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To cite this article: Yingcui Xu *et al* 2021 *J. Phys.: Conf. Ser.* **1983** 012113

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Joint optimization for product warranty and preventive maintenance in service supply chain

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Abstract. Industrial competition has been extended from the traditional price and quality competition to warranty competition, from the single manufacturer competition to the whole supply chain system competition. Existing research mainly focuses on the decision-making problems of warranty period and price. However, with the increasing complexity and safety requirements of CNC machine tools and other engineering equipment, customers have put forward higher requirements for product quality and equipment reliability. In this paper, the equipment product deteriorating with aging is taken as the research object, and a two-level service supply chain profit model under Stackelberg game with strong manufacturer bundled sales of "product + warranty" is constructed. Moreover, the impact of warranty period and preventive maintenance frequency on the equilibrium price of product-service package is discussed. This paper provides a new and scientific approach for the optimization of product service strategy.

1. Introduction

Derived service has become an important means for manufacturers to achieve competitive advantages. The way of providing service derivative is output in the form of product-service packages (PSP), which not only increases the reliability of products, but also integrates a series of activities from pre-sale to after-sale. Bundling "product + warranty" is a typical PSP, which can enlarge the customer value and improve the manufacturer's performance at the same time. This strategy is called "servitization" [1]. Closely related to servitization is the service supply chain (SSC), which manages the important service contents of equipment products [2]. In the SSC, warranty service can improve product demand, but it also leads to high costs. Therefore, there is a dynamic correlation between warranty service and product price, and joint decision-making is required to maximize the manufacturer's profit.

In recent years, some scholars have been focused on SSC. The optimization of warranty strategy mainly focuses on the reactive service supply. For example, Huang et al. [3] analyzed the manufacturer's optimal decision under two stages, and pointed out that the longer the warranty period, the lower the price and the higher the sales rate. Fang [4] analyzed the joint decision-making problem of products price and warranty period in the non-cooperative game between a technology supplier and two duopoly manufacturers. Liu [5] pointed that the equipment availability is determined by the joint

efforts of the equipment manufacturer's effort on quality improvement and the customer enterprise's effort on maintenance services.

Few scholars try to quantify the maintenance demand through the source of demand. An effective service supply process should be driven by demand, and its success depends on correct and accurate demand planning [6]. This requires further integration of industrial maintenance processes based on product degradation. Hum [7] and other scholars pointed out that the key standard of supply chain optimization is responsiveness, which is defined as the probability of meeting customer demand in the shortest time. In a sense, predicting customers' needs and providing them with proactive services will increase the probability. Preventive maintenance (PM) is exactly an effective measure to reduce equipment failures, warranty costs and downtime risks.

In the field of reliability engineering, some scholars have carried out in-depth research on PM and warranty period optimization. Chang et al. [8] formulated the optimal PM scheme and the optimal extended warranty period to improve the manufacturer's profit. Su and Cheng [9] periodically optimized the PM plan during the warranty period to minimize the cost on the premise of ensuring the availability within the warranty period.

To sum up, the existing research has carried out in-depth research on the warranty service integrating PM from different perspectives, but there is still a lack of research on the optimization of manufacturer's pricing and warranty service joint strategy in the context of SSC. In the SSC, the cooperation or game behavior among different members (including manufacturers and retailers) will have an important impact on their decision-making. Therefore, aiming at PSP, this paper constructs and solves a two-stage Stackelberg game model from the perspective of warranty strategy affecting product demand, and uses grid search algorithm to analyze the impact of different warranty strategies on price, so as to achieve the goal of profit maximization.

2. Problem description

The structure of SSC is shown in Figure 1, in which a two-level supply chain composed of a manufacturer and a retailer is considered. After the retailer sells PSP to customers, the manufacturer provides warranty service. Suppose that the production cost of the product is c, the wholesale price is w, and the retailer sells PSP at the price p. The warranty strategy in PSP is (L, N), that is, during the warranty period L, N times of PM is carried out by the manufacturer at the same time interval to reduce the frequency of failure. The manufacturer is the only supplier of the retailer and plays a dominant role in the simple supply chain. Both the manufacturer and the retailer are risk neutral and have complete information, so the decision goal is to maximize their own profits.

