Illustrations for the course 'Probability Theory and Mathematical Statistic'

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Used packages:

```
\usepackage{pgfplots}
\usepackage{mathtools,amssymb}
\usepackage{tikz}
\usepackage{pgfplots}
\usepackage{listings}
\usepackage{tkz-euclide}
```

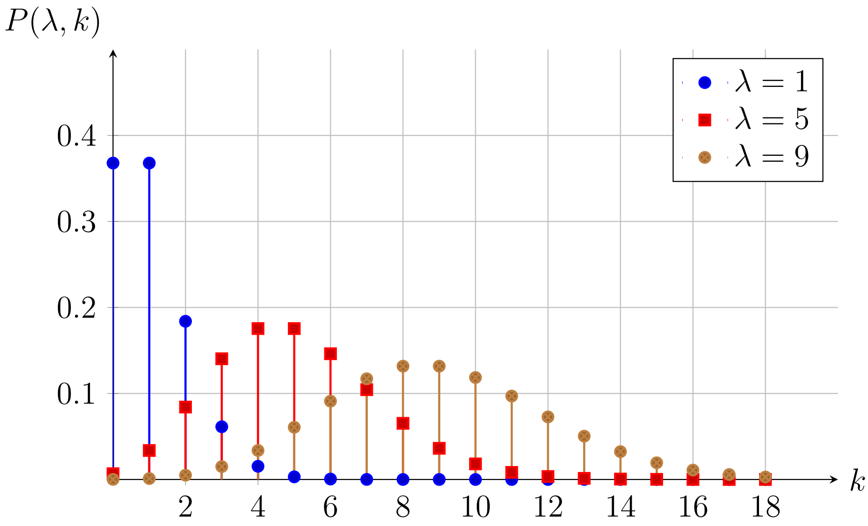
Contents

1 Discrete Distribution

1.1 Poisson distribution

Density function

$$P(\lambda, k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

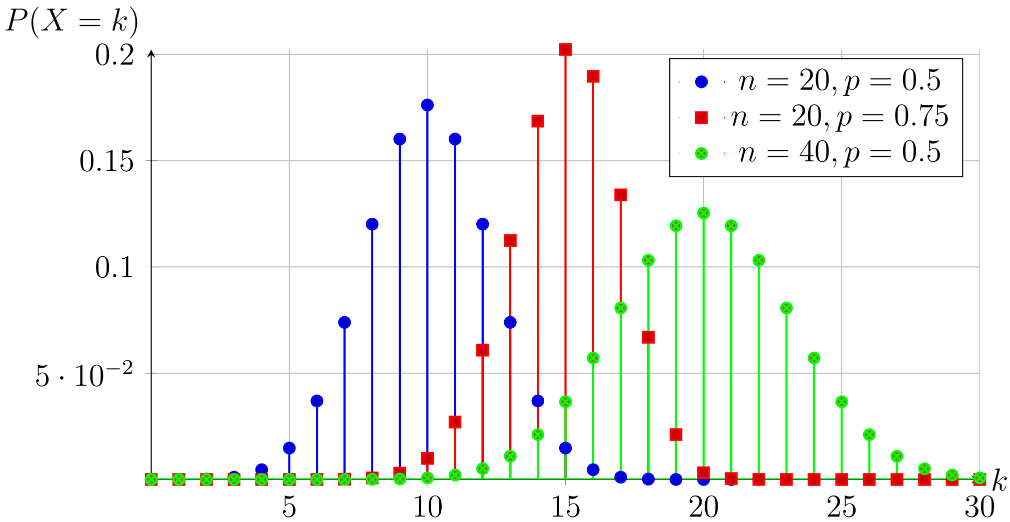



```
\pgfmathdeclarefunction{poiss}{1}{%
  \pgfmathparse{(#1^x)*exp(-#1)/(x!)}}
\begin{tikzpicture}
\begin{axis}[
              axis x line=center,
              axis y line=center,
              xtick={0,2,...,19},
              ytick={0.1,0.2,...,0.4},
                domain = 0:18,
                samples = 19,
                xlabel={$k$},
                ylabel={$P(\lambda,k)$},
                xlabel style={right},
                ylabel style={above left},
                ymax=0.5,xmax=20,x post scale=1.4,grid = major]
                \addplot+[ycomb,blue,thick] {poiss(1)};
                \addlegendentry{$\lambda = 1$}
                \addplot+[ycomb,red,thick] {poiss(5)};
                 \addlegendentry{$\lambda = 5$}
                \addplot+[ycomb,brown,thick] {poiss(9)};
                 \addlegendentry{$\lambda = 9$};
            \end{axis}
\end{tikzpicture}
```

