Final Project

Operations Research 2024-2025

Group 14

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Smartphone company:

A tech company produces three models of smartphones: basic, standard, and premium. The number of components and labour hours required for manufacturing these three models are summarized in the following table:

Resources and components	MODEL		
	BASIC	*STANDARD*	*PREMIUM*
Processors	1	1	2
Screens	1	1	1
Batteries	1	2	3
Labour hours	3	5	8

The company has a weekly availability of 500 processors, 400 screens, 600 batteries, and 1200 labour hours.

The profit per unit sold varies among the models: €50 for the basic model, €80 for the standard model, and €120 for the premium model. To ensure market presence, the company must produce at least 50 units of each model weekly. Additionally, the number of premium phones cannot exceed 40% of the total phones produced.

The company has engaged your team to determine the number of phones of each model to produce in order to maximize weekly profit.

1. **Formulate the mathematical model**

x_j: The units of mobile phones produced by the company on a weekly basis. Where j = 1,2,3

- x_1: Number of basic smartphones produced weekly. .
- x_2: Number of standard smartphones produced weekly.
- x_3: Number of premium smartphones produced weekly.

$$max\ f(x) = 50x_1 + 80x_2 + 120x_3$$

$$s.t. \quad x_1 + x_2 + 2x_3 \leq 500$$

$$x_1 + x_2 + x_3 \leq 400$$

$$x_1 + 2x_2 + 3x_3 \leq 600$$

$$3x_1 + 5x_2 + 8x_3 \leq 1200$$

$$x_3 \leq 0.4\,(x_1 + x_2 + x_3) \ --> \ -0.4\,x_1 - 0.4x_2 + 0.6x_3 \leq 0$$

$$x_1,\ x_2,\ x_3 \geq 50$$

$$x_1,\ x_2,\ x_3 \geq 0$$

2. **Write the model in compact form and matrix form.**

COMPACT FORM:

$$OF : \max f(x) = \sum_{j=1}^{3} c_j x_j$$

$$s.t. \quad \sum_{j=1}^{3} a_{ij} x_j \leq b_i \ \ \forall i \in \{1,2,3,4\}$$

$$\mathrm{x_j} \geq 50 \quad \forall \mathrm{j} \in \{1,2,3\}$$

$$x_3 \leq 0.4 \sum_{j=1}^{3} x_j$$
$$x_j \geq 0 \; \forall j \in \{1,2,3\}$$

MATRIX FORM:

MAX f(x)= c'x

st: Ax ≤ b →

x ≥ o

$$max\, f(x) = 50x_1 + 80x_2 + 120x_3$$

st:

$$x_1 + x_2 + 2x_3 \leq 500$$
$$x_1 + x_2 + x_3 \leq 400$$
$$x_1 + 2x_2 + 3x_3 \leq 600$$
$$3x_1 + 5x_2 + 8x_3 \leq 1200$$
$$-x_1, -x_2, -x_3 \leq -50$$
$$-0.4x_1 - 0.4x_2 + 0.6x_3 \leq 0$$
$$x_1, x_2, x_3 \geq 0$$

$$c_j^T = [\,50\ 80\ 120]_{1x3}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 5 & 8 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -0.4 & -0.4 & 0.6 \end{bmatrix}$$

$$b = \begin{pmatrix} 500 \\ 400 \\ 600 \\ 1200 \\ -50 \\ -50 \\ -50 \\ 0 \end{pmatrix}$$

3. Indicate the number of constraints and real variables in the problem

In total, we have 8 constraints including the four constraints regarding the resources (processors, batteries, screens, labor hours), one constraint regarding the premium model limit, and finally the three constraints regarding the minimum production requirements as shown in the previous question. When it comes to the real variables, we have three variables (x_1; x_2; x_3): the number of basic, premium, and standard smartphones produced as shown in question 1.

