GROUP ASSIGNMENT

BSD3143 OPERATIONAL RESEARCH

SECTION 03G

SEMESTER II 2022/2023

TITLE: OPTIMIZATION OF DELIVERY COST USING VAM AND IMPROVISED STEPPING STONE METHOD

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Problem Description

As the competition within the business world is growing rapidly, one of the crucial aspects that must be considered so that a company can survive is cost optimization. One of the important things of cost optimization is the efficient management of transportation costs. To mitigate the problem, a method, namely the method of transportation is designed. The transformation method is developed to regulate the distribution of goods from sources to destinations optimally and ensuring the effective delivery of goods. Various methods of transportation models can be employed to solve transportation problems, for example the North West Corner (NWC), Least Cost or Vogel's Approximation Method (VAM) as the initial solution while Improvised Stepping Stone or Modified Distribution (MODI) methods as the optimum solution.

In this specific research conducted by Ardhyani, the focus is on optimizing the transportation cost incurred, by choosing the optimum distribution channel using the transportation method. The methods employed to enhance cost efficiency in transportation are North West Corner Method, Least Cost Method, Vogel's Approximation Method, optimal test using Improvised Stepping Stone and Modified Distribution methods. This research aims to identify the most cost-effective distribution strategy for UD Yosarita in April 2019, May 2019, and June 2019. The Vogel's Approximation Method as the initial solution and the Stepping Stone method as the optimum solution has been used as the transportation method and the results of the study for each month which were Rp.4,615,000, Rp.4,810,000, and Rp. 4,910,000.

UD Yosarita is a company specializing in the production of household furniture, including cabinets, beds, doors, windows, and so on. UD Yosarita purchase wood form various sectors in Toba Samosir Regency and transform it into furniture for households to get selling value and profit. UD Yosarita utilizes three vehicles for distributing their products to several places in Toba Samosir Regency, namely Balige, Lagu Boti, Porsea, Lumban Lobu, Indorayon, Sihiong and Jangga. The company possesses two pickup trucks and one medium-sized pickup truck, and the distribution carry out three to four times a week.

This research contributes to the field of cost optimization in transportation by providing insights into effective delivery route planning. Through the implementation of a transportation model and the optimizing delivery costs, companies like UD Yosarita can enhance their operational efficiency, minimize expenses, and remain competitive in the market.

UD Yosarita produces household furniture, which is distributed to several locations in Toba Samosir Regency. Utilizing pick-up trucks, the company collects data on warehouse capacity, destination demand and distribution costs. Three warehouses operate in Par-parean, Indorayon, and Balige areas. The table below presents the warehouse capacities at UD Yosarita for the months of April, May, and June in 2019:

Table 1: Warehouse Capacity in April 2019

Warehouse	Capacity(unit)
1	43
2	30
3	34
Quantity	107

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Table 2: Warehouse Capacity in May 2019

Warehouse	Capacity(unit)
1	44
2	35
3	32
Quantity	111

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Table 3: Warehouse Capacity in June 2019

Warehouse	Capacity(unit)
1	48
2	37
3	39
Quantity	124

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Information:

1 = Par-parean

2 = Indorayon

3 = Balige

UD Yosarita's products distributed includes several destination areas, which are Lagu Boti, Narumonda, Porsea, Lumban Lobu, Sipitu-pitu, Dolok Nauli, Lumban Julu. The table below shows the distribution of UD Yosarita for April, May, and June 2019:

Table 4: Demand from Destinations in April 2019

Purpose	Demand(unit)
1	19
2	7
3	11
4	23
5	14
6	28
7	5
Quantity	107

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Table 5: Demand from Destinations in May 2019

Purpose	Demand(unit)
1	16
2	11
3	14
4	21
5	17
6	24
7	8
Quantity	111

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Table 6: Demand from Destinations in June 2019

Purpose	Demand(unit)
1	21
2	14
3	12
4	18
5	20
6	27
7	12
Quantity	124

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency

Information:

1 = Lagu Boti

2 = Narumonda

3 = Porsea

4 = Lumban Lobu

5 = Sipitu-pitu

6 = Dolok Nauli

7 = Lumban Julu

Distribution costs can be seen in the following table:

Table 7: Distribution Costs from Source to Destination

To From	1 Rp/Unit	2 Rp/Unit	3 Rp/Unit	4 Rp/Unit	5 Rp/Unit	6 Rp/Unit	7 Rp/Unit
1	50	35	25	55	70	60	85
2	75	55	50	45	60	30	60
3	20	60	75	100	85	150	125

Source: UD Yosarita Pasar Baru Village, Porsea District, Toba Samosir Regency.

Description: distribution costs in thousands

Method

In this case study, methods of transportation model are utilized to solve the problem. The main objective of the transportation model is to transport certain loads of product from various sources to various destinations while optimizing cost. Essentially, the transportation model links the supply from a number of sources to fulfil the demand of a number of destinations. The number of sources and the number of destinations must first be determined in order to collect two pieces of important information for the model, which are the corresponding cost, c_{ij} and resource, x_{ij}. The transportation problem can be divided into two phases of solutions, first is to determine the standard basic feasible solution and the second is the optimization and improving the solution. Before solving for the initial solution, the transportation model must be balanced. A balanced transportation model signifies that the sum of demand must be equivalent to the sum of supply. In case an unbalanced model occurs, a dummy source needs to be added if the sum of demand is greater than the sum of supply, on the contrary, a dummy destination needs to be added when the sum of demand is less than the sum of supply. In this study, a total of three methods are applied to find the starting basic feasible solution, which are the Northwest Corner Method, Least-Cost Method and Vogel Approximation Method (VAM). The initial solution help to figure out the allocation of items from the source to each destination. Subsequently, the Improvised Stepping-Stone is used to obtain the final optimal solution. The final step of improving the solution is extremely important because the initial solution cannot guarantee to obtain the minimum cost. The suitable allocation of product from sources to locations are calculated in order to obtain a more efficient delivery cost and to lower operational costs of the business.