1.2 Binomial distribution

Density function

$$P(X = k) = C_n^k \cdot p^k \cdot (1 - p)^{n-k}$$

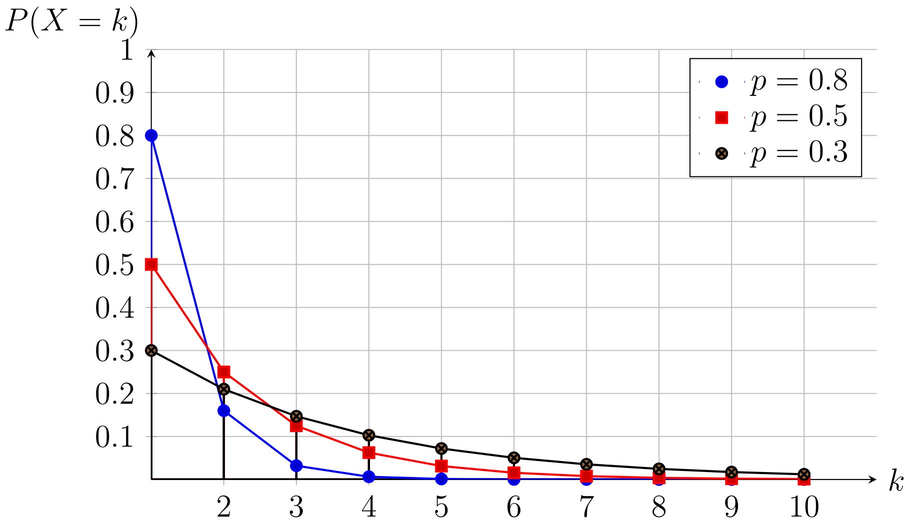



```
\pgfmathdeclarefunction{binom}{2}{\pgfmathparse{
(((#2)!)/(((#2-x)!)*(x!))) * ((#1)^x) * (1-#1)^(#2-x)}%
}

\begin{tikzpicture}

\begin{axis}[axis x line=center,
              axis y line=center,
                domain = 0:30,
                samples = 31,
                ytick = {0,0.05,0.1,...,0.5},
                xlabel={$k$},
                ylabel={$P(X = k)$},
                xlabel style={right},
                ylabel style={above left},
                x post scale=1.6,
                grid = major]
\addplot+[ycomb,blue,thick]  {binom(0.5, 20)} \closedcycle;
\addlegendentry{$n=20, p=0.5$};
\addplot+[ycomb,red,thick]  {binom(0.75, 20)} \closedcycle;
\addlegendentry{$n=20, p=0.75$};
\addplot+[ycomb,green,thick]  {binom(0.5, 40)} \closedcycle;
\addlegendentry{$n=40, p=0.5$};
\end{axis}
\end{tikzpicture}
```

