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Reproduction of "Fast and Accurate Matrix Completion via Truncated Nuclear Norm Regularization"

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Dec. 09, 2022

Contests

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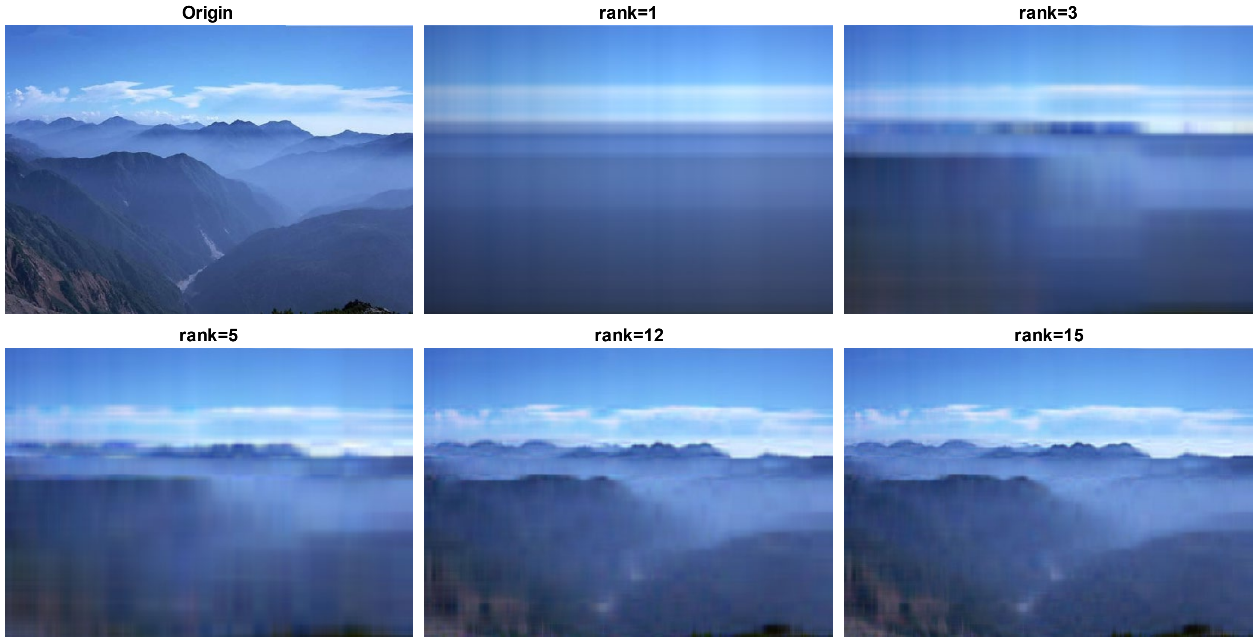
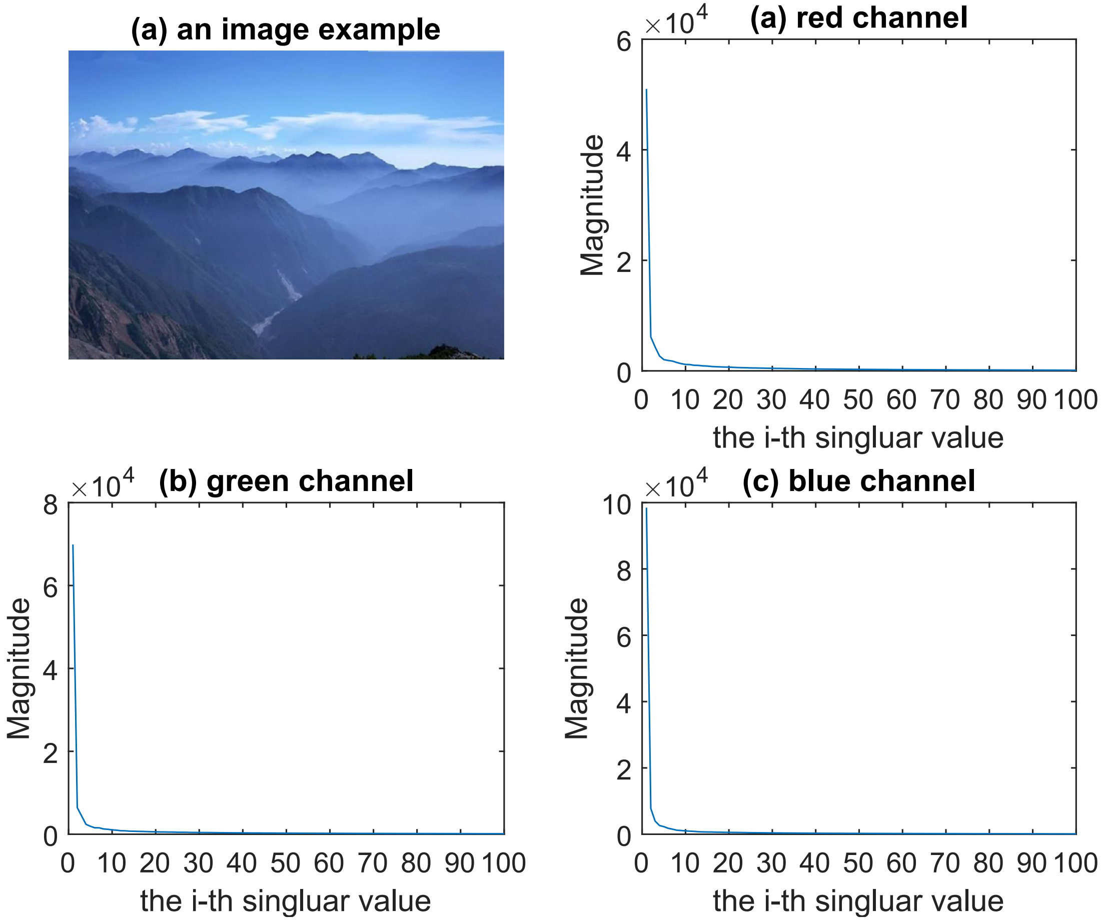
• 1. Background – rank