Figure 1. Structure of SSC.
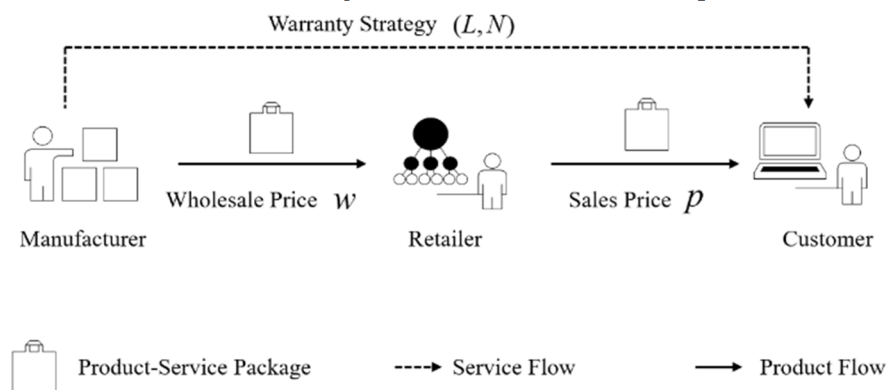
This paper focuses on the manufacture's decision, considers the comprehensive influence of warranty strategy and market strategy and establishes a two-stage Stackelberg game model. The dynamic output includes the overall scheme of warranty period L, PM frequency N and wholesale price w of PSP. Through the analysis of the interaction between the price, warranty strategy and the

profit of supply chain members, the theoretical reference is provided for manufacturing enterprises to implement the marketing strategy of warranty service.

3. Model

As a marketing tool, the warranty strategy provided by manufacturer is important in the sales of PSP. Different warranty strategies lead to different warranty periods, preventive maintenance frequency, warranty costs and product demands. The game between manufacturer and retailer in sales is a research hotspot. Because of the individual rationality and inequality in the SSC, if the manufacturer only sets the wholesale price based on its own profit maximization, it will restrict the retailer's pricing space for PSP, resulting in the reduction of market demand. Therefore, the difference of warranty period and preventive maintenance frequency will lead to the fluctuation of wholesale price, sales price and demand. To sum up, the relationship model between warranty strategy, market strategy and profit of supply chain enterprises can be established as shown in Figure 2.

The decision-making model comprehensively considers the influence of warranty strategy and market strategy. By analyzing the effect of warranty period, preventive maintenance frequency, wholesale price and retail price, as well as the profit of supply chain enterprises, the optimal joint strategy is selected to provide theoretical reference for manufacturing enterprises to bundle products and warranty.
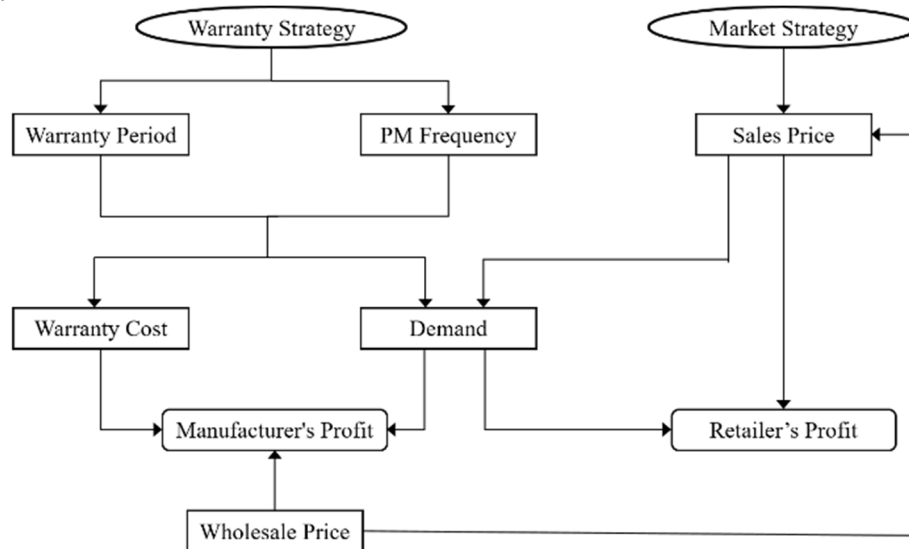

Figure 2. Relationship model between warranty strategy, market strategy and profit.

