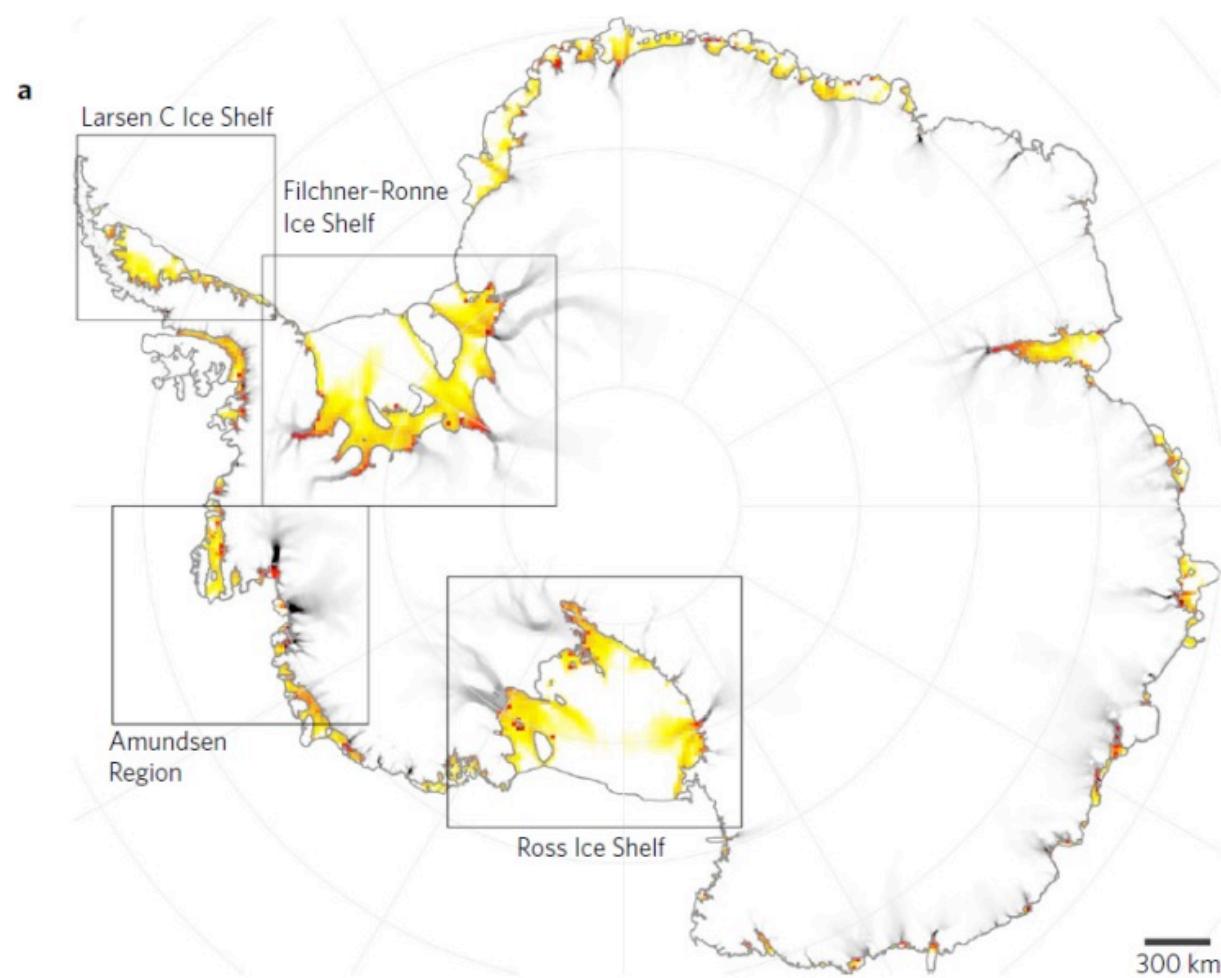


# Lecture 9: Review + Ice Shelves

Review of SIA practical

Review of first ½ of semester

Ice shelves



# SIA practical: surface mass balance

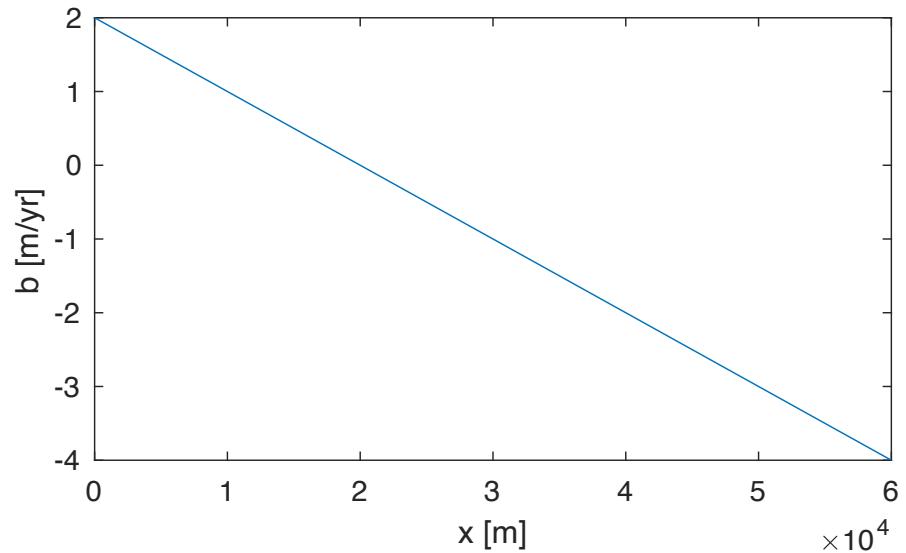
$$\frac{\partial H}{\partial t} + \frac{\partial q}{\partial x} = b_i$$

$$q = \frac{2A}{n+2} (\rho g \alpha)^n H^{n+2}$$

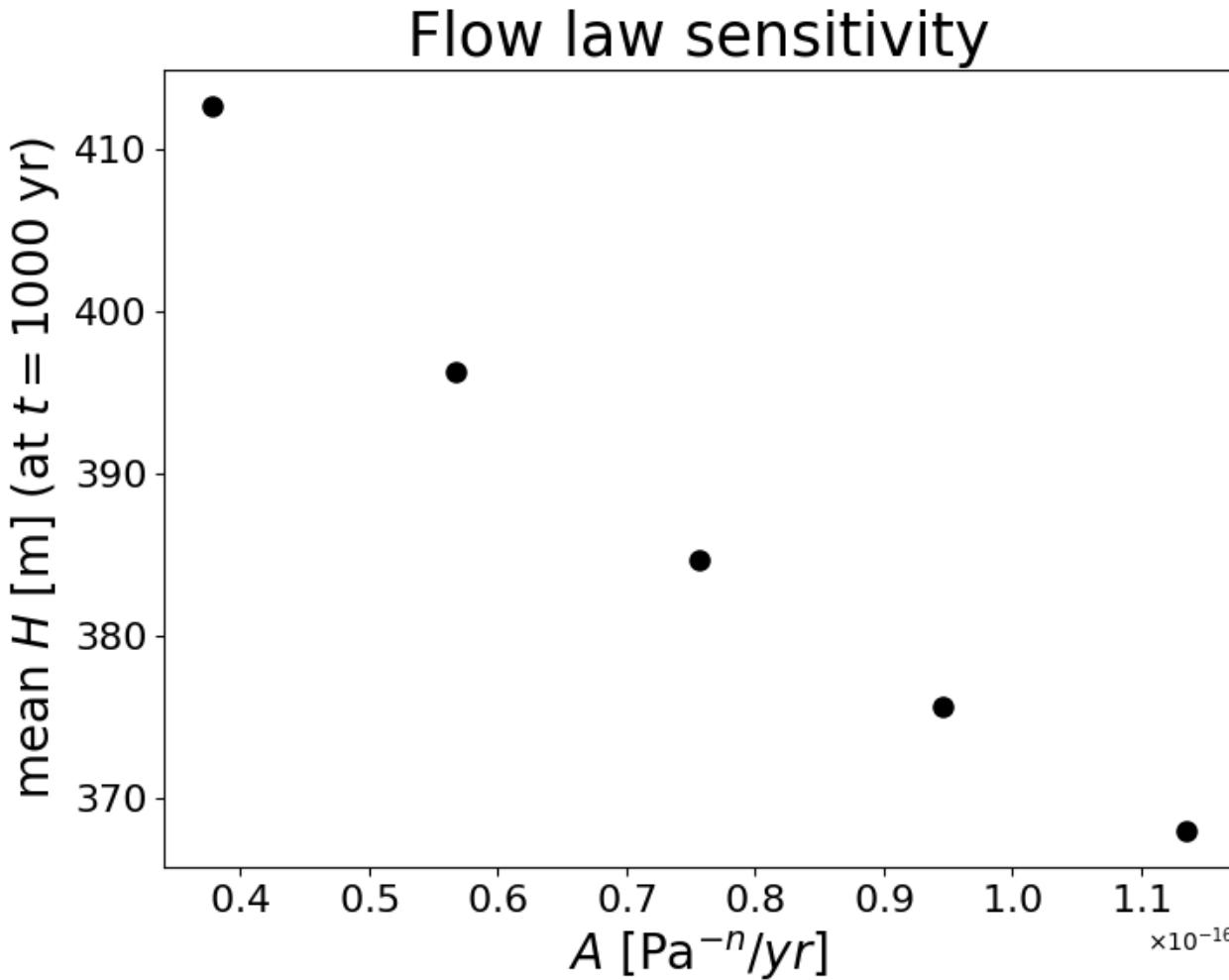
$$q(0) = 0$$

$$ELx = X/3$$

$$b = 10^{-4}(X/3 - x)$$

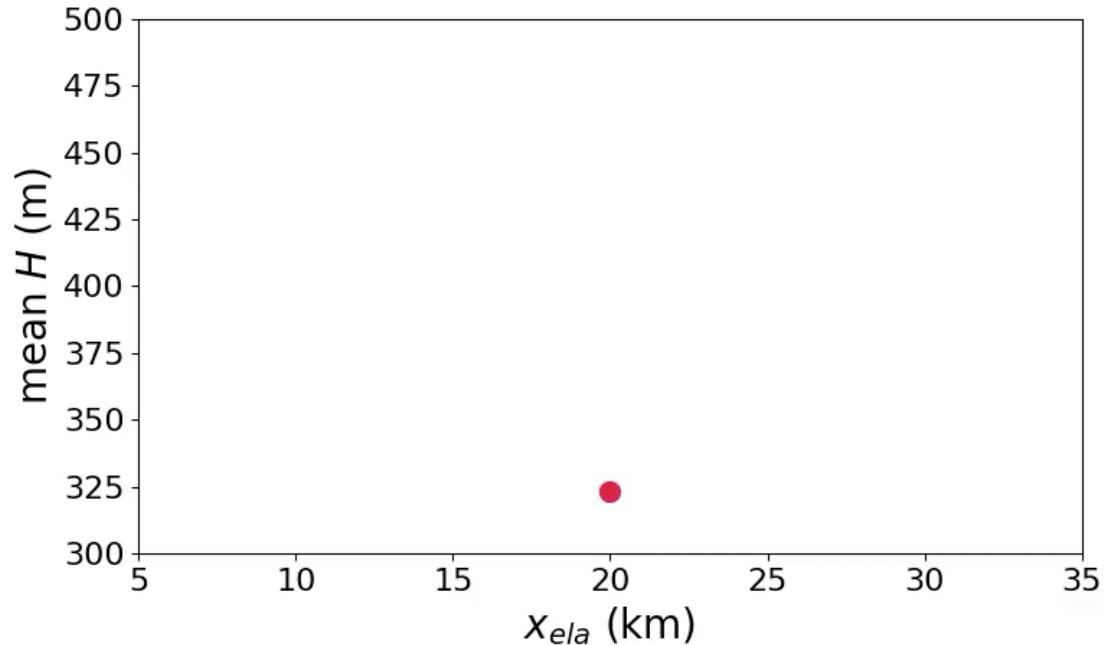
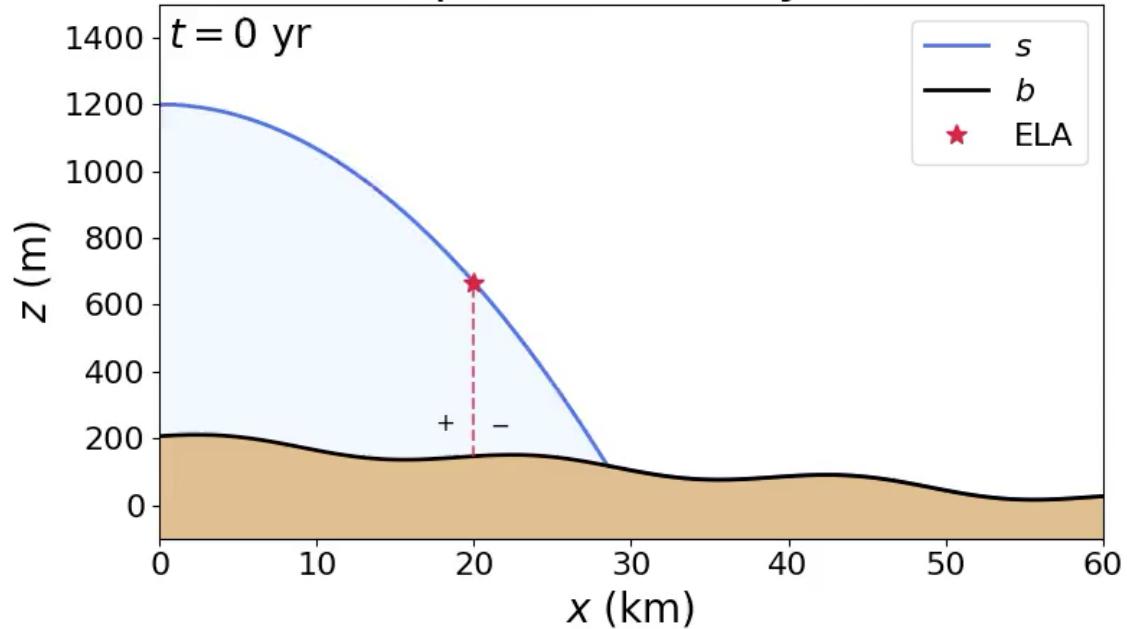