Using `svd` function decompose the image (Channels separately)

- The first few singular values are much larger than others
- For r > 20, r-th singular value σ_r close to 0



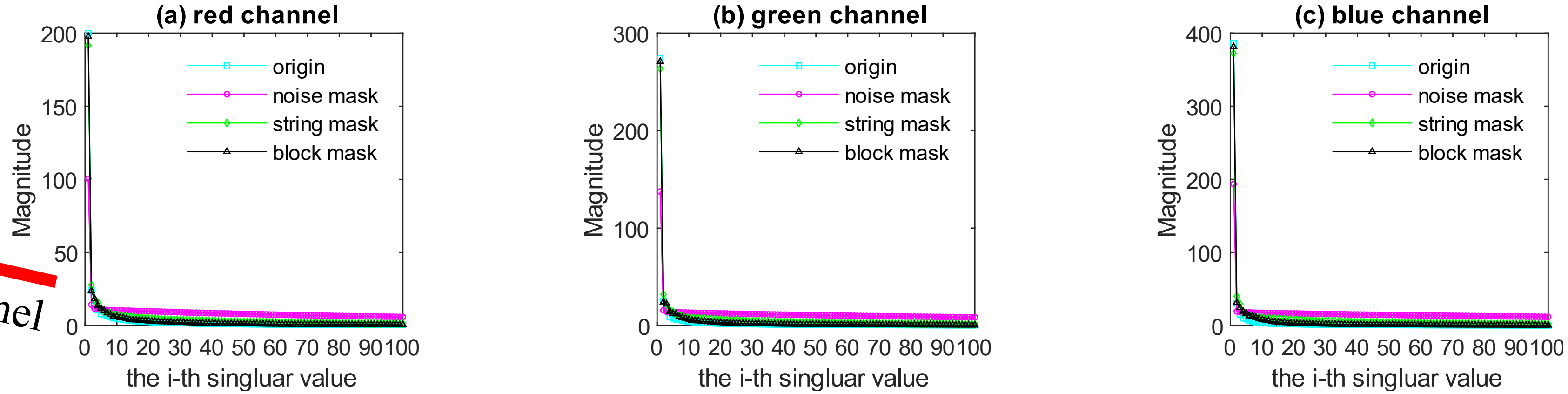


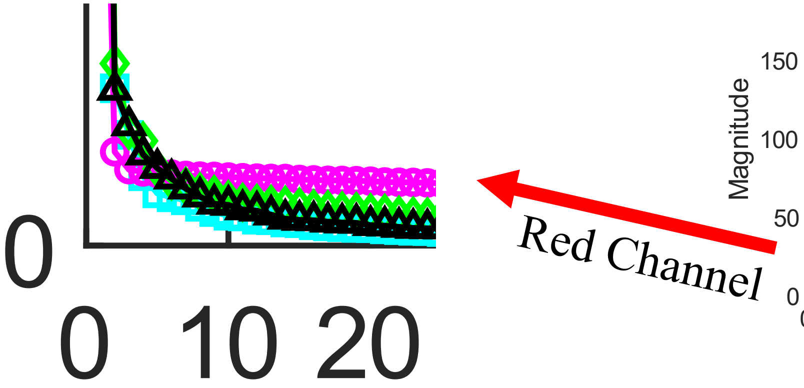
First Few values contain main information → Low rank approximate is reasonable → Truncated svd (svds)

• 1. Background – with mask

We mask the figure with three ways:

- Random mask (50% missing)
- String mask (add string)
- Block mask (add block)





$$\min_{\mathbf{X}} \quad \mathrm{rank}(\mathbf{X}) \qquad \qquad \min_{\mathbf{X}} \quad \|\mathbf{X}\|_*$$
$$s.t. \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}) \quad \Rightarrow \quad s.t. \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}) \quad [1]$$

$\mathbf{M} \in \mathbb{R}^{m \times n}$, $\mathbf{X} \in \mathbb{R}^{m \times n}$, $\mathbf{\Omega}$ observed entries, Orthogonal projection operator $\mathcal{P}_\Omega$

[1] M. Fazel, "Matrix Rank Minimization with Applications," PhD thesis, Stanford Univ., 2002.