1.3 Geom distribution

Density function

$$P(X = k) = p \cdot (1 - p)^{k-1}$$



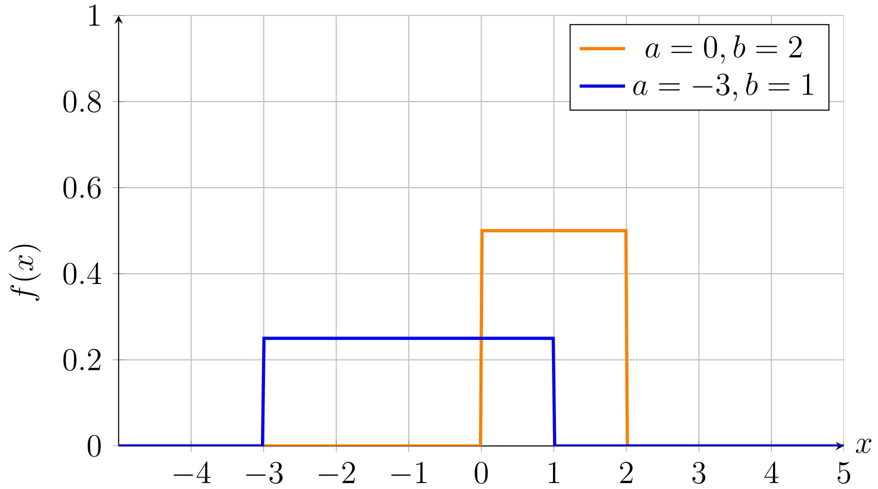

```
\pgfmathdeclarefunction{geom}{1}{\pgfmathparse{( 1 - #1 ) ^ ( x - 1) * #1}}
\begin{tikzpicture}
\begin{axis}[axis x line=center,
             axis y line=center,
             xtick={1,2,...,10},
             ytick={0.1, 0.2,...,1},
               domain = 1:10,
               samples = 10,
               xlabel={$k$},
               ylabel={$P(X = k)$},
               xlabel style={right},
               ylabel style={above left},
               ymax=1,
               xmax=11,
               x post scale=1.4,
               grid = major]
\addplot+[ycomb,blue,thick]  {geom(0.8)} \closedcycle;
\addlegendentry{$p = 0.8$};
\addplot+[ycomb,red,thick]  {geom(0.5)} \closedcycle;
\addlegendentry{$p = 0.5$};
\addplot+[ycomb,black,thick]  {geom(0.3)} \closedcycle;
\addlegendentry{$p = 0.3$};
\addplot[blue,thick]  {geom(0.8)};
\addplot[red,thick]  {geom(0.5)};
\addplot[black,thick]  {geom(0.3)};
\end{axis}
\end{tikzpicture}
```

2 Continuous distribution

2.1 Uniform distribution

2.1.1 Uniform density function

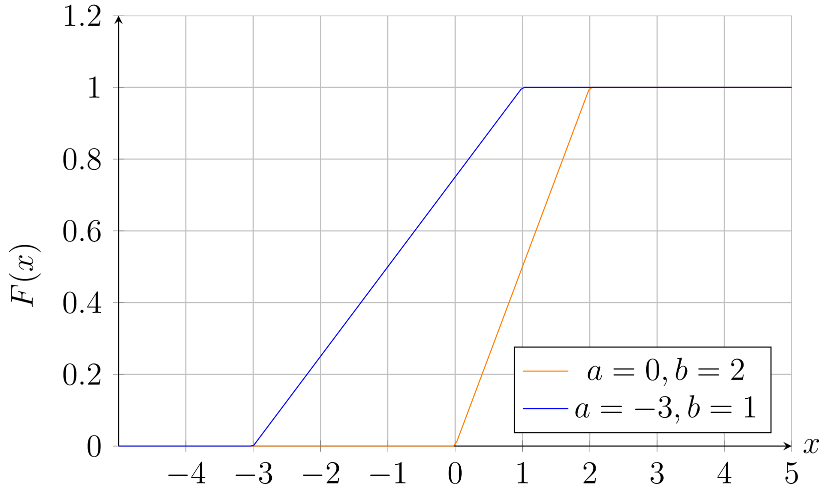
$$f(x) = \begin{cases} \dfrac{1}{b-a} & , x \in [a,b] \\ 0 & , x \notin [a,b] \end{cases}$$




```
\begin{tikzpicture}[
    declare function={unipdf(\x,\xl,\xu)= (\x>=\xl)*(\x<\xu)*1/(\xu-\xl);}
]

\begin{axis}[axis x line=center,
            axis y line = left,
                ymin=0,ymax=1,
                xmin=-5, xmax=5,
                samples = 500,
                xlabel={$x$},
                ylabel={$f(x)$},
                xlabel style={right},
                ylabel style={above left},
                x post scale=1.4,
                grid = major
]
\addplot [very thick, orange] {unipdf(x,0,2)};
\addlegendentry{$a=0, b=2$};
\addplot [very thick,blue] {unipdf(x,-3,1)};
\addlegendentry{$a=-3, b=1$};
\end{axis}
\end{tikzpicture}
```

