QF622 Credit Risk Models Part I

Assignment

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1. A one-year bond with a face value of $100 is trading at $95. Assume the recovery rate to be 40%, and the one-year continuously compounded risk-free rate to be 4%, determine
   a. the one-year probability of default.
   b. the spread.

a) We have r = 4%, P = 95, and recovery = 40%.
   95 = e^{-r} * ((1-P) * 100 + P * 100*40%)
   ⇨ **P = 0.0187**

b) 95 = 100* e^{-r} where r in this case equates to risk-free + credit spread.
   r = ln(100/95) = 0.0513.
   The credit spread is thus **S = 0.0513 – 0.04 = 0.0113**

2. A zero-coupon bond with 6m to expiry is trading at $94.50 in the market. The face value is $100. The continuously compounded interest rate is 2%. Assuming a recovery rate of 30%, determine the default probability of the issuer.

T = 0.5, r = 2%

94.5 = e^{-r*T} ((1-P)*100 + P*30%*100)

=> P = 0.065

3. A random variable X has a probability density function given by

$$f(x) = Ae^{-x},\ 0 < x < \infty$$

   a. Find A.
   b. Calculate the probability that X lies in the interval 1 < X <= 2.

a) integrating $\int_0^\infty Ae^{-x}$ dx = 1, -A e^{-x} (0-- ∞) = 1
   => **A = 1**

b) the probability for the range 1 < x <= 2 can be broken down to:
   P(x<=2) - P(x<=1) = (1-e^{-2}) – (1-e^{-1}) = 0.2325

4. Consider 1000 investors who, lacking any skill, invest at random. The probability that they will achieve a positive return for each year is 0.5. Assume that they invest for 10 consecutive years. How many of them are expected to make 8 or more positive results?

P(n = 10,x >= 8) = P(n = 10,x = 8) + P(n = 10,x = 9) + P(n = 10,x = 10) = 0.055

where p = 0.5 the probability P follows the binomial distribution

For 1000 people, the expected number of people earning at least 8 positive returns is **0.055 * 1000 = 55 people**

5. In an economy, there is on average 1 corporate bond default per half-year interval. Assuming that corporate bond defaults arrive randomly in time (i.e. Poisson distribution), find the probabilities that within any particular year we observe

   a. 0 corporate bond default.
   b. 3 corporate bond defaults.

λ = 1 * 2 = 2 for one year. The probabilities are:

a) P(X = 0) = $\frac{e^{-2}2^0}{0!}$ = 0.1353 or **13.53%**

b) P(X = 3) = $\frac{e^{-2}2^3}{3!}$ = 0.1804 or **18.04%**

6. Companies default following a Poisson distribution at an average of 4 per month (λ=4)
   a. Calculate the probability of more than 5 defaults in any one month.
   b. What is the probability that at least two months will elapse without any default event? Hint: The waiting time follows an exponential distribution.

λ = 4 for one month and λ = 4*2 = 8 for two months

a) P(X>5) = 1 – (P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5))

$= 1 - \frac{e^{-4}4^0}{0!} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!} - \frac{e^{-2}2^3}{3!} - \frac{e^{-2}2^4}{4!} - \frac{e^{-2}2^5}{5!}$ = **0.2149**

b) P(T>2) = $\int_2^{\infty} \lambda e^{-\lambda T} dT = e^{-2\lambda}$ ≈ **0.0003**

7. Suppose the estimated linear probability model used by a FI to predict business loan applicant default probabilities is **PD = .03X$_1$ + .02X$_2$ - .05X$_3$ + error**, where X_1 is the borrower's debt/equity ratio, X_2 is the volatility of borrower earnings, and X_3 is the borrower's profit ratio. For a loan applicant, X_1 = 0.6, X_2 = 0.25, and X_3 = 0.1,

   a. What is the projected probability of default for the borrower?
   b. What is the projected probability of repayment if the debt/equity ratio is 2.4?
   c. What is a major weakness of the linear probability model?

Given the input parameters:

a) the projected **PD = 0.03*0.6 + 0.02*0.25-0.05*0.1 = 0.018**

b) PD = 0.03*2.4+0.02*0.25-0.05*0.1 = 0.072

   ⇨ **Probability of repayment = 1-0.072 = 0.928**

c) The major disadvantages:
   - output of PD can be outside of the [0,1] range
   - it suffers from bias arising from violations of OLS assumptions, example: the error term is not normally distributed
   - it does not consider correlation between predictors
   - predictors are usually based on historical data, this may not be a correct reflection of what will happen in the future

8. There are two bonds: one-year AA-rated bond yielding 9.5% and one-year BB-rated bond yielding 13.5%. The rate on one-year T-Bills currently is 6%.

   a. What is the repayment probability for each of these two bonds?
   b. Assume that if the loan is defaulted, no payments are expected. What is the market-determined risk premium for the corresponding probability of default for each bond?

a) for AA bond, k = 9.5%, the repayment probability:

   p*0 + (1-p)(1+9.5%) = (1+6%) >>(1-p) = 1.06/1.095 **= 0.968 for the AA rated bond**

   p*0 + (1-p)(1+13.5%) = (1+6%) >>(1-p) = 1.06/1.135 = **0.934 for the BB rated bond**

b) The market determined risk premium for the two bonds are simply the credit spread:
For AA bond = 9.5%-6% = **3.5%**
For BB bond = 13.5% - 6% = **7.5%**

9. Assume that there is no recovery if a corporate bond is defaulted. Calculate the term structure of default probabilities over two years using the following spot rates from the Treasury strip and corporate bond yield curves.