• 2. Related Work

- Singular Value Thresholding (SVT)[1]: Time complexity $\mathcal{O}(\frac{1}{N})$

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* + \alpha\|\mathbf{X}\|_F^2 \\ s.t. \quad & \mathcal{P}_{\mathbf{\Omega}}(\mathbf{X}) = \mathcal{P}_{\mathbf{\Omega}}(\mathbf{M}) \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \|\mathbf{X}\|_* + \alpha\|\mathbf{X}\|_F^2 + \langle \mathbf{Y}, \mathcal{P}_{\mathbf{\Omega}}(\mathbf{M}-\mathbf{X})\rangle$$

- Singular Value Projection (SVP)[2]: Solve the rank minimization problem

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{P}_{\mathbf{\Omega}}(\mathbf{X}) - P_{\mathbf{\Omega}}(\mathbf{M})\|_F^2 \\ s.t. \quad & \mathrm{rank}(\mathbf{X}) \le r \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \min_{\mathbf{S}\in\mathbb{R}^{r\times r}} \|\mathcal{A}(\mathbf{X}) - \mathbf{y}\|_F^2 \Rightarrow \begin{aligned} \mathbf{Y}_{k+1} &= \mathbf{X}_k - \gamma_k \mathcal{A}^*(\mathcal{A}(\mathbf{X}_k) - y)) \\ \mathbf{X}_{k+1} &= \text{Trancated SVD}_r(\mathbf{Y}_{k+1}) \end{aligned}$$

- OptSpace[3]:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{P}_{\mathbf{\Omega}}(\mathbf{X}) - P_{\mathbf{\Omega}}(\mathbf{M})\|_F \\ s.t. \quad & \mathrm{rank}(\mathbf{X}) \le r \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \min_{\mathbf{S}\in\mathbb{R}^{r\times r}} L(\mathbf{X},\mathbf{Y},\mathbf{S}) = \frac{1}{2}\|\mathcal{P}_{\mathbf{\Omega}}(\mathbf{M}-\mathbf{XSY}^T)\|_F^2 + \frac{\lambda}{2}\|\mathcal{P}_{\mathbf{\Omega}^c}(\mathbf{M}-\mathbf{XSY}^T)\|_F^2$$

$\mathbf{Y} \in \mathbb{R}^{m\times n}$ is the Lagrange multiplier matrix, inner produce $\langle \mathbf{X},\mathbf{Y}\rangle = \sum_{i,j} X_{ij}Y_{ij}$

[1] J.F. Cai, E.J. Candès, and Z. Shen, “A Singular Value Thresholding Algorithm for Matrix Completion,” SIAM J. Optimization, vol. 20, pp. 1956-1982, 2010.
[2] P. Jain, R. Meka, and I. Dhillon, “Guaranteed Rank Minimization via Singular Value Projection,” in Advances in Neural Information Processing Systems, 2010, vol. 23
[3] R. H. Keshavan and S. Oh, “A Gradient Descent Algorithm on the Grassman Manifold for Matrix Completion,” Transportation Research Part C: Emerging Technologies, vol. 28, pp. 15–27, Mar. 2013.

• 3. Truncated Nuclear Norm Regularization (TNNR)

- Truncated nuclear norm: $$\|\mathbf{X}\|_r = \sum_{i=r+1}^{\min(m,n)} \sigma_i(\mathbf{X}) \ (\mathbf{X} \in \mathbb{R}^{m\times n})$$

• 3. Truncated Nuclear Norm Regularization (TNNR)

