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Reproduction of "Fast and Accurate Matrix Completion via Truncated Nuclear Norm Regularization"

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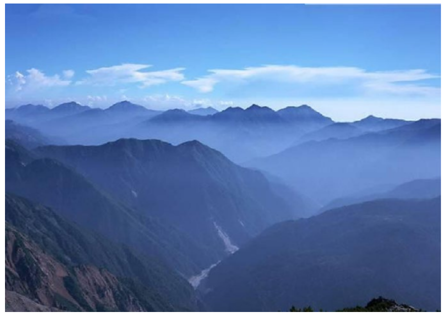
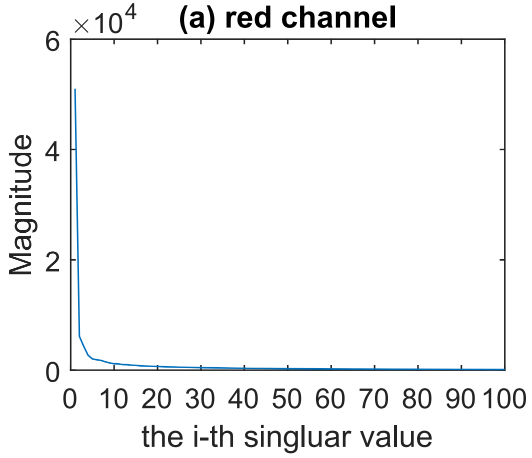
Contests

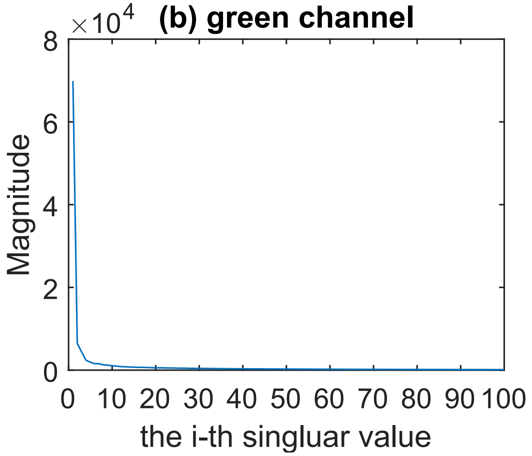
• 1. Background – rank

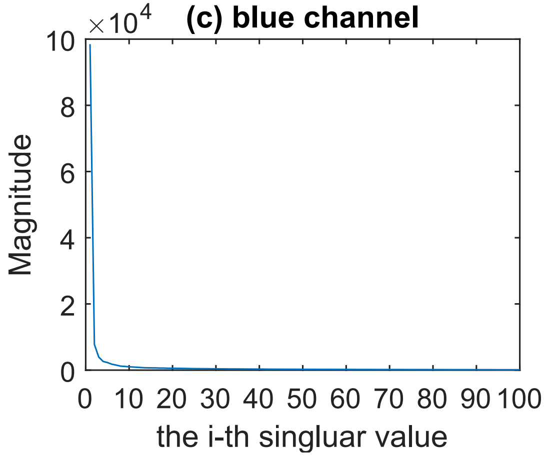
Using `svd` function decompose the image (Channels separately)

- The first few singular values are much larger than others
- For r > 20, r-th singular value σ_r close to 0