2.1.2 Uniform distribution function

$$F(x) = \begin{cases} 1 & , x > b \\ \dfrac{x-a}{b-a} & , x \in [a,b] \\ 0 & , x < a \end{cases}$$

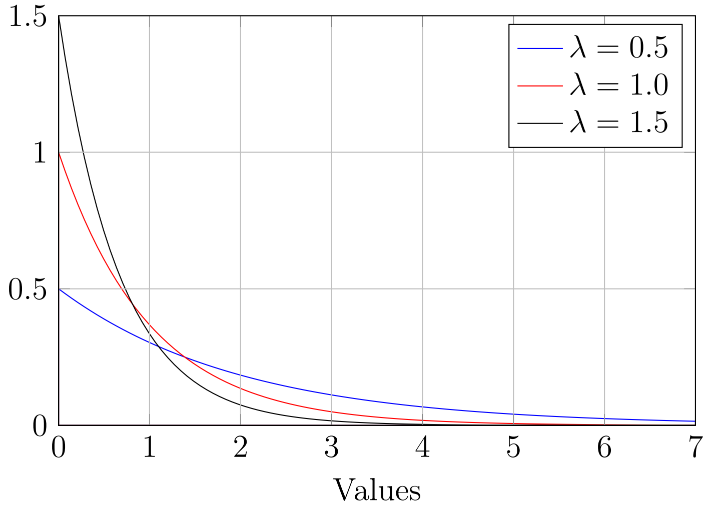



```
\begin{tikzpicture}[
    declare function={unip(\x,\xl,\xu)= (\x>=\xl)*(\x<\xu)*(\x-\xl)/(\xu-\xl
       ) + (\x>\xu);}
]

\begin{axis}[axis x line=center,
              axis y line=left,
                samples = 200,
                ymax = 1.2,
                xlabel={$x$},
                ylabel={$F(x)$},
                xlabel style={right},
                ylabel style={above left},
                x post scale=1.3,
                grid = major, legend pos=south east
]
\addplot [orange] {unip(x,0,2)};
\addlegendentry{$a=0, b=2$};
\addplot [blue] {unip(x,-3,1)};
\addlegendentry{$a=-3, b=1$};
\end{axis}
\end{tikzpicture}
```

2.2 Exponential distribution

2.2.1 Exponential density function

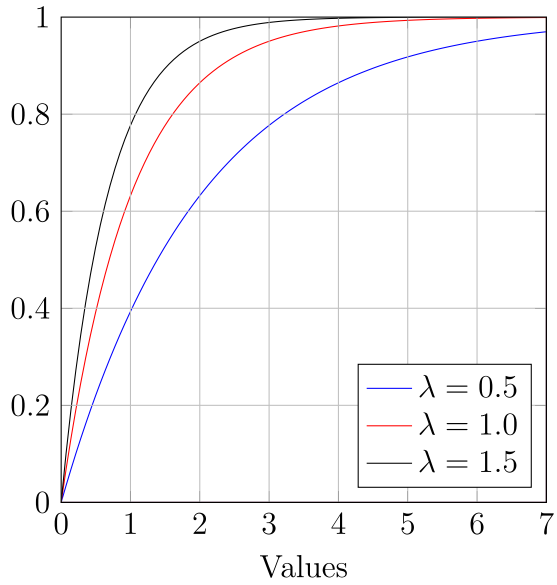
$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$$




```
\pgfmathdeclarefunction{exp}{1}{\pgfmathparse{exp(-#1 * x)}%}

\begin{tikzpicture}
\begin{axis}
[no markers, domain=0:10,
samples=100, xlabel={Values}, enlargelimits=false,
height=7cm, width=7cm,
clip=false, axis on top, grid = major]
   \addplot [domain=1:10, color = blue] {expon(0.5)} \closedcycle;
     \addlegendentry{$ \lambda = 0.5$};
  \addplot [domain=1:10, color = red] {expon(1,0)} \closedcycle;
     \addlegendentry{$ \lambda = 1.0$};
  \addplot [domain=1:10] {expon(1.5)} \closedcycle;
     \addlegendentry{$ \lambda = 1.5$};
\end{axis}
\end{tikzpicture}
```

