Glaciology
EESCGU4220

Lecture 10:
Heat flow in glaciers

Why do we care?

Glacier thermal structure

General heat equation derivation

Two examples:
1. Heat flow near the surface

2. Full thickness temperature profiles

Why do we care about heat flow in ice?

--> Because it controls ice-sheet thermal structure.

But why do we care about that?

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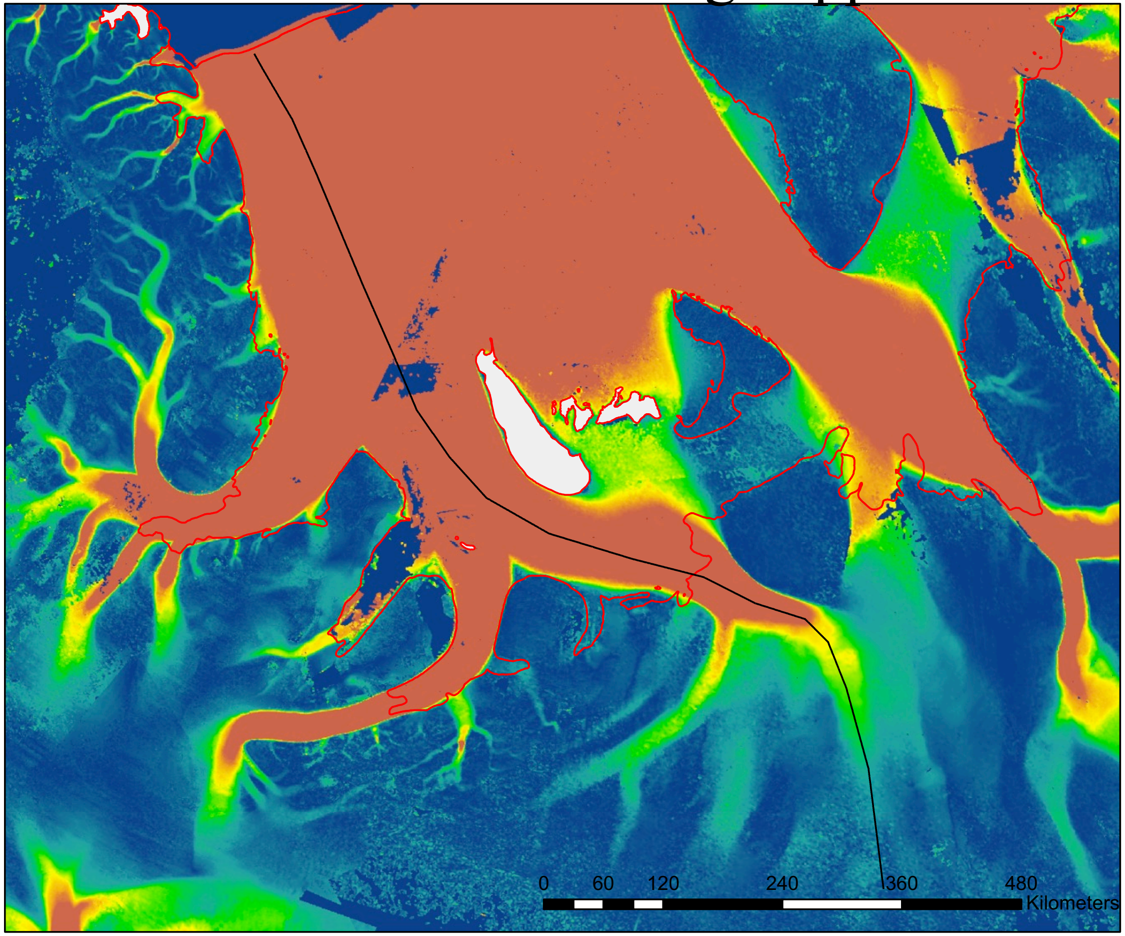
But why do we care about that?