3.1. Warranty strategy

Assume that the PM provided by the manufacturer is imperfect maintenance. In order to describe the dynamic change of products condition before and after maintenance, age reduction factor α is introduced, and the reliability after maintenance is changed by modifying the equivalent age with age reduction factor. Suppose that the PM interval is T, that is $T = \dfrac{L}{N+1}$. After the i th PM, the service life time is back to the moment $(T - \alpha T)$. Assuming that the initial failure law of the product is Weibull distribution, the hazard rate function $\lambda_i(t)$ is expressed as follows [10]:

$$\lambda_i(t) = \begin{cases} \dfrac{m}{\eta}\left(\dfrac{t}{\eta}\right)^{m-1} & i = 0 \\ \lambda_{i-1}[t + (1-\alpha)T] & i \geq 1 \end{cases} \tag{1}$$

Corrective maintenance (CM) is done when product fails. This kind of maintenance is minimal repair. The number of failures in the warranty period is equivalent to the corresponding CM demands. In each PM interval, the product failure follows non-homogeneous Poisson process (NHPP), so the number of CM in each PM interval is as follows:

$$n_i = \int_0^T \lambda_i(t)\mathrm{d}t = \int_0^T \lambda\left[t + i\left(T - \alpha T\right)\right]\mathrm{d}t \quad i = 0,1,2,\cdots,N \tag{2}$$

As stated above, the total warranty cost C_1 is composed of two parts. The average cost of each PM is C_{PM}, and the average cost of CM is C_{CM}. As a result, for each PSP, the warranty cost imposed on the manufacturer is given by:

$$C_1 = C_{CM}\sum_{i=1}^{N} n_i + C_{PM}N \tag{3}$$

3.2. Market strategy

In the sales phase, the order quantity of the retailer is equal to the market demand quantity. Customers can generally evaluate the perceived quality of products based on the warranty information, which will affect the satisfaction of products, and then affect the sales price and demand [11]. On the other hand, sales price is the only profit-generating factor in decision-making, which will affect the demand of PSP [12]. Therefore, the length of warranty period and the frequency of PM often lead to the fluctuation of sales price and demand. By extending the function of Banker [13], the demand function in this paper is a linear function of the sales price, warranty period and PM frequency, which is $Q(L,N,\mathrm{p}) = Q_0 - \theta_1 p + \theta_2 L + \theta_3 N$. Among them, Q_0 is the initial market demand of PSP, and $\theta_1, \theta_2, \theta_3$ respectively represents the sensitivity of price, warranty period and PM frequency to PSP demand, all of which are nonnegative constant coefficients.

The cost of retailer is the wholesale price of PSP, while the cost of manufacturer is the production cost and the cost of repair and PM in warranty service. In this paper, subscript m denotes manufacturer, subscript r denotes retailer, and superscript $*$ denotes optimal.

3.3. Decision-making model

In the actual market environment, most supply chain members are unable to adopt the centralized strategy, and each supply chain member makes their own decisions to maximize their own profits. The manufacturer and the retailer form a Stackelberg game model, with the manufacturer as the leader and the retailer as the follower. In the SSC in which the manufacturer provides warranty service, the manufacturer first determines the wholesale price w, the length of warranty period L and the frequency of PM N; then, the retailer determines the sales price p according to the manufacturer's decision.

The manufacturer's profit is:

$$\begin{aligned}\pi_m(L,N,w) &= (w - c - C_1) \times Q(L,N,\mathrm{p}) \\ &= (w - c - C_{CM}\sum_{i=1}^{N} n_i - C_{PM}N) \times (Q_0 - \theta_1 p + \theta_2 L + \theta_3 N)\end{aligned} \tag{4}$$

Accordingly, the retailer's profit is:

$$\pi_r(p) = (p - w) \times (Q_0 - \theta_1 p + \theta_2 L + \theta_3 N) \tag{5}$$

According to the principle of Stackelberg game, the inverse order method is used to solve the problem. First, the response function of p can be obtained by deriving Equation (5):

$$p(w) = \frac{Q_0 + \theta_2 L + \theta_3 N + \theta_1 w}{2\theta_1} \tag{6}$$

Substitute (6) into (4) to get:

$$\pi_m(L, N, w) = \frac{1}{2}(w - c - C_{CM}\sum_{i=1}^{N} n_i - C_{PM}N) \times (Q_0 - \theta_1 w + \theta_2 L + \theta_3 N) \tag{7}$$