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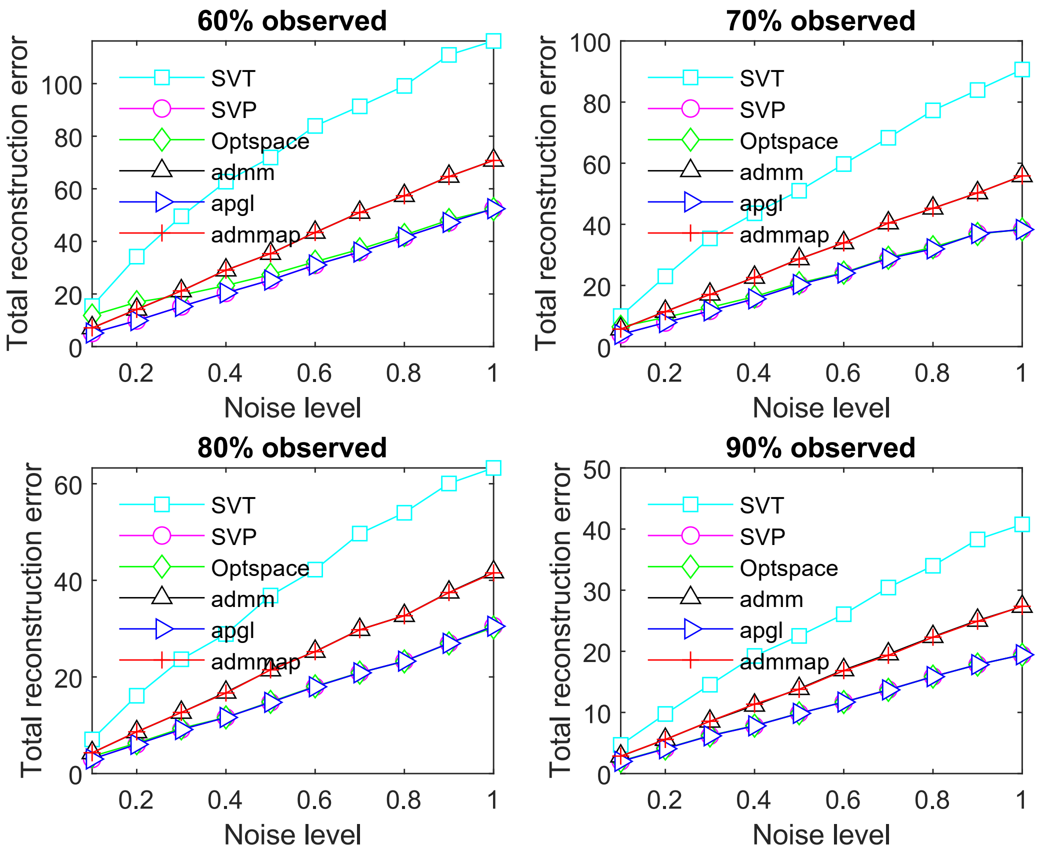
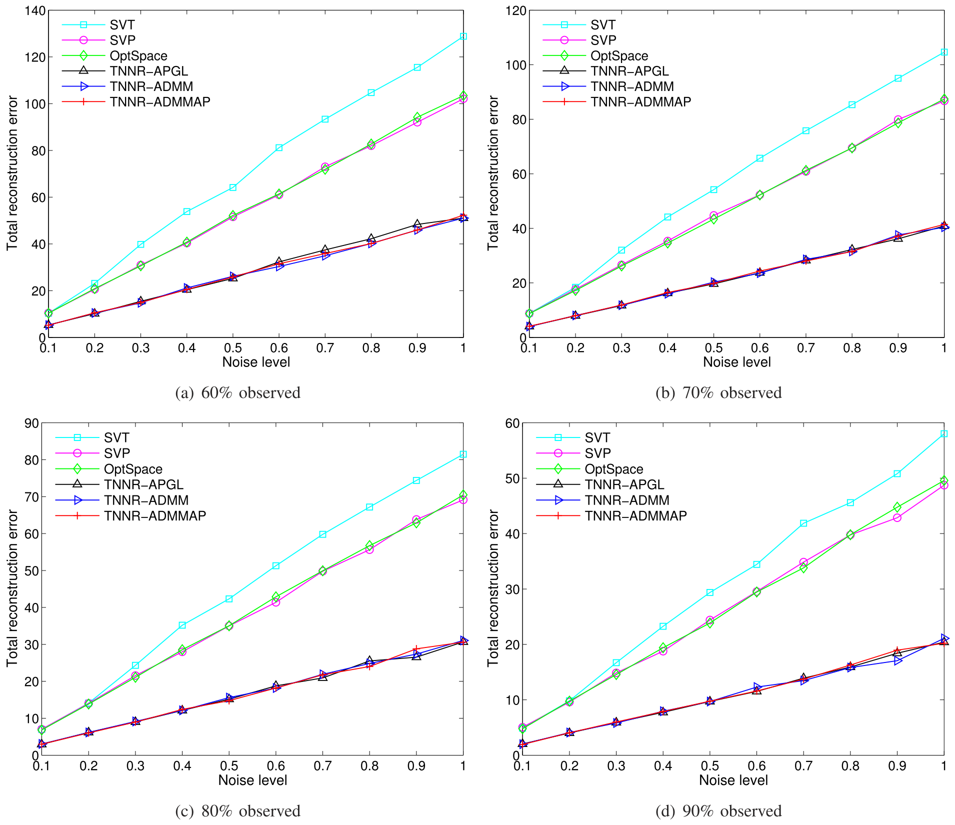
$$
\begin{aligned}
\min_{\mathbf{X}} \quad & \|\mathbf{X}\|_r \\
s.t. \quad & \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M})
\end{aligned}
\Rightarrow
\begin{aligned}
\min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* - \max_{\mathbf{A}\mathbf{A}^T=\mathbf{I},\mathbf{B}\mathbf{B}^T=\mathbf{I}} \mathrm{Tr}(\mathbf{A}\mathbf{X}\mathbf{B}^T) \\
s.t. \quad & \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M})
\end{aligned}
\Rightarrow
\begin{aligned}
\min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* - \mathrm{Tr}(\mathbf{A}\mathbf{X}\mathbf{B}^T) \\
s.t. \quad & \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M})
\end{aligned}
$$