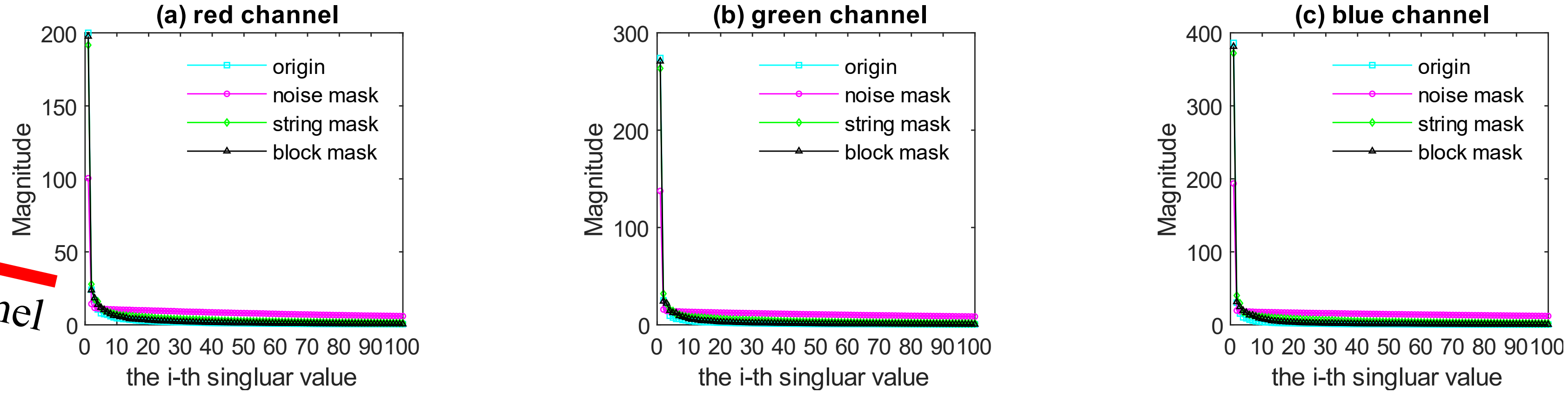


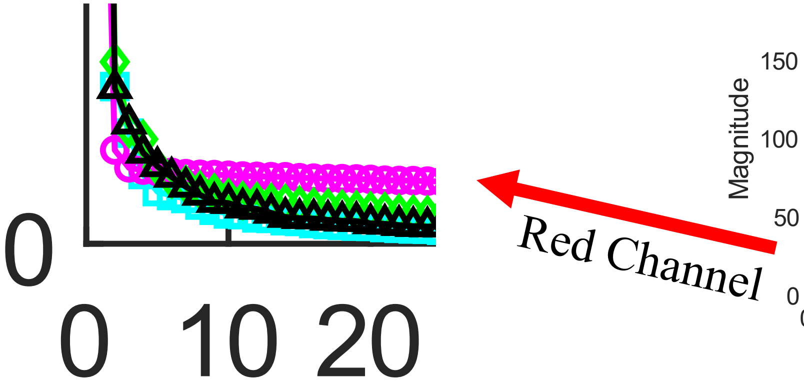
First Few values contain main information → Low rank approximate is reasonable → Truncated svd (svds)

• 1. Background – with mask

We mask the figure with three ways:

- Random mask (50% missing)
- String mask (add string)
- Block mask (add block)





$$\min_{\mathbf{X}} \quad \mathrm{rank}(\mathbf{X}) \qquad \qquad \min_{\mathbf{X}} \quad \|\mathbf{X}\|_*$$

$$s.t. \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}) \quad \Rightarrow \quad s.t. \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}) \quad [1]$$

$\mathbf{M} \in \mathbb{R}^{m\times n}$, $\mathbf{X} \in \mathbb{R}^{m\times n}$, $\mathbf{\Omega}$ observed entries, Orthogonal projection operator $\mathcal{P}_\Omega$

[1] M. Fazel, “Matrix Rank Minimization with Applications,” PhD thesis, Stanford Univ., 2002.

• 2. Related Work

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- Singular Value Thresholding (SVT)[1]: Time complexity $\mathcal{O}(\frac{1}{N})$

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* + \alpha\|\mathbf{X}\|_F^2 \\ s.t. \quad & \mathcal{P}_{\mathbf{\Omega}}(\mathbf{X}) = \mathcal{P}_{\mathbf{\Omega}}(\mathbf{M}) \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \|\mathbf{X}\|_* + \alpha\|\mathbf{X}\|_F^2 + \langle \mathbf{Y}, \mathcal{P}_{\mathbf{\Omega}}(\mathbf{M}-\mathbf{X})\rangle$$