2.2.2 Exponential distribution function

$$f(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$$

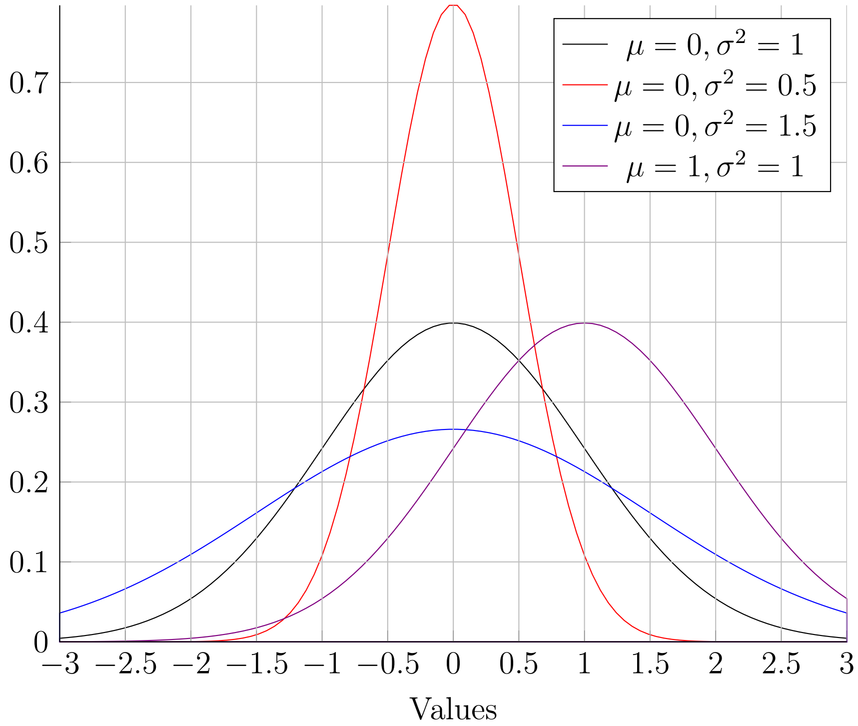



```
\pgfmathdeclarefunction{expondist}{1}{\pgfmathparse{ 1 -  exp(- (#1 * x))}%}

\begin{tikzpicture}
\begin{axis}[no markers,
domain=0:10, samples=100, xlabel={Values}, enlargelimits=false,
height=8cm, width=8cm, clip=false, axis on top,
grid = major]
  \addplot [domain=0:7, color = blue] {expondist(0.5)} \closedcycle;
    \addlegendentry{$ \lambda = 0.5$};
  \addplot [domain=0:7, color = red] {expondist(1.0)} \closedcycle;
    \addlegendentry{$ \lambda = 1.0$};
  \addplot [domain=0:7] {expondist(1.5)} \closedcycle;
    \addlegendentry{$ \lambda = 1.5$};
\end{axis}
\end{tikzpicture}
```

2.3 Normal distribution

Density function

$$f(x, \mu, \sigma) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\dfrac{-(x-\mu)^2}{2\sigma^2}}$$