- 4. Optimization ways -- ADMM & APGL & ADMMAP

• 5. Results



The reproduction of Fig. 2.



The original of Fig. 2.

Data Generation: $\mathbf{B} = \mathbf{M} + \sigma \mathbf{Z},\ B_{ij} = M_{ij} + \sigma Z_{ij},\ (i,j) \in \mathbf{\Omega}$

$\mathbf{M} = \mathbf{M}_L \mathbf{M}_R,\ \mathbf{M}_L \in \mathbb{R}^{m \times r_0},\ \mathbf{M}_R \in \mathbb{R}^{r_0 \times n}$

$\mathbf{M}_L,\ \mathbf{M}_R,\ \mathbf{Z}$ is gaussian.

Metric:

Total reconstruction error: $\|\mathcal{P}_{\mathbf{\Omega}^c}(\mathbf{X}_{sol} - \mathbf{X}_{full})\|_F$

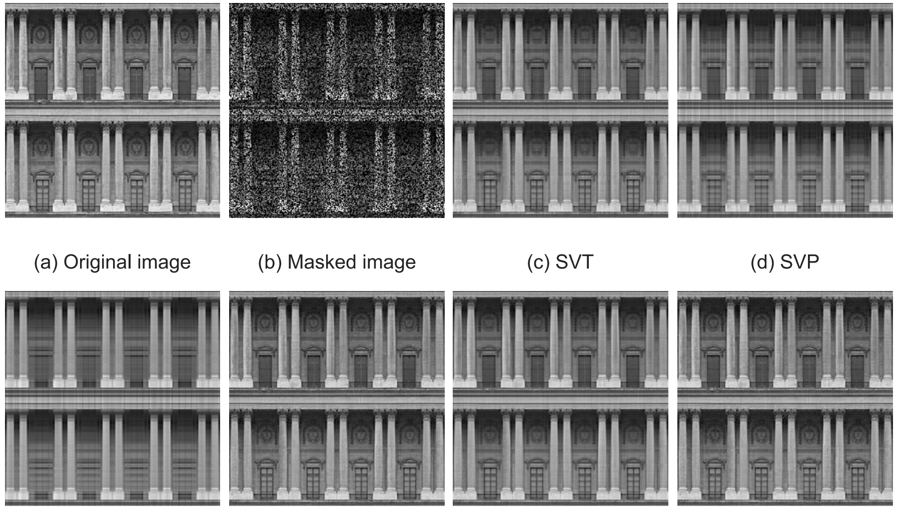
• 5. Results

β	1e-3	r	5
$\beta_{\max}$	1e10	ϵ_0	1e-3
ρ_0	1.1	ϵ	1e-4
κ	1e-3	λ	1e-2
p_{SVT}	0.87	τ_{opt}	1e-3
δ_{2k}	0.2	$iter_{\max}$	100

Total number of missing pixels: T

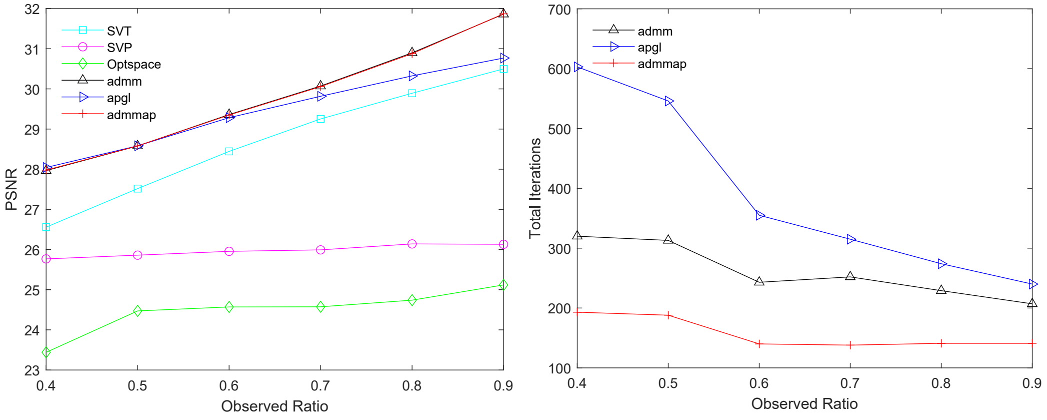
$$\text{MSE} = \frac{error_r^2 + error_g^2 + error_b^2}{3T}$$