Northwest Corner is a method started by allocating initial value with the maximum amount that can be allowed by supply and demand in northwest-corner which is x_{11}. By subtracting the initial value from supply and demand, there may have three possibilities. The first possibility is that the supply subtracts the initial value is equal to zero, moving down to the next cell. The second is if the demand subtracts the initial value is equal to zero, move to the right of the cell. Third, if both the supply and demand subtract initial value is equal to zero, moving diagonally to the next cell. The steps are repeated until both supply and demand are zero as it indicates no further assignment.

Besides, Least-cost Method is also applied in this study to obtain the initial solution. It is considered to produce more optimal results compared to the Northwest Corner Method because Least-Cost Method takes the shipping cost into consideration while making the allocation, whereas the inferior Northwest Corner Method only considers the availability and supply requirements. The general idea of Least-Cost Method is to allocate as much resource as possible to routes which require the least amount of cost, hence producing an overall low cost solution. By using subtraction, adjust the amount from capacity to demand accordingly until none is left. The method may cause some special situations to happen, where there exists two or more minimum costs. In this case, the row and the column corresponding to the lower-numbered row should be selected. However, if the minimum costs are on the same row, then the lower-numbered column will be chosen.

Vogel Approximation Method (VAM) is a step-by-step procedure to obtain the initial solution of transportation costs. The VAM algorithm will require more calculations when compared to the previous methods, however the advantage is that it results in a more desirable minimum solution. The VAM algorithm starts by identifying a penalty for every row and column. The penalty is calculated by subtracting the smallest unit cost element from the next smallest unit cost element. The largest penalty is selected, whether it is row or column. If there exist two largest penalties, then either one of them can be chosen. The method shows that allocating as much as possible to the cell with least cost from the selected largest row or column penalty will finally produce the solution. Multiple iterations of the solution is required until both demand and supply reaches zero.

Furthermore, the initial solution obtained will be further optimized to finalize the result. The Improvised Stepping-stone Method starts by obtaining the initial solution from the Northwest Corner Method, Least-cost Method or VAM, whichever that produces the best Starting Basic Feasible Solution. After that, the basic solutions and non-basic solutions are listed appropriately. The implicit cost of the basic variables are determined by using the formula $u_i + v_j = c_{ij}, \forall x_{ij}$ under the assumption of $u_i = 0$. Once the value of all u_i and v_j are determined, the opportunity cost of non-basic solutions are calculated using $u_i + v_j - c_{ij}, \forall x_{ij}$. If the opportunity costs of the non-basic variables are contains positive values, then the given solution is not optimal. The calculation starts over with a new iteration involving the movement of leaving and entering variable, while abiding to the optimality conditions of simplex. The method requires a entering variable which has the most positive opportunity cost and a leaving variable adjacent to it with the smallest value. Alternatively, if all the opportunity costs of the non-basic variables are non-positive, then it indicates that the solution is optimal and ready to be used. Furthermore, choosing the unoccupied cell with the greatest positive value as the entering variable and choosing another suitable basic variable as leaving variable.

SOLUTION

NORTHWEST-CORNER METHOD

April 2019

	1	2	3	4	5	6	7	Capacity
1	50 19	35 7	25 11	55 6	70 0	60 0	85 0	43
2	75 0	55 0	50 0	45 17	60 13	30 0	60 0	30
3	20 0	60 0	75 0	100 0	85 1	150 28	125 5	34
Demand	19	7	11	23	14	28	5	107

∴ Basic solution: $x_{11} = 19, x_{12} = 7, x_{13} = 11, x_{14} = 6, x_{24} = 17, x_{25} = 13, x_{35} = 1,\ x_{36} = 28, x_{37} = 5$

∴ Non basic solution: $x_{15} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{26} = x_{27} = x_{31} = x_{32} = x_{33} = x_{34} = 0$

∴ Min z = [19(50)+7(35)+11(25)+6(55)+17(45)+13(60)+1(85)+28(150)+5(125)]x1000

= Rp.8255000

Based on the results of Northwest-corner method (NW) in April 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 19 units to Lagu Boti, 7 units to Narumonda, 11 units to Porsea, 6 units to Lumban Lobu.

2. Indorayon distributed 17 units to Lumban Lobu and 13 units to Sipitu-pitu.

3. Balige distributed 1 units to Sipitu-pitu, 28 units to Dolok Nauli and 5 units to Lumban Julu.

May 2019

	1	2	3	4	5	6	7	Capacity
1	50 16	35 11	25 14	55 3	70 0	60 0	85 0	44
2	75 0	55 0	50 0	45 18	60 17	30 0	60 0	35
3	20 0	60 0	75 0	100 0	85 0	150 24	125 8	32
Demand	16	11	14	21	17	24	8	111

∴ Basic solution: $x_{11} = 16, x_{12} = 11, x_{13} = 14, x_{14} = 3, x_{24} = 18, x_{25} = 17, x_{36} = 24,\ x_{37} = 8$

∴ Non basic solution: $x_{15} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{26} = x_{27} = x_{31} = x_{32} = x_{33} = x_{34} = x_{35} = 0$

∴ Min z = [16(50)+11(35)+14(25)+3(55)+18(45)+17(60)+24(150)+8(125)]x1000

= Rp.8130000

Based on the results of Northwest-corner method (NW) in May 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 16 units to Lagu Boti, 11 units to Narumonda, 14 units to Porsea, 3 units to Lumban Lobu.

2. Indorayon distributed 18 units to Lumban Lobu and 17 units to Sipitu-pitu.

3. Balige distributed 24 units to Dolok Nauli and 8 units to Lumban Julu.

June 2019

	1	2	3	4	5	6	7	Capacity
1	50 21	35 14	25 12	55 1	70 0	60 0	85 0	48
2	75 0	55 0	50 0	45 17	60 20	30 0	60 0	37
3	20 0	60 0	75 0	100 0	85 0	150 27	125 12	39
Demand	21	14	12	18	20	27	12	124

∴ Basic solution: $x_{11} = 21, x_{12} = 14, x_{13} = 12, x_{14} = 1, x_{24} = 17, x_{25} = 20, x_{36} = 27,\ x_{37} = 12$

∴ Non basic solution: $x_{15} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{26} = x_{27} = x_{31} = x_{32} = x_{33} = x_{34} = x_{35} = 0$

∴ Min z = [21(50)+14(35)+12(25)+1(55)+17(45)+20(60)+27(150)+12(125)]x1000

= Rp.9410000

Based on the results of Northwest-corner method (NW) in June 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 21 units to Lagu Boti, 14 units to Narumonda, 12 units to Porsea, 1 unit to Lumban Lobu.