- Singular Value Projection (SVP)[2]: Solve the rank minimization problem

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{P}_{\mathbf{\Omega}}(\mathbf{X}) - P_{\mathbf{\Omega}}(\mathbf{M})\|_F^2 \\ s.t. \quad & \mathrm{rank}(\mathbf{X}) \leq r \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \min_{\mathbf{S}\in\mathbb{R}^{r\times r}} \|\mathcal{A}(\mathbf{X}) - \mathbf{y}\|_F^2 \Rightarrow \begin{aligned} \mathbf{Y}_{k+1} &= \mathbf{X}_k - \gamma_k \mathcal{A}^*(\mathcal{A}(\mathbf{X}_k) - y)) \\ \mathbf{X}_{k+1} &= \text{Trancated SVD}_r(\mathbf{Y}_{k+1}) \end{aligned}$$

- OptSpace[3]:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{P}_{\mathbf{\Omega}}(\mathbf{X}) - P_{\mathbf{\Omega}}(\mathbf{M})\|_F \\ s.t. \quad & \mathrm{rank}(\mathbf{X}) \leq r \end{aligned} \Rightarrow L(\mathbf{X},\mathbf{Y}) = \min_{\mathbf{S}\in\mathbb{R}^{r\times r}} L(\mathbf{X},\mathbf{Y},\mathbf{S}) = \frac{1}{2}\|\mathcal{P}_{\mathbf{\Omega}}(\mathbf{M} - \mathbf{XSY}^T)\|_F^2 + \frac{\lambda}{2}\|\mathcal{P}_{\mathbf{\Omega}^c}(\mathbf{M} - \mathbf{XSY}^T)\|_F^2$$

$\mathbf{Y} \in \mathbb{R}^{m\times n}$ is the Lagrange multiplier matrix, inner produce $\langle \mathbf{X}, \mathbf{Y}\rangle = \sum_{i,j} X_{ij}Y_{ij}$

[1] J.F. Cai, E.J. Candès, and Z. Shen, "A Singular Value Thresholding Algorithm for Matrix Completion," SIAM J. Optimization, vol. 20, pp. 1956-1982, 2010.
[2] P. Jain, R. Meka, and I. Dhillon, "Guaranteed Rank Minimization via Singular Value Projection," in Advances in Neural Information Processing Systems, 2010, vol. 23
[3] R. H. Keshavan and S. Oh, "A Gradient Descent Algorithm on the Grassman Manifold for Matrix Completion," Transportation Research Part C: Emerging Technologies, vol. 28, pp. 15–27, Mar. 2013.