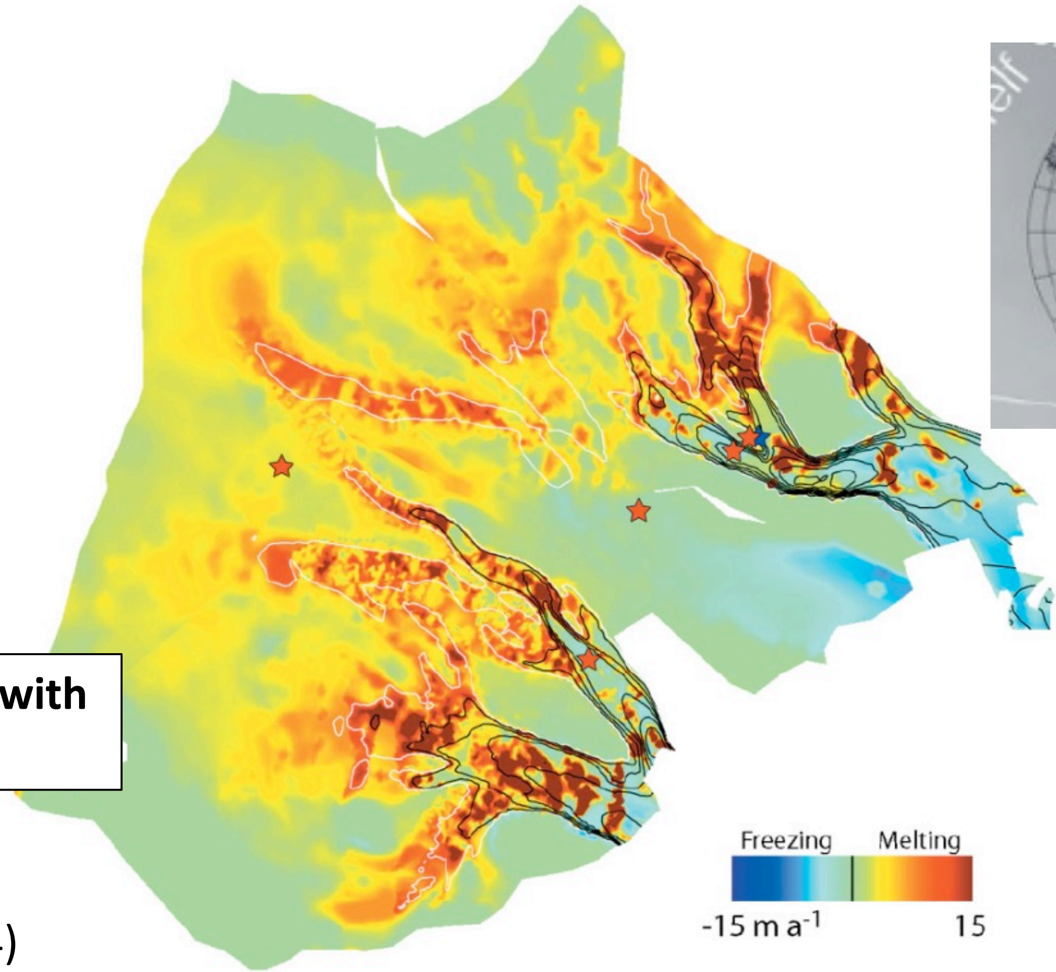
It controls where sliding happens

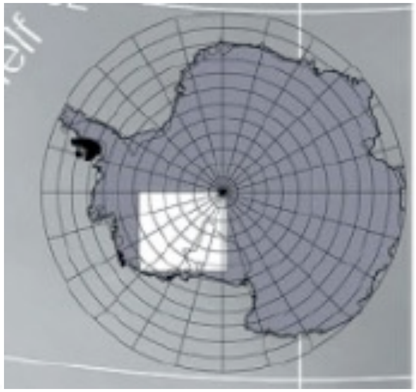


http://bslmagb.nerc-bas.ac.uk/mosaicviewer/

Why do we care about heat flow in ice?

It controls basal melt and re-freezing rates





What is wrong with this plot? (!!!)