Find the first derivative of $\pi_m(L, N, w)$ with respect to the wholesale price w as follows:

$$\frac{\partial \pi_m(L, N, w)}{\partial w} = \frac{1}{2}[Q_0 - \theta_1 w + \theta_2 L + \theta_3 N - \theta_1(w - c - C_{CM}\sum_{i=1}^{N} n_i - C_{PM}N)] \tag{8}$$

Then find the second derivative of $\pi_m(L, N, w)$ with respect to the wholesale price w as follows:

$$\frac{\partial^2 \pi_m(L, N, w)}{\partial w^2} = -\theta_1 < 0 \tag{9}$$

It can be seen that the manufacturer's profit function is a concave function of the wholesale price w, and there is a maximum value. Therefore, the response function of the optimal wholesale price w^* about warranty period L and PM frequency N is obtained as follows:

$$w^*(L, N) = \frac{Q_0 + \theta_2 L + \theta_3 N + \theta_1 c + \theta_1 C_{CM}\sum_{i=1}^{N} n_i + \theta_1 C_{PM}N}{2\theta_1} \tag{10}$$

4. Case study

In order to verify the effectiveness of the above optimization model, the following case is analyzed. Suppose that a CNC machine manufacturer produces a specific type of machine with unit production cost of 20000. Besides, suppose that the hazard rate function is Weibull distribution [14], the shape parameter is $m = 2$ and scale parameter is $\eta = 1.4$. The relevant parameter settings are shown in Table 1.

Table 1. Related parameter settings.

Parameter	Value Setting	Explanation
C_{CM}	400	CM cost
C_{PM}	640	PM cost
Q_0	500	Market Capacity
θ_1	0.005	Sensitivity Coefficient of Sales Price
θ_2	5	Sensitivity Coefficient of Warranty Period
θ_3	2	Sensitivity Coefficient of PM Frequency

Based on the proposed optimization model, the optimal wholesale price w and manufacturer's profit $\pi_m(L, N, w)$ can be obtained under different L and N. The dimension reduction analysis of the optimization results is shown in Figure 3 and Figure 4. Figure 3 shows the impact of warranty period L on manufacturer's profit; Figure 4 shows the impact of PM frequency N on manufacturer's profit. It can be seen from the change trend of the two graphs that when either of the variables L and N is fixed, there is an optimal decision value to maximize the manufacturer's profit.
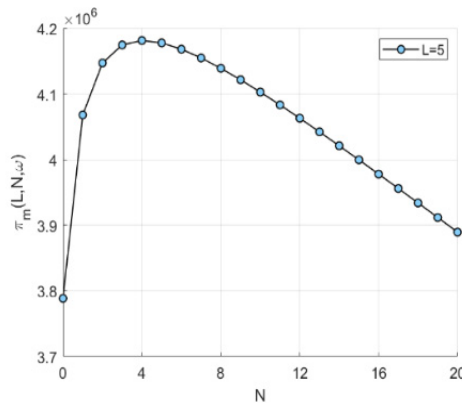
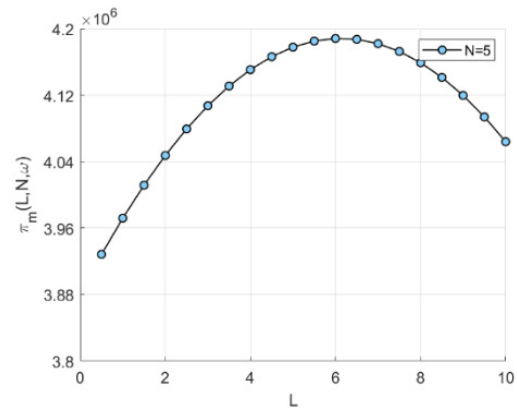**Figure 3.** Trend chart of $L-\pi_m(L,N,w)$ when $N=5$.

Figure 4. Trend chart of $N-\pi_m(L,N,w)$ when $L=5$.