# SIA practical: parameter sensitivity

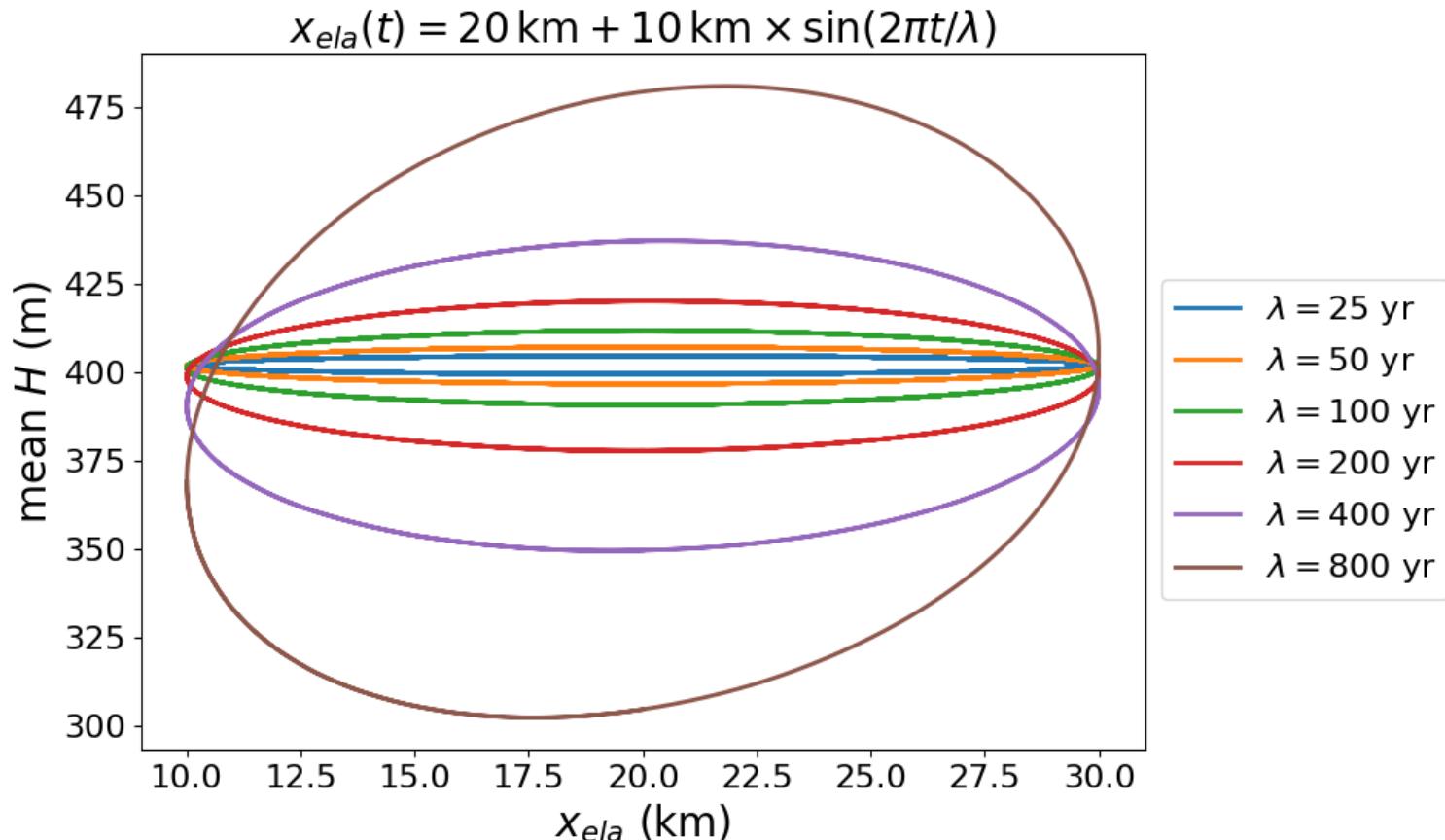


# SIA practical: Transient evolution

SIA with time-varying ELA  
(period = 400 yr.)



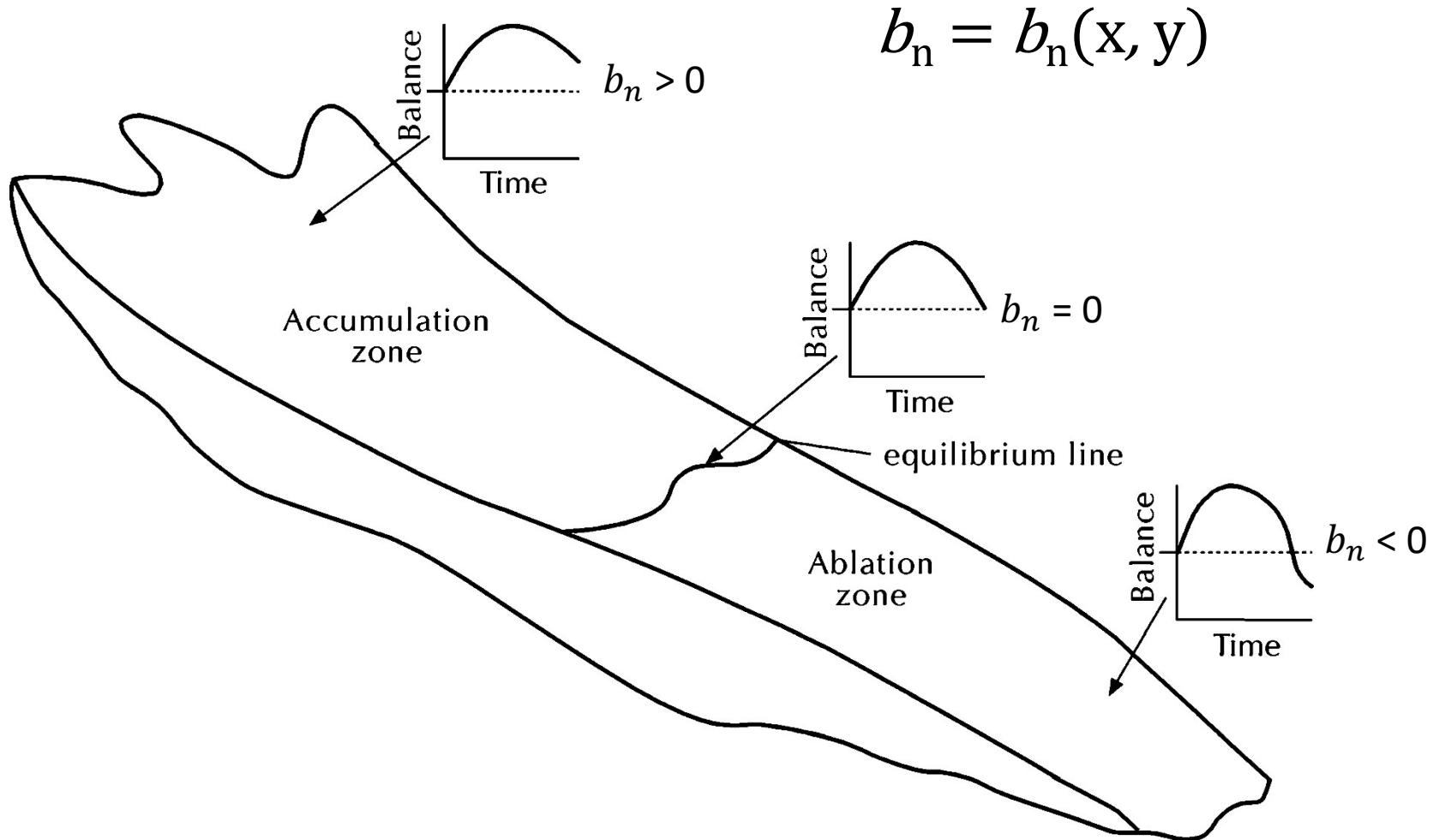
# SIA practical: Transient evolution



# Review of first half of semester

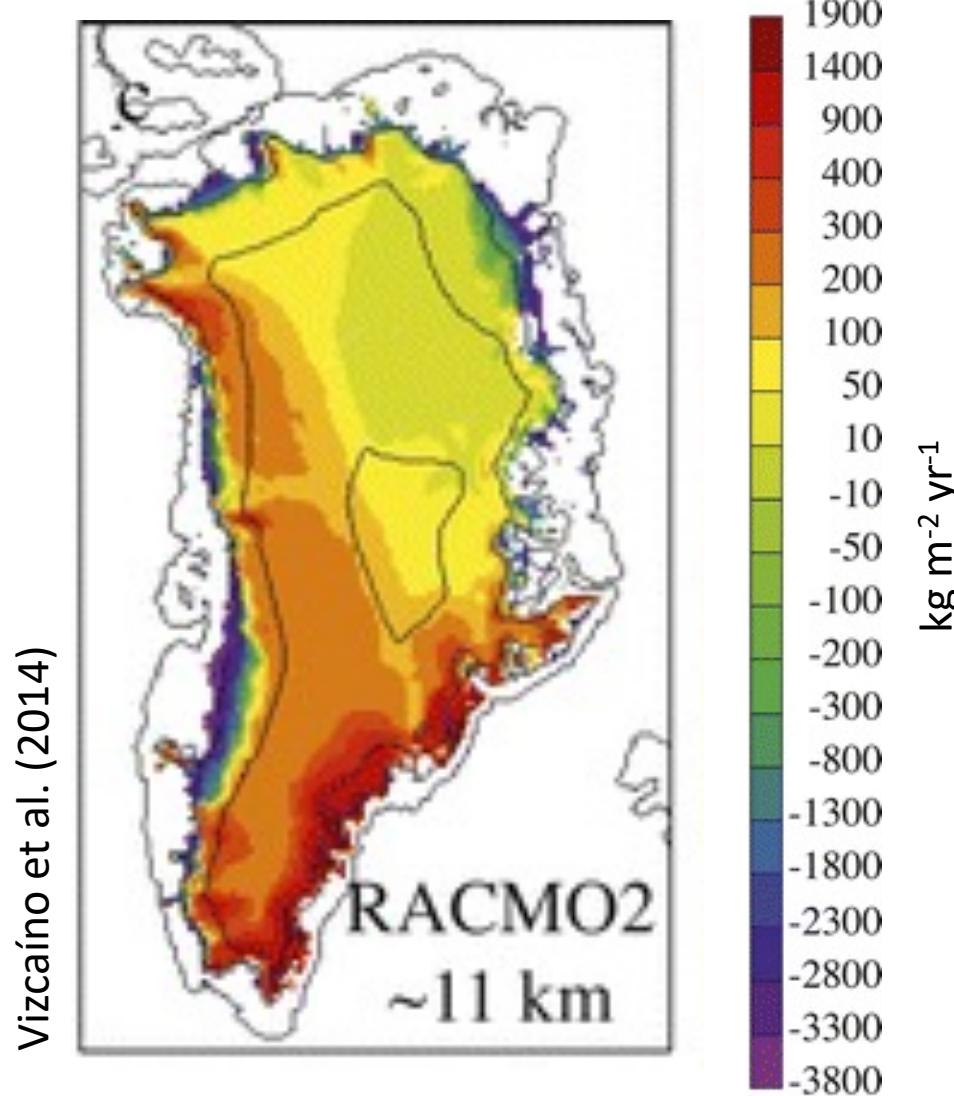
- Surface mass balance
  - Accumulation zone processes
  - Surface energy balance
  - Basal melting
- Ice flow
  - Rheology
  - Stress balance
  - SIA model
  - Basal motion
  - Ice divides
  - Grounding lines

# Spatially-varying net specific balance



Cuffey and Paterson (2010)