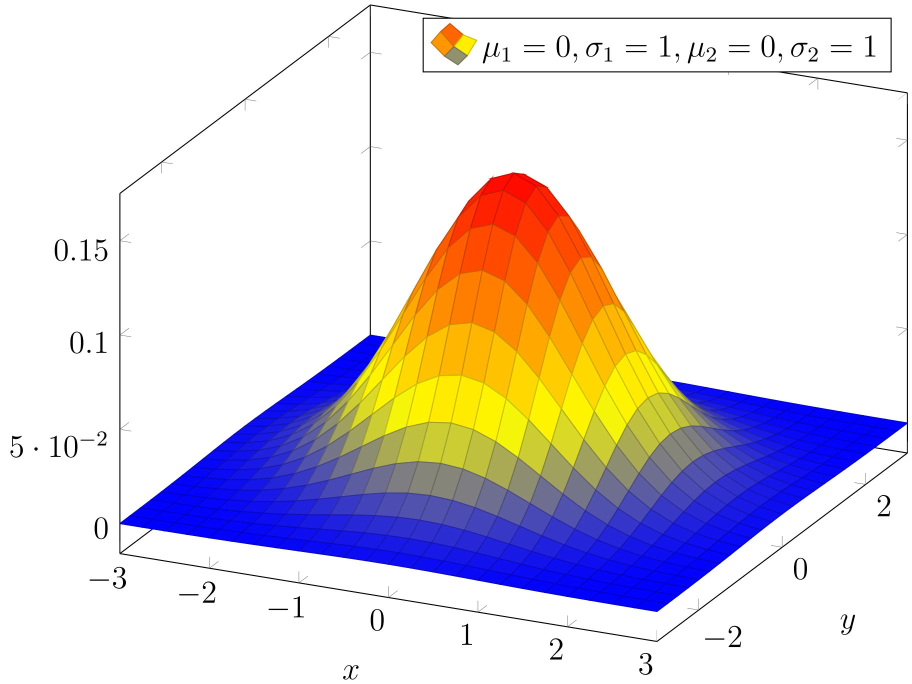



```
\pgfmathdeclarefunction{gauss}{2}{\pgfmathparse{1/(#2*sqrt(2*pi))*exp(-((x
    -#1)^2)/(2*#2^2))}}
\begin{tikzpicture}
\begin{axis}[no markers, domain=0:10, samples=100,axis lines*=left, xlabel=
    Values,
height=10cm, width=12cm, ytick={0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,
     0.9, 1.0},enlargelimits=false, clip=false, axis on top, grid = major]
\addplot [domain=-3:3] {gauss(0,1)};
\addlegendentry{$\mu = 0, \sigma^2 = 1$};
\addplot [domain=-3:3, color = red] {gauss(0,0.5)};
\addlegendentry{$\mu = 0, \sigma^2 = 0.5$};
\addplot [domain=-3:3, color = blue] {gauss(0,1.5)};
\addlegendentry{$\mu = 0, \sigma^2 = 1.5$};
\addplot [domain=-3:3, color = violet] {gauss(1,1)};
\addlegendentry{$\mu = 1, \sigma^2 = 1$};
\end{axis}
\end{tikzpicture}
```

2.4 Multidimensional Normal Distribution

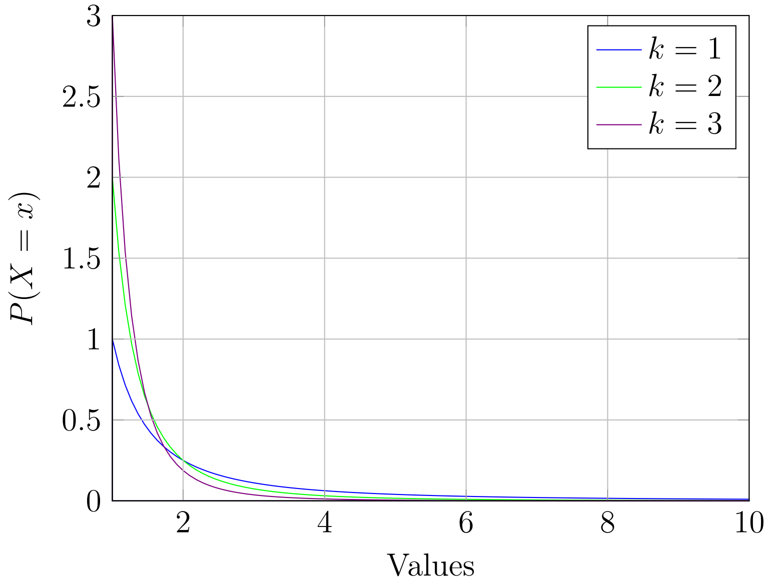
Density function

$$f(x,y) = \frac{1}{2\pi \cdot \sigma_x \cdot \sigma_y} \cdot e^{\frac{-1}{2} \cdot \left(\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right)}$$