2. Indorayon distributed 17 units to Lumban Lobu and 20 units to Sipitu-pitu.

3. Balige distributed 27 units to Dolok Nauli and 12 units to Lumban Julu.

LEAST-COST METHOD

April 2019

	1	2	3	4	5	6	7	Capacity
1	50 0	35 7	25 11	55 21	70 4	60 0	85 0	43
2	75 0	55 0	50 0	45 2	60 0	30 28	60 0	30
3	20 19	60 0	75 0	100 0	85 10	150 0	125 5	34
Demand	19	7	11	23	14	28	5	107

∴ Basic solution: $x_{12} = 7, x_{13} = 11, x_{14} = 21, x_{15} = 4, x_{24} = 2, x_{26} = 28, x_{31} = 19,\ x_{35} = 10,\ x_{37} = 5$

∴ Non basic solution: $x_{11} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{25} = x_{27} = x_{32} = x_{33} = x_{34} = x_{36} = 0$

∴ Min z = [7(35)+11(25)+21(55)+4(70)+2(45)+28(30)+19(20)+10(85)+5(125)]x1000

= Rp.4740000

Based on the results of Least-Cost method in April 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 7 units to Narumonda, 11 units to Porsea, 21 units to Lumban Lobu and 4 units to Sipitu-pitu.

2. Indorayon distributed 2 units to Lumban Lobu and 28 units to Dolok Nauli.

3. Balige distributed 19 units to Lagu Boti, 10 units to Sipitu-pitu and 5 units to Lumban Julu.

May 2019

	1	2	3	4	5	6	7	Capacity
1	50 0	35 11	25 14	55 10	70 9	60 0	85 0	44
2	75 0	55 0	50 0	45 11	60 0	30 24	60 0	35
3	20 16	60 0	75 0	100 0	85 8	150 0	125 8	32
Demand	16	11	14	21	17	24	8	111

∴ Basic solution: $x_{12} = 11, x_{13} = 14, x_{14} = 10, x_{15} = 9, x_{24} = 11, x_{26} = 24, x_{31} = 16,\ x_{35} = 8,\ x_{37} = 8$

∴ Non basic solution: $x_{11} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{25} = x_{27} = x_{32} = x_{33} = x_{34} = x_{36} = 0$

∴ Min z = [11(35)+14(25)+10(55)+9(70)+11(45)+24(30)+16(20)+8(85)+8(125)]x1000

= Rp.5130000

Based on the results of Least-Cost method in May 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 11 units to Narumonda, 14 units to Porsea, 10 units to Lumban Lobu and 9 units to Sipitu-pitu.

2. Indorayon distributed 11 units to Lumban Lobu and 24 units to Dolok Nauli.

3. Balige distributed 16 units to Lagu Boti, 8 units to Sipitu-pitu and 8 units to Lumban Julu.

June 2019

	1	2	3	4	5	6	7	Capacity
1	$\boxed{50}$ 0	$\boxed{35}$ 14	$\boxed{25}$ 12	$\boxed{55}$ 8	$\boxed{70}$ 14	$\boxed{60}$ 0	$\boxed{85}$ 0	48
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 10	$\boxed{60}$ 0	$\boxed{30}$ 27	$\boxed{60}$ 0	37
3	$\boxed{20}$ 21	$\boxed{60}$ 0	$\boxed{75}$ 0	$\boxed{100}$ 0	$\boxed{85}$ 6	$\boxed{150}$ 0	$\boxed{125}$ 12	39
Demand	21	14	12	18	20	27	12	124

∴ Basic solution: $x_{12} = 14, x_{13} = 12, x_{14} = 8, x_{15} = 14, x_{24} = 10, x_{26} = 27, x_{31} = 21,\ x_{35} = 6,\ x_{37} = 12$

∴ Non basic solution: $x_{11} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{25} = x_{27} = x_{32} = x_{33} = x_{34} = x_{36} = 0$

∴ Min z = [14(35)+12(25)+8(55)+14(70)+10(45)+27(30)+21(20)+6(85)+12(125)]x1000

= Rp.5900000

Based on the results of Least-Cost method in June 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 14 units to Narumonda, 12 units to Porsea, 8 units to Lumban Lobu and 14 units to Sipitu-pitu.

2. Indorayon distributed 10 units to Lumban Lobu and 27 units to Dolok Nauli.

3. Balige distributed 21 units to Lagu Boti, 6 units to Sipitu-pitu and 12 units to Lumban Julu.

VOGEL'S APPROXIMATION METHOD (VAM)

April 2019

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60	85	43	35-25=10
2	75 0	55	50	45	60	30	60	30	45-30=15
3	20 19	60	75	100	85	150	125	34	60-20=40*
Demand	19	7	11	23	14	28	5	107	
Column penalty	50-20=30	55-35=20	50-25=25	55-45=10	70-60=10	60-30=30	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60 0	85	43	35-25=10
2	75 0	55	50	45	60	30 28	60	30	45-30=15
3	20 19	60	75	100	85	150 0	125	34	75-60=15
Demand	19	7	11	23	14	28	5	107	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10	60-30=30*	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60 0	85	43	35-25=10
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	50-45=5
3	20 19	60	75	100	85	150 0	125	34	75-60=15
Demand	19	7	11	23	14	28	5	107	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10		85-60=25*		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25 11	55	70	60 0	85	43	35-25=10
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	
3	20 19	60	75 0	100	85	150 0	125	34	75-60=15
Demand	19	7	11	23	14	28	5	107	
Column penalty		60-35=25	75-25=50*	100-55=45	85-70=15		125-85=40		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25 11	55 23	70	60 0	85	43	55-35=20
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	
3	20 19	60	75 0	100 0	85	150 0	125	34	85-60=25
Demand	19	7	11	23	14	28	5	107	
Column penalty		60-35=25		100-55=45*	85-70=15		125-85=40		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25 11	55 23	70	60 0	85 3	43	70-35=35
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	
3	20 19	60	75 0	100 0	85	150 0	125 0	34	85-60=25
Demand	19	7	11	23	14	28	5	107	
Column penalty		60-35=25			85-70=15		125-85=40*		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35 6	25 11	55 23	70 0	60 0	85 3	43	70-35=35*
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	
3	20 19	60	75 0	100 0	85	150 0	125 0	34	85-60=25
Demand	19	7	11	23	14	28	5	107	
Column penalty		60-35=25			85-70=15				