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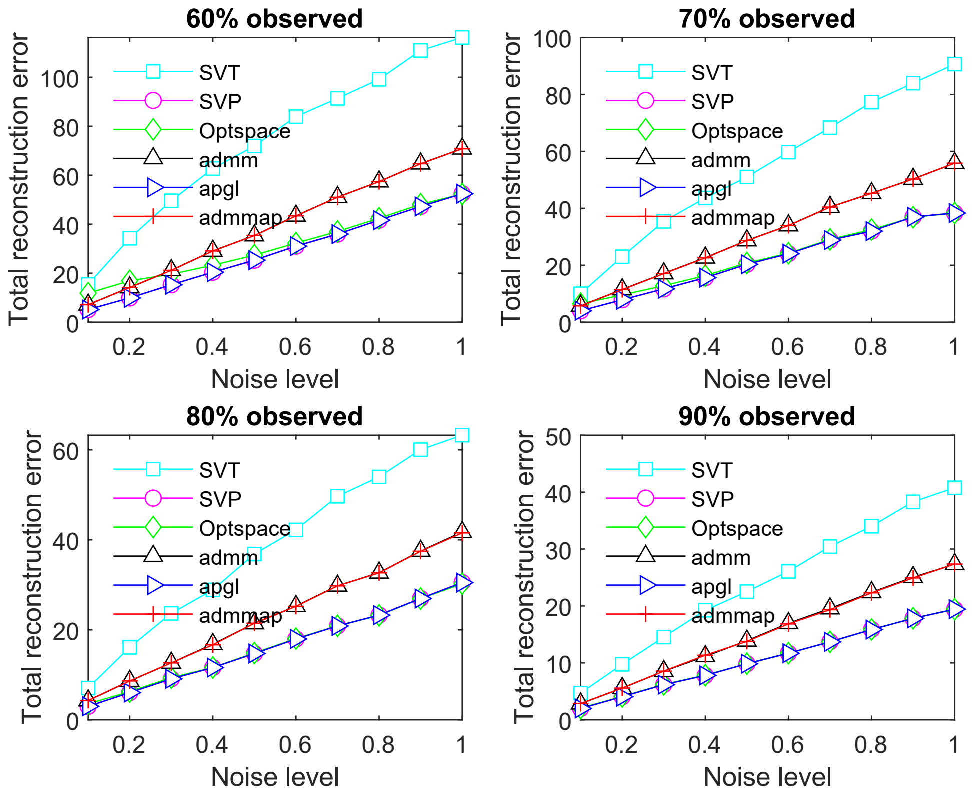
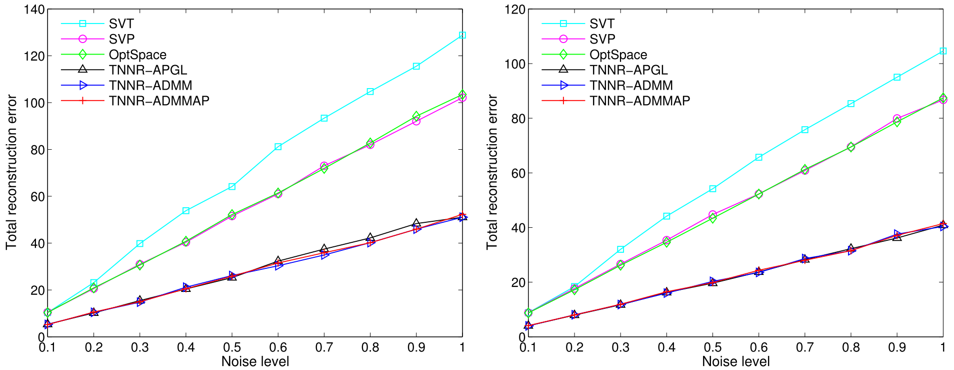
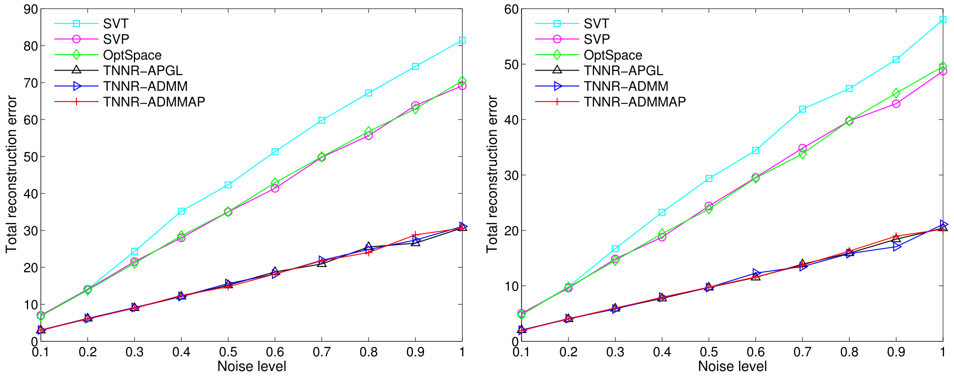
• 5. Results