Joughin et al. (2004)

Fig. 8. Estimated basal melt/freeze rates. Flow-speed contours at 100 m a^{-1} intervals (black) and at 50 m a^{-1} (white) show locations of the ice streams. Stars show borehole locations discussed in the text.

Why do we care about heat flow in ice?

It controls ice viscosity

Water flows into a crevasse and warms up the ice, leading to faster flow

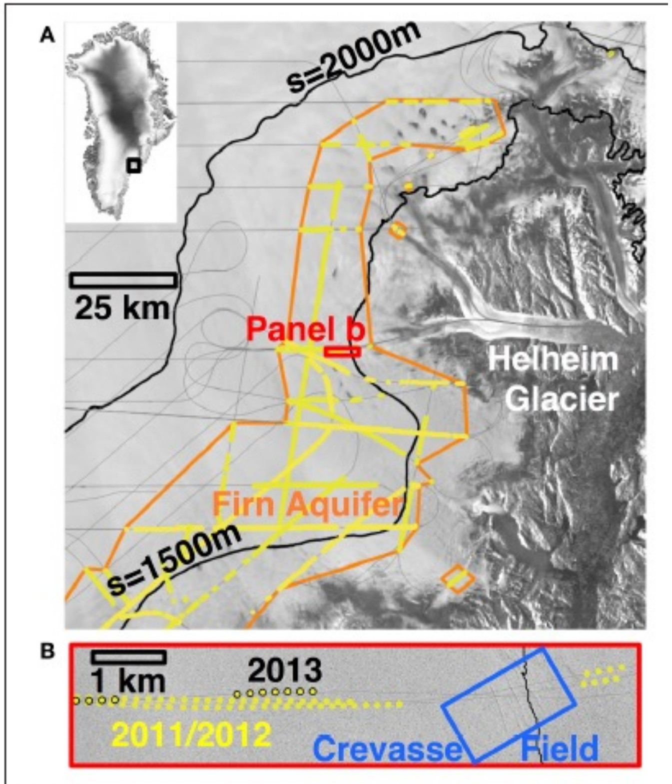
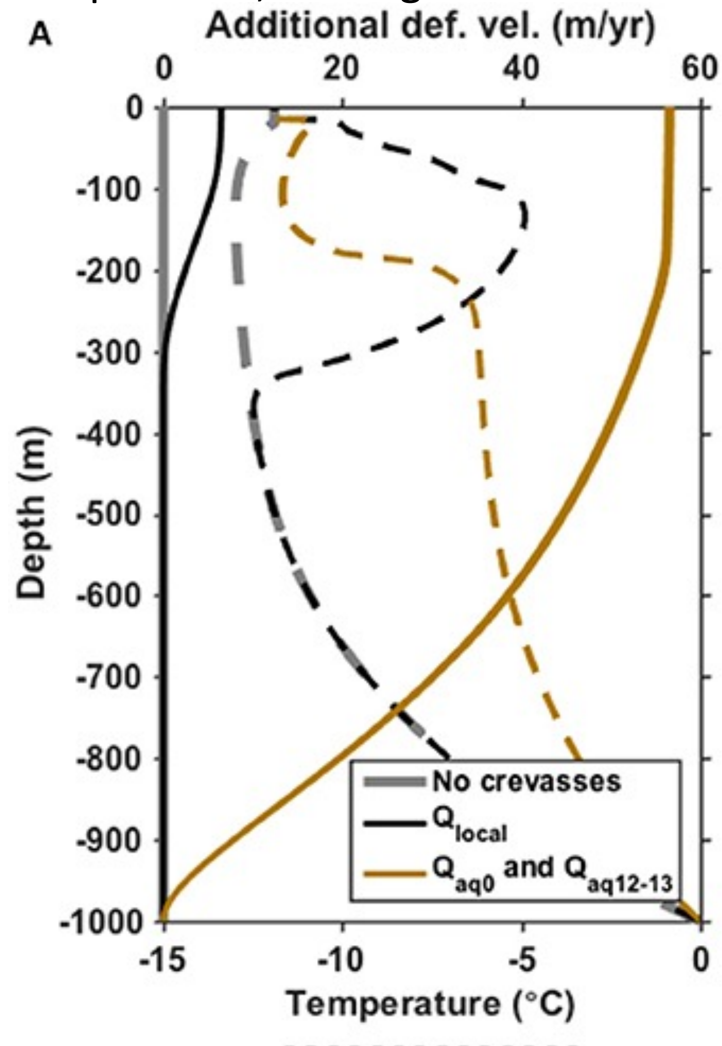