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35 6	25 11	55 23	70 0	60 0	85 3	43	
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30	
3	20 19	60 1	75 0	100 0	85 14	150 0	125 0	34	85-60=25*
Demand	19	7	11	23	14	28	5	107	
Column penalty									

∴ **April 2019:**

	1	2	3	4	5	6	7	Capacity
1	50 0	35 6	25 11	55 23	70 0	60 0	85 3	43
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30
3	20 19	60 1	75 0	100 0	85 14	150 0	125 0	34
Demand	19	7	11	23	14	28	5	107

∴ Basic solution: $x_{12} = 6, x_{13} = 11, x_{14} = 23, x_{17} = 3, x_{26} = 28, x_{27} = 2, x_{31} = 19,\ x_{32} = 1, x_{35} = 14$

∴ Non basic solution: $x_{11} = x_{15} = x_{16} = x_{21} = x_{22} = x_{23} = x_{24} = x_{25} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

∴ Min z = [6(35)+11(25)+23(55)+3(85)+28(30)+2(60)+19(20)+1(60)+14(85)]x1000

= Rp.4595000

Based on the results of Vogel's Aproximation Method (VAM) in April 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 6 units to Narumonda, 11 units to Porsea, 23 units to Lumban Lobu and 3 units to Lumban Julu.

2. Indorayon distributed 28 units to Dolok Nauli and 2 units to Lumban Julu.

3. Balige distributed 19 units to Lagu Boti, 1 unit to Narumonda and 14 units to Sipitu-pitu.

May 2019

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60	85	44	35-25=10
2	75 0	55	50	45	60	30	60	35	45-30=15
3	20 16	60	75	100	85	150	125	32	60-20=40*
Demand	16	11	14	21	17	24	8	111	
Column penalty	50-20=30	55-35=20	50-25=25	55-45=10	70-60=10	60-30=30	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60 0	85	44	35-25=10
2	75 0	55	50	45	60	30 24	60	35	45-30=15
3	20 16	60	75	100	85	150 0	125	32	75-60=15
Demand	16	11	14	21	17	24	8	111	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10	60-30=30*	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60 0	85 0	44	35-25=10
2	75 0	55	50	45	60	30 24	60 8	35	50-45=5
3	20 16	60	75	100	85	150 0	125 0	32	75-60=15
Demand	16	11	14	21	17	24	8	111	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10		85-60=25*		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25 14	55	70	60 0	85 0	44	35-25=10
2	75 0	55	50 0	45	60	30 24	60 8	35	50-45=5
3	20 16	60	75 0	100	85	150 0	125 0	32	75-60=15
Demand	16	11	14	21	17	24	8	111	
Column penalty		55-35=20	50-25=25*	55-45=10	70-60=10				

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35 0	25 14	55	70	60 0	85 0	44	55-35=20
2	75 0	55 0	50 0	45	60	30 24	60 8	35	55-45=10
3	20 16	60 11	75 0	100	85	150 0	125 0	32	85-60=25*
Demand	16	11	14	21	17	24	8	111	
Column penalty		55-35=20		55-45=10	70-60=10				

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35 0	25 14	55	70	60 0	85 0	44	70-55=15
2	75 0	55 0	50 0	45	60	30 24	60 8	35	60-45=15
3	20 16	60 11	75 0	100 0	85 5	150 0	125 0	32	100-85=15*
Demand	16	11	14	21	17	24	8	111	
Column penalty				55-45=10	70-60=10				

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35 0	25 14	55 21	70 9	60 0	85 0	44	70-55=15*
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35	60-45=15
3	20 16	60 11	75 0	100 0	85 5	150 0	125 0	32	
Demand	16	11	14	21	17	24	8	111	
Column penalty				55-45=10	70-60=10				

∴ **May 2019:**

	1	2	3	4	5	6	7	Capacity
1	50 0	35 0	25 14	55 21	70 9	60 0	85 0	44
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35
3	20 16	60 11	75 0	100 0	85 5	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

∴ Basic solution: $x_{13} = 14, x_{14} = 21, x_{15} = 9, x_{25} = 3, x_{26} = 24, x_{27} = 8, x_{31} = 16,\ x_{32} = 11,\ x_{35} = 5$

∴ Non basic solution: $x_{11} = x_{12} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{24} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

∴ Min z = [14(25)+21(55)+9(70)+3(60)+24(30)+8(60)+16(20)+11(60)+5(85)]x1000

= Rp. 4920000

Based on the results of Vogel's Aproximation Method (VAM) in May 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 14 units to Porsea, 21 units to Lumban Lobu and 9 units to Sipitu-pitu.
2. Indorayon distributed 3 units to Sipitu-pitu, 24 units to Dolok Nauli and 8 units to Lumban Julu.
3. Balige distributed 16 units to Lagu Boti, 11 unit to Narumonda and 5 units to Sipitu-pitu.