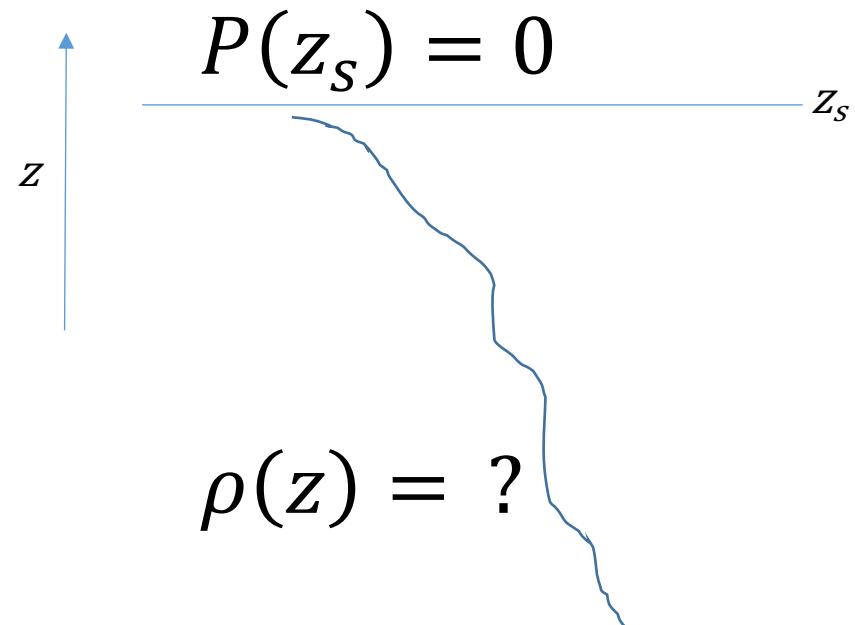
# Long-term SMB of Greenland



# Overburden pressure

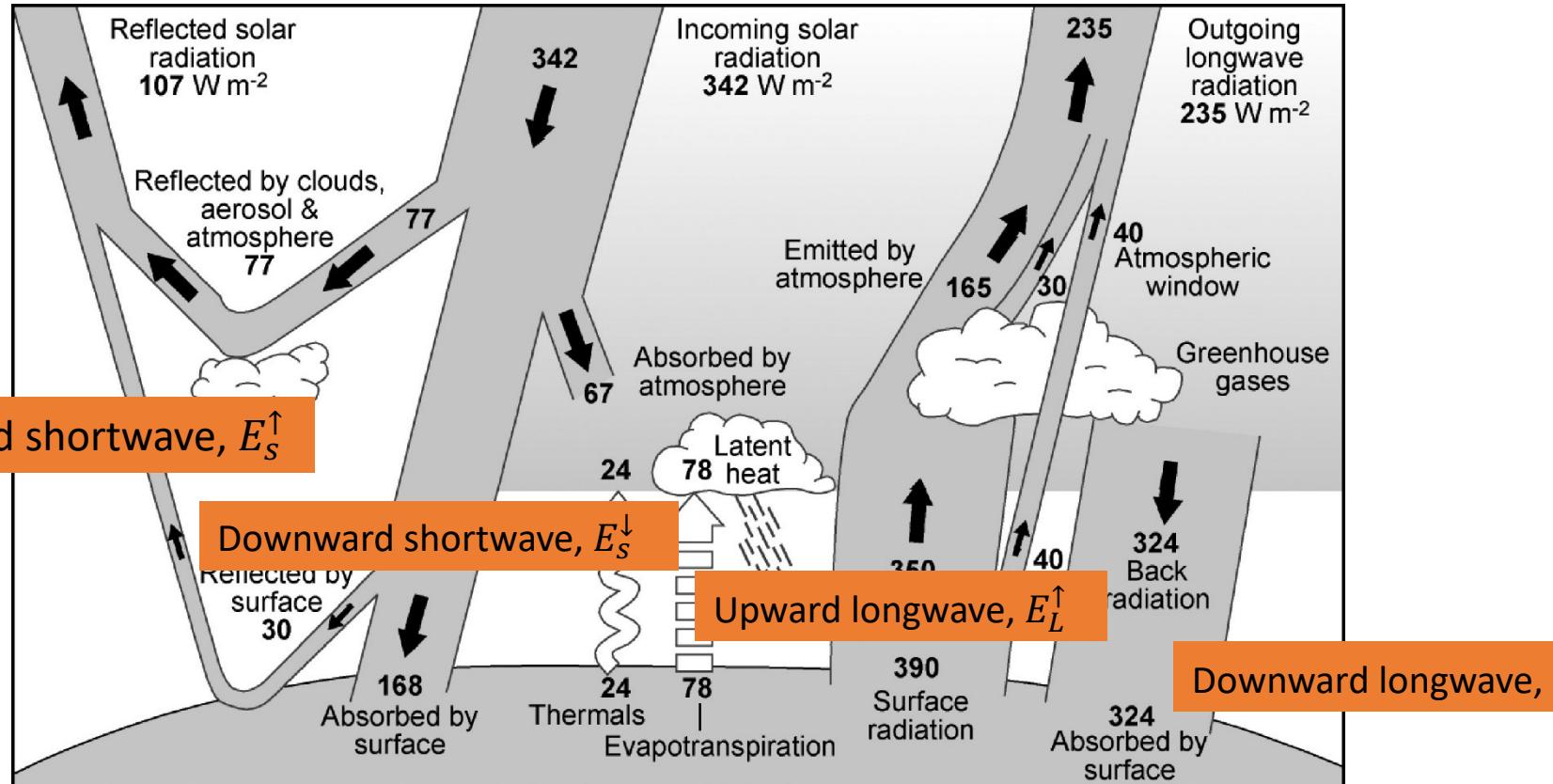
We want an equation for how pressure  $P$  varies with depth.

$$\frac{dP}{dz} = \rho(z)g$$



# Surface energy balance

$$E_N = E_S^\downarrow + E_S^\uparrow + E_L^\downarrow + E_L^\uparrow + E_H + E_E$$



# Stress, strain, rheology, mass balance

$$\dot{\boldsymbol{\epsilon}} = \begin{bmatrix} \dot{\epsilon}_{xx} & \dot{\epsilon}_{xy} & \dot{\epsilon}_{xz} \\ \dot{\epsilon}_{yx} & \dot{\epsilon}_{yy} & \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{zx} & \dot{\epsilon}_{zy} & \dot{\epsilon}_{zz} \end{bmatrix} \quad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right]$$

$$\boldsymbol{\sigma} = \boldsymbol{\tau} + \boldsymbol{\sigma}_m$$

$$\dot{\epsilon}_{ij} = A \tau_E^{n-1} \tau_{ij} \quad (i,j = x, y, z)$$

$$\tau_E^2 = \frac{1}{2} [\tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2] + \tau_{xz}^2 + \tau_{xy}^2 + \tau_{yz}^2$$

$$\frac{\partial H}{\partial t} + \frac{\partial q}{\partial x} = b_i$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho g$$

# Ice flux in the Shallow Ice Approximation

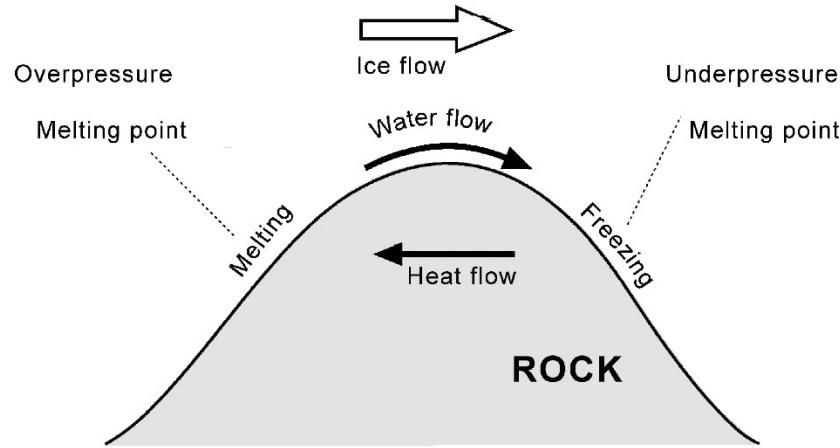
Depth-integrated horizontal velocity per unit width:

$$\hat{u} = 2A(\rho g \alpha H)^n \left( \frac{H}{n+2} \right)$$

Depth-integrated volume-flux per unit width:

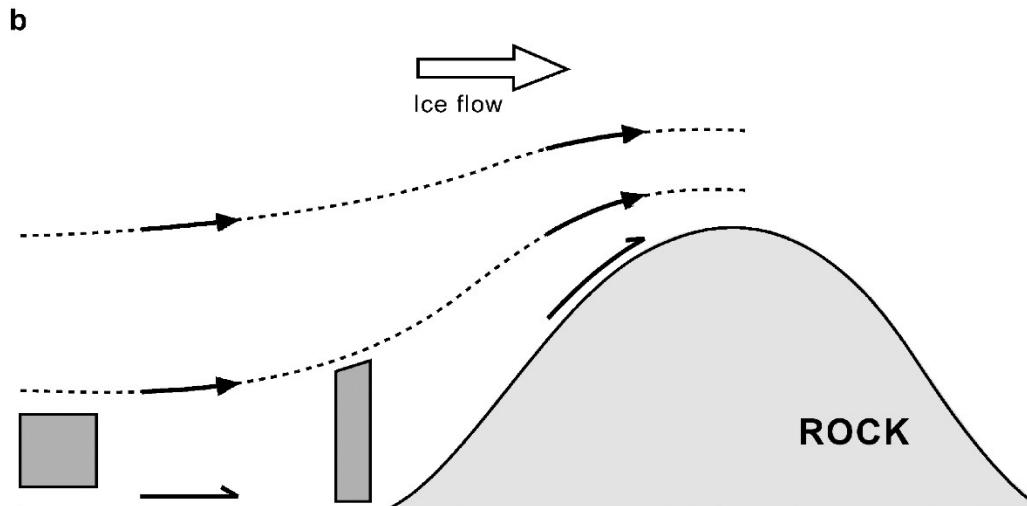
$$q = \hat{u}H = \frac{2A}{n+2} (\rho g \alpha)^n H^{n+2}$$

# Weertman's theory for hard beds



Two mechanisms:  
**Regelation &**  
**enhanced creep.**

$$u_b \propto \left(\frac{\sqrt{\tau_b}}{R}\right)^{n+1}$$

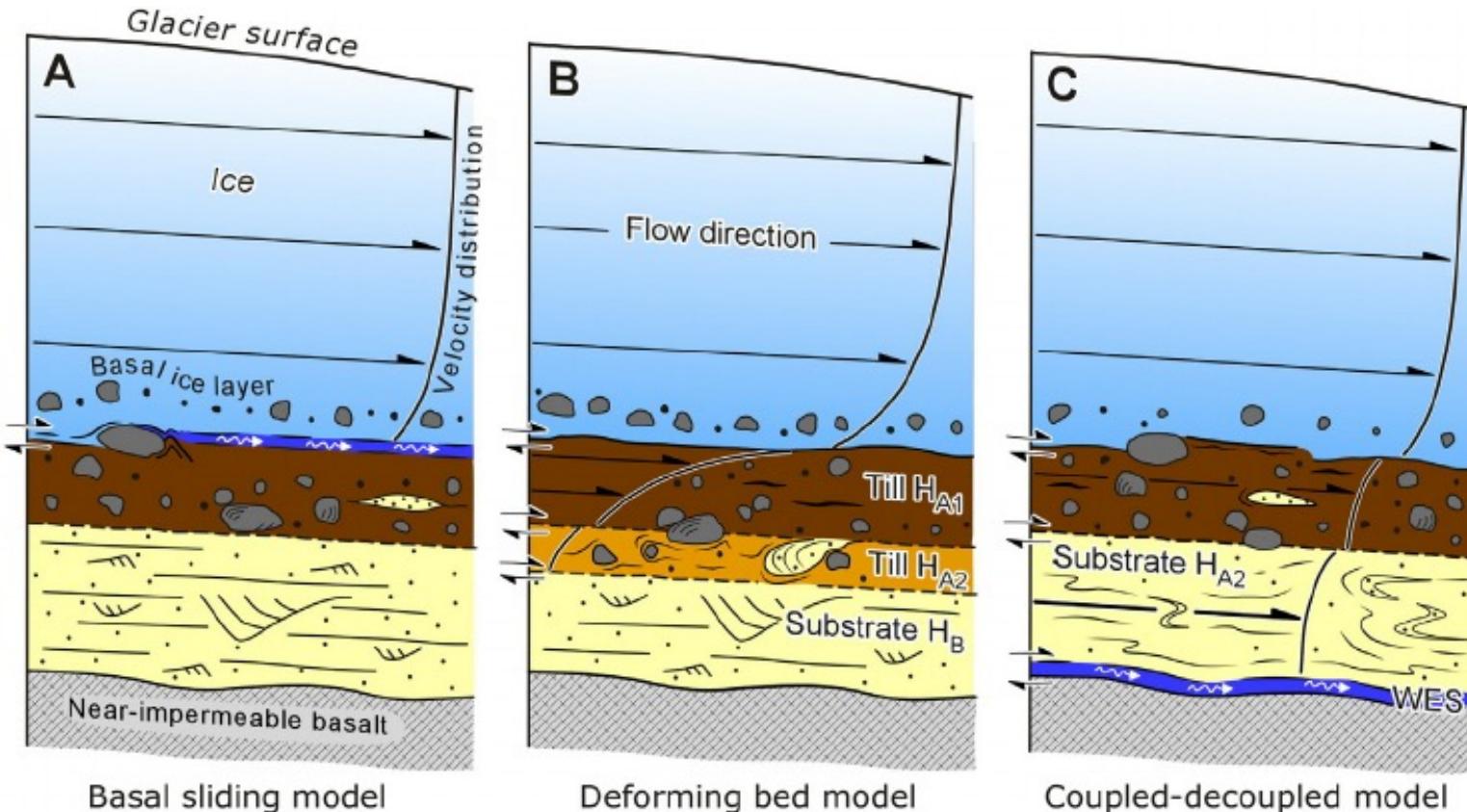


This is a crude model, but it captures some essence of the processes.

Extension of the theory to include cavitation: Iken (1981)