4. Write the LINGO code using the @sum and @for commands

```
! Define the sets;

SETS:
MODELS /1..3/: ProfitXUnit, MinProd, Phone_model;  ! Models: Basic (1), Standard (2), Premium (3) -->j;
RESOURCES /1..4/: Availability;  ! Recursos: Processors (1), Screens (2), Batteries (3), Labor hours (4)-->i;
MATRIX(RESOURCES, MODELS): A;
ENDSETS

! Input the profit values, minimum production requirements, and resource data;
DATA:
ProfitXUnit= 50 80 120;       ! Profit per unit for Basic, Standard, Premium;
MinProd = 50 50 50;      ! Minimum production required for each model;
Availability = 500 400 600 1200; ! Available resources: processors, screens, batteries, labor hours;
A = 1 1 2   ! Processors: A(1, Basic), A(1, Standard), A(1, Premium);

    1 1 1   ! Screens: A(2, Basic), A(2, Standard), A(2, Premium);

    1 2 3   ! Batteries: A(3, Basic), A(3, Standard), A(3, Premium);

    3 5 8;  ! Labor hours: A(4, Basic), A(4, Standard), A(4, Premium);
ENDDATA

! Objective function: Maximize profit;
MAX = @SUM(MODELS(j): ProfitXUnit(j) * Phone_model(j));

! Resource constraints;
@FOR(RESOURCES(i):  [Resources_slack]@SUM(MODELS(j): A(i,j) * Phone_model(j)) <= Availability(i));

! Minimum production constraints;
@FOR(MODELS(j):  [Min_prod_slack]Phone_model(j) >= MinProd(j));

! Premium model production limit (40% of total production);
Phone_model(3) <= 0.4 *@SUM(MODELS(j):Phone_model(j));

END
```

The output using 'Solve' in LONGO is:

Variable	Value	Reduced Cost
PROFITXUNIT(1)	50.00000	0.000000
PROFITXUNIT(2)	80.00000	0.000000
PROFITXUNIT(3)	120.0000	0.000000
MINPROD(1)	50.00000	0.000000
MINPROD(2)	50.00000	0.000000
MINPROD(3)	50.00000	0.000000
PHONE_MODEL(1)	183.3333	0.000000
PHONE_MODEL(2)	50.00000	0.000000
PHONE_MODEL(3)	50.00000	0.000000
AVAILABILITY(1)	500.0000	0.000000
AVAILABILITY(2)	400.0000	0.000000
AVAILABILITY(3)	600.0000	0.000000
AVAILABILITY(4)	1200.000	0.000000
A(1, 1)	1.000000	0.000000
A(1, 2)	1.000000	0.000000
A(1, 3)	2.000000	0.000000
A(2, 1)	1.000000	0.000000
A(2, 2)	1.000000	0.000000
A(2, 3)	1.000000	0.000000
A(3, 1)	1.000000	0.000000
A(3, 2)	2.000000	0.000000
A(3, 3)	3.000000	0.000000
A(4, 1)	3.000000	0.000000
A(4, 2)	5.000000	0.000000
A(4, 3)	8.000000	0.000000

Row	Slack or Surplus	Dual Price
1	19166.67	1.000000
RESOURCES_SLACK(1)	166.6667	0.000000
RESOURCES_SLACK(2)	116.6667	0.000000
RESOURCES_SLACK(3)	166.6667	0.000000
RESOURCES_SLACK(4)	0.000000	16.66667
MIN_PROD_SLACK(1)	133.3333	0.000000
MIN_PROD_SLACK(2)	0.000000	-3.333333
MIN_PROD_SLACK(3)	0.000000	-13.33333
9	63.33333	0.000000

5. Determine the maximum profit the company can achieve and the number of devices of each model required to achieve it.

After analyzing the production and market requirements and using LINGO we have enough information to know the optimal production plan to maximize the company's weekly profit, which is 19166.67€. This profit is obtained by producing 183 units of the basic model, 50 units of the standard model and 50 units of the premium model. Following this allows us to use all available resources and follows all the restrictions of the model.

