Glaciology
EESCGU4220

Practical 7:
Non-steady heat equation

Aims:

Learn about boundary conditions

Learn more about "parameter space" and how to *search* it.

Understand what controls temperature in cold-based ice sheets.

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}}$$

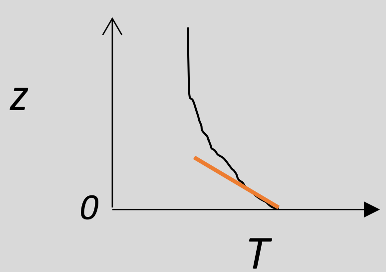
Boundary conditions

- Surface energy balance (Lecture 4)
- Basal energy balance

--- **geothermal heat flux**, friction: $\tau_b u_b$, hydrology

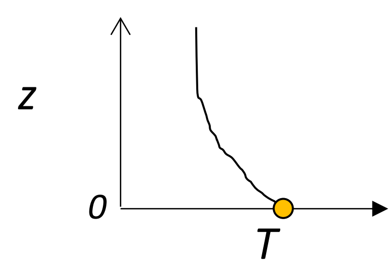
Cold-based ice:
Geothermal heat flux is set at the base.

Neumann boundary condition
(one that fixes the derivative of T)



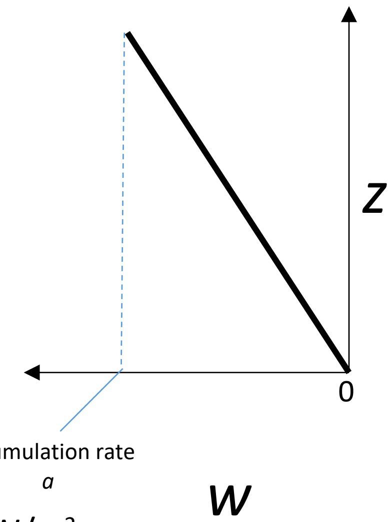
Warm-based ice:
T_b = melting point

Dirichlet boundary condition
(one that fixes the value of T)



1. Solve the heat equation:

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}}$$



basal boundary condition:

dT/dz = G/k at the bed
where G is geothermal heat flux and k is thermal conductivity in the ice: 0.060 W/m^2

Steady-state T (parameter space)

2. Plot steady-state basal *T* against ice thickness

3. Plot steady-state basal *T* against accumulation rate

4. Plot steady-state basal *T* against ice thickness AND accumulation rate on the same plot (2-D parameter space)

$$\boxed{T(0,t) = \mathrm{A}\sin\left(\frac{2\pi}{\lambda}t\right)}$$

Time-varying T (phase space)
5. Phase space plot (surface *T* vs. basal *T*) and how that varies with λ

Hints

- Be careful of the units [do everything in seconds]
- Vectorize
- Finite difference version of second derivative is:

$$\left.\frac{d^2T}{dz^2}\right|_j = \frac{T(j+1) - 2T(j) + T(j-1)}{\Delta z^2}$$

- think carefully about the basal boundary condition
- When you have a script to evolve T forward in time, make another one that calls the first script with different values of the parameters.
- Make many versions: ctrl+a, ctrl+c, ctrl+n, ctrl+v, ctrl+s
- Structure your code sensibly
- Comment