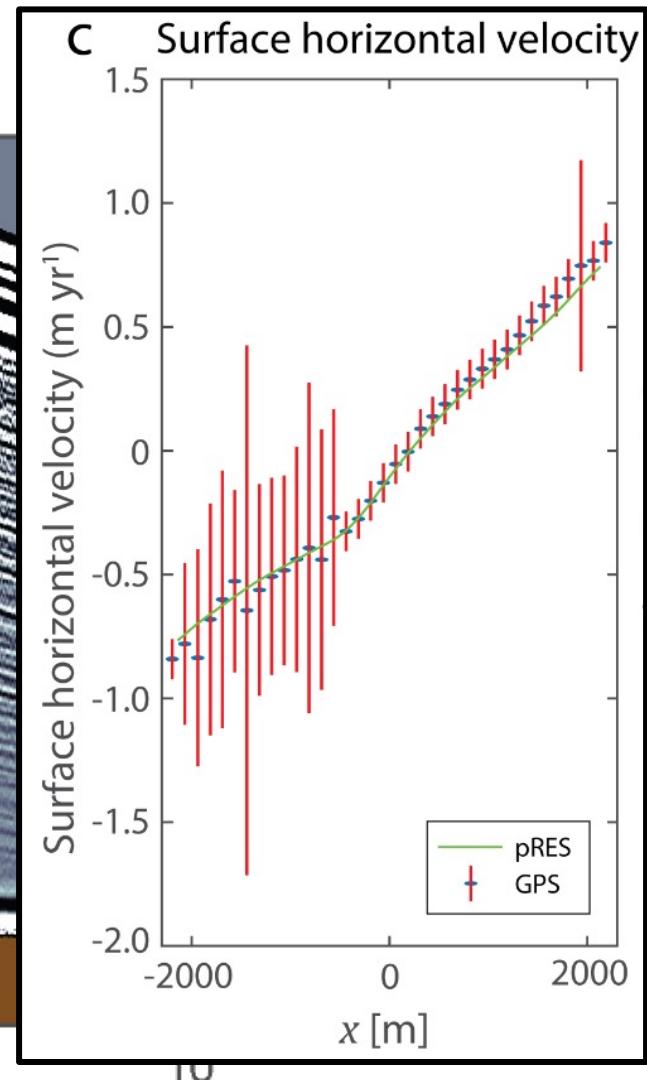
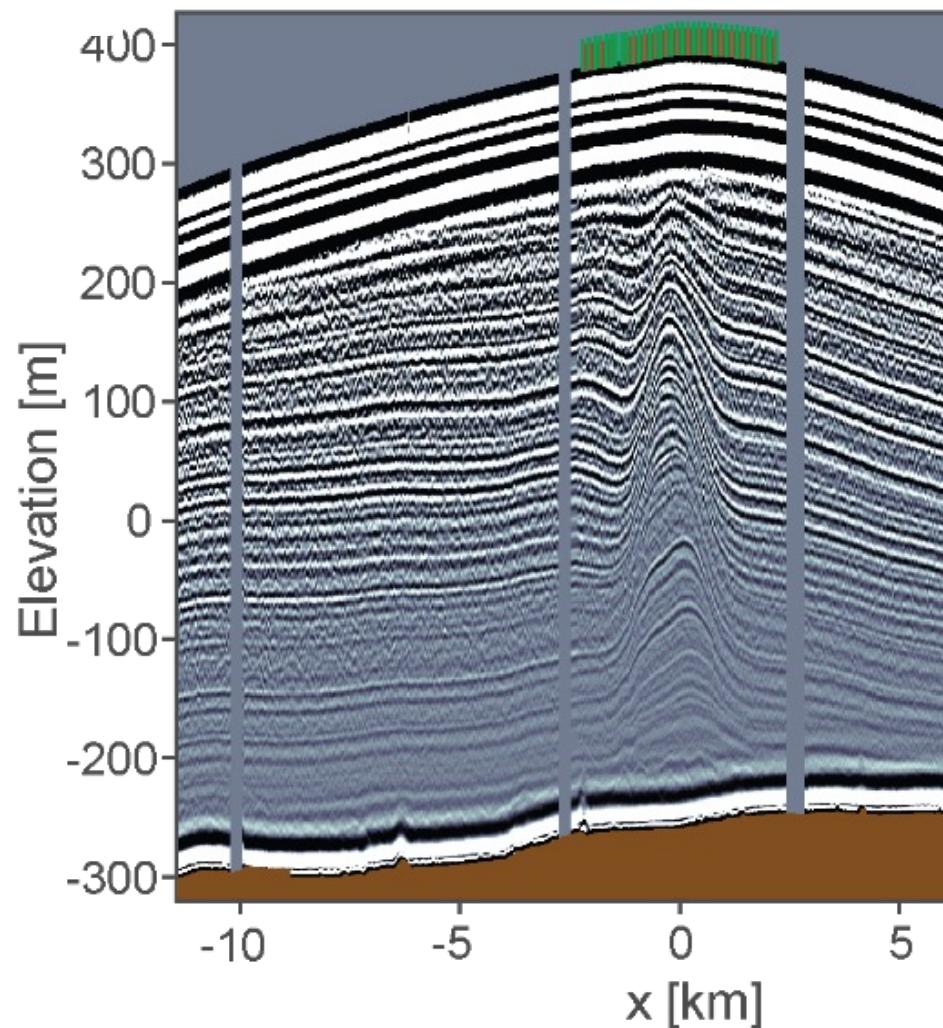
# “Soft beds”

deformable material under the ice.



Modified from Kjær et al. (2006)

# Ice divides

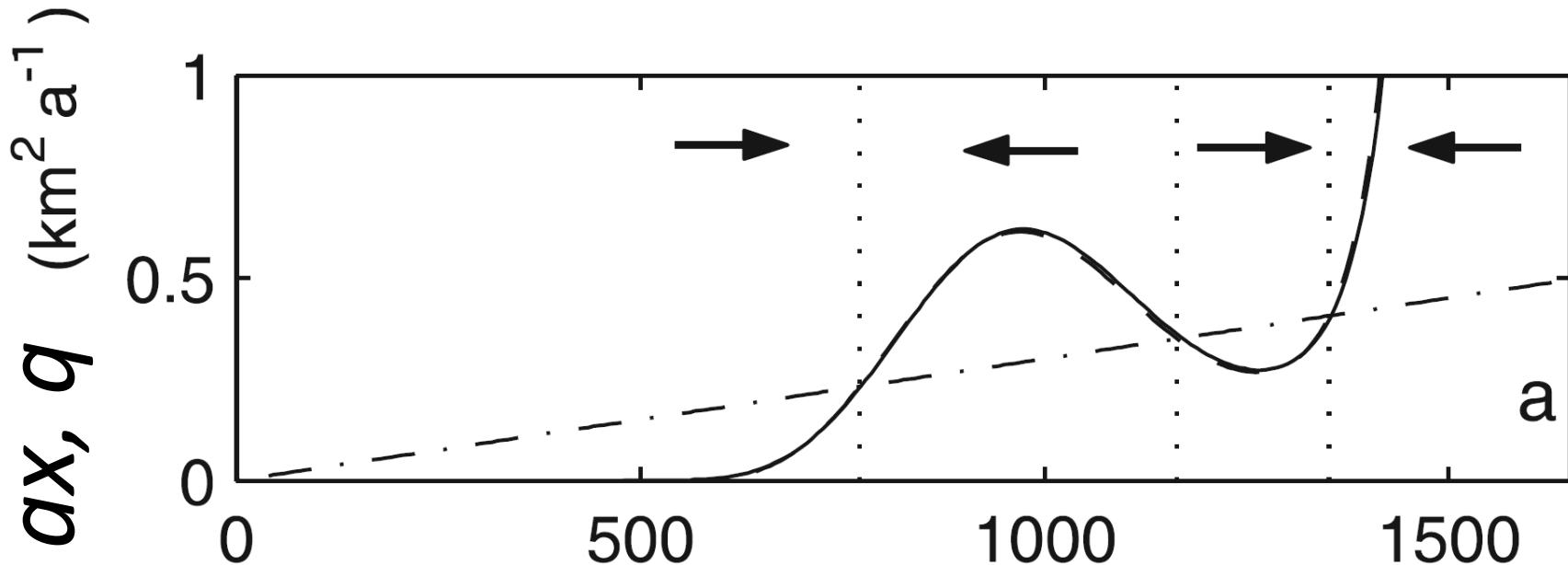
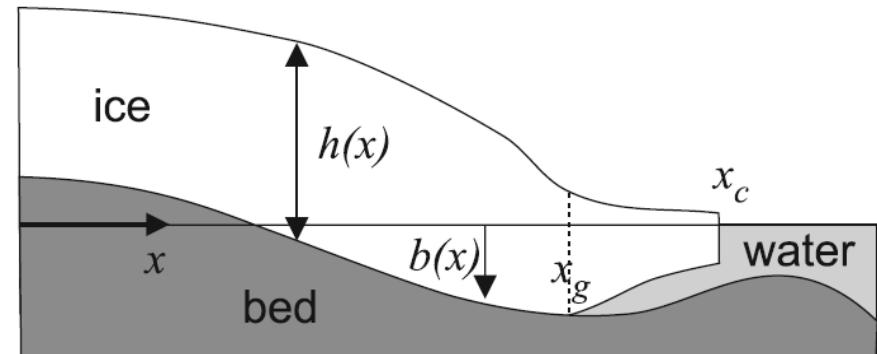


Kingslake et al. (2015)

# Schoof's (2007) theory

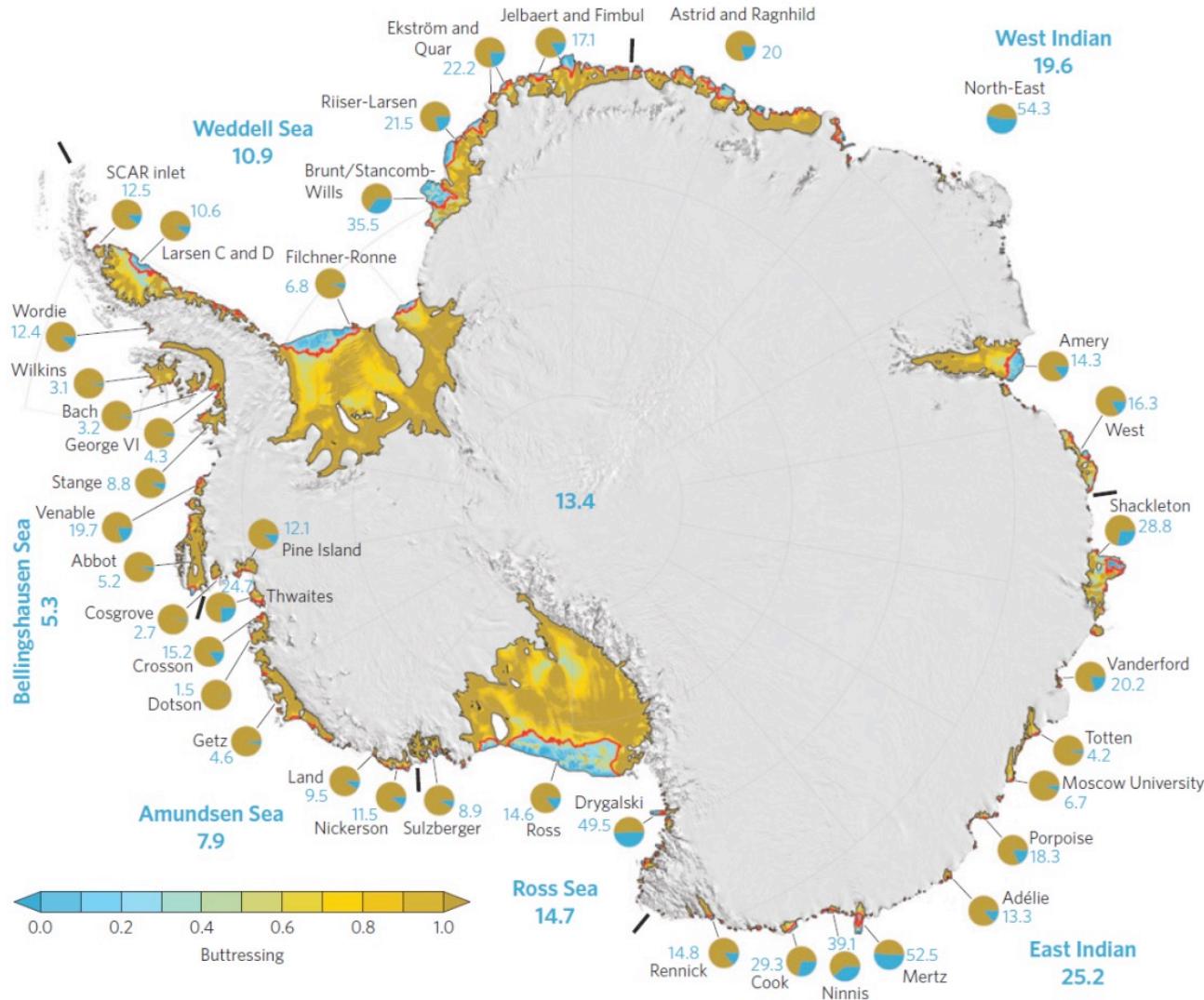
$$q(x_g) \propto h(x_g)^{4.75}$$

$$q = ax$$



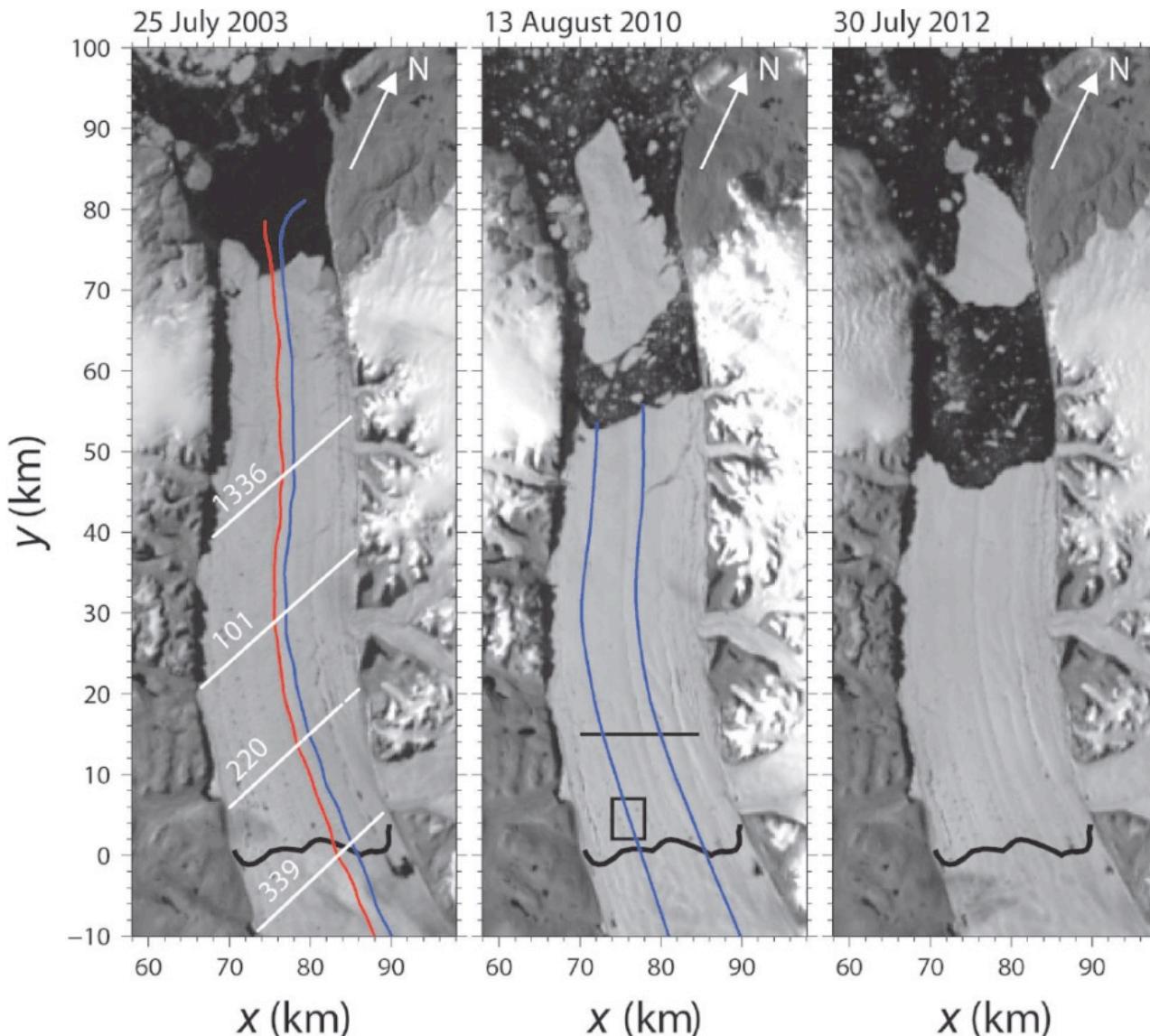
# Ice shelves

# Antarctic ice shelves



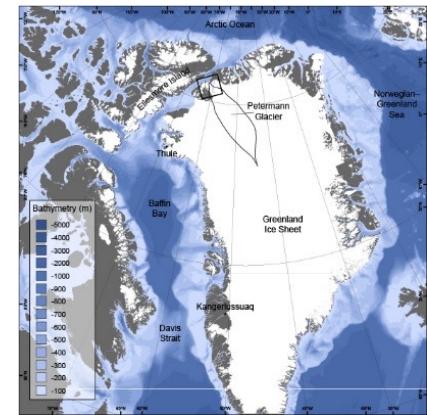
Fürst et al. (2016)