$$\text{PSNR} = 10 \times \log_{10}\left(\frac{255^2}{MSE}\right)$$



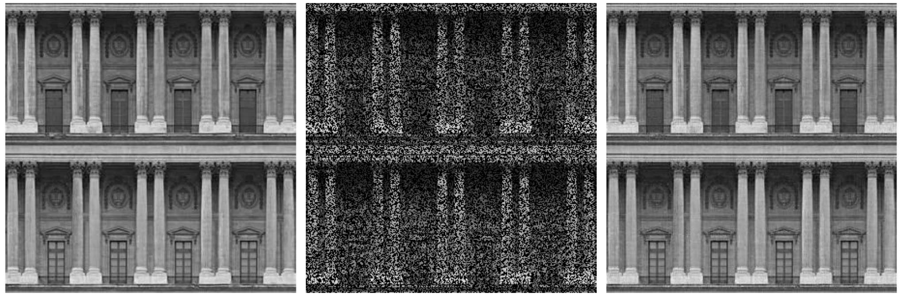
(a) Original image (b) Masked image (c) SVT (d) SVP

(e) Opt (f) tnnr-admm (g) tnnr-apgl (h) tnnr-admmap



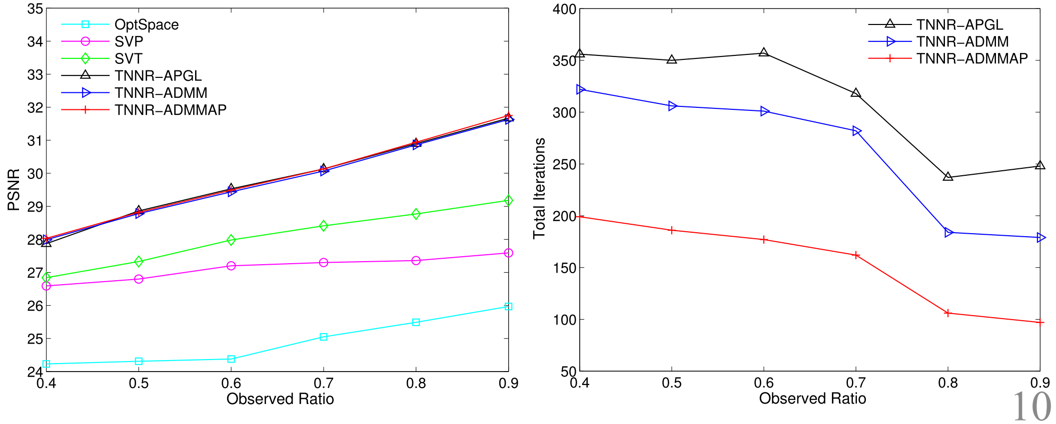
The reproduction of Fig. 4, Fig. 5, Fig. 6.

β	1e-3	r	1~20
$\beta_{\max}$	1e10	ϵ_0	1e-3
ρ_0	1.9	ϵ	1e-4
κ	1e-3	λ	0.06



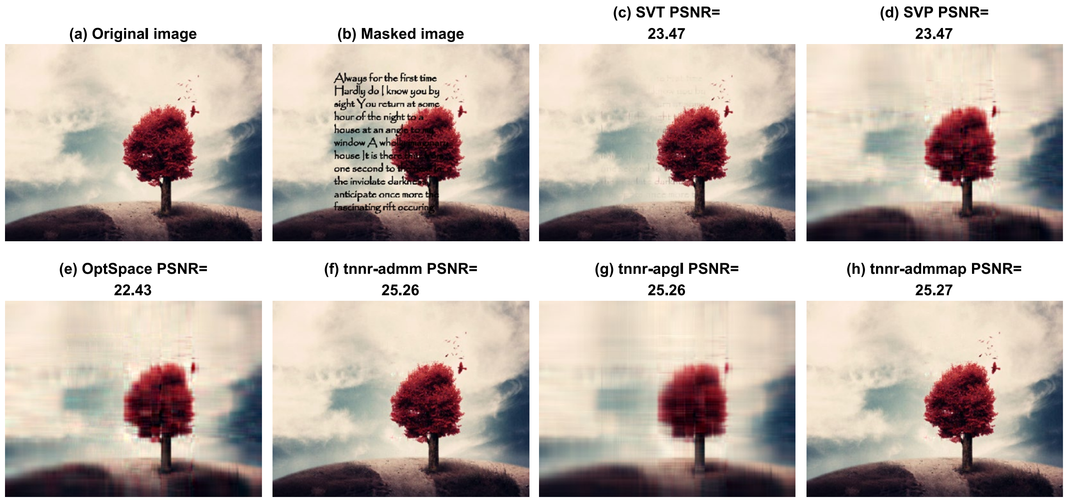
(a) Original image (b) Masked image (c) Recovered image by TNNR-ADMMAP