FIGURE 1 | The region in Southeast Greenland where the firn aquifer overlies the upper reaches of Helheim Glacier. (A) Operation IceBridge



Poinar et al. (2017)

Glacier thermal structure

- Temperate/warm

  at the melting-point everywhere
- Polar/cold

  below the melting point everywhere
- Polythermal

  A mixture

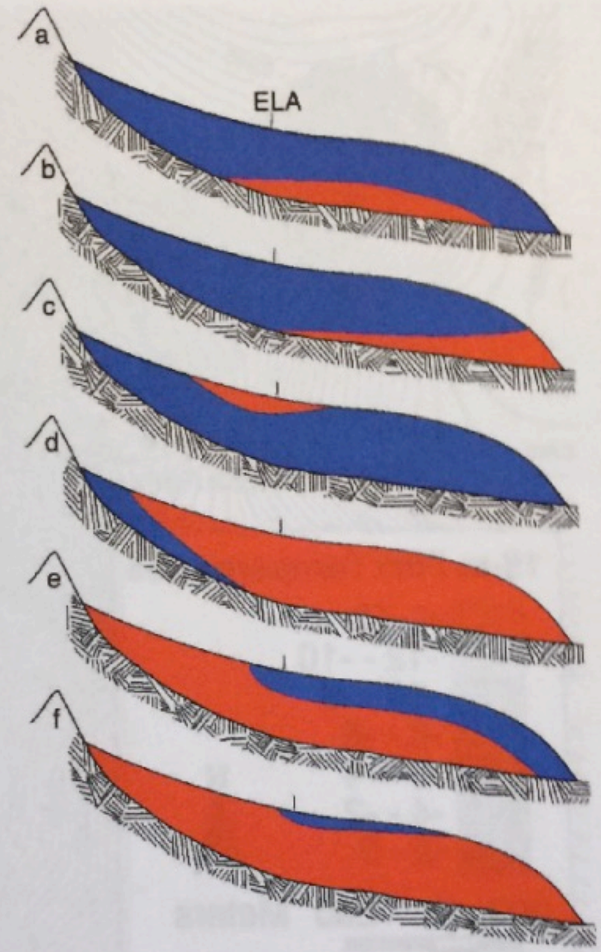


Figure 2.6 Polythermal glacier types, with warm ice shown in red and cold ice in blue (Modified from Pettersson, 2004).

Benn and Evans (2010)

General heat flow equation.

Plan for derivation:

1. Setup the problem
2. dT/dt term
3. Diffusion term
4. Advection term
5. Bring it all together

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5. Bring it all together

T is temperature
α is thermal diffusivity
x, y, z are dimensions
u, v, w are components of velocity in x, y and z directions

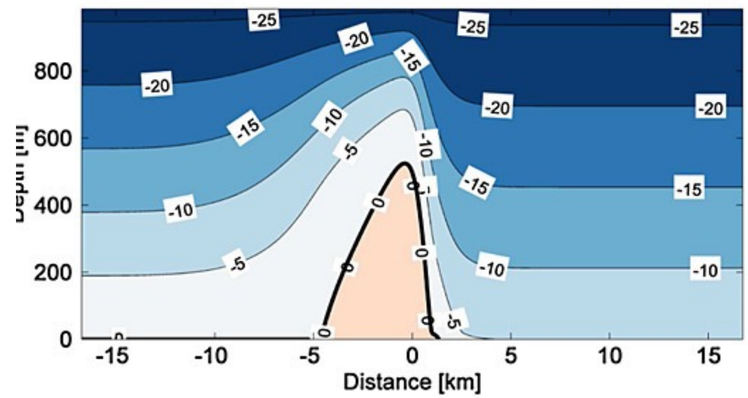
$$\boxed{\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right)}$$

What about sources of heat within the ice? (so-called source terms).