(a) 60% observed

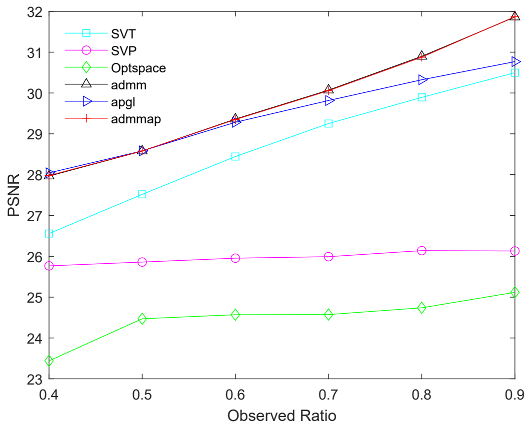
(b) 70% observed

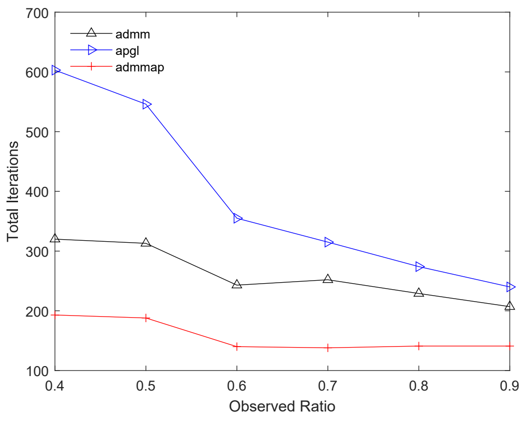


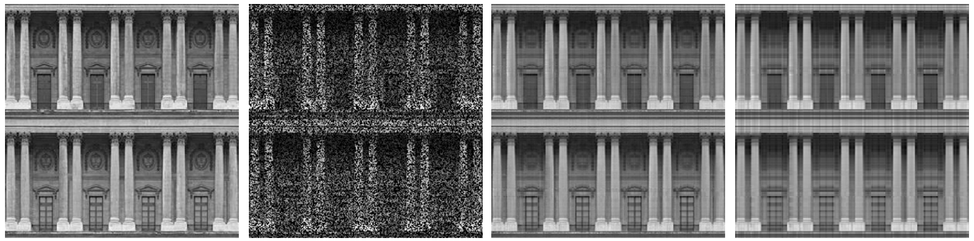
(c) 80% observed

(d) 90% observed

• 5. Results

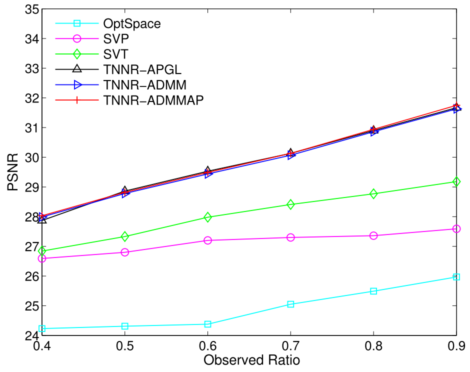


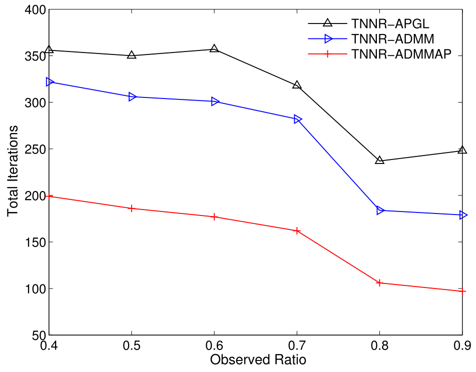


