# 1819-108-C1-W6-FirstExam-Final

Anna Bogachova

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(a)

| | 1 | 5 | 7 | 11 |
|---|---|---|---|---|
| 1 | 1 | 5 | 7 | 11 |
| 5 | 5 | 1 | 11 | 7 |
| 7 | 7 | 11 | 1 | 5 |
| 11 | 11 | 7 | 5 | 1 |

(b)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 4 | 1 | 3 |
| 3 | 3 | 1 | 4 | 2 |
| 4 | 4 | 3 | 2 | 1 |

Table 28.2 (a) The multiplication table for the group $S' = \{1, 5, 7, 11\}$ under multiplication (mod 24). (b) The multiplication table for the group $S'' = \{1, 2, 3, 4\}$ under multiplication (mod 5).

| | $I$ | $A$ | $B$ | $C$ |
|---|---|---|---|---|
| $I$ | $I$ | $A$ | $B$ | $C$ |
| $A$ | $A$ | $I$ | $C$ | $B$ |
| $B$ | $B$ | $C$ | $I$ | $A$ |
| $C$ | $C$ | $B$ | $A$ | $I$ |

Table 28.3 The common structure exemplified by tables 28.1 and 28.2(a).

| | $1$ | $i$ | $-1$ | $-i$ |
|---|---|---|---|---|
| $1$ | $1$ | $i$ | $-1$ | $-i$ |
| $i$ | $i$ | $-1$ | $-i$ | $1$ |
| $-1$ | $-1$ | $-i$ | $1$ | $i$ |
| $-i$ | $-i$ | $1$ | $i$ | $-1$ |

Table 28.4 The group table for the set $\{1, i, -1, -i\}$ under ordinary multiplication of complex numbers.

is multiplication (mod 24). However, the really important point is that the two groups $S$ and $S'$ have equivalent group multiplication tables – they are said to be *isomorphic*, a matter to which we will return more formally in section 28.5.

►*Determine the behaviour of the set of four elements*

$$\{1, i, -1, -i\}$$

*under the ordinary multiplication of complex numbers. Show that they form a group and determine whether the group is isomorphic to either of the groups $S$ (itself isomorphic to $S'$) and $S''$ defined above.*

That the elements form a group under the associative operation of complex multiplication is immediate; there is an identity (1), each possible product generates a member of the set and each element has an inverse (1, $-i$, $-1$, $i$, respectively). The group table has the form shown in table 28.4.

We now ask whether this table can be made to look like table 28.3, which is the standardised form of the tables for $S$ and $S'$. Since the identity element of the group (1) will have to be represented by $I$, and '1' only appears on the leading diagonal twice whereas $I$ appears on the leading diagonal four times in table 28.3, it is clear that no

(a)

| | 1 | 5 | 7 | 11 |
|---|---|---|---|---|
| 1 | 1 | 5 | 7 | 11 |
| 5 | 5 | 1 | 11 | 7 |
| 7 | 7 | 11 | 1 | 5 |
| 11 | 11 | 7 | 5 | 1 |

(b)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 4 | 1 | 3 |
| 3 | 3 | 1 | 4 | 2 |
| 4 | 4 | 3 | 2 | 1 |

Table 28.2 (a) The multiplication table for the group $S' = (1, 5, 7, 11)$ under multiplication (mod 24). (b) The multiplication table for the group $S' = (1, 2, 3, 4)$ under multiplication (mod 5).

| | 1 | A | B | C |
|---|---|---|---|---|
| 1 | 1 | A | B | C |
| A | A | 1 | C | B |
| B | B | C | 1 | A |
| C | C | B | A | 1 |

Table 28.3 The common structure exemplified by tables 28.1 and 28.2(a).

| | 1 | $i$ | -1 | -$i$ |
|---|---|---|---|---|
| 1 | 1 | $i$ | -1 | -$i$ |
| $i$ | $i$ | -1 | -$i$ | 1 |
| -1 | -1 | -$i$ | 1 | $i$ |
| -$i$ | -$i$ | 1 | $i$ | -1 |

Table 28.4 The group table for the set (1,$i$,-1,-$i$) under ordinary multiplication of complex numbers.

is multiplication (mod 24). However, the really important point is that the two groups $S$ and S' have equivalent group multiplication tables - they are said to be *isomorphic*, a matter to which we will return more formally in section 28.5.

► *Determine the behaviour of the set of four elements*

(1,$i$,-1,-$i$)

*under the ordinary multiplication of complex numbers.Show that they form a group and determine whether the group is isomorphic to either of the groups S (itself isomorphic to S') and S" defined above.*

That the elements form a group under the associative operation of complex multiplication is immediate; there is an identity (1), each possible product generates a member of the set and each element has an inverse (1,$i$,-1,-$i$, respectively). The group table has the form shown in table 28.4.