June 2019

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60	85	48	35-25=10
2	75 0	55	50	45	60	30	60	37	45-30=15
3	20 21	60	75	100	85	150	125	39	60-20=40*
Demand	21	14	12	18	20	27	12	124	
Column penalty	50-20=30	55-35=20	50-25=25	55-45=10	70-60=10	60-30=30	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 0	35	25	55	70	60 0	85	48	35-25=10
2	75 0	55	50	45	60	30 27	60	37	45-30=15
3	20 21	60	75	100	85	150 0	125	39	75-60=15
Demand	21	14	12	18	20	27	12	124	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10	60-30=30*	85-60=25		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	$\boxed{50}$ 0	$\boxed{35}$	$\boxed{25}$	$\boxed{55}$	$\boxed{70}$	$\boxed{60}$ 0	$\boxed{85}$	48	35-25=10
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 0	$\boxed{60}$ 0	$\boxed{30}$ 27	$\boxed{60}$ 10	37	50-45=5
3	$\boxed{20}$ 21	$\boxed{60}$	$\boxed{75}$	$\boxed{100}$	$\boxed{85}$	$\boxed{150}$ 0	$\boxed{125}$	39	75-60=15
Demand	21	14	12	18	20	27	12	124	
Column penalty		55-35=20	50-25=25	55-45=10	70-60=10		85-60=25*		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	$\boxed{50}$ 0	$\boxed{35}$	$\boxed{25}$ 12	$\boxed{55}$	$\boxed{70}$	$\boxed{60}$ 0	$\boxed{85}$	48	35-25=10
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 0	$\boxed{60}$ 0	$\boxed{30}$ 27	$\boxed{60}$ 10	37	
3	$\boxed{20}$ 21	$\boxed{60}$	$\boxed{75}$ 0	$\boxed{100}$	$\boxed{85}$	$\boxed{150}$ 0	$\boxed{125}$	39	75-60=15
Demand	21	14	12	18	20	27	12	124	
Column penalty		60-35=25	75-25=50*	100-55=45	85-70=15		125-85=40		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	$\boxed{50}$ 0	$\boxed{35}$	$\boxed{25}$ 12	$\boxed{55}$ 18	$\boxed{70}$	$\boxed{60}$ 0	$\boxed{85}$	48	55-35=20
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 0	$\boxed{60}$ 0	$\boxed{30}$ 27	$\boxed{60}$ 10	37	
3	$\boxed{20}$ 21	$\boxed{60}$	$\boxed{75}$ 0	$\boxed{100}$ 0	$\boxed{85}$	$\boxed{150}$ 0	$\boxed{125}$	39	85-60=25
Demand	21	14	12	18	20	27	12	124	
Column penalty		60-35=25		100-55=45*	85-70=15		125-85=40		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 / 0	35	25 / 12	55 / 18	70	60 / 0	85 / 2	48	70-35=35
2	75 / 0	55 / 0	50 / 0	45 / 0	60 / 0	30 / 27	60 / 10	37	
3	20 / 21	60	75 / 0	100 / 0	85	150 / 0	125 / 0	39	85-60=25
Demand	21	14	12	18	20	27	12	124	
Column penalty		60-35=25			85-70=15		125-85=40*		

	1	2	3	4	5	6	7	Capacity	Row penalty
1	50 / 0	35 / 14	25 / 12	55 / 18	70 / 2	60 / 0	85 / 2	48	70-35=35*
2	75 / 0	55 / 0	50 / 0	45 / 0	60 / 0	30 / 27	60 / 10	37	
3	20 / 21	60 / 0	75 / 0	100 / 0	85 / 18	150 / 0	125 / 0	39	85-60=25
Demand	21	14	12	18	20	27	12	124	
Column penalty		60-35=25			85-70=15				

∴ **June 2019:**

	1	2	3	4	5	6	7	Capacity
1	50 0	35 14	25 12	55 18	70 2	60 0	85 2	48
2	75 0	55 0	50 0	45 0	60 0	30 27	60 10	37
3	20 21	60 0	75 0	100 0	85 18	150 0	125 0	39
Demand	21	14	12	18	20	27	12	124

∴ Basic solution: $x_{12} = 14, x_{13} = 12, x_{14} = 18, x_{15} = 2, x_{17} = 2, x_{26} = 27, x_{27} = 10,\ x_{31} = 21,\ x_{35} = 18$

∴ Non basic solution: $x_{11} = x_{16} = x_{21} = x_{22} = x_{23} = x_{24} = x_{25} = x_{32} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

∴ Min z = [14(35)+12(25)+18(55)+2(70)+2(85)+27(30)+10(60)+21(20)+18(85)]x1000

= Rp.5450000

Based on the results of Vogel's Aproximation Method (VAM) in June 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 14 units to Narumonda, 12 units to Porsea, 18 units to Lumban Lobu, 2 units to Sipitu-pitu and 2 units to Lumban Julu.

2. Indorayon distributed 27 units to Dolok Nauli and 10 units to Lumban Julu.

3. Balige distributed 21 units to Lagu Boti and 18 units to Sipitu-pitu.

IMPROVISED STEPPING-STONE METHOD

April 2019

	1	2	3	4	5	6	7	Capacity
1	50 0	35 6	25 11	55 23	70 0	60 0	85 3	43
2	75 0	55 0	50 0	45 0	60 0	30 28	60 2	30
3	20 19	60 1	75 0	100 0	85 14	150 0	125 0	34
Demand	19	7	11	23	14	28	5	107

∴ Basic solution: $x_{12} = 6, x_{13} = 11, x_{14} = 23, x_{17} = 3, x_{26} = 28, x_{27} = 2, x_{31} = 19,\ x_{32} = 1, x_{35} = 14$

∴ Non basic solution: $x_{11} = x_{15} = x_{16} = x_{21} = x_{22} = x_{23} = x_{24} = x_{25} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

Basic	u_i			v_i			c_{ij}	
x_{12}	u_1	0		v_2	35		c_{12}	35
x_{13}	u_1	0		v_3	25		c_{13}	25
x_{14}	u_1	0		v_4	55		c_{14}	55
x_{17}	u_1	0		v_7	85		c_{17}	85
x_{26}	u_2	-25	+	v_6	55	=	c_{26}	30
x_{27}	u_2	-25		v_7	85		c_{27}	60
x_{31}	u_3	25		v_1	-5		c_{31}	20
x_{32}	u_3	25		v_2	35		c_{32}	60
x_{35}	u_3	25		v_5	60		c_{35}	85

∴ $u_1 = 0, u_2 = -25, u_3 = 25$

∴ $v_1 = -5, v_2 = 35, v_3 = 25, v_4 = 55, v_5 = 60, v_6 = 55, v_7 = 85$

Non-Basic	u_i			v_i			c_{ij}		=	$u_i + v_i - c_{ij}$
x_{11}	u_1	0		v_1	-5		c_{11}	50		-55
x_{15}	u_1	0		v_5	60		c_{15}	70		-10
x_{16}	u_1	0		v_6	55		c_{16}	60		-5
x_{21}	u_2	-25	+	v_1	-5	-	c_{21}	75		-105
x_{22}	u_2	-25		v_2	35		c_{22}	55		-45
x_{23}	u_2	-25		v_3	25		c_{23}	50		-50
x_{24}	u_2	-25		v_4	55		c_{24}	45		-15
x_{25}	u_2	-25		v_5	60		c_{25}	60		-25
x_{33}	u_3	25		v_3	25		c_{33}	75		-25

x_{34}	u_3	25		v_4	55		c_{34}	100		-20
x_{36}	u_3	25		v_6	55		c_{36}	150		-70
x_{37}	u_3	25		v_7	85		c_{37}	125		-15

Since all is non-positive value, therefore the solution is optimal.