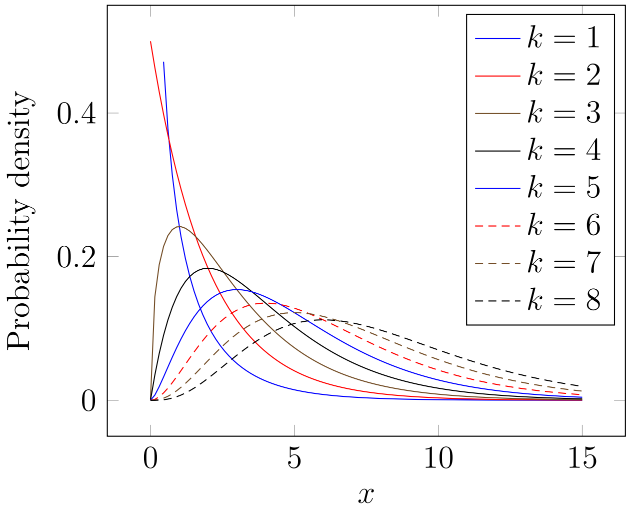

```
\pgfmathdeclarefunction{norm2}{4}{%
  \pgfmathparse{(1/(2*pi*#2*#4))* exp((-0.5)*(((x-#1)/#2)^2+((y-#3)/#4)^2))}
      %
}

\begin{tikzpicture}
  \begin{axis}[
  height=10cm, width=12cm,
  xlabel=$x$,
  ylabel=$y$]
  \addplot3[surf,domain=-3:3] {norm2(0,1,0,1)};
  \addlegendentry{$\mu_1 = 0, \sigma_1 = 1,\mu_2=0, \sigma_2=1$};
  \end{axis}
\end{tikzpicture}
```

2.5 Pareto distribution



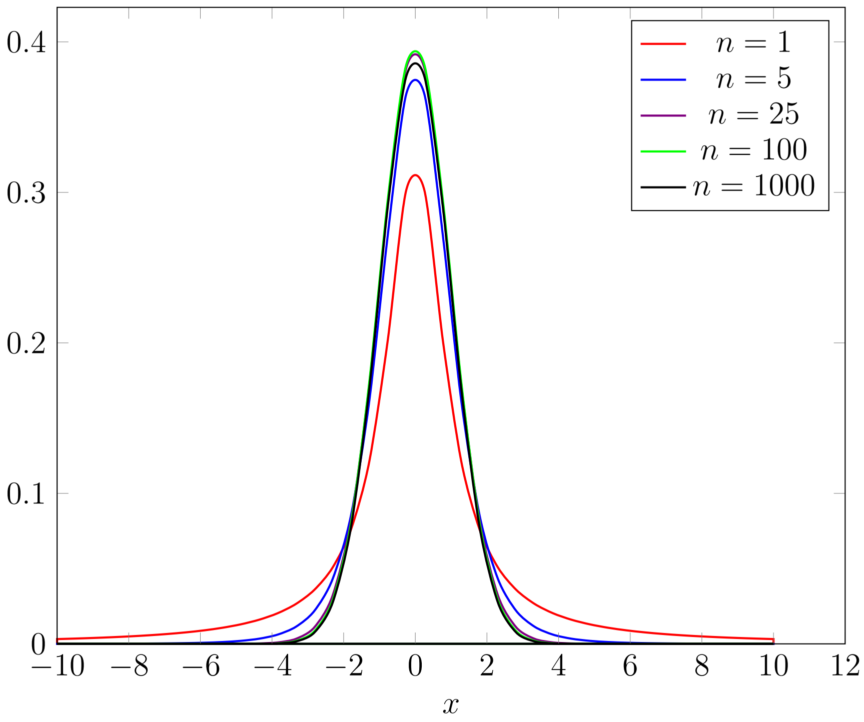

```
\pgfmathdeclarefunction{pareto}{1}{\pgfmathparse{( #1 * 1 ^ ( #1 ) / ( x ^
    (#1 + 1)))}%
}
\begin{center}
\begin{tikzpicture}

\begin{axis}[no markers, domain=0:10, samples=100, xlabel={Values}, ylabel
    ={$P(X = x)$}, enlargelimits=false, ytick = {0, 0.5, ..., 3},
height=8cm, width=10cm, clip=false, axis on top,
grid = major]
\addplot [domain=1:10, color = blue] {pareto(1)} \closedcycle;
\addlegendentry{$k = 1$};
\addplot [domain=1:10, color = green] {pareto(2)} \closedcycle;
\addlegendentry{$k = 2$};
\addplot [domain=1:10, color = violet] {pareto(3)} \closedcycle;
\addlegendentry{$k = 3$};

\end{axis}
\end{tikzpicture}
\end{center}
```