6. **The company is considering a collaboration with a supplier who offers the option to increase the weekly availability of one resource. The company can only choose one option and requests your team to select the one that maximizes net profit:**
    - **Processors: increase by 10 units at a cost of 80€**
    - **Labour Hours: Increase by 20 hours at a cost of 120€**

The company has a limited amount of resources, and to understand how much are we able to increase the resources and how much first we need to calculate the ranges of the resources available each week. Thus allow us to study the sensibility of the objective function with the changes of the available resources.

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
RESOURCES_SLACK(1)	500.0000	INFINITY	166.6667
RESOURCES_SLACK(2)	400.0000	INFINITY	116.6667
RESOURCES_SLACK(3)	600.0000	INFINITY	166.6667
RESOURCES_SLACK(4)	1200.000	350.0000	400.0000
MIN_PROD_SLACK(1)	50.00000	133.3333	INFINITY
MIN_PROD_SLACK(2)	50.00000	80.00000	50.00000
MIN_PROD_SLACK(3)	50.00000	38.00000	50.00000
9	0.000000	INFINITY	63.33333

<u>i) Processors increase by 10 units at a cost of 80€</u>

- The sensibility of restriction 1 [Processors], it could decrease up to 166.6 units and it could increase up to infinity. In this case in which we aim to increase the units of processors by 10, we confirm that it is possible due 10 is within the range [-166.6, ∞).
    - The dual price of processors is 0, thus the profit does not change when increasing the units of processors used. Therefore if we increase by 10 units the profit will decrease 80€, which is the cost of this increase.
- CONCLUSION: Increasing 10 units of the processors leads to a negative profit, meaning that it is not beneficial.

<u>ii) Labour hours increase by 20 hours at a cost of 120€</u>

- The sensibility of restriction 4 [labour hours]], it could decrease up to 400 units and it could increase up to 350. In this case in which we aim to increase the hours of labour up to 20, we confirm that it is possible due 20 is within the range [-400,350].
    - The dual price of processors is 16.67, which would lead to an increase in profit of 20*16.67 = 333.5€. But we have to take into consideration that to increase the labour we will have an extra cost of 120€. Therefore the net profit is 213.4€.
- CONCLUSION: In this case increasing labour 20 hours is profitable, this the net profit increases up to 213.4€

7. **If the selling price of the standard model increases to 90€, would the optimal solution change? What if the selling price of the premium model increases to 130€? In both cases, specify the resulting profit.**

In order to study the feasibility to change the price per unit of the phones we have to conduct the sensitivity analysis on the objective coefficients. LINGO program provided this output:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
PHONE_MODEL(1)	50.00000	INFINITY	2.000000
PHONE_MODEL(2)	80.00000	3.333333	INFINITY
PHONE_MODEL(3)	120.0000	13.33333	INFINITY

<u>i) Increasing the selling price of the standard model to 90€.</u>

- The original price of the standard model is 80€, this increase would imply a rise of 10€ from the initial price. To study whether it is possible we have to contempt the objective coefficient ranges of X2(standard model). The table provided by LINGO allows us to conclude that the price of X2 can decrease up to infinity or increase 3.33€. Therefore, the proposed increase of 10€ is not within the range of (∞,3.34], meaning that in order to change the price from 80 to 90€ we would have to change the production plan.
- We consider to code the new production plan using LINGO, by changing the syntax of the original model:
    - ProfitxUnit = 50 ***90*** 120