	Spot 1 year	Spot 2 year
Treasury strip	4.65%	5.5%
Corporate bond	8.5%	10.25%

a. Calculate the one-year forward rate on the Treasury strip and the corporate bond.
b. Using the current and forward one-year rates, calculate the probability of default on the corporate bond in years 1 and year 2, respectively.
c. Calculate the cumulative probability of default on the corporate bond over the next two years.

Using the spot curve, given i_1 = 4.65% and i_2 = 5.5% and k_1 = 8.5% and k_2 = 10.25%

a) The one-year forward rate on treasuries:
$1.055^2 = 1.0465(1+f_1)$, f_1 = **6.36%**
For corporate bonds:
$1.1025^2 = 1.085(1+g_1)$, g_1 = **12.03%**

b) assuming no recovery, the 1 year default probability:
$p*0 + (1-p) * (1+k) = (1+i)$
$(1-p) = 1.0465/1.085$, p_1 = **3.55%**
the 2-year default probability:
$(1-p_2)*(1+12.03\%) = (1+6.36\%)$
$(1-p_2) = 1.0636/1.1203$, thus p_2 = **5.06%**

c) The cumulative default probability is:
C = 1-(1-0.0355)(1-0.0506) = **8.43%**

10. Calculate the cumulative default probabilities over 3 years using the following spot rates from the Treasury strip and corporate bond yield curves.

	Spot 1 Year	Spot 2 Year	Spot 3 Year
Treasury strip	5.0%	6.1%	7.0%
BBB-rated bond	7.0%	8.2%	9.3%

f1 = 1.0612/1.05 - 1 =7.21%
g1 = 1.0822/1.07 – 1 = 9.41%
f2 = 1.073/1.0612 - 1 = 8.82%
g2 = 1.0933/1.0822 – 1 = 11.53%

The default probabilities are:

(1-p1) = 1.05/1.07, p1 = 1.87%
(1-p2) = 1.0721/1.0941 = 2.01%
(1-p3) = 1.0882/1.1153 = 2.43%

The cumulative default probability over three years is:
C = 1-(1-1.87%)(1-2.01%)(1-2.43%) = **6.18%**

11. If the face value of the debt of a company is $1, and the asset is $1. Assume the volatility of the asset process is 0.5, time to maturity of the debt is 10 years, and the risk-free interest rate is 1%. What is the probability of default of this company?

$A_0 = 1$, $Dp = 1$, $\sigma_A = 0.5$, $r = 0.01$, $T = 10$

$DD^Q = \frac{LN\left(\frac{1}{1}\right)+(0.01-0.5(0.5^2))10}{0.5\sqrt{10}} = -0.73$

PD = 1-N(-0.73) = 0.77

12. In Merton's structural model, If $A_0 = 100$, $r = 5\%$, $\sigma = 10\%$, $T = 1$, and $DP = 110$,
    a. calculate the default probability of this company on the expiry of the debt at the end of the year,
    b. Find the implied credit spread.

a)

$DD^Q = \frac{LN\left(\frac{100}{110}\right)+(0.05-0.5(0.1^2))}{0.1\sqrt{1}} = -0.503$

PD = 1-N(-0.503) = 0.69

b)

$d1 = = \frac{LN\left(\frac{100}{110}\right)+(0.05+0.5(0.1^2))}{0.1\sqrt{1}} = -0.403$

d2 = -0.503

$y = -\frac{1}{1}ln(\frac{100}{110}N(0.403) + e^{-0.05}N(-0.503)) = 0.1173 = 11.73\%$

credit spread = 11.73%-5% = 6.73%

13. If the face value of debt of a company is $700,000, while the asset is $1,000,000. Assume the volatility of the asset process is 20%, time to maturity of the debt is 5 years, and risk-free interest rate is 2%. Calculate distance to default and the probability of default of this company using Merton's structural model.

$A_0 = 1{,}000{,}000$, $Dp = 700{,}000$, $\sigma_A = 0.2$, $r = 0.02$, $T = 5$

$\mathbf{DD^Q = \frac{LN\left(\frac{1000000}{700000}\right)+(0.02-0.5(0.2^2))5}{0.2\sqrt{5}} = 0.7975}$

PD = 1-N(0.7975) = 0.2126

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14. Assume a firm has asset value, A_0= 100, and asset volatility = 0.20. The firm has outstanding one year (T = 1) debt with a face value = 80. The risk free rate for a year is 4% continuously compounded. The market risk premium is 6%. The firm's asset beta is estimated as 1.5 and market price of risk is 0.9. What is the implied correlation of the firm's asset value with the market?