- Ice deformation (including firn compaction).

$$\epsilon_{xx}\sigma_{xx}, \epsilon_{yy}\sigma_{yy}$$

- Refreezing of meltwater



Suckale et al. (2014)

$$\boxed{\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + S}$$

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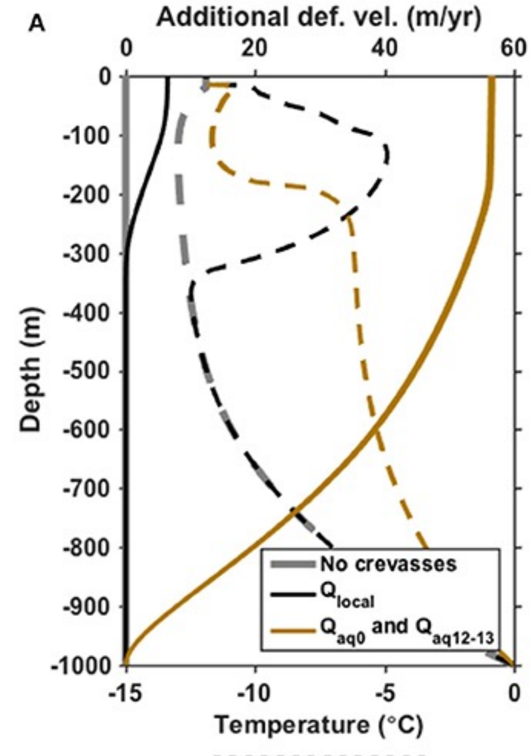

Poinar et al. (2017)

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A simple example:

it is time dependent, but ignores advection.

Example 1

Temperature variations near the surface

$$\boxed{\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + S}$$

Example 1

Temperature variations near the surface

Let's ignore advection

$$\boxed{\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \cancel{- \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right)} + S}$$

Example 1

Temperature variations near the surface

Let's ignore advection and source terms

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Temperature variations near the surface

Let's ignore advection and source terms

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and *x* and *y* directions

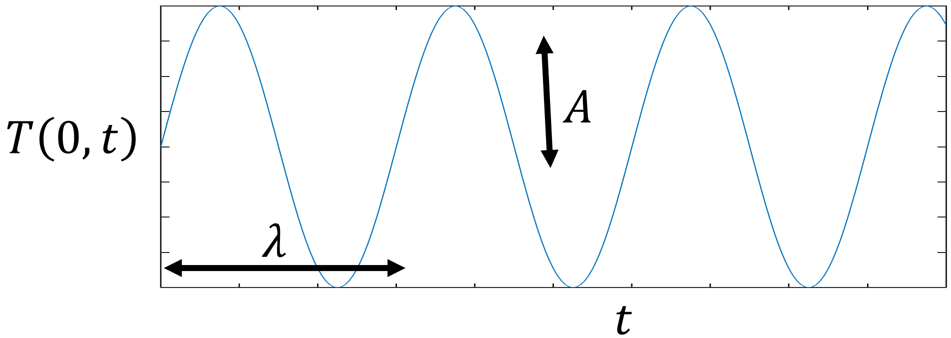
$$\boxed{\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial z^2}}$$

Example 1

Assume surface energy balance produces a sinusoidally-varying surface temperature, *T*(0,*t*)