Variable	Value	Reduced Cost
PROFITXUNIT(1)	50.00000	0.000000
PROFITXUNIT(2)	90.00000	0.000000
PROFITXUNIT(3)	120.0000	0.000000
MINPROD(1)	50.00000	0.000000
MINPROD(2)	50.00000	0.000000
MINPROD(3)	50.00000	0.000000
PHONE_MODEL(1)	50.00000	0.000000
PHONE_MODEL(2)	130.0000	0.000000
PHONE_MODEL(3)	50.00000	0.000000
AVAILABILITY(1)	500.0000	0.000000
AVAILABILITY(2)	400.0000	0.000000
AVAILABILITY(3)	600.0000	0.000000
AVAILABILITY(4)	1200.000	0.000000
A(1, 1)	1.000000	0.000000
A(1, 2)	1.000000	0.000000
A(1, 3)	2.000000	0.000000
A(2, 1)	1.000000	0.000000
A(2, 2)	1.000000	0.000000
A(2, 3)	1.000000	0.000000
A(3, 1)	1.000000	0.000000
A(3, 2)	2.000000	0.000000
A(3, 3)	3.000000	0.000000
A(4, 1)	3.000000	0.000000
A(4, 2)	5.000000	0.000000
A(4, 3)	8.000000	0.000000

Row	Slack or Surplus	Dual Price
1	20200.00	1.000000
RESOURCES_SLACK(1)	220.0000	0.000000
RESOURCES_SLACK(2)	170.0000	0.000000
RESOURCES_SLACK(3)	140.0000	0.000000
RESOURCES_SLACK(4)	0.000000	18.00000
MIN_PROD_SLACK(1)	0.000000	-4.000000
MIN_PROD_SLACK(2)	80.00000	0.000000
MIN_PROD_SLACK(3)	0.000000	-24.00000
9	42.00000	0.000000

Considering the new updated LINGO table, we obtain a new production plan of 50 units of basic phone model, 130 units of standard phone model and lastly 50 units of premium phone model.

The profit of this new model, in which we have increased the selling price of the standard model up to 90€, is of 20200€.

```
Global optimal solution found.
Objective value:                              20200.00
Infeasibilities:                              0.000000
Total solver iterations:                             2
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                     3
Nonlinear variables:                 0
Integer variables:                   0

Total constraints:                   9
Nonlinear constraints:               0

Total nonzeros:                     21
Nonlinear nonzeros:                  0
```

<u>ii) Increase the selling price of the premium model to 130€.</u>

- The current price of the premium model is 120€, this represents an increase of 10€ from the initial price. To study whether it is feasible we have to contempt the objective coefficient ranges of X3 (premium model). The sensitivity analysis provided by LINGO allows us to conclude that the price of the premium model can decrease up to infinity or increase 13.34€, without requiring a change in production. Therefore, the proposed increase of 10€ is within the range of (∞,13.34]. Thus, we keep the production plan of 183 units of basic model, 50 units of standard model and 50 units of premium model, but the profit changed: 183*50 + 50*80 + 50*130 = 1966.67€. It has increased by 500€.

8. **If the availability of processors is reduced to 450 units, how would this affect the solution? Would it still be possible to produce the same quantities of each model? Indicate how the solution vector for all variables would change.**

In this case we aim to reduce the available processors up to 450 units, which represent a 50 units decrease of the original model. In order to conclude whether it is feasible or not we have to consider the LINGO table we previously studied in part 6. The sensitivity analysis allows us to conclude that the quantity of processors available per week can decrease up to 183 units or increase up to infinity.. Therefore, the proposed decrease of 50 units falls within the range [183,∞). Meaning that this change would not affect the production plan nor the profit.

9. **The company's partners are considering easing the restriction that limits premium phones to no more than 40% of total production, arguing that this regulation is negatively impacting profits. Do you agree with their perspective?**

After eliminating the 40% restriction, we can observe that the profit remains 19166.67€..

With restriction:

```
Global optimal solution found.
Objective value:                              19166.67
Infeasibilities:                              0.000000
Total solver iterations:                             3
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                      3
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    9
Nonlinear constraints:                0

Total nonzeros:                      21
Nonlinear nonzeros:                   0
```

Without restriction:

```
Global optimal solution found.
Objective value:                              19166.67
Infeasibilities:                              0.000000
Total solver iterations:                             3
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                      3
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      18
Nonlinear nonzeros:                   0
```

Hence removing the 40% restriction has no effect on the solution or profits of the current model, so either keeping or removing the restriction has no effect on the commercial strategy.