∴ Min z = [6(35)+11(25)+23(55)+3(85)+28(30)+2(60)+19(20)+1(60)+14(85)]x1000

= Rp.4595000

Based on the results of Improvised Stepping-Stone for Vogel's Aproximation Method (VAM) in April 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 6 units to Narumonda, 11 units to Porsea, 23 units to Lumban Lobu and 3 units to Lumban Julu.

2. Indorayon distributed 28 units to Dolok Nauli and 2 units to Lumban Julu.

3. Balige distributed 19 units to Lagu Boti, 1 unit to Narumonda and 14 units to Sipitu-pitu.

May 2019

	1	2	3	4	5	6	7	Capacity
1	$\boxed{50}$ 0	$\boxed{35}$ 0	$\boxed{25}$ 14	$\boxed{55}$ 21	$\boxed{70}$ 9	$\boxed{60}$ 0	$\boxed{85}$ 0	44
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 0	$\boxed{60}$ 3	$\boxed{30}$ 24	$\boxed{60}$ 8	35
3	$\boxed{20}$ 16	$\boxed{60}$ 11	$\boxed{75}$ 0	$\boxed{100}$ 0	$\boxed{85}$ 5	$\boxed{150}$ 0	$\boxed{125}$ 0	32
Demand	16	11	14	21	17	24	8	111

∴ Basic solution: $x_{13} = 14, x_{14} = 21, x_{15} = 9, x_{25} = 3, x_{26} = 24, x_{27} = 8, x_{31} = 16,\ x_{32} = 11,\ x_{35} = 5$

∴ Non basic solution: $x_{11} = x_{12} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{24} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

Basic	u_i			v_i			c_{ij}	
x_{13}	u_1	0	+	v_3	25	=	c_{12}	25
x_{14}	u_1	0		v_4	55		c_{13}	55
x_{15}	u_1	0		v_5	70		c_{14}	70
x_{25}	u_2	-10		v_5	70		c_{17}	60
x_{26}	u_2	-10		v_6	40		c_{26}	30
x_{27}	u_2	-10		v_7	70		c_{27}	60
x_{31}	u_3	15		v_1	5		c_{31}	20
x_{32}	u_3	15		v_2	45		c_{32}	60
x_{35}	u_3	15		v_5	70		c_{35}	85

$\therefore\ u_1 = 0, u_2 = -10, u_3 = 15$

$\therefore\ v_1 = 5, v_2 = 45, v_3 = 25, v_4 = 55, v_5 = 70, v_6 = 40, v_7 = 70$

Non-Basic	u_i			v_i			c_{ij}		=	$u_i + v_i - c_{ij}$
x_{11}	u_1	0		v_1	5		c_{11}	50		-45
x_{12}	u_1	0		v_2	45		c_{12}	35		10
x_{16}	u_1	0		v_6	40		c_{16}	60		-20
x_{17}	u_1	0		v_7	70		c_{17}	85		-15
x_{21}	u_2	-10	+	v_1	5	-	c_{21}	75		-80
x_{22}	u_2	-10		v_2	45		c_{22}	55		-20
x_{23}	u_2	-10		v_3	25		c_{23}	50		-35
x_{24}	u_2	-10		v_4	55		c_{24}	45		0
x_{33}	u_3	15		v_3	25		c_{33}	75		-35
x_{34}	u_3	15		v_4	55		c_{34}	100		-30
x_{36}	u_3	15		v_6	40		c_{36}	150		-95
x_{37}	u_3	15		v_7	70		c_{37}	125		-40

Entering variable: x_{12}
Leaving variable: x_{15}

May 2019 - Iteration 1

	1	2	3	4	5	6	7	Capacity
1	50 0	35 0	25 14	55 21	70 9	60 0	85 0	44
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35
3	20 16	60 11	75 0	100 0	85 5	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

$\therefore$ May 2019 (Iteration 1) :

	1	2	3	4	5	6	7	Capacity
1	50 0	35 9	25 14	55 21	70 0	60 0	85 0	44
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35
3	20 16	60 2	75 0	100 0	85 14	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

$\therefore$ Basic solution: $x_{12} = 9, x_{13} = 14, x_{14} = 21, x_{25} = 3, x_{26} = 24, x_{27} = 8, x_{31} = 16,\ x_{32} = 2,\ x_{35} = 14$

∴ Non basic solution: $x_{11} = x_{15} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{24} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

Basic	u_i			v_i			c_{ij}	
x_{12}	u_1	0		v_2	35		c_{12}	35
x_{13}	u_1	0		v_3	25		c_{13}	25
x_{14}	u_1	0		v_4	55		c_{14}	55
x_{25}	u_2	0	+	v_5	60	=	c_{25}	60
x_{26}	u_2	0		v_6	30		c_{26}	30
x_{27}	u_2	0		v_7	60		c_{27}	60
x_{31}	u_3	25		v_1	-5		c_{31}	20
x_{32}	u_3	25		v_2	35		c_{32}	60
x_{35}	u_3	25		v_5	60		c_{35}	85

