

Practical 7:

Non-steady heat equation

Aims:

Learn about boundary conditions

Learn more about “parameter space” and how to *search* it.

Understand what controls temperature in cold-based ice sheets.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}$$

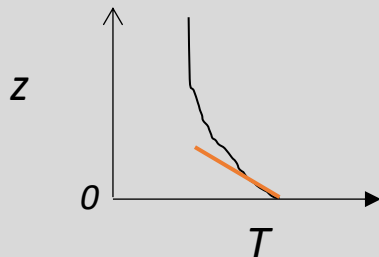
Boundary conditions

- Surface energy balance (Lecture 4)
- Basal energy balance
- **geothermal heat flux**, friction: $\tau_b u_b$, hydrology

Cold-based ice:

Geothermal heat flux is set at the base.

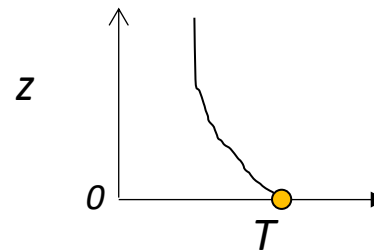
Neumann boundary condition
(one that fixes the derivative of T)



Warm-based ice:

$T_b = \text{melting point}$

Dirichlet boundary condition
(one that fixes the value of T)



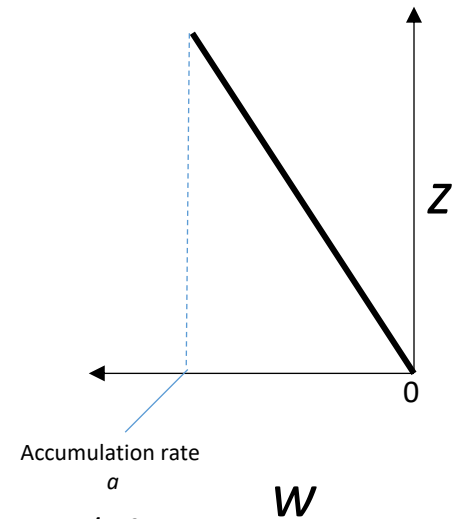
1. Solve the heat equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}$$

basal boundary condition:

$$dT/dz = G/k \text{ at the bed}$$

where G is geothermal heat flux and k is thermal conductivity in the ice: 0.060 W/m^2



Steady-state T (parameter space)

2. Plot steady-state basal T against ice thickness
3. Plot steady-state basal T against accumulation rate
4. Plot steady-state basal T against ice thickness AND accumulation rate on the same plot (2-D parameter space)

$$T(0, t) = A \sin\left(\frac{2\pi}{\lambda} t\right)$$

Time-varying T (phase space)

5. Phase space plot (surface T vs. basal T) and how that varies with λ