10. **To define the new commercial strategy, the partners want to explore what the profit would be if the restriction of being present in all markets with at least 50 units were removed:**

a) **Formulate the new problem.**

$$max\, f(x) \; = \; 50x_1 + 80x_2 + 120x_3$$

s. $t.$ $\quad x_1 + x_2 + 2x_3 \leq 500$

$$x_1 + x_2 + x_3 \quad \leq 400$$

$x_1 + 2x_2 + 3x_3 \leq 600$

$3x_1 + 5x_2 + 8x_3 \leq 1200$

$x_3 \leq 0.4\,(x_1 + x_2 + x_3)$ ---> $-0.4\,x_1 - 0.4x_2 + 0.6x_3 \leq 0$

$x_1,\ x_2,\ x_3 \geq 0$

In compact form:

$$OF : \max f(x) = \sum_{j=1}^{3} c_j x_j$$

$$s.t. \quad \sum_{j=1}^{3} a_{ij} x_j \leq b_i \ \ \forall i \in \{1,2,3,4\}$$

$$x_3 \leq 0.4 \sum_{j=1}^{3} x_j$$

$$x_j \geq 0 \ \forall j \in \{1,2,3\}$$

where $c_j^T = [50\ 80\ 120]$

$b_i^T = [500\ \ 400\ \ 600\ \ 1200]$

$a_{ij} = coefficients\ of\ matrix\ A$

Matrix form:

$$OF : \max \mathrm{c^T x}$$
$$s.t.\ \ \mathrm{Ax \leq b}$$
$$x \geq 0$$

where

$$c_j^T = [\,50\ 80\ 120]_{1x3}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3x1}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 5 & 8 \\ -0.4 & -0.4 & 0.6 \end{bmatrix}_{5x3}$$

$$b = \begin{bmatrix} 500 \\ 400 \\ 600 \\ 1200 \\ 0 \end{bmatrix}_{5x1}$$

The updated LINGO code is:

```
SETS:
MODELS /1..3/: ProfitXUnit, MinProd, Phone_model;  ! Models: Basic (1), Standard (2), Premium (3) -->j;
RESOURCES /1..4/: Availability;  ! Recursos: Processors (1), Screens (2), Batteries (3), Labor hours (4)-->i;
MATRIX(RESOURCES, MODELS): A;
ENDSETS
! Input the profit values, minimum production requirements, and resource data;
DATA:
ProfitXUnit= 50 80 120;        ! Profit per unit for Basic, Standard, Premium;
Availability = 500 400 600 1200; ! Available resources: processors, screens, batteries, labor hours;
A = 1 1 2   ! Processors: A(1, Basic), A(1, Standard), A(1, Premium);
    1 1 1   ! Screens: A(2, Basic), A(2, Standard), A(2, Premium);
    1 2 3   ! Batteries: A(3, Basic), A(3, Standard), A(3, Premium);
    3 5 8;  ! Labor hours: A(4, Basic), A(4, Standard), A(4, Premium);
ENDDATA
! Objective function: Maximize profit;
MAX = @SUM(MODELS(j): ProfitXUnit(j) * Phone_model(j));
! Resource constraints;
@FOR(RESOURCES(i):  [Resources_slack]@SUM(MODELS(j): A(i,j) * Phone_model(j)) <= Availability(i));
! Premium model production limit (40% of total production);
Phone_model(3) <= 0.4 * @SUM(MODELS(j):Phone_model(j));
END
```

b) **Express it in standard form.**

Since there are five ≤ constraints, in order to standardise the Linear Problem and have all equalities, we need to add five slack variables, i.e. x_4, x_5, x_6, x_7, x_8.