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}}$$

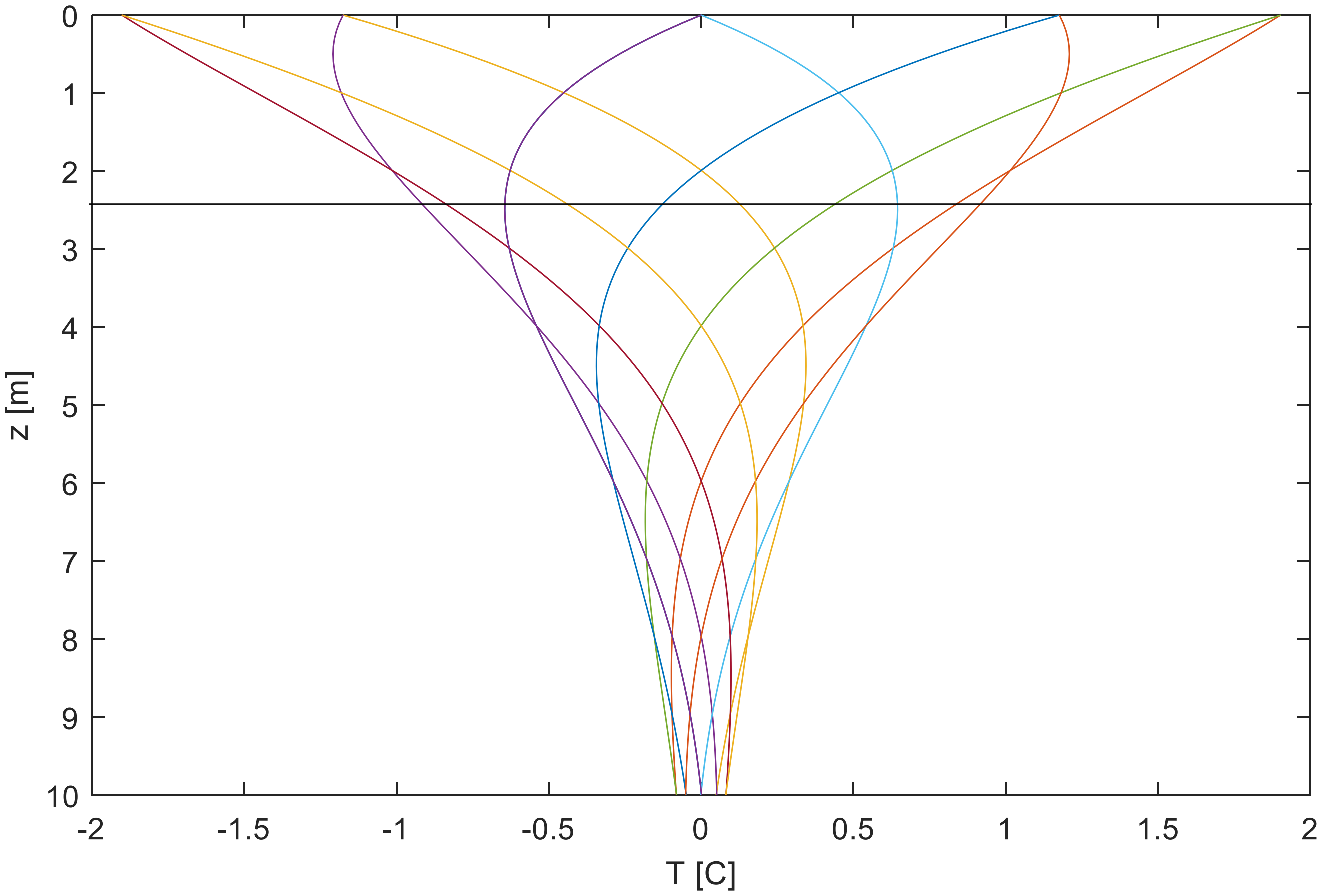
$$\boxed{T(0,t) = \mathrm{A}\sin\left(\frac{2\pi}{\lambda}t\right)}$$



$$\boxed{T(z,t) = \mathrm{A}e^{-z\sqrt{\frac{\pi}{\lambda\alpha}}}\sin\left(\frac{2\pi}{\lambda}t - z\sqrt{\frac{\pi}{\lambda\alpha}}\right)}$$