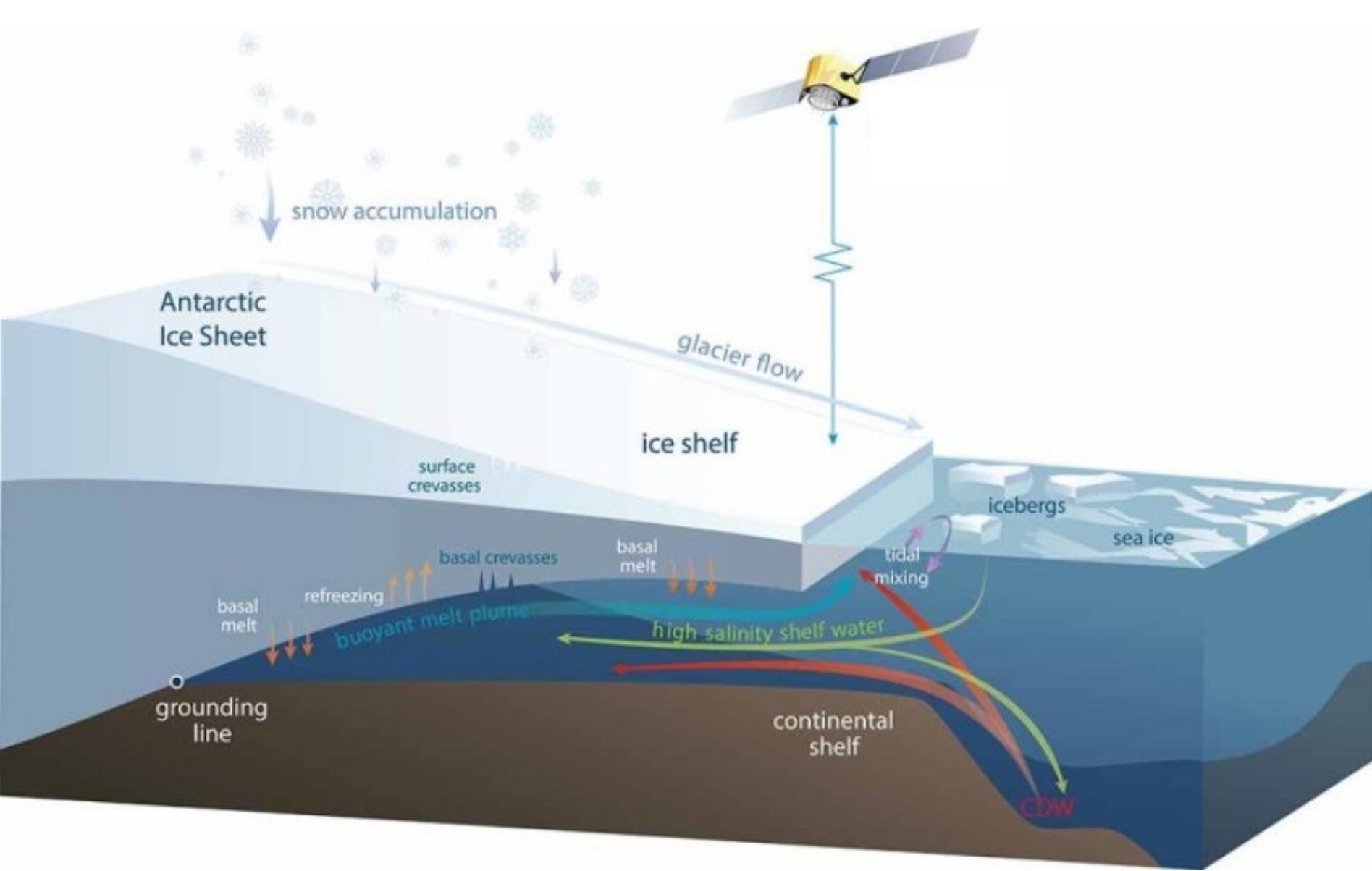
# Greenlandic ice shelves



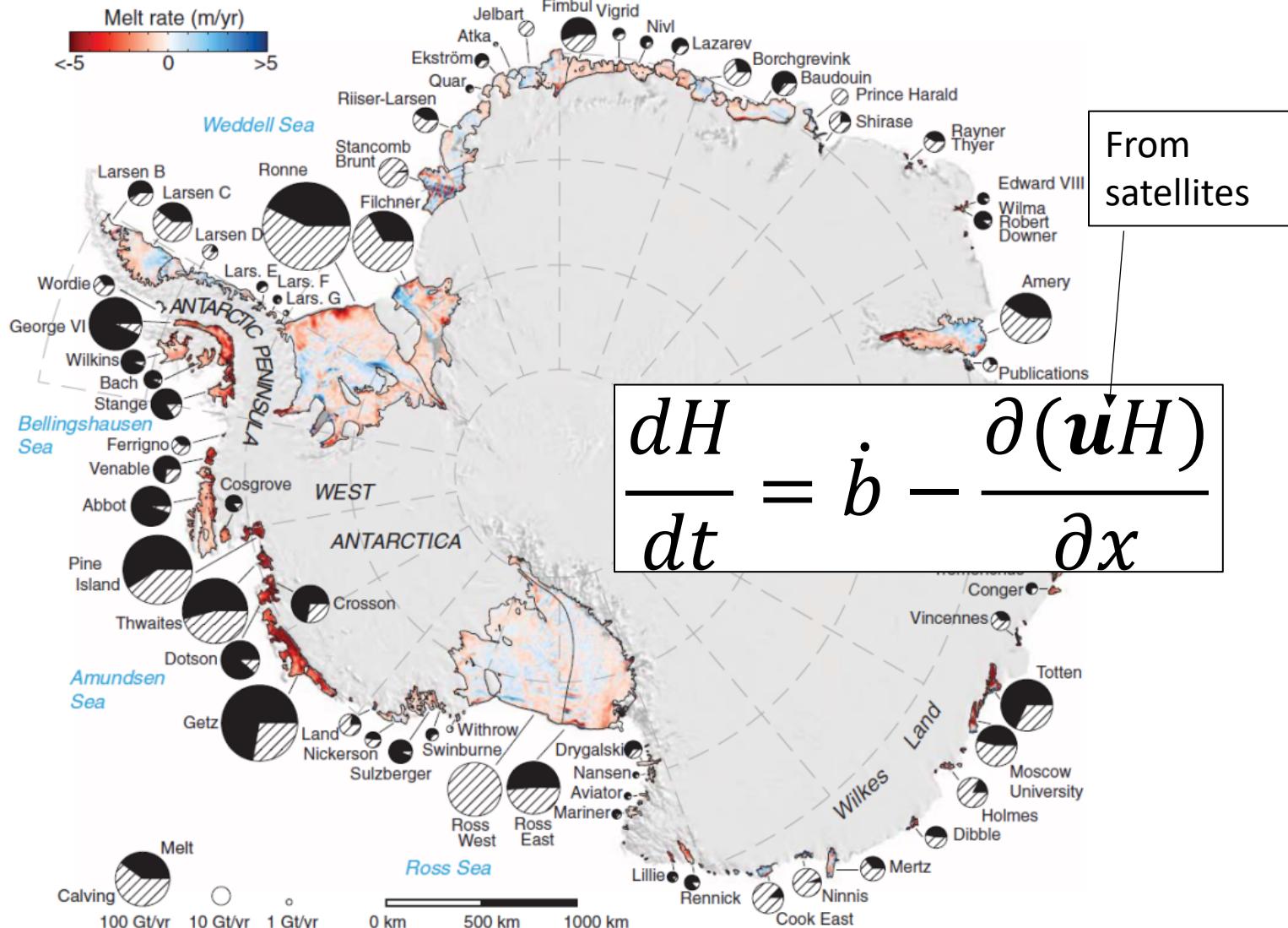
Münchow et al. (2014)

Ice Shelves + review





# Ice Shelf mass balance



Rignot et al. (2013)

$\frac{dH}{dt}$   
and  
 $H$   
from satellite  
altimetry.

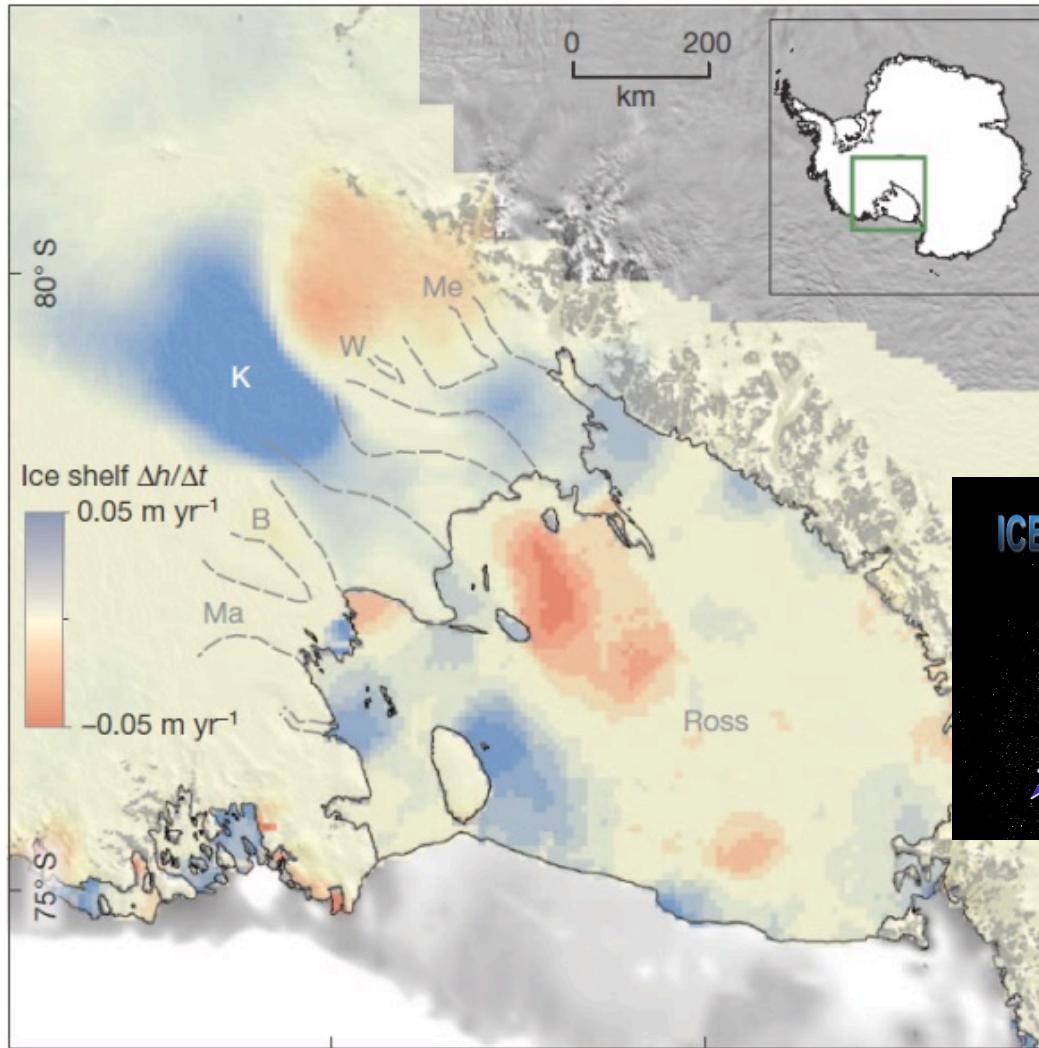
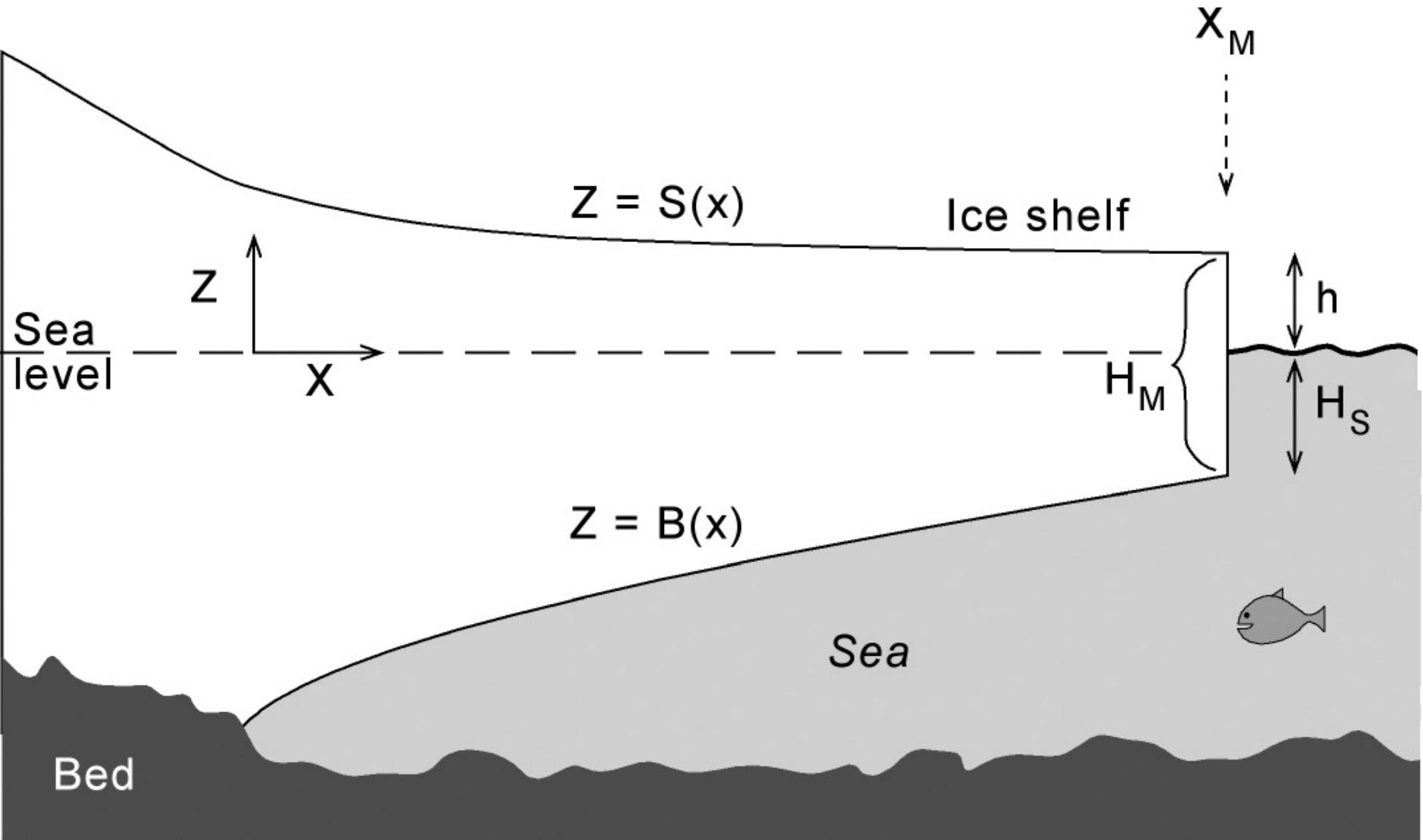