$$
\begin{aligned}
&OF: max f(x) = 50x_1 + 80x_2 + 120x_3 \\
&s.t.\ x\_1 + x\_2 + 2x\_3 + x\_4 && = 500 \\
&\quad x\_1 + x\_2 + x\_3 \quad + x\_5 && = 400 \\
&\quad x\_1 + 2x\_2 + 3x\_3 \quad + x\_6 && = 600 \\
&\quad 3x\_1 + 5x\_2 + 8x\_3 \quad + x\_7 && = 1200 \\
&-0.4x\_1 - 0.4x\_2 + 0.6x\_3 \quad + x_8 && = 0 \\
&\quad x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0
\end{aligned}
$$

c) **What is the optimal combination of phone models to produce?**

After removing the restriction of being present in all markets with at least 50 units, the optimal combination to produce is 400 units of the basic model and 0 of the other phone models as can be seen from the image below in light blue. Following this allows us to use all available resources and follows all the other restrictions of the model.

Variable	Value	Reduced Cost
PROFITXUNIT(1)	50.00000	0.000000
PROFITXUNIT(2)	80.00000	0.000000
PROFITXUNIT(3)	120.0000	0.000000
MINPROD(1)	0.000000	0.000000
MINPROD(2)	0.000000	0.000000
MINPROD(3)	0.000000	0.000000
PHONE_MODEL(1)	400.0000	0.000000
PHONE_MODEL(2)	0.000000	3.333333
PHONE_MODEL(3)	0.000000	13.33333
AVAILABILITY(1)	500.0000	0.000000
AVAILABILITY(2)	400.0000	0.000000
AVAILABILITY(3)	600.0000	0.000000
AVAILABILITY(4)	1200.000	0.000000
A(1, 1)	1.000000	0.000000
A(1, 2)	1.000000	0.000000
A(1, 3)	2.000000	0.000000
A(2, 1)	1.000000	0.000000
A(2, 2)	1.000000	0.000000
A(2, 3)	1.000000	0.000000
A(3, 1)	1.000000	0.000000
A(3, 2)	2.000000	0.000000
A(3, 3)	3.000000	0.000000
A(4, 1)	3.000000	0.000000
A(4, 2)	5.000000	0.000000
A(4, 3)	8.000000	0.000000

Row	Slack or Surplus	Dual Price
1	20000.00	1.000000
RESOURCES_SLACK(1)	100.0000	0.000000
RESOURCES_SLACK(2)	0.000000	0.000000
RESOURCES_SLACK(3)	200.0000	0.000000
RESOURCES_SLACK(4)	0.000000	16.66667
6	160.0000	0.000000

d) **What would be the weekly profit obtained?**

After removing the restriction, the optimal production plan to maximize the company's weekly profit is now 20,000€.

```
Global optimal solution found.
Objective value:                              20000.00
Infeasibilities:                              0.000000
Total solver iterations:                             3
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                      6
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    6
Nonlinear constraints:                0

Total nonzeros:                      18
Nonlinear nonzeros:                   0
```

e) **What type of solution is it?**

After removing the required constraint, there are 6 left, as can be seen from the image above, one of which is the non-negativity constraint. We can then conclude that there are 5 constraints, so m=5, and we have 5 reduced cost nulls, as highlighted in yellow in the image above. So the problem has a unique optimal feasible solution.

The basic variables are five (m) and they are those that have reduced cost zero, i.e. x_1, x_4, x_5, x_6, x_8, while the non-basic variables are 3 (n - m = 8 - 5) , i.e. x_2, x_3, x_7 .

11. **Determine the minimum price at which premium phones should be sold to make their production viable under this new context. What type of solution would the problem have if the model reached this price?**

    Checking the Reduced Cost we can see that the price would need to increase at least 13.3333 in order to make the production of premium phones viable.