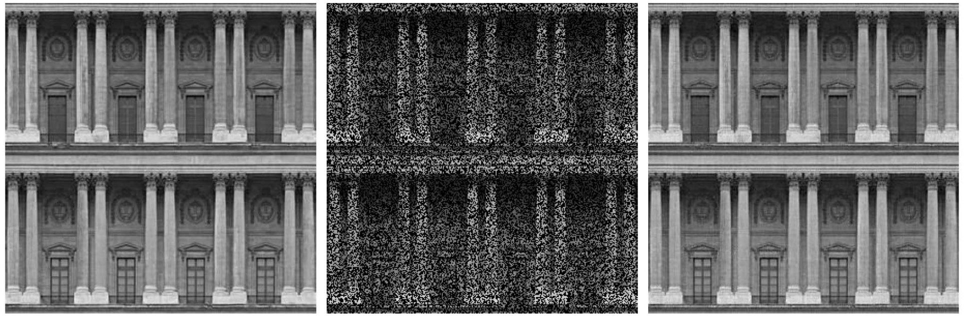


(a) Original image (b) Masked image (c) SVT (d) SVP

(e) Opt (f) tnnr-admm (g) tnnr-apgl (h) tnnr-admmap







(a) Original image (b) Masked image (c) Recovered image by TNNR-ADMMAP

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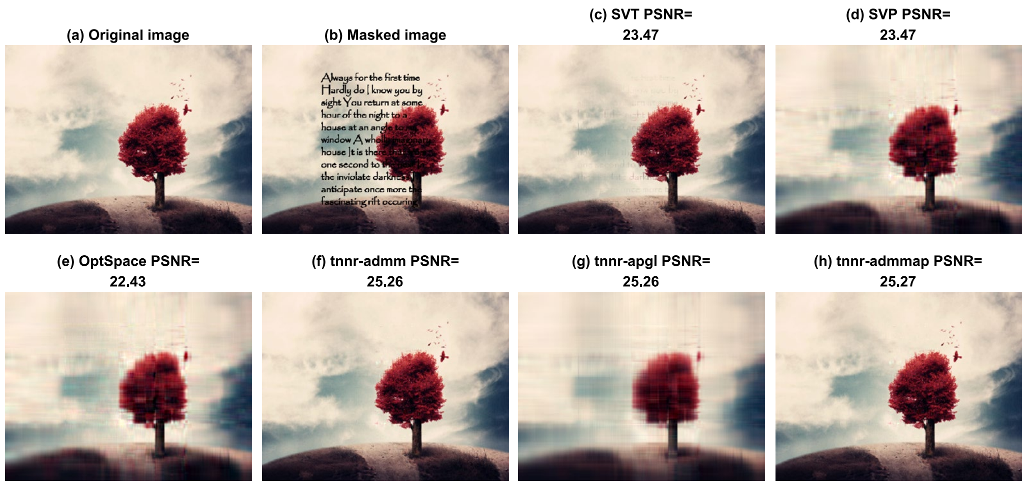