Figure 1 | Surface  $\Delta h/\Delta t$  of Ross Ice Shelf, 2003–2008.

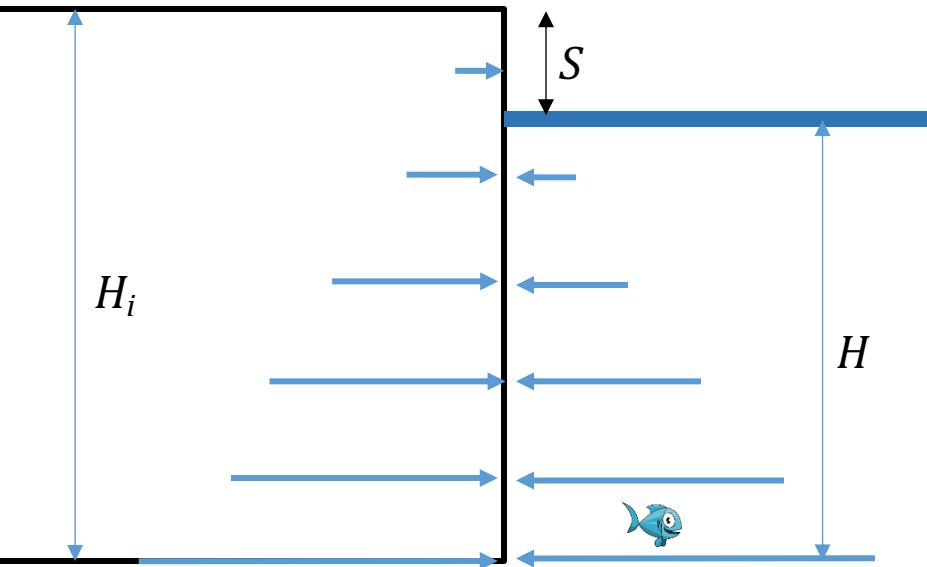
Pritchard et al. (2012)

# Ice shelf dynamics



# Ice shelf dynamics

## Driving stresses



Hydrostatic equilibrium

$$\rho_i g H_i = \rho_w g H$$

$$H = \frac{\rho_i}{\rho_w} H_i$$

**There are two sources of driving stress:**

1. Surface slope (same as for grounded ice)

$$\rho_i g H_i \frac{dS}{dx}$$

2. Stress imbalance at the front

$$= \int_0^{H_i} \rho_i g z \, dz - \int_0^H \rho_w g z \, dz$$

$$= \frac{1}{2} \rho_i g H_i^2 - \frac{1}{2} \rho_w g H^2$$

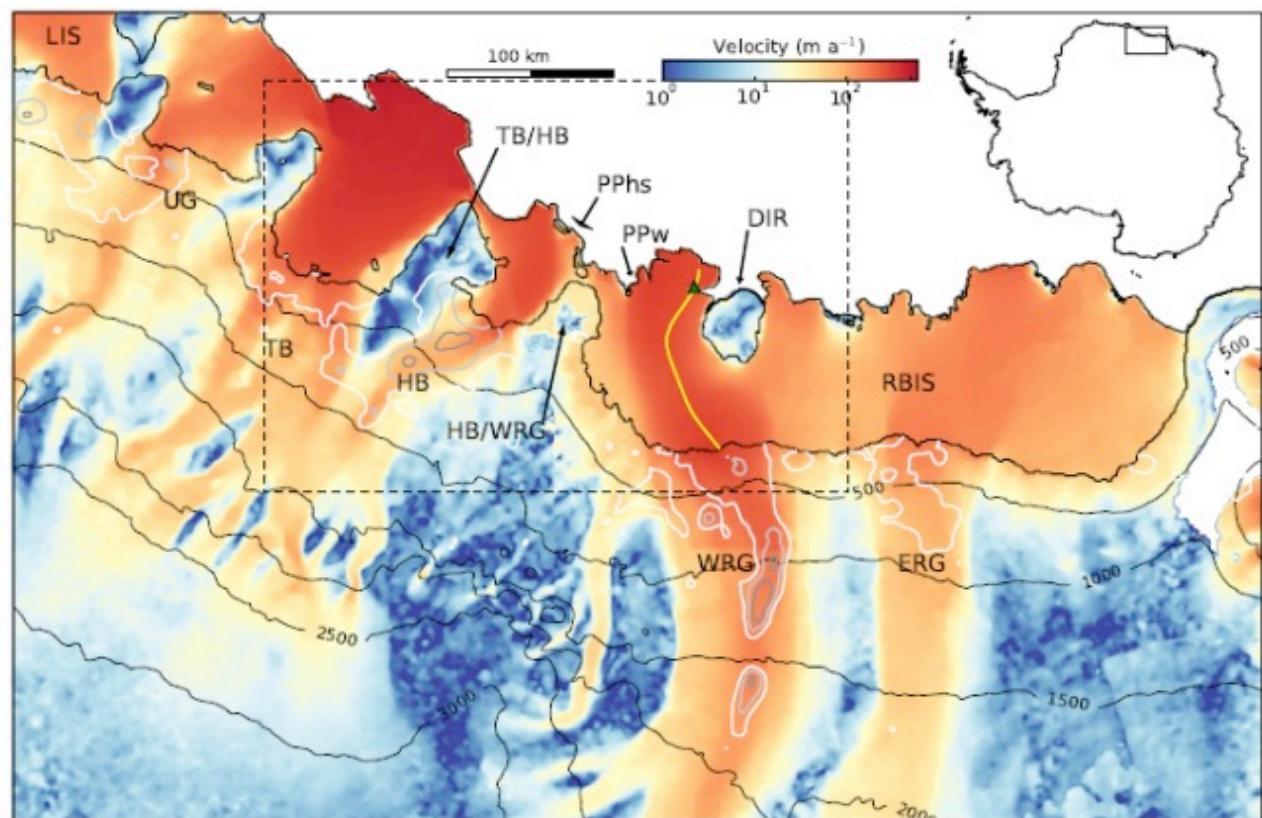
$$= \frac{1}{2} \rho_i g H_i^2 \left( 1 - \frac{\rho_i}{\rho_w} \right)$$

# Ice shelf dynamics

## Resistive stresses

There are three sources of resistive stress:

1. Extensional stresses in the ice
2. Lateral shear stresses
3. Basal shear stress where the ice temporarily grounds.



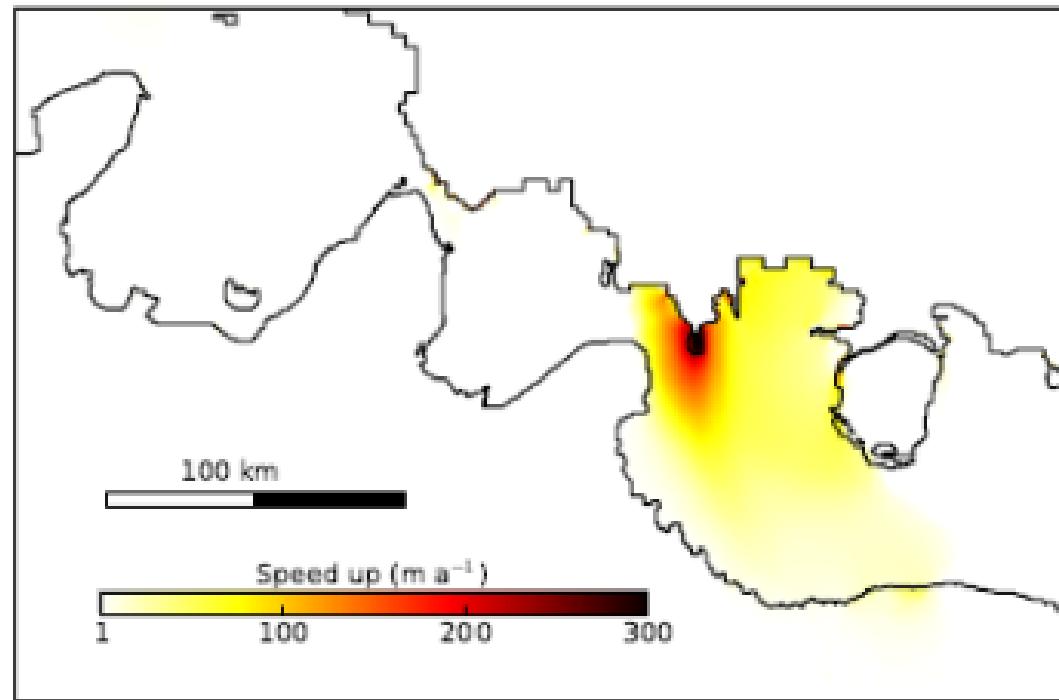
Favier et al. (2016)

# Ice shelf dynamics

## Resistive stresses

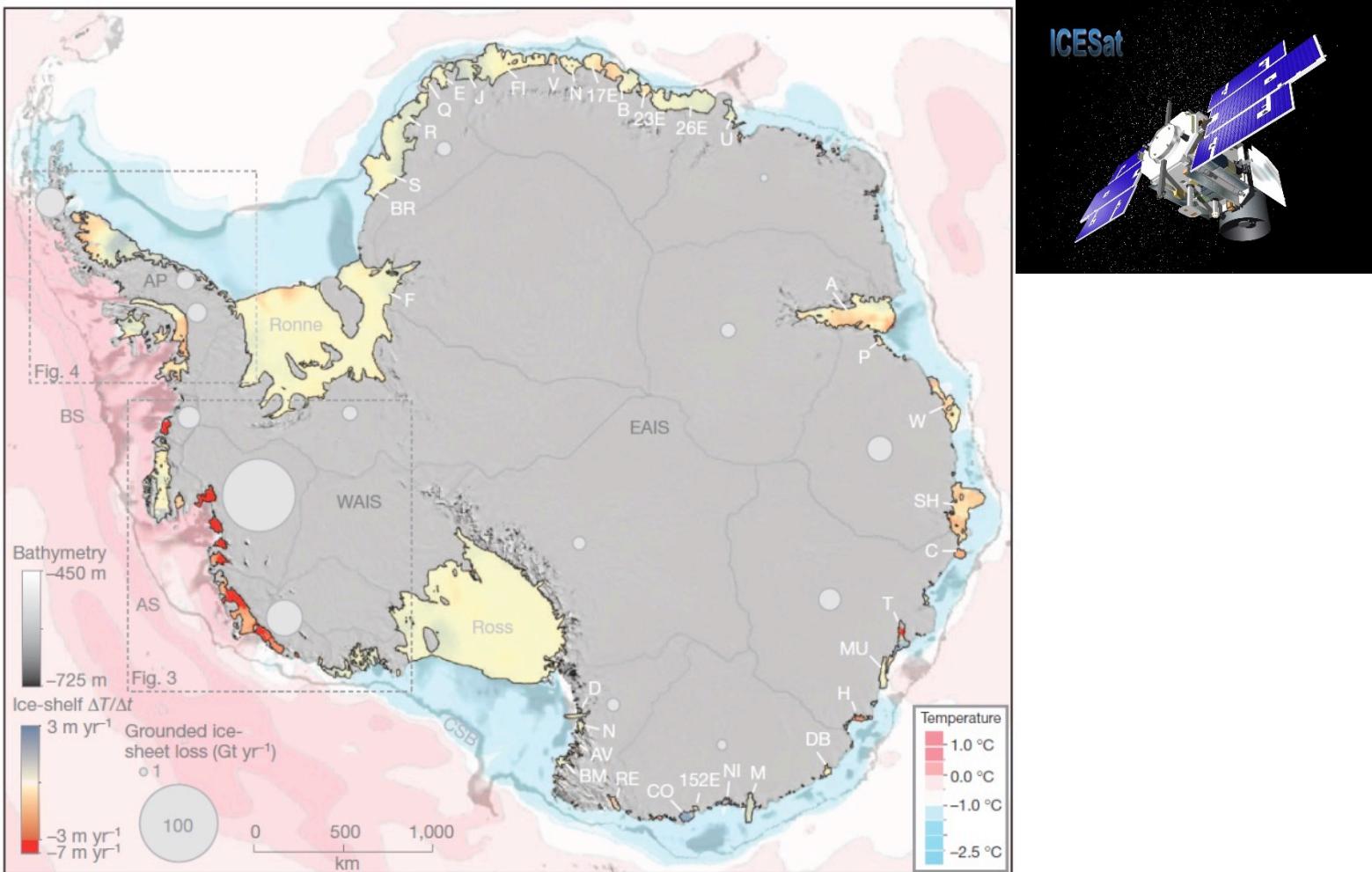
**There are three sources of resistive stress:**