Variable	Value	Reduced Cost
PROFITXUNIT(1)	50.00000	0.000000
PROFITXUNIT(2)	80.00000	0.000000
PROFITXUNIT(3)	120.0000	0.000000
MINPROD(1)	0.000000	0.000000
MINPROD(2)	0.000000	0.000000
MINPROD(3)	0.000000	0.000000
PHONE_MODEL(1)	400.0000	0.000000
PHONE_MODEL(2)	0.000000	3.333333
PHONE_MODEL(3)	0.000000	13.33333
AVAILABILITY(1)	500.0000	0.000000
AVAILABILITY(2)	400.0000	0.000000
AVAILABILITY(3)	600.0000	0.000000
AVAILABILITY(4)	1200.000	0.000000
A(1, 1)	1.000000	0.000000
A(1, 2)	1.000000	0.000000
A(1, 3)	2.000000	0.000000
A(2, 1)	1.000000	0.000000
A(2, 2)	1.000000	0.000000
A(2, 3)	1.000000	0.000000
A(3, 1)	1.000000	0.000000
A(3, 2)	2.000000	0.000000
A(3, 3)	3.000000	0.000000
A(4, 1)	3.000000	0.000000
A(4, 2)	5.000000	0.000000
A(4, 3)	8.000000	0.000000

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
MINPROD(1)	0.000000	0.000000	INFINITY
MINPROD(2)	0.000000	0.000000	INFINITY
MINPROD(3)	0.000000	0.000000	INFINITY
PHONE_MODEL(1)	50.00000	INFINITY	2.000000
PHONE_MODEL(2)	80.00000	3.333333	INFINITY
PHONE_MODEL(3)	120.0000	13.33333	INFINITY

Since the Reduced Cost is 13.33€ and the value of Premium phones is 0, to economically justify the production of these phones their price must increase from the initial 120€.

Now, let's say we increase the price up to 133.34€ (price > 120+13.33), so ProfitxUnit = 50 80 133.34, the new optimal would be (144 0 96) as can be seen here:

Variable	Value	Reduced Cost
PROFITXUNIT(1)	50.00000	0.000000
PROFITXUNIT(2)	80.00000	0.000000
PROFITXUNIT(3)	133.3400	0.000000
MINPROD(1)	0.000000	0.000000
MINPROD(2)	0.000000	0.000000
MINPROD(3)	0.000000	0.000000
PHONE_MODEL(1)	144.0000	0.000000
PHONE_MODEL(2)	0.000000	3.334400
PHONE_MODEL(3)	96.00000	0.000000
AVAILABILITY(1)	500.0000	0.000000
AVAILABILITY(2)	400.0000	0.000000
AVAILABILITY(3)	600.0000	0.000000
AVAILABILITY(4)	1200.000	0.000000
A(1, 1)	1.000000	0.000000
A(1, 2)	1.000000	0.000000
A(1, 3)	2.000000	0.000000
A(2, 1)	1.000000	0.000000
A(2, 2)	1.000000	0.000000
A(2, 3)	1.000000	0.000000
A(3, 1)	1.000000	0.000000
A(3, 2)	2.000000	0.000000
A(3, 3)	3.000000	0.000000
A(4, 1)	3.000000	0.000000
A(4, 2)	5.000000	0.000000
A(4, 3)	8.000000	0.000000

Row	Slack or Surplus	Dual Price
1	20000.64	1.000000
RESOURCES_SLACK(1)	164.0000	0.000000
RESOURCES_SLACK(2)	160.0000	0.000000
RESOURCES_SLACK(3)	168.0000	0.000000
RESOURCES_SLACK(4)	0.000000	16.66720
6	0.000000	0.4000000E-02

With a new optimal value of 20000.64€ (this would increase as the price of premium phones increase).

```
Global optimal solution found.
Objective value:                              20000.64
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                  6
Nonlinear variables:              0
Integer variables:                0

Total constraints:                6
Nonlinear constraints:            0

Total nonzeros:                  18
Nonlinear nonzeros:               0
```

The solution we are looking at now is a unique optimal solution since this is the only combination that would increase our profits, in fact we've 5 constraints and 5 reduced cost nulls.