We now ask whether this table can be made to look like table 28.3, which is the standardised form of the tables for $S$ and $S'$. Since the identity elementnof the group (1) will have to be represented by I, and '1' only appears on the leading diagonal twice whereas $I$ appears on the leading diagonal four times in table 28.3, it is clear that no

```
\documentclass[9pt]{extarticle}
\usepackage[utf8]{inputenc}

\usepackage{ragged2e}
\usepackage{subcaption}
\usepackage{mdframed}
\usepackage{xcolor}
\usepackage[paperheight=247mm,paperwidth=178mm,margin=20mm,heightrounded]{geometr
\usepackage{tikz}
\usetikzlibrary{patterns}
\usepackage{scrextend}
\def\changemargin#1#2{\list{}{\rightmargin#2\leftmargin#1}\item[]}
\let\endchangemargin=\endlist

\begin{document}


\title {\color{black}{1819-108-C1-W6-FirstExam-Final}}
\author{\color{black}{Anna Bogachova}}
\date{\color{black}{March 2019}}

\maketitle

\pagebreak

\includegraphics[width=\textwidth]{IMG_0694}

\usetikzlibrary{patterns}

\pagebreak

\centerline{28.2 FINITE GROUPS}
\rule{13.5cm}{0.4pt}
\vspace*{10px}



 \begin{table}[!htb]
    \begin{subtable}{.5\linewidth}
      \centering
  \begin{enumerate}
          \item[$(a)$]
          \hspace{5px}
         \begin{tabular}{c|c c c c}
          \hline\hline
            &  1 & 5 & 7 & 11 \\
          \hline
          1  &  1 & 5 & 7 & 11 \\
          5  &  5 & 1 & 11 & 7 \\
          7  &  7 & 11 & 1 & 5 \\
         11  &  11 & 7 & 5 & 1 \\
          \hline\hline
         \end{tabular}
  \end{enumerate}
```

```
    \end{subtable}%
    \begin{subtable}{.5\linewidth}
      \centering
  \begin{enumerate}
        \item[$(b)$]
        \hspace{5px}
        \begin{tabular}{c|c c c c}
        \hline\hline
           &  1 & 2 & 3 & 4 \\
        \hline
        1  &  1 & 2 & 3 & 4 \\
        2  &  2 & 4 & 1 & 3 \\
        3  &  3 & 1 & 4 & 2 \\
        4  &  4 & 3 & 2 & 1 \\
        \hline\hline
        \end{tabular}
 \end{enumerate}
    \end{subtable}
 \end{table}


\parindent=0in


\begin{FlushLeft}
\begin{addmargin}[7em]{2em}
Table 28.2  (a) The multiplication table for the group $S'=(1,5,7,11)$ under mult
\end{addmargin}
\end{FlushLeft}


\hfill


\begin{center}
\begin{tabular}{c|c c c c}
\hline\hline
    &  1 & A & B & C \\
\hline
  1  &  1 & A & B & C \\
  A  &  A & 1 & C & B \\
  B  &  B & C & 1 & A \\
  C  &  C & B & A & 1 \\
\hline\hline
\end{tabular}
\end{center}


\begin{addmargin}[7em]{1em}
Table 28.3  \hspace{10px}The common structure exemplified by tables 28.1 and 28.2
\end{addmargin}
```

```
\hfill

\begin{center}
\begin{tabular}{c|c c c c}
\hline\hline
    &  1 & \textit{i} & −1 & \textit{−i} \\
\hline
1  &  1 & \textit{i} & −1 & \textit{−i} \\
  \textit{i}  &  \textit{i} & −1 & \textit{−i} & 1 \\
  −1  &  −1 & \textit{−i} & 1 & \textit{i} \\
  \textit{−i}  &  \textit{−i} & 1 & \textit{i} & −1 \\
\hline\hline
\end{tabular}
\end{center}


\begin{FlushLeft}
\begin{addmargin}[7em]{1em}
Table 28.4 \hspace{10px}The group table for the set (1,\textit{i},−1,\textit{−i})
\end{addmargin}
\end{FlushLeft}

is multiplication (mod 24). However, the really important point is that the two g

\hfill

 \begin{mdframed}[backgroundcolor=grey!20]
   \begin{tikzpicture}
    \draw (0,0) node[anchor=north]{}
    −− (0.15,0.05) node[anchor=north]{}
    −− (0,0.1) node[anchor=south]{}
    −− cycle;
    \fill[pattern color=blsck ](0,0) −− (0.15,0.05) −− (0,0.1) −− cycle;
   \end{tikzpicture}
   \textit{Determine the behaviour of the set of four elements}
   \begin{center}
       (1,\textit{i},−1,\textit{−i})
   \end{center}
   \textit{under the ordinary multiplication of complex numbers.Show that they for
 \end{mdframed}


That the elements form a group under the associative operation of complex multipl

\setlength{\parindent}{1em}

We now ask whether this table can be made to look like table 28.3, which is the s

\end{document}
```