Example 1

$$T(z,t) = Ae^{-z\sqrt{\frac{\pi}{\lambda\alpha}}} \sin\left(\frac{2\pi}{\lambda}t - z\sqrt{\frac{\pi}{\lambda\alpha}}\right)$$

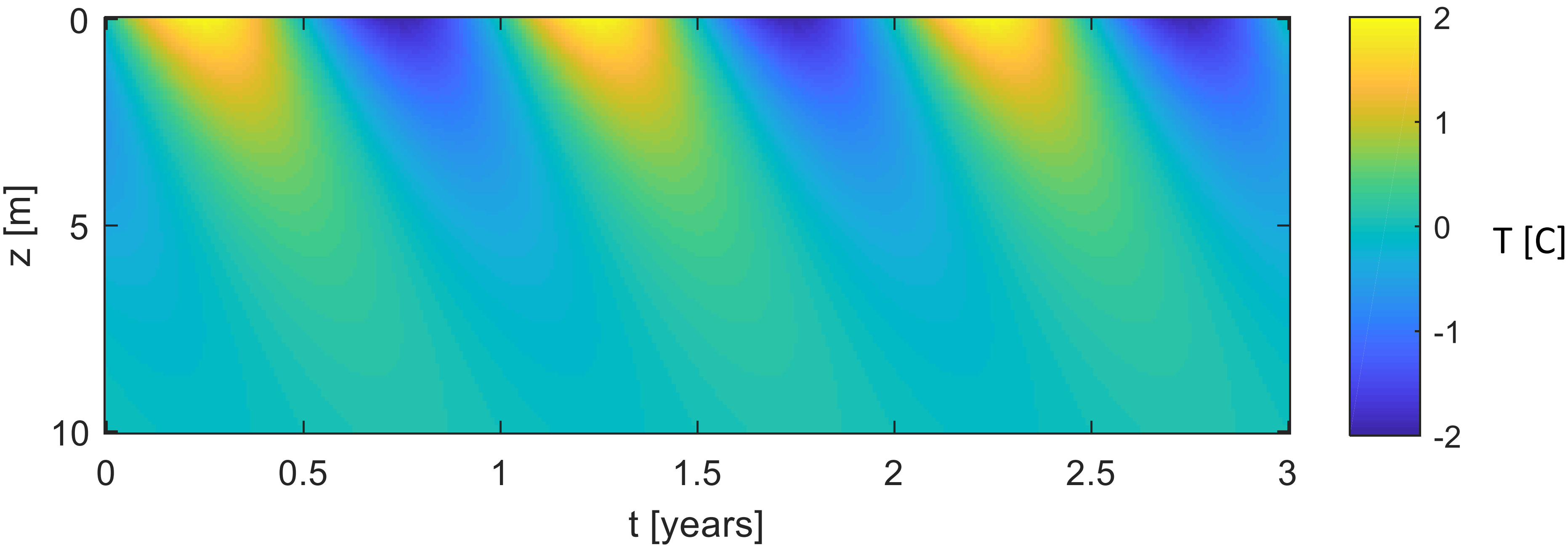


$$\sqrt{\frac{\pi}{\lambda\alpha}} = 0.3 \text{ m}^{-1}$$

Amplitude of annual variations is ~0.05A at 10 m.

Example 1

$$T(z,t) = \mathrm{A}e^{-z\sqrt{\frac{\pi}{\lambda\alpha}}}\sin\left(\frac{2\pi}{\lambda}t - z\sqrt{\frac{\pi}{\lambda\alpha}}\right)$$



Summary

- Temperature controls where sliding happens, ice deformation, and basal mass balance.
- Heat moves around through advection and diffusion.
- Internal heat sources include strain heating, refreezing.
- Boundary conditions set by environment.
- Seasonal waves penetrate 10-20 m.