1. Extensional stresses in the ice
2. Lateral shear stresses
3. Basal shear stress where the ice temporarily grounds.



Favier et al. (2016)

# Ice shelf mass loss: thinning

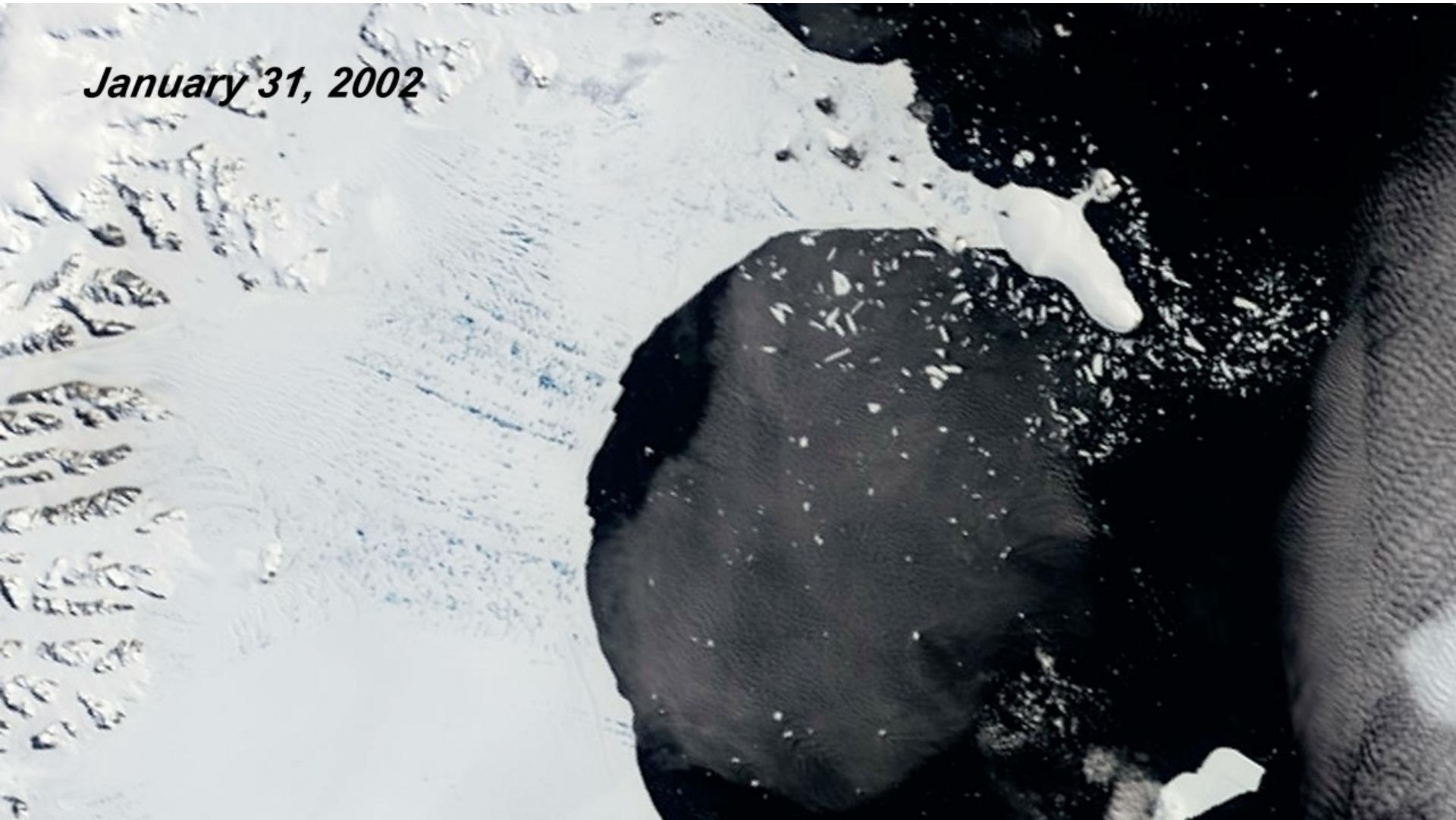


Pritchard et al. (2012)

# Ice shelf mass loss: collapse

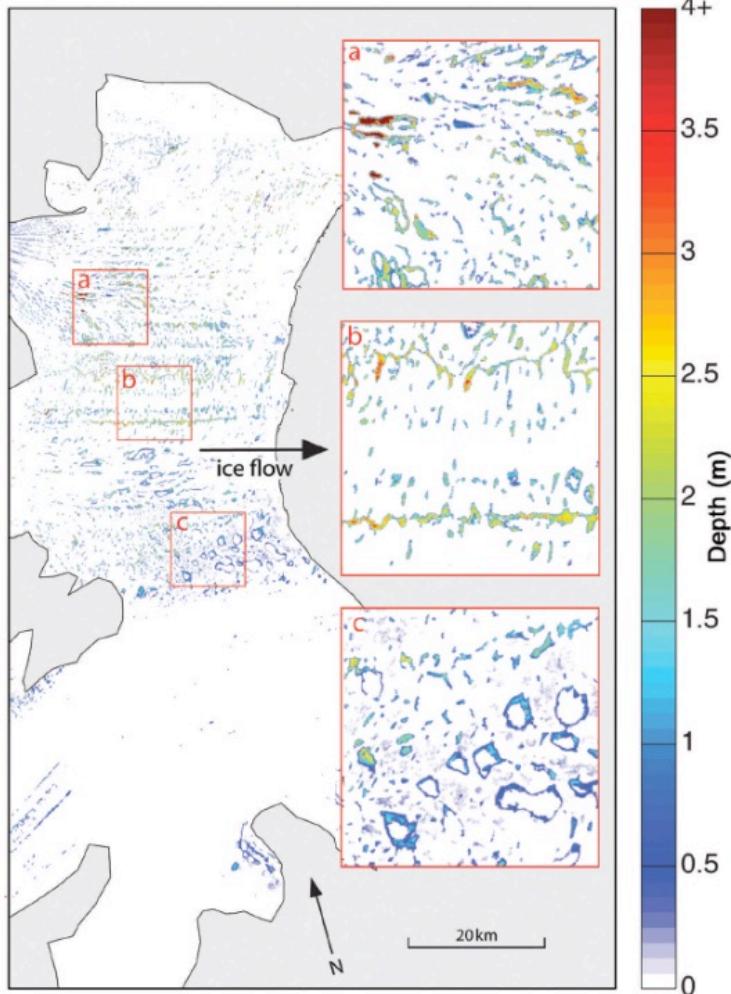
Larsen B Ice Shelf

*January 31, 2002*

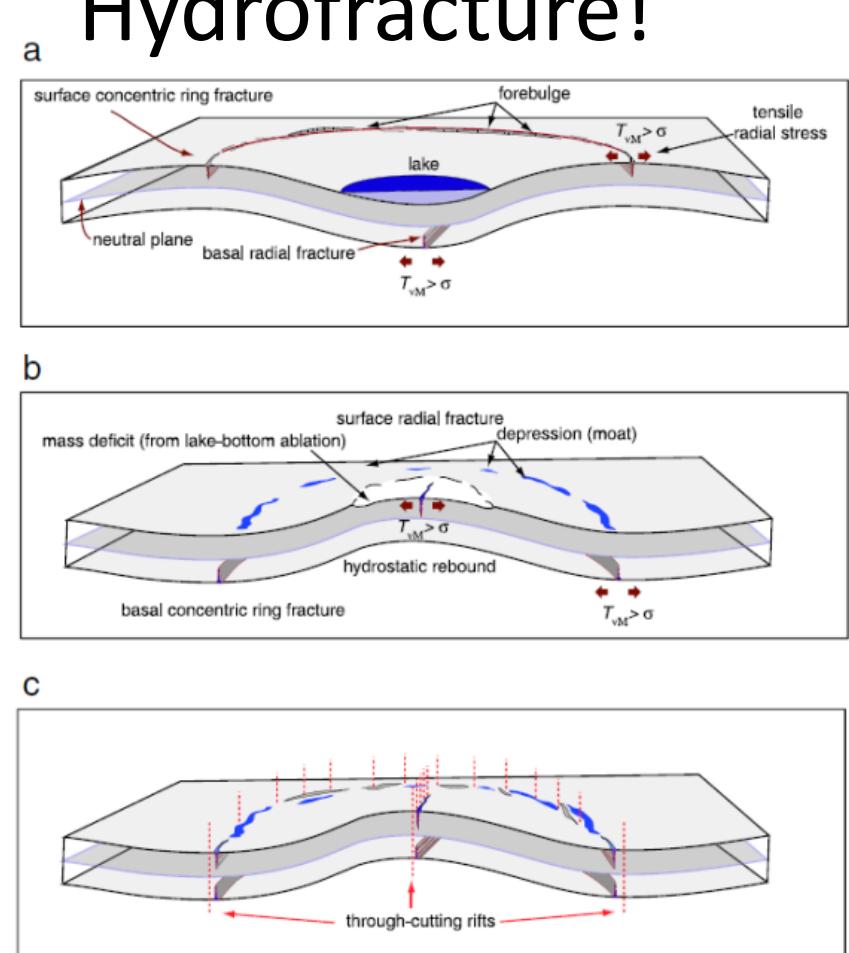


# Causes of ice shelf collapse

## Hydrofracture!

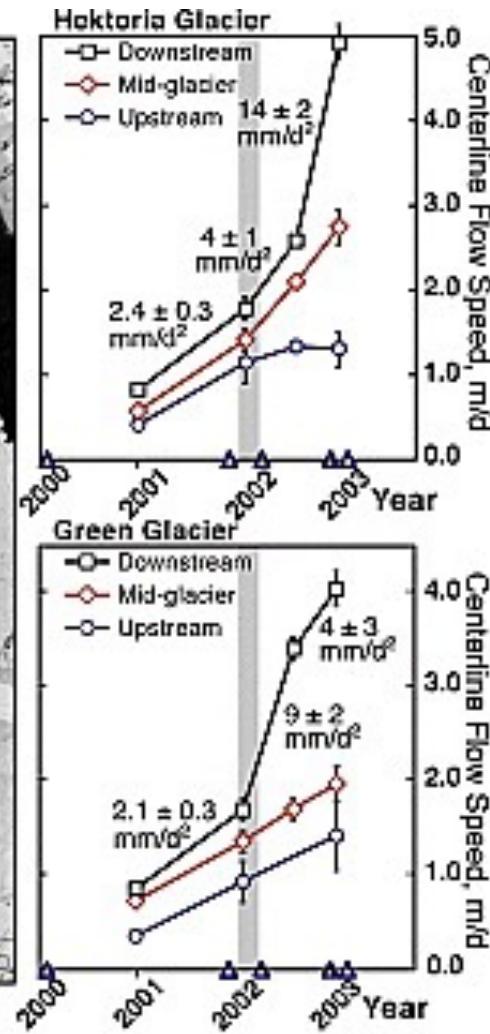
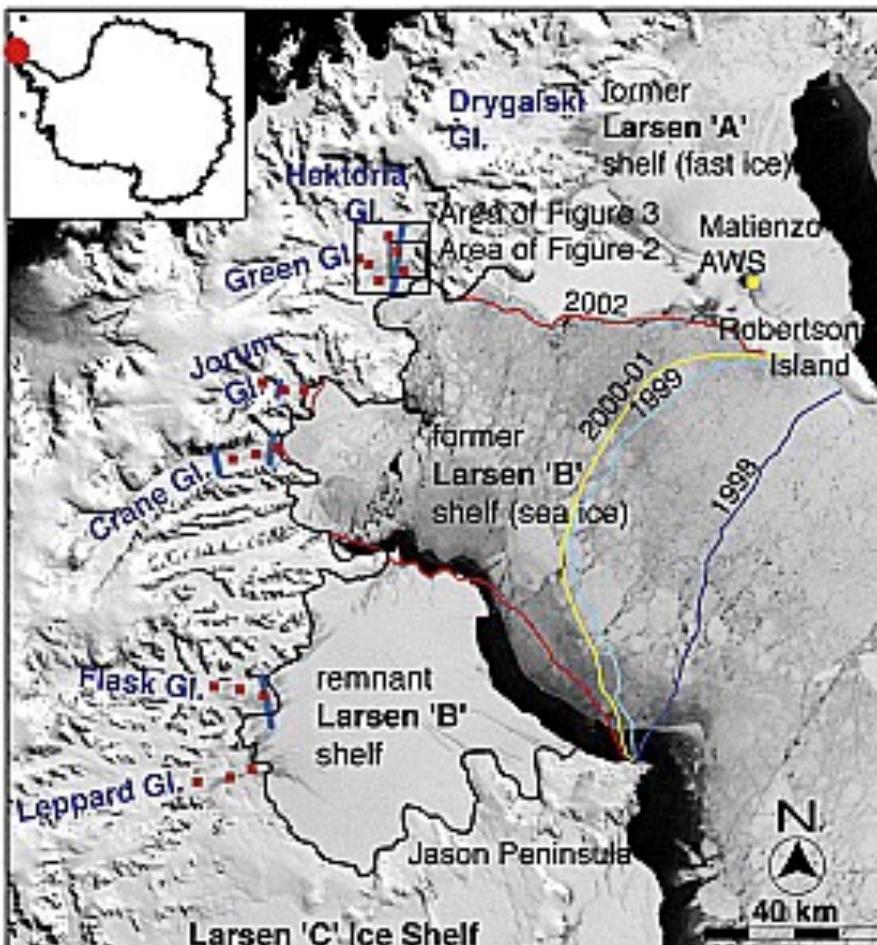
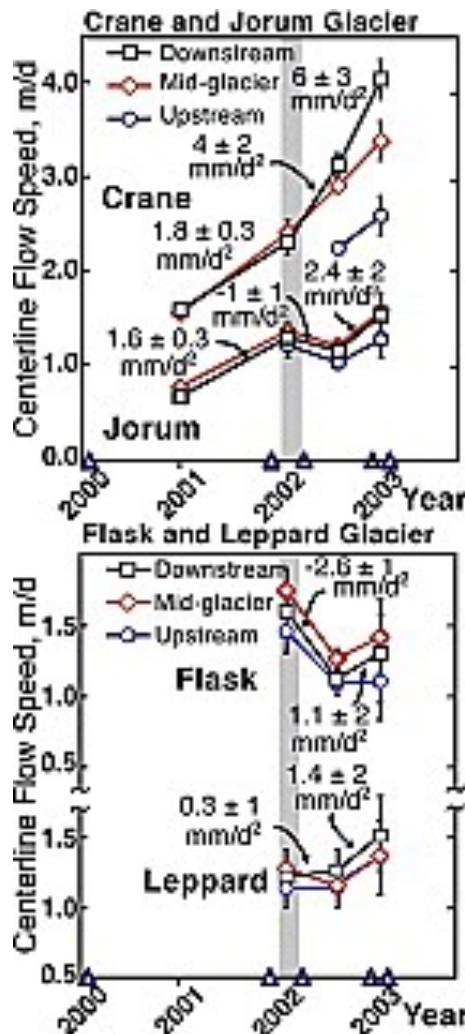