∴ $u_1 = 0, u_2 = 0, u_3 = 25$

∴ $v_1 = -5, v_2 = 35, v_3 = 25, v_4 = 55, v_5 = 60, v_6 = 30, v_7 = 60$

Non-Basic	u_i			v_i			c_{ij}			$u_i + v_i - c_{ij}$
x_{11}	u_1	0		v_1	-5		c_{11}	50		-55
x_{15}	u_1	0		v_5	60		c_{15}	70		-10
x_{16}	u_1	0		v_6	30		c_{16}	60		-30
x_{17}	u_1	0		v_7	60		c_{17}	85		-25
x_{21}	u_2	0		v_1	-5		c_{21}	75		-80
x_{22}	u_2	0	+	v_2	35	-	c_{22}	55	=	-20
x_{23}	u_2	0		v_3	25		c_{23}	50		-25
x_{24}	u_2	0		v_4	55		c_{24}	45		10
x_{33}	u_3	25		v_3	25		c_{33}	75		-25
x_{34}	u_3	25		v_4	55		c_{34}	100		-20
x_{36}	u_3	25		v_6	30		c_{36}	150		-95
x_{37}	u_3	25		v_7	60		c_{37}	125		-40

i. Entering variable: x_{24}
   Leaving variable: x_{25}

ii. Entering variable: x_{24}
   Leaving variable: x_{32}

May 2019-Iteration 2

i. When Entering variable: x_{24} , Leaving variable: x_{25}

	1	2	3	4	5	6	7	Capacity
1	50 0	35 9	25 14	55 21	70 0	60 0	85 0	44
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35
3	20 16	60 2	75 0	100 0	85 14	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

∴ May 2019 (Iteration 2)

When Entering variable: x_{24} , Leaving variable: x_{25}

	1	2	3	4	5	6	7	Capacity
1	50 3	35 9	25 14	55 18	70 0	60 0	85 0	44
2	75 0	55 0	50 0	45 3	60 0	30 24	60 8	35
3	20 13	60 2	75 0	100 0	85 17	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

∴Min z =[3(50)+9(35)+14(25)+18(55)+3(45)+24(30)+8(60)+13(20)+2(60)+17(85)] x1000

= Rp.4965000 (reject)

By using Improvised Stepping Stone method, we have calculated the min z = Rp.4965000. Since the min z =Rp.4965000 is bigger than the min z =Rp. 4920000 which we calculated by using VAM method, therefore we reject the leaving variable x_{25}.

May 2019-Iteration 2

ii. When Entering variable: x_{24} ,Leaving variable: x_{32}

	1	2	3	4	5	6	7	Capacity
1	50 0	35 9	25 14	55 21	70 0	60 0	85 0	44
2	75 0	55 0	50 0	45 0	60 3	30 24	60 8	35
3	20 16	60 2	75 0	100 0	85 14	150 0	125 0	32
Demand	16	11	14	21	17	24	8	111

∴ May 2019 (Iteration 2)

When Entering variable: x_{24} ,Leaving variable: x_{32}

	1	2	3	4	5	6	7	Capacity
1	$\boxed{50}$ 0	$\boxed{35}$ 11	$\boxed{25}$ 14	$\boxed{55}$ 19	$\boxed{70}$ 0	$\boxed{60}$ 0	$\boxed{85}$ 0	44
2	$\boxed{75}$ 0	$\boxed{55}$ 0	$\boxed{50}$ 0	$\boxed{45}$ 2	$\boxed{60}$ 1	$\boxed{30}$ 24	$\boxed{60}$ 8	35
3	$\boxed{20}$ 16	$\boxed{60}$ 0	$\boxed{75}$ 0	$\boxed{100}$ 0	$\boxed{85}$ 16	$\boxed{150}$ 0	$\boxed{125}$ 0	32
Demand	16	11	14	21	17	24	8	111

∴ Basic solution: $x_{12} = 11, x_{13} = 14, x_{14} = 19, x_{24} = 2, x_{25} = 1, x_{26} = 24, x_{27} = 8,\ x_{31} = 16,\ x_{35} = 15$

∴ Non basic solution: $x_{11} = x_{15} = x_{16} = x_{17} = x_{21} = x_{22} = x_{23} = x_{32} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

Basic	u_i			v_i			c_{ij}	
x_{12}	u_1	0	+	v_2	35	=	c_{12}	35
x_{13}	u_1	0		v_3	25		c_{13}	25
x_{14}	u_1	0		v_4	55		c_{14}	55
x_{24}	u_2	-10		v_4	55		c_{24}	45
x_{25}	u_2	-10		v_5	70		c_{25}	60
x_{26}	u_2	-10		v_6	40		c_{26}	30
x_{27}	u_2	-10		v_7	70		c_{27}	60
x_{31}	u_3	15		v_1	5		c_{31}	20
x_{35}	u_3	15		v_5	70		c_{35}	85

∴ $u_1 = 0, u_2 = -10, u_3 = 15$

∴ $v_1 = 5, v_2 = 35, v_3 = 25, v_4 = 55, v_5 = 70, v_6 = 40\ , v_7 = 70$

Non-Basic	u_i			v_i			c_{ij}			$u_i + v_i - c_{ij}$
x_{11}	u_1	0	+	v_1	5	-	c_{11}	50	=	-45
x_{15}	u_1	0		v_5	70		c_{15}	70		0
x_{16}	u_1	0		v_6	40		c_{16}	60		-20
x_{17}	u_1	0		v_7	70		c_{17}	85		-15
x_{21}	u_2	-10		v_1	5		c_{21}	75		-80
x_{22}	u_2	-10		v_2	35		c_{22}	55		-30
x_{23}	u_2	-10		v_3	25		c_{23}	50		-35
x_{32}	u_3	15		v_2	35		c_{32}	60		-10
x_{33}	u_3	15		v_3	25		c_{33}	75		-35

x_{34}	u_3	15		v_4	55		c_{34}	100		-30
x_{36}	u_3	15		v_6	40		c_{36}	150		-95
x_{37}	u_3	15		v_7	70		c_{37}	125		-40

Since all is non-positive value, therefore the solution is optimal.

∴ Min z = [11(35)+14(25)+19(55)+2(45)+1(60)+24(30)+8(60)+16(20)+16(85)]x1000

= Rp.4810000

By using Improvised stepping stone method, the cost for May 2019 has been reduced from Rp.4920000 to Rp. 4810000.

Based on the results of Improvised Stepping-Stone for Vogel's Aproximation Method (VAM) in May 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 11 units to Narumonda, 14 units to Porsea, 19 units to Lumban Lobu.

2. Indorayon distributed 2 units to Lumban Lobu, 1 unit to Sipitu-pitu, 24 units to Dolok Nauli and 8 units to Lumban Julu.

