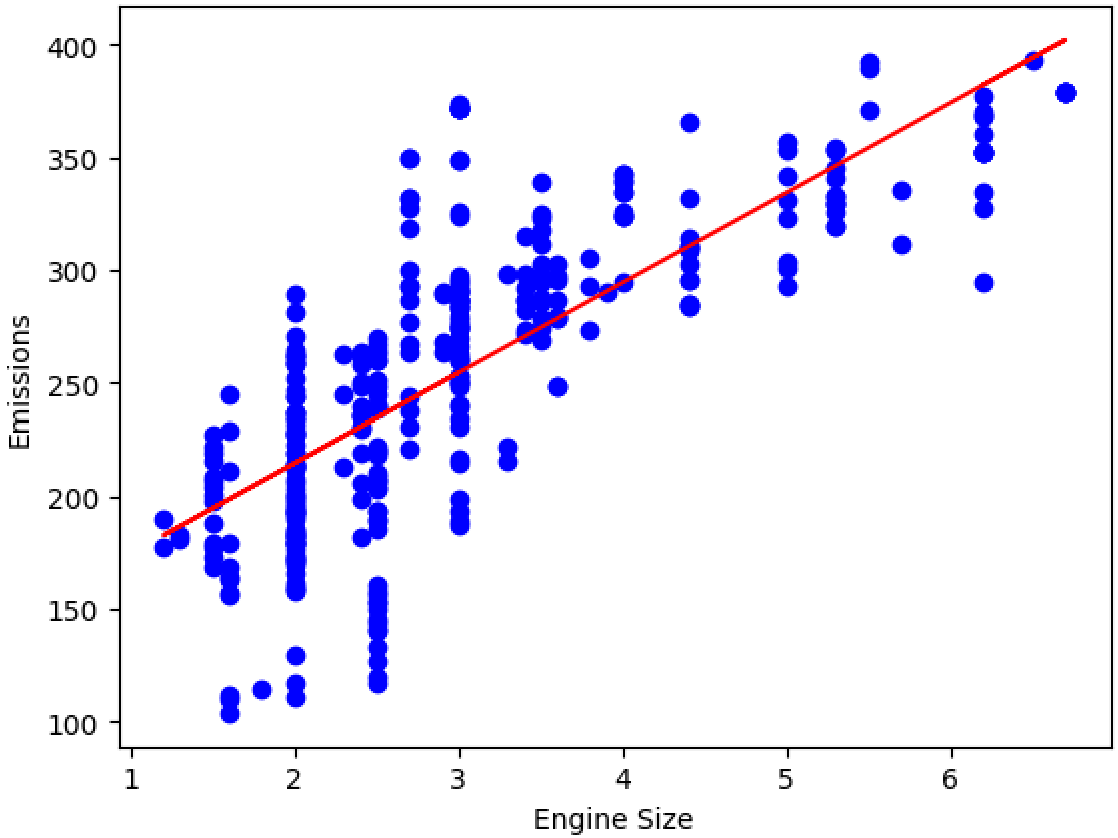
# Visualize Model Outputs

Plot Goodness-of-fit of the model to the training data.

The regression model is the line given by y = intercept + coefficient* x.

In [20]:
```
plt.scatter(X_train, y_train,  color='blue')
plt.plot(X_train, regress.coef_ *X_train + regress.intercept_, '-r')
plt.xlabel("Engine Size")
plt.ylabel("Emissions")
```

Out[20]:
```
Text(0, 0.5, 'Emissions')
```




# Model Evaluation

Compare the actual values and predicted values to calculate the accuracy of the regression model.

- **Mean Absolute Error (MAE):** Average of the absolute differences between actual and predicted values.
- **Mean Squared Error (MSE):** Average of squared differences between actual and predicted values; measures model error.
- **Root Mean Squared Error (RMSE):** Square root of MSE; in the same units as the data, easier to interpret.
- **R-squared ($R^2$):** Measures how well the model fits the data; closer to 1 means better fit.

In [24]:
```
from sklearn.metrics import mean_absolute_error, mean_squared_error, root_mean_squared_error, r2_score

#Use the predict method to make test prediction
y_test_ = regress.predict(X_test.reshape(-1,1))
```

```
print("Mean absolute error: %.2f" % mean_absolute_error(y_test_, y_test))
print("Mean squared error: %.2f" % mean_squared_error(y_test_, y_test))
print("Root mean squared error: %.2f" % root_mean_squared_error(y_test_, y_test))
 print("R2-score: %.2f" % r2_score(y_test_, y_test))
```

```
Mean absolute error: 30.08
Mean squared error: 1583.53
Root mean squared error: 39.79
R2-score: 0.53
```

- **Mean Absolute Error (MAE = 30.08):** On average, the model's predicted $CO_2$ emissions are about 30 g/km off from the actual values.
- **Mean Squared Error (MSE = 1583.53):** Average of squared errors $(g/km)^2$; larger mistakes contribute more, highlighting big prediction errors.
- **Root Mean Squared Error (RMSE = 39.79):** Square root of MSE; gives average deviation in original units, so predictions are about39.79 g/km off on average.
- **$R^2$-score (0.53):** The model explains 53% of the variance in $CO_2$ emissions, indicating a moderate fit.

**Summary:** The model moderately fits the $CO_2$ emissions data, capturing some patterns but with typical prediction errors around 40 g/km. Large deviations have a bigger impact due to squaring in MSE.