

A small note about the error function

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In mathematics, the error function (also called the Gauss error function) is a special function (non-elementary) of sigmoid shape that occurs in probability, statistics, and partial differential equations describing diffusion. It is defined as:

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_a^b e^{-t^2} dt \quad (1)$$

In statistics, for nonnegative values of x , the error function has the following interpretation: for a random variable Y that is normally distributed with mean 0 and variance 1/2, $\operatorname{erf}(x)$ describes the probability of Y falling in the range $[-x, x]$

When the results of a series of measurements are described by a normal distribution with *standard deviation* σ and expected value 0, then $\operatorname{erf}(\frac{a}{\sigma\sqrt{2}})$ is the probability that the error of a single measurement lies between $-a$ and $+a$, for positive a . This is useful, for example, in determining the bit error rate of a digital communication system.

The error and complementary error functions occur, for example, in solutions of the heat equation when boundary conditions are given by the Heaviside step function.

The error function and its approximations can be used to estimate results that hold with high probability.

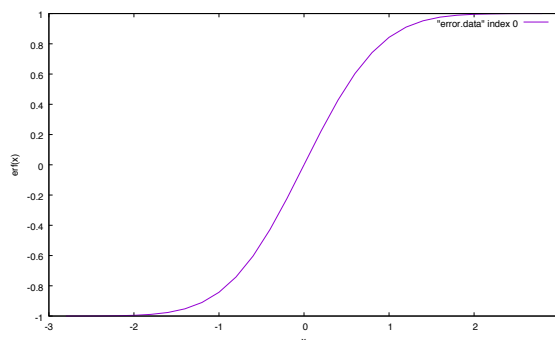


Figure 1: Graph of the error function in the range $x = [-3,3]$