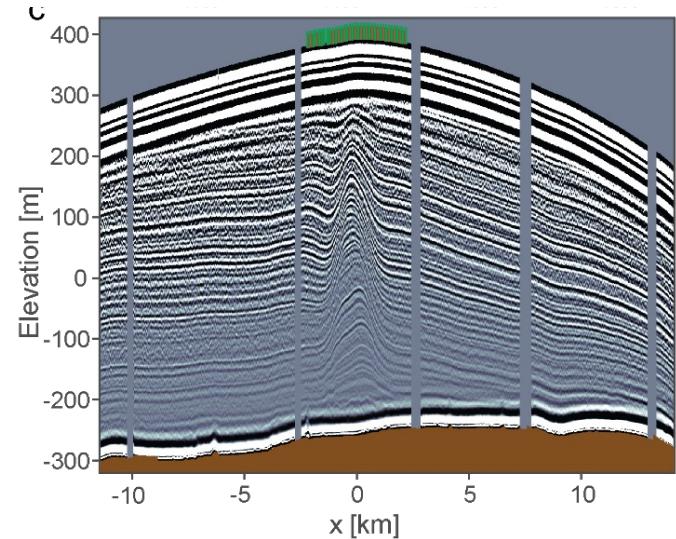
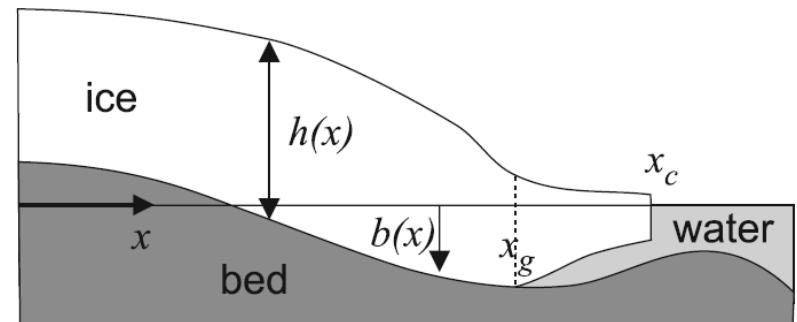
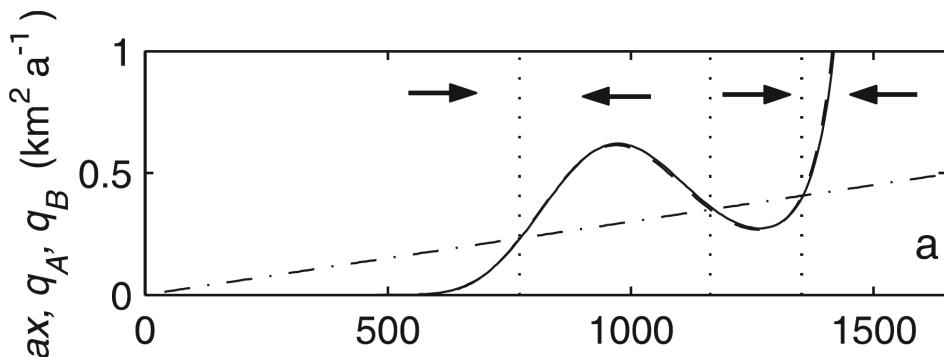