3. Balige distributed 16 units to Lagu Boti and 16 units to Sipitu-pitu.

June 2019

	1	2	3	4	5	6	7	Capacity
1	50 0	35 14	25 12	55 18	70 2	60 0	85 2	48
2	75 0	55 0	50 0	45 0	60 0	30 27	60 10	37
3	20 21	60 0	75 0	100 0	85 18	150 0	125 0	39
Demand	21	14	12	18	20	27	12	124

∴ Basic solution: $x_{12} = 14, x_{13} = 12, x_{14} = 18, x_{15} = 2, x_{17} = 2, x_{26} = 27, x_{27} = 10,\ x_{31} = 21,\ x_{35} = 18$

∴ Non basic solution: $x_{11} = x_{16} = x_{21} = x_{22} = x_{23} = x_{24} = x_{25} = x_{32} = x_{33} = x_{34} = x_{36} = x_{37} = 0$

Basic	u_i		+	v_i		=	c_{ij}	
x_{12}	u_1	0		v_2	35		c_{12}	35
x_{13}	u_1	0		v_3	25		c_{13}	25
x_{14}	u_1	0		v_4	55		c_{14}	55
x_{15}	u_1	0		v_5	70		c_{15}	70
x_{17}	u_1	0		v_7	85		c_{17}	85
x_{26}	u_2	-25		v_6	55		c_{26}	30
x_{27}	u_2	-25		v_7	85		c_{27}	60
x_{31}	u_3	15		v_1	5		c_{31}	20
x_{35}	u_3	15		v_5	70		c_{35}	85

$\therefore\ u_1 = 0, u_2 = -25, u_3 = 15$

$\therefore\ v_1 = 5, v_2 = 35, v_3 = 25, v_4 = 55, v_5 = 70, v_6 = 55, v_7 = 85$

Non-Basic	u_i		+	v_i		-	c_{ij}		=	$u_i + v_i - c_{ij}$
x_{11}	u_1	0		v_1	5		c_{11}	50		-45
x_{16}	u_1	0		v_6	55		c_{16}	60		-5
x_{21}	u_2	-25		v_1	5		c_{21}	75		-95
x_{22}	u_2	-25		v_2	35		c_{22}	55		-45
x_{23}	u_2	-25		v_3	25		c_{23}	50		-50
x_{24}	u_2	-25		v_4	55		c_{24}	45		-15
x_{25}	u_2	-25		v_5	70		c_{25}	60		-15
x_{32}	u_3	15		v_2	35		c_{32}	60		-10
x_{33}	u_3	15		v_3	25		c_{33}	75		-35
x_{34}	u_3	15		v_4	55		c_{34}	100		-30
x_{36}	u_3	15		v_6	55		c_{36}	150		-80
x_{37}	u_3	15		v_7	85		c_{37}	125		-25

Since all is non-positive value, therefore the solution is optimal.

∴ Min z = [14(35)+12(25)+18(55)+2(70)+2(85)+27(30)+10(60)+21(20)+18(85)]x1000

= Rp.5450000

Based on the results of Improvised Stepping-Stone for Vogel's Aproximation Method (VAM) in June 2019, it can be described that the distribution of household furniture carried out by UD Yosarita as follows:

1. Par-parean distributed 14 units to Narumonda, 12 units to Porsea, 18 units to Lumban Lobu, 2 units to Sipitu-pitu and 2 units to Lumban Julu.

2. Indorayon distributed 27 units to Dolok Nauli and 10 units to Lumban Julu.

3. Balige distributed 21 units to Lagu Boti and 18 units to Sipitu-pitu.

CONCLUSION

Indubitably, transportation model for cost optimization holds a critical aspect of business management in the modern era, where companies strive to maximize profitability and remain competitive in a dynamic marketplace. Cost optimization is extremely significant in risk management by helping companies maintain stability and sustainability in the long term. By reducing costs, companies can make smart decisions to build resilience and better persist in any economic crisis, industry disruptions, and market fluctuations. It provides a buffer against potential financial challenges and enables companies to adapt to changing market conditions more effectively. Essentially, cost optimization practices is a fundamental aspect of effective financial management and strategic decision-making in companies. By analyzing costs, implementing efficient practices, and continuously seeking opportunities for improvement, companies can enhance profitability, operational efficiency, and long-term sustainability. In the strategic process of identifying and implementing measures to reduce expenses while maintaining or enhancing productivity and profitability, the company must thoroughly analyze and grasp elements of logistics such as warehouse capacity, supply of materials and also market demand to fully utilize cost optimization methods.

In this study, several methods of transportation model are executed to compare and determine the best method to help push the efficiency and productivity of UD Yosarita. The methods include Northwest-Corner Method, Least-Cost Method, Vogel's Approximation Method (VAM) and Improvised Stepping-Stone Method. From the study, it is shown that the Northwest-Corner Method and Least-Cost Method are considerably easy to work with. Least-Cost Method obtains an overall better solution compared to Northwest-Corner Method, however both are unable to produce results which are as efficient as VAM. Despite VAM demanding a more complex calculation process, it obtains a much efficient solution compared to the previous two methods. Thus, results from VAM are selected for the Stepping-Stone Method to create further cost optimization. It is important to note that the more optimized result out of the three methods should be selected as the initial iteration for the Improvised Stepping-Stone Method. This is to decrease the number of iterations which will occur and hence reducing prolonged calculations. As seen from the results in May 2019, Improvised Stepping-Stone Method is extremely reliable in improving the results, where a total of Rp.110000 has been reduced from the total cost. In addition, it is also highly possible that the answers obtain are already fully optimized before Improvised Stepping-Stone Method is carried out, as shown in the results from April 2019 and June 2019. Finally, it can be concluded that the optimized cost for UD Yosarita is Rp.4595000 in April 2019, Rp.4810000 in May 2019 and Rp.5450000 in June 2019.

From the above findings, UD Yosarita has received a substantial boost in productivity and competency as a company. By implementing cost optimization methods, the company has enabled a more efficient allocation of warehouse capacity and higher capability of meeting market demands. UD Yosarita has analyzed their demand patterns and space utilization to make informed decisions about how to allocate their warehouse capacity effectively to avoid underutilization or overutilization. By leveraging data-driven insights, the company can make informed decisions about inventory management and picking strategies. Overall, this has lead to cost savings, increased efficiency, improved customer satisfaction, and better alignment of warehouse operations with market demands.

REFERENCE