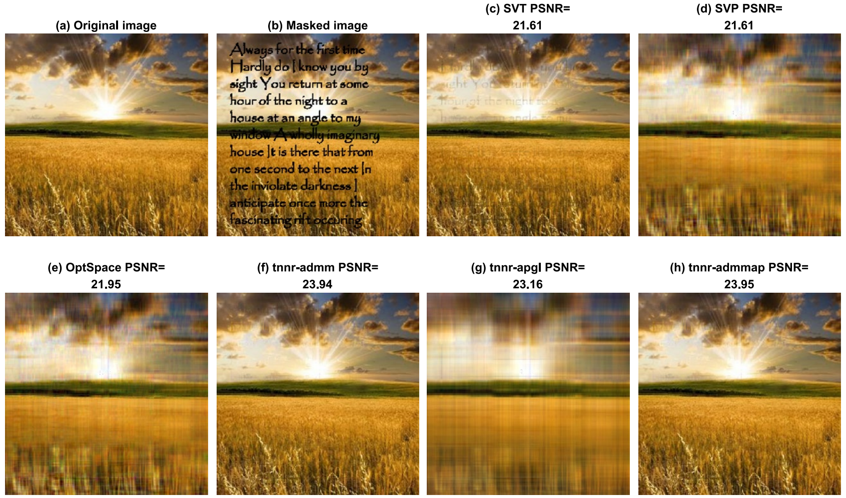

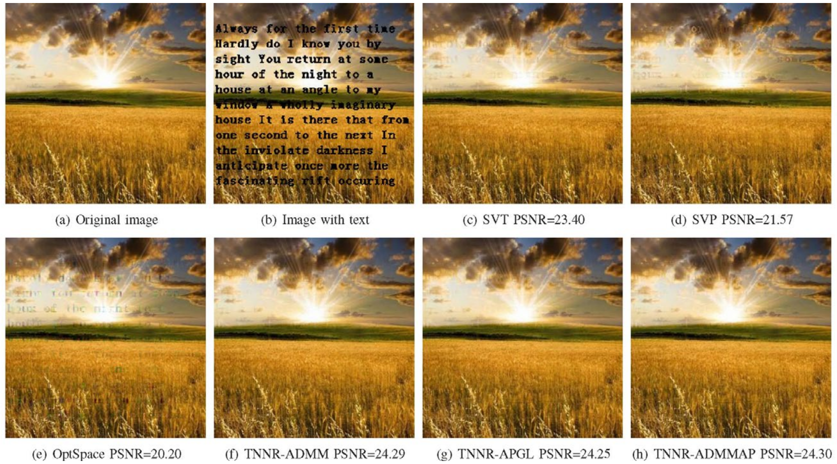

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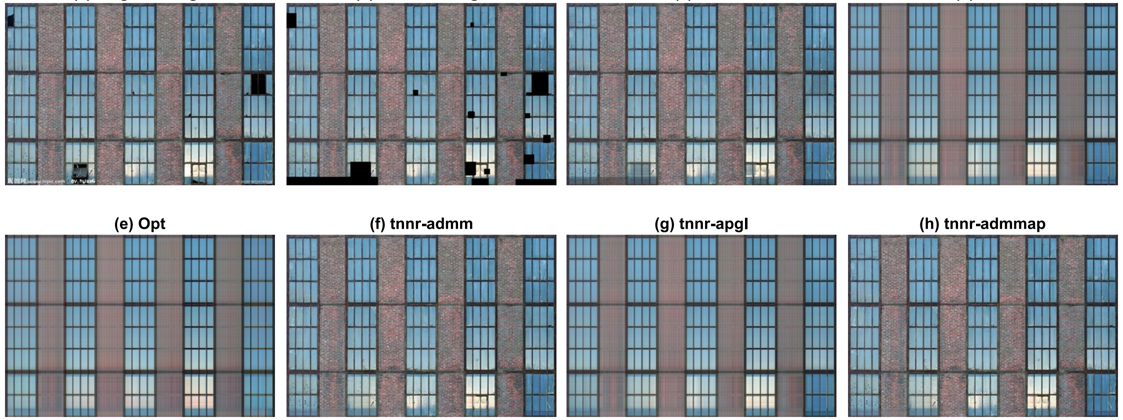
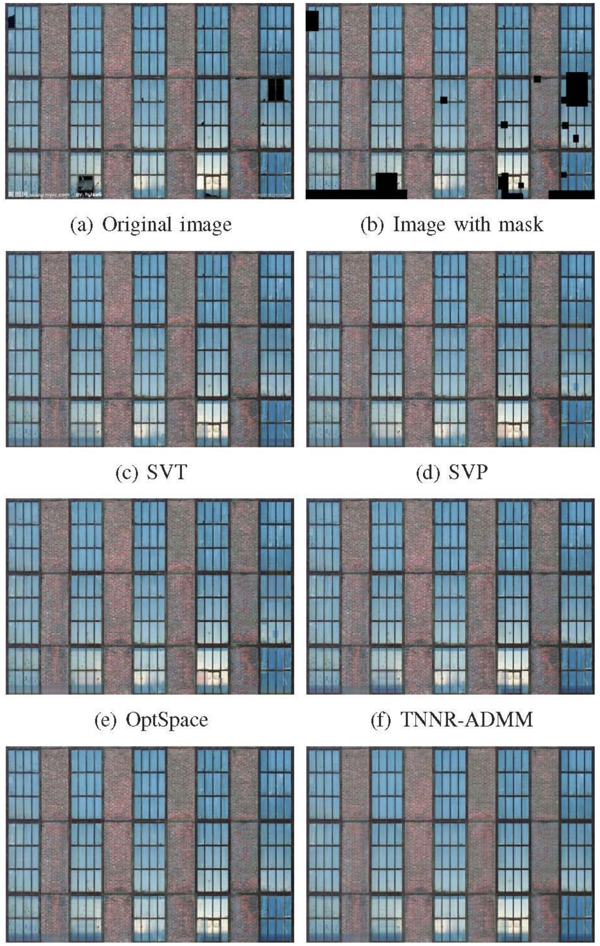


(a) Original image (b) Masked image (c) SVT (d) SVP
(e) Opt (f) tnnr-admm (g) tnnr-apgl (h) tnnr-admmap



(a) Original image (b) Image with mask
(c) SVT (d) SVP
(e) OptSpace (f) TNNR-ADMM
(g) TNNR-APGL (h) TNNR-ADMMAP

Thanks for listening