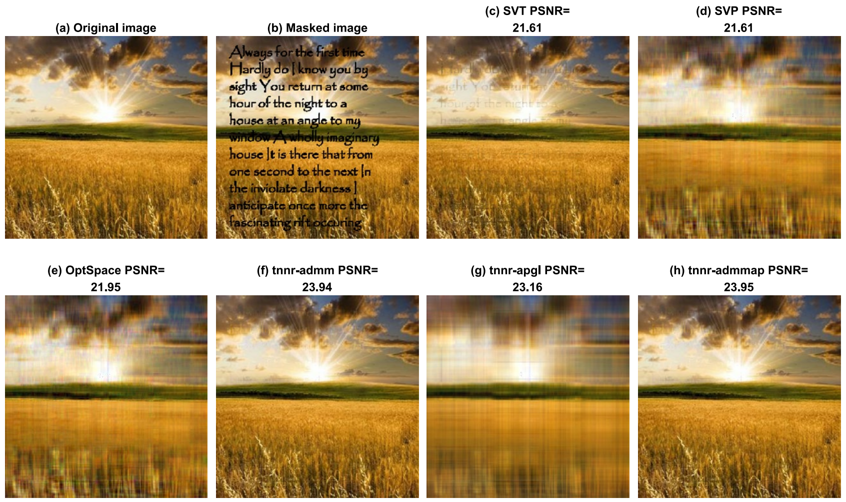
The original of Fig. 4, Fig. 5, Fig. 6.



• 5. Results

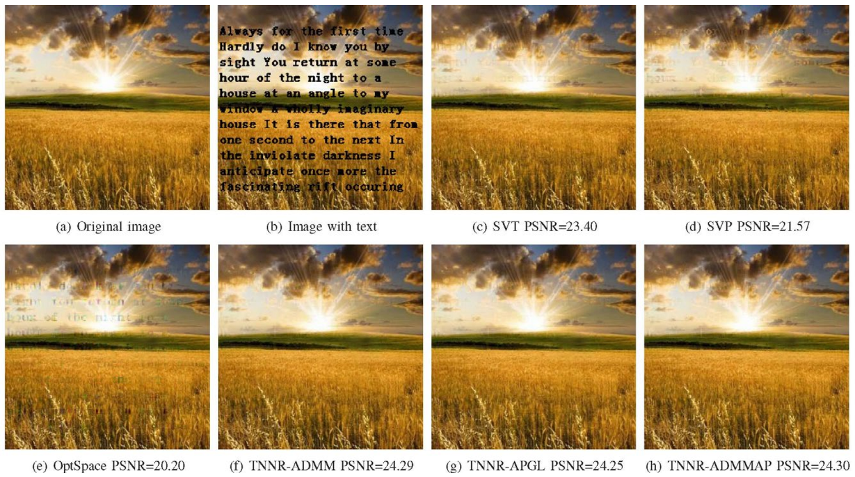
β	1e-3	r	3~15
$\beta_{\max}$	1e10	ϵ_0	1e-3
ρ_0	1.01	ϵ	1e-4
κ	1e-3	λ	1e-3
p_{SVT}	0.8	τ_{opt}	1e-3
δ_{2k}	0.2	$iter_{\max}$	1e2/1e3