2.6 Chi-square distribution




```
\begin{tikzpicture}
  \begin{axis}[%
    xlabel = $x$,
    ylabel = {Probability density},
    samples = 200,
    restrict y to domain = 0:0.5,
    domain = 0.01:15]
    \foreach \k in {1,...,8} {%
      \addplot+[mark={}] gnuplot[raw gnuplot] {%
        isint(x) = (int(x)==x);
        log2 = 0.693147180559945;
        chisq(x,k)=k<=0||!isint(k)?1/0:x<=0?0.0:exp((0.5*k-1.0)*log(x)-0.5*x
            -lgamma(0.5*k)-k*0.5*log2);
        set xrange [1.00000e-5:15.0000];
        set yrange [0.00000:0.500000];
        samples=200;
        plot chisq(x,\k)};
    \addlegendentryexpanded{$k = \k$}}
  \end{axis}
\end{tikzpicture}
```

2.7 Student distribution




```
\begin{center}
\begin{tikzpicture}[
    declare function={gamma(\z)=
    2.506628274631*sqrt(1/\z)+ 0.20888568*(1/\z)^(1.5)+ 0.00870357*(1/\z)
       ^(2.5)- (174.2106599*(1/\z)^(3.5))/25920- (715.6423511*(1/\z)^(4.5))
       /1244160)*exp((-ln(1/\z)-1)*\z;},
    declare function={student(\x,\n)= gamma((\n+1)/2.)/(sqrt(\n*pi) *gamma(\
       n/2.)) *((1+(\x*\x)/\n)^(-(\n+1)/2.));}
]

\begin{axis}[xlabel = $x$, height=10cm, width=12cm,
ytick={0.0, 0.1, 0.2, 0.3, 0.4, 0.5},
enlargelimits=false, clip=false, axis on top,
grid = major
    axis lines=left,
    enlargelimits=upper,
    samples=50
]

\addplot [thick, smooth, domain=-10:10, color = red]{student(x,1)} \
   closedcycle;
\addlegendentry{$n = 1$};
\addplot [thick, smooth, domain=-10:10, color = blue]{student(x,5)} \
   closedcycle;
\addlegendentry{$n = 5$}
\addplot [thick, smooth, domain=-10:10, color = violet]{student(x,25)}
\closedcycle;
\addlegendentry{$n = 25$}
\addplot [thick, smooth, domain=-10:10, color = green]{student(x,100)}
\closedcycle;
\addlegendentry{$n = 100$}
\addplot [thick, smooth, domain=-10:10, color = black]{student(x,1000)}
\closedcycle;
\addlegendentry{$n = 1000$}
```

```
\end{axis}
\end{tikzpicture}
\end{center}
```

3 Theorems

3.1 Illustration of the law of big numbers

The law of large numbers is a theorem that describes the result of performing the same experiment a large number of times. According to the law, the average of the results obtained from a large number of trials should be close to the expected value and will tend to become closer to the expected value as more trials are performed. As we can see we after tossing the cube for a huge enough number of times average becames closer to the expected value.