Steady-state T(z) at an ice divide

- Steady-state

$$\boxed{\cancel{\frac{\partial T}{\partial t}} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + S}$$

And again ignore source terms

$$\boxed{0 = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + \cancel{S}}$$

and *x* and *y* directions

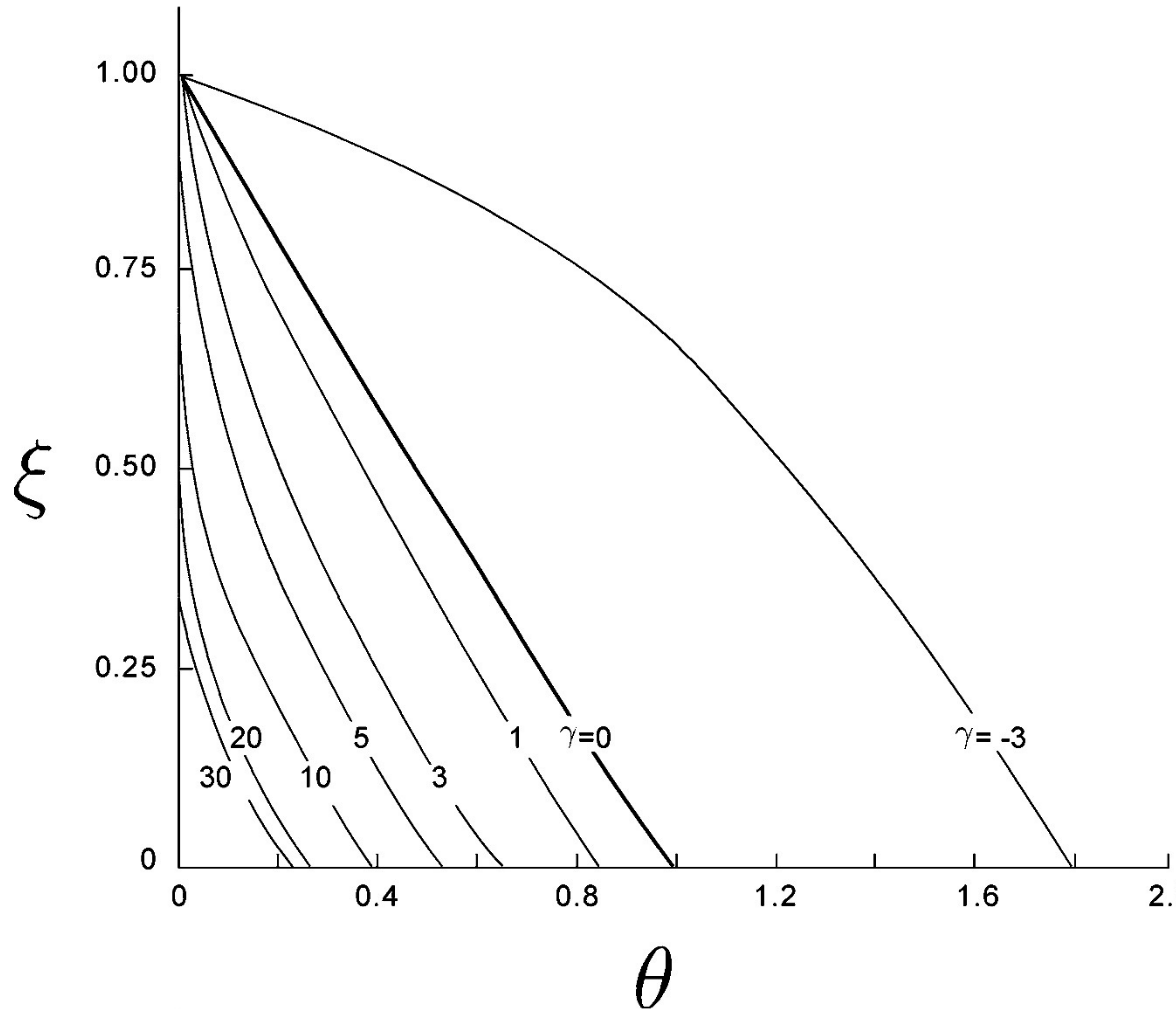
$$\boxed{w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2}}$$

Example 2

Steady-state T(z) at an ice divide

$$\boxed{w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2}}$$

"non-dimensional" variables



References

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