Banwell et al. (2014)



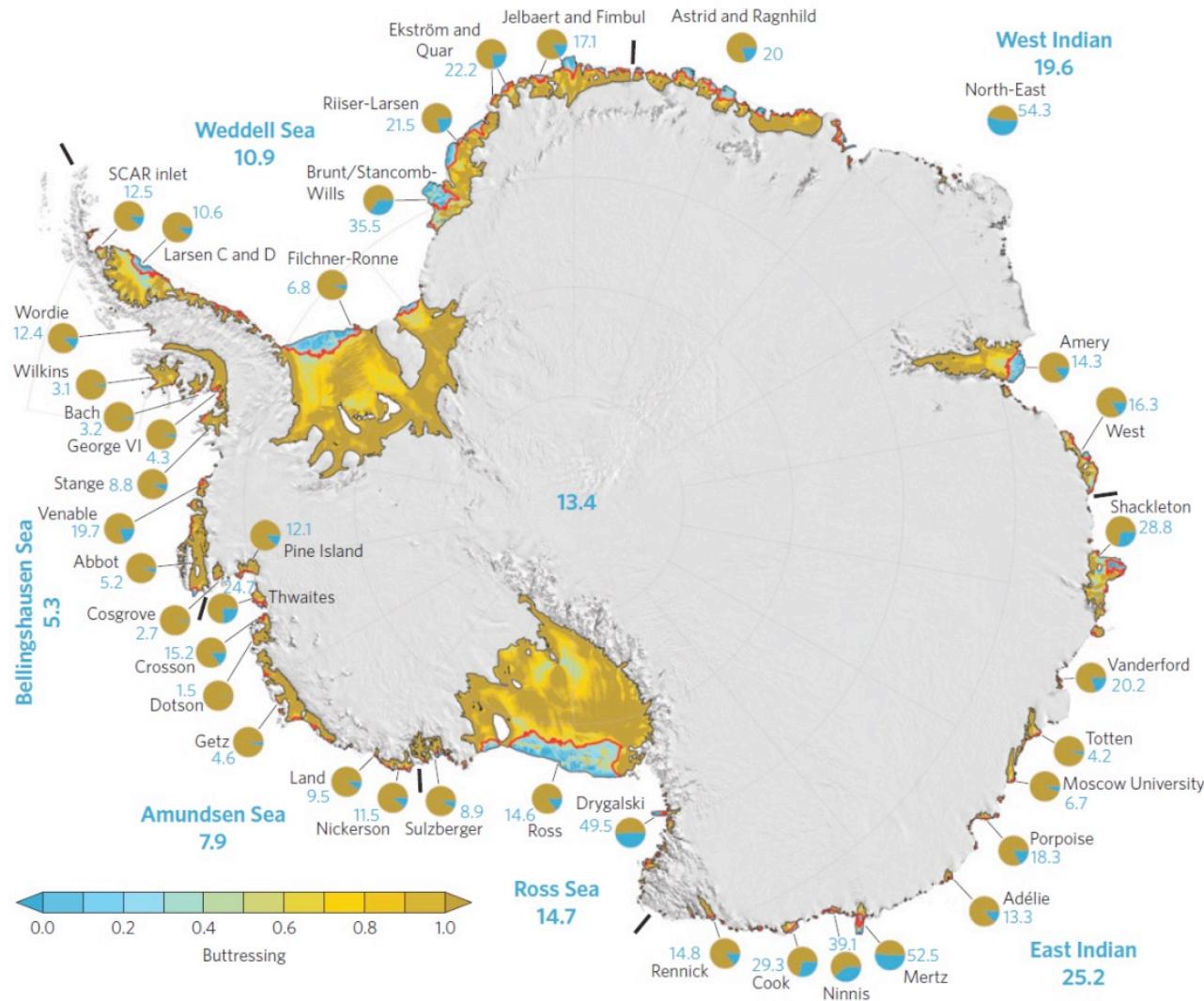
Banwell et al. (2013)

# Implications of ice shelf loss

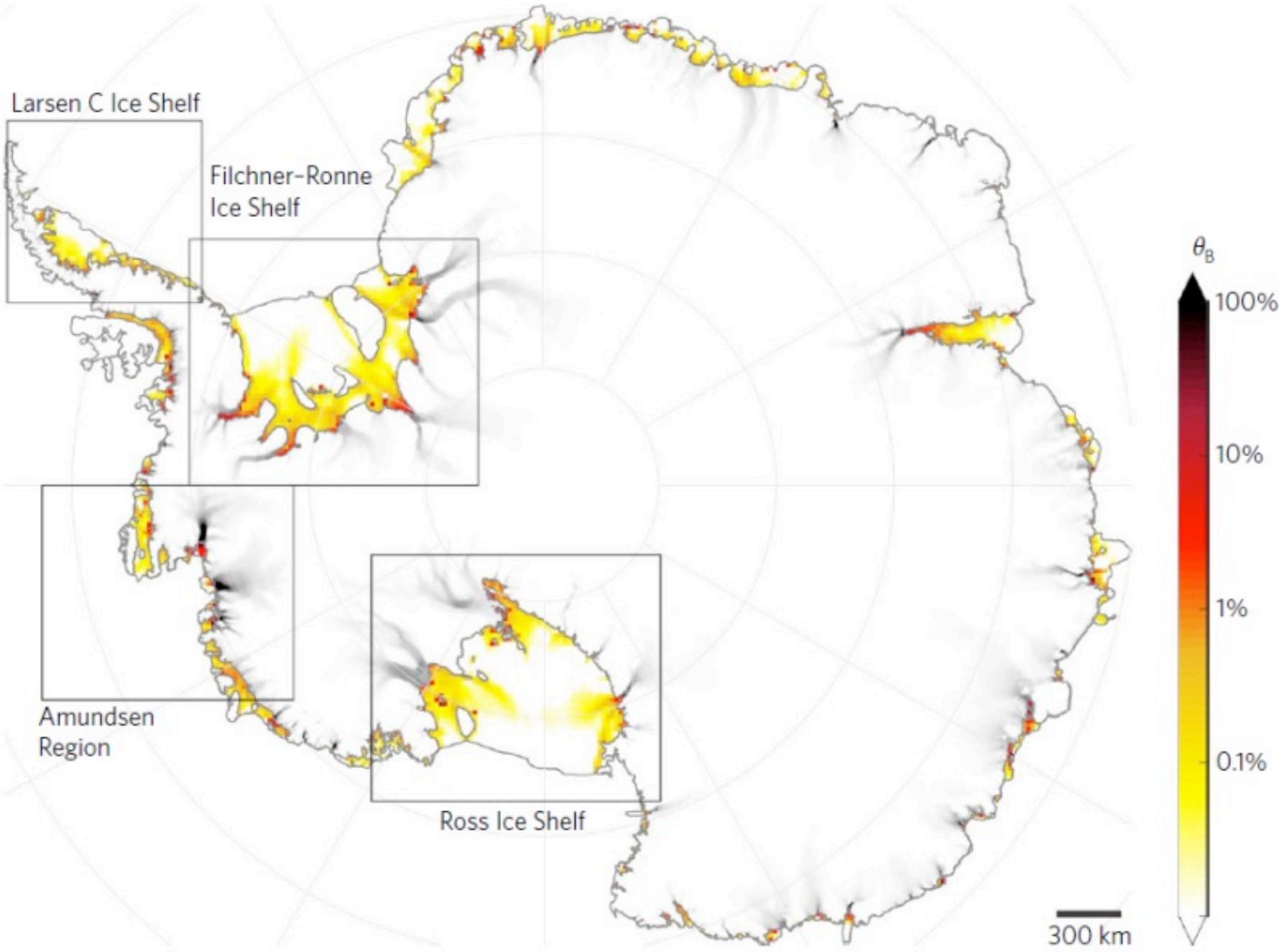


Scambos et al. (2004)

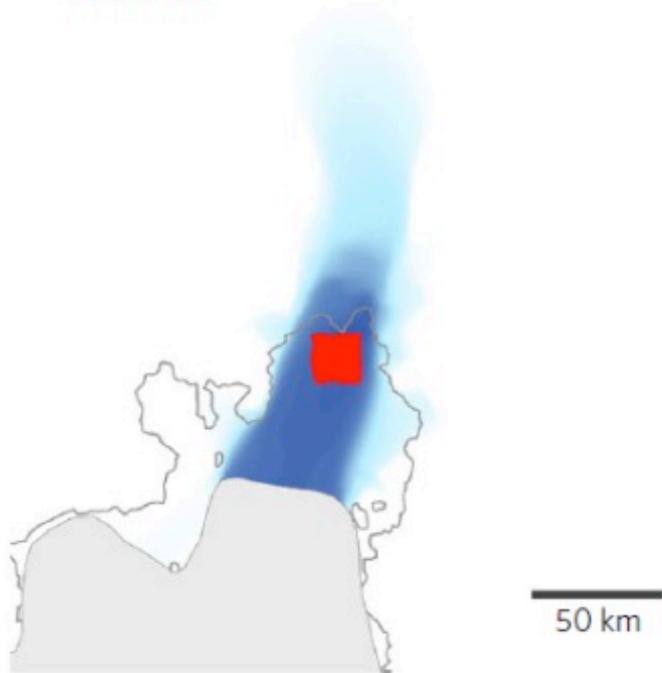
# Implications of ice shelf loss



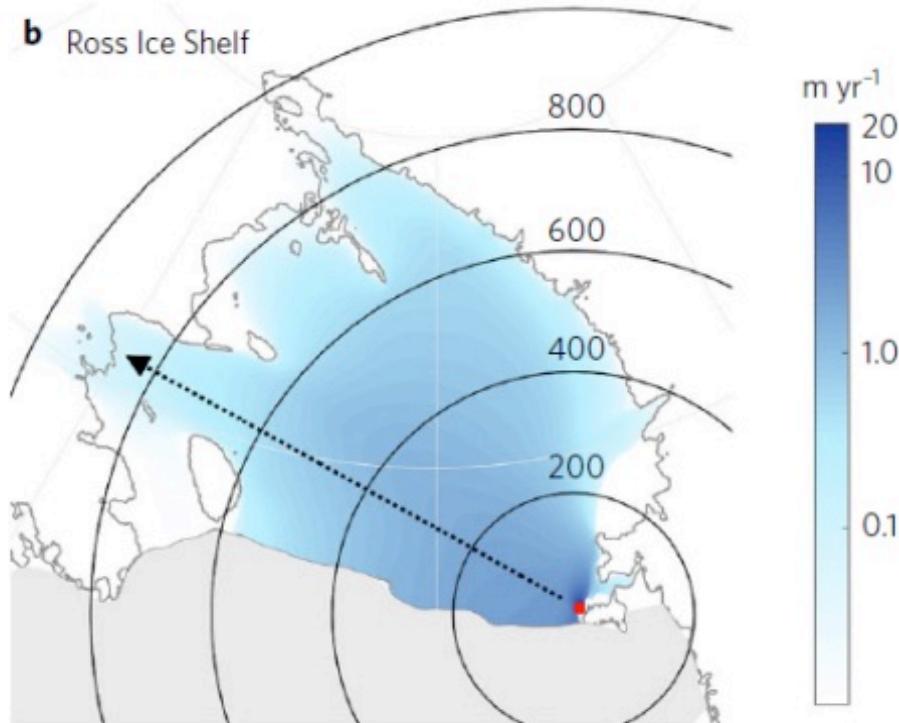
Fürst et al. (2016)

**a**

**a** Pine Island



**b** Ross Ice Shelf



**Fig. 2 | Examples of changes in speed resulting from 1m thinning.**

# Summary

- Ice shelves are common in Antarctica, rarer in Greenland.
- Satellite-based observations suggests Antarctic ice shelves lose half their mass via basal melting, half from calving.
- Driving forces are due to (1) surface slope and (2) stress imbalance at the front (both would be zero if  $\rho_i = \rho_w$ ).
- Resistive forces are from shear against embayment sides and extensional stresses within ice shelves.
- When ice shelves lose mass or collapse completely, ice upstream accelerates.
- Ice shelves are thinning due to increased ocean melting.
- Collapse appears to be related to surface meltwater through processes such as hydrofracturing.

# Next half of semester:

- Heat flow
- Hydrology
- Isochrones and age structure
- Start thinking about your projects

# References

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