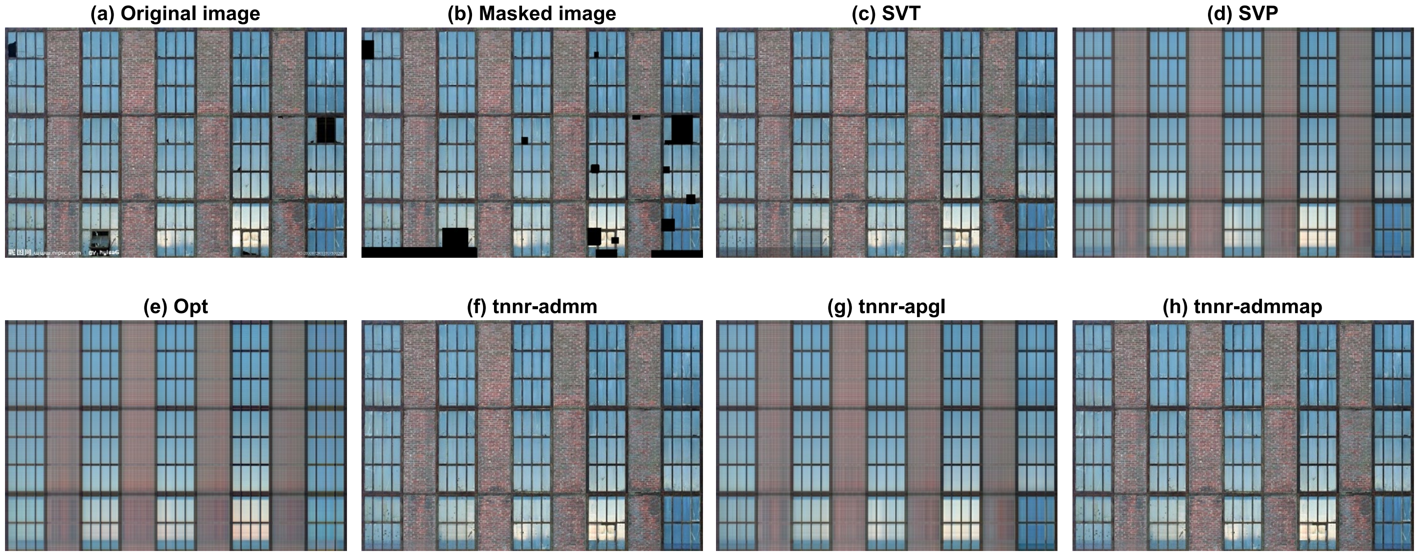
The reproduction of Fig. 7, Fig. 8.

The original Parameter Setting is equal to Fig. 4-6.



The original of Fig. 7, Fig. 8.

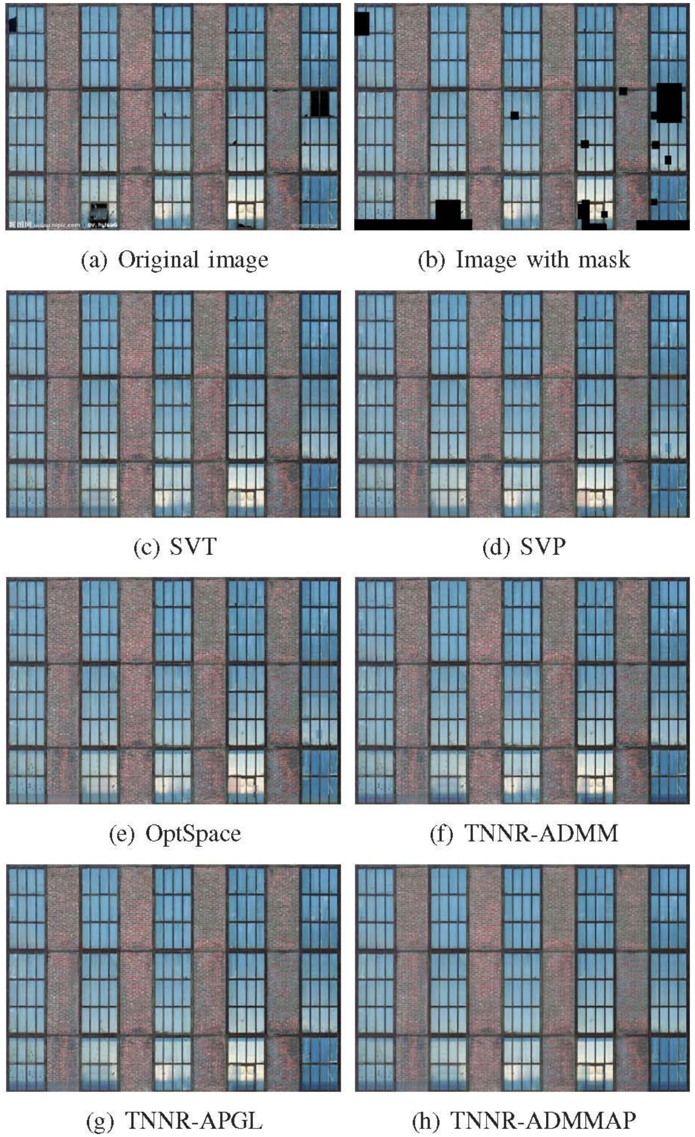
• 5. Results



The reproduction of Fig. 9.

β	1e-3	r	3	p_{SVT}	0.8
$\beta_{\max}$	1e10	ϵ_0	1e-3	τ_{opt}	1e-2
ρ_0	1.01	ϵ	1e-4	δ_{2k}	0.2
κ	1e-3	λ	1e-3	$iter_{\max}$	1e2/1e3

The original Parameter Setting is equal to Fig. 4-6.



The original of Fig. 9.

Source code available at: https://github.com/AnnLIU15/matrixTheory

Thanks for listening