Thus, the optimal value of manufacturer's profit under different warranty periods L can be obtained, as shown in Figure 5 With L increasing, the manufacturer's optimal profit first increases and then decreases, and reaches the maximum value when the warranty period is 6 years.
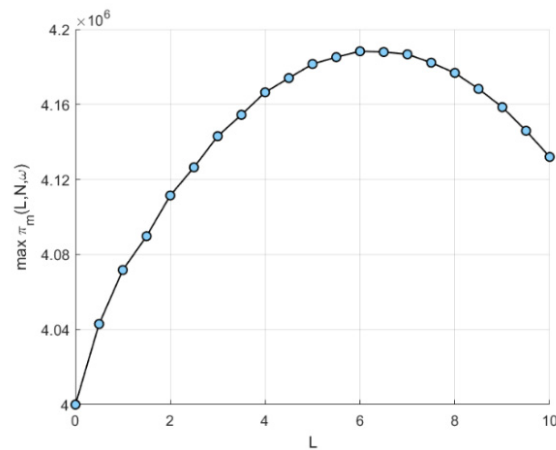

Figure 5. Trend chart of $L-\max\pi_m(L,N,w)$.

Table 2. Joint decision of optimal price and warranty.

L	N^*	$w^*\times10^4$	$p^*\times10^4$	$\pi_m^*\times10^4$	$\pi_r^*\times10^4$
2	1	6.18	8.21	411.15	205.57
2.5	2	6.27	8.30	412.65	206.32
3	2	6.31	8.34	414.31	207.15
3.5	3	6.39	8.43	415.45	207.72
4	3	6.44	8.48	416.64	208.32
4.5	4	6.52	8.57	417.40	208.70
5	4	6.57	8.62	418.15	209.08
5.5	5	6.66	8.70	418.52	209.26
6	5	6.71	8.75	418.83	209.41
6.5	6	6.80	8.84	418.79	209.39
7	6	6.85	8.89	418.67	209.34

The detailed optimization results are shown in Table 2. With the increase of the warranty period L, the overall trend of the optimal PM frequency N^* is increasing, and the wholesale price w^* and sales price p^* are also on the rise. This is because with the growth of warranty period, the failure probability is rising. At this time, the increase of PM can more effectively reduce the number of failures and reduce the maintenance cost; longer warranty period and more frequent PM mean more warranty cost and better warranty service, so the wholesale price and sales price are also on the rise.

When the length of warranty period is $L = 6$ and the frequency of PM is $N = 5$, the manufacturer will get the maximum profit. The relevant parameters of the decision scheme are shown in Table 3.

Table 3. Optimal decision while providing PM.

L^*	N^*	$w^* \times 10^4$	$p^* \times 10^4$	$\pi_m^* \times 10^4$	$\pi_r^* \times 10^4$
6	5	6.71	8.75	418.83	209.41

When the manufacturer does not provide PM, it can be seen from Equation (4) that the optimal warranty decision in the manufacturer's PSP is the warranty period is $L = 2$, and the specific decision scheme is shown in Table 4.

Table 4. Optimal decision while not providing PM.

L^*	N^*	$w^* \times 10^4$	$p^* \times 10^4$	$\pi_m^* \times 10^4$	$\pi_r^* \times 10^4$
2	0	6.16	8.18	408.62	204.31

When the manufacturer does not bundle the warranty service, that is $L = 0$ and $N = 0$, the relevant decision variables are shown in Table 5.

Table 5. Optimal decision while not providing warranty service.

L^*	N^*	$w^* \times 10^4$	$p^* \times 10^4$	$\pi_m^* \times 10^4$	$\pi_r^* \times 10^4$
0	0	6.00	8.00	400.00	200.00

Compared with the case that only CM is provided in the PSP, the manufacturer's profit increases by 2.499% and the retailer's profit increases by 2.498% when PM is provided; compared with the case that no warranty service is provided, the manufacturer's profit increases by 4.708% and the retailer's profit increases by 4.705% when PM is provided. It reveals that the scientific and reasonable "product + warranty" bundling strategy meets the profit needs of the manufacturer and the retailer.

5. Conclusions

In this paper, PM is introduced into warranty strategy, and the perspective of joint optimization is extended from single product to PSP. In this paper, profit model of supply chain enterprise is established, considering that a two-level SSC is composed of a strong manufacturer and a retailer, and the warranty strategy in PSP and the sales price will affect the market demand. In order to discuss the influence of warranty period and PM frequency on the equilibrium price of PSP, the joint decision of warranty period, PM frequency and wholesale price is made. The result shows that for the products with accelerated degradation over time, bundling a certain warranty service can significantly improve the manufacturer's profit, which provides a basis for the manufacturer to formulate the warranty strategy of PSP and its wholesale price, thus verifying the significant advantages of the proposed model in terms of effectiveness and economy.

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