$A_0 = 100$, $\sigma_A = 0.2$, $Dp = 80$, $r = 0.04$, $\theta = 0.9$, $\beta = 1.5$, $\mu_m - r = 0.06$

$\mu_a = 0.05*1.5+0.04 = 0.13$

$$DD = = \frac{LN\left(\frac{100}{80}\right)+(0.13-0.5(0.2^2))1}{0.2\sqrt{1}} = 1.6657$$

$$DD^Q = \frac{LN\left(\frac{100}{80}\right)+(0.04-0.5(0.2^2))1}{0.2\sqrt{1}} = 1.2155$$

1.2155 = 1.6657 – R(0.9)(1)

R = 0.5002

15. A bank has $20 million in assets, with risk-adjusted assets of $10 million. Tier I capital is $600,000 (including CET1 and additional Tier 1) and Tier II capital is $400,000.

a. What is the current values of the Tier I ratio and the total ratio? Does the bank meet the Basel III capital requirements?
b. If the bank repurchases $90,000 of common stock with cash, what will the new value of each ratio be?
c. If the bank issues $800,000 in common stock and lends it to help finance a new shopping mall (commercial loan). What will the new value of each ratio be?

a)
Tier I:
600,000 / 10,000,000 = 6%
Total ratio:
(600,000 +400,000) / 10,000,000 = 10%
➔ It meets the Basel II requirement.

b)
repurchase common stock => Tier I: 600,000 – 90,000 = 510,000
Tier I ratio:
510,000/10,000,000 = 5.1%
Total ratio:
(510,000 + 400,000) / 10,000,000 = 9.1%
➔ it does not meet Basel II requirement.

c)
Tier I ratio:
(600,000 + 800,000) / (10,000,000 + 800,000) = 12.96%
Total ratio:
(600,000 + 800,000 + 400,000) / (10,000,000 + 800,000) = 16,67 %
➔ It meets requirement but more than enough.

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16. A Bank has the following balance sheet (in millions), with the risk weights in parentheses.

Assets		**Liabilities and Equity**	
Cash (0%)	\$21	Deposits	\$176
OECD interbank deposits (20%)	25	Subordinated debt (5 years)	2
Mortgage loans (50%)	70	Cumulative preferred stock	2
Consumer loans (100%)	70	Equity	5
Reserve for loan losses	(1)		
Total assets	\$185	Total liabilities and equity	\$185

Equity is CET1. Cumulative preferred stock is additional Tire I. Subordinated debt (5 years) and Reserve for loan losses are Tier II.

In addition, the bank has the following:

\$30 m in performance-related standby letters of credit (LCs) to a public corporation,
\$40 m in two-year forward FX contracts that are currently in the money by \$1 million,
\$300 m in six-year interest rate swaps that are currently out of the money by \$2 million.

Credit conversion factors:

Performance-related standby LCs	50%
1 to 5-year foreign exchange contracts	5%
1 to 5-year interest rate swaps	0.5%
5 to 10-year interest rate swaps	1.5%

a. What are the risk-adjusted on-balance-sheet assets of the bank as defined under the Basel Accord (in dollar)?
b. Disregarding the capital conservation buffer, to be adequately capitalized, what are the CET1, Tier I, and total capital required for both off- and on-balance-sheet assets (risk weight: 100% for OBS)? Does the bank have enough capital to meet the Basel requirements?
c. Including the capital conservation buffer requirement, what are the CET1, Tier I, and total capital required for both off- and on-balance-sheet assets (risk weight: 100% for OBS)? Does the bank have enough capital to meet the Basel requirements?

a)
The risk-adjusted on -balance-sheet assets:
0*21+0.2*25+0.5*70+1*70 = 110

b)
on-balance-sheet:

Capital requirement
CEIT: 110 * 0.045 = 4.85 mil --> meet requirement
Tier I: 110* 0.06 = 6.6 mil --> meet requirement
Total capital: 110 * 0.08 = 8.8 mil --> meet requirement

OFF-balance-sheet:

Total risk-weighted assets:
110 + (30*0.5) + (40*0.05) + (300*0.015+2) = 133.5

Capital requirement
CEIT: 133.5 * 0.045 = 6.0075 mil --> NOT meet requirement
Tier I: 133.5* 0.06 = 8.01 mil --> NOT meet requirement
Total capital: 133.5 * 0.08 = 10.68 mil --> NOT meet requirement

c) consider capital conservation buffer requirement

on-balance-sheet:

Capital requirement
CEIT: 110 * 0.07 = 7.7 mil --> NOT meet requirement
Tier I: 110* 0.085 = 9.35 mil --> NOT meet requirement
Total capital: 110 * 0.105 = 11.55 mil --> NOT meet requirement

OFF-balance-sheet:

Capital requirement
CEIT: 133.5 * 0.07 = 9.345 mil --> NOT meet requirement
Tier I: 133.5* 0.085 = 11.3475 mil --> NOT meet requirement
Total capital: 133.5 * 0.105 = 14.0175 mil --> NOT meet requirement