Test Task for the "Science Researcher" Position

December 13, 2023

A detailed explanation for each problem is expected. The anticipated completion time is 1.5 hours.

Problem 1. Find all solutions of the following congruences:

$$\begin{aligned} &\text{a.} \quad x^4 + x^3 + 2 \equiv 0 \mod 7 \\ &\text{b.} \quad x^7 + x + 1 \equiv 0 \mod 343 \end{aligned}$$

Problem 2. Find all solutions of the following system of congruences:

$$\begin{aligned} 2x &\equiv 1 \mod 5 \\ 3x &\equiv 9 \mod 6 \\ 4x &\equiv 1 \mod 7 \\ 5x &\equiv 9 \mod 11 \end{aligned}$$

Problem 3. Write an algorithm for finding a solution to the system of congruences

$$\begin{aligned} x &\equiv x_1 \mod m_1 \\ x &\equiv x_2 \mod m_2 \end{aligned}$$

assuming that $x_1 \equiv x_2 \mod \gcd(m_1, m_2)$

Problem 4. Write an algorithm for multiplying two polynomials, implicitly based on a recursive use of splitting formulas:

$$(a_1x + a_0)(b_1x + b_0) = c_2x^2 + c_1x + c_0$$

where c_0, c_1, c_2 are calculated

$$\begin{aligned} c_0 &= a_0b_0 \\ c_2 &= a_1b_1 \\ c_1 &= c_0 + c_2 - (a_1 - a_0)(b_1 - b_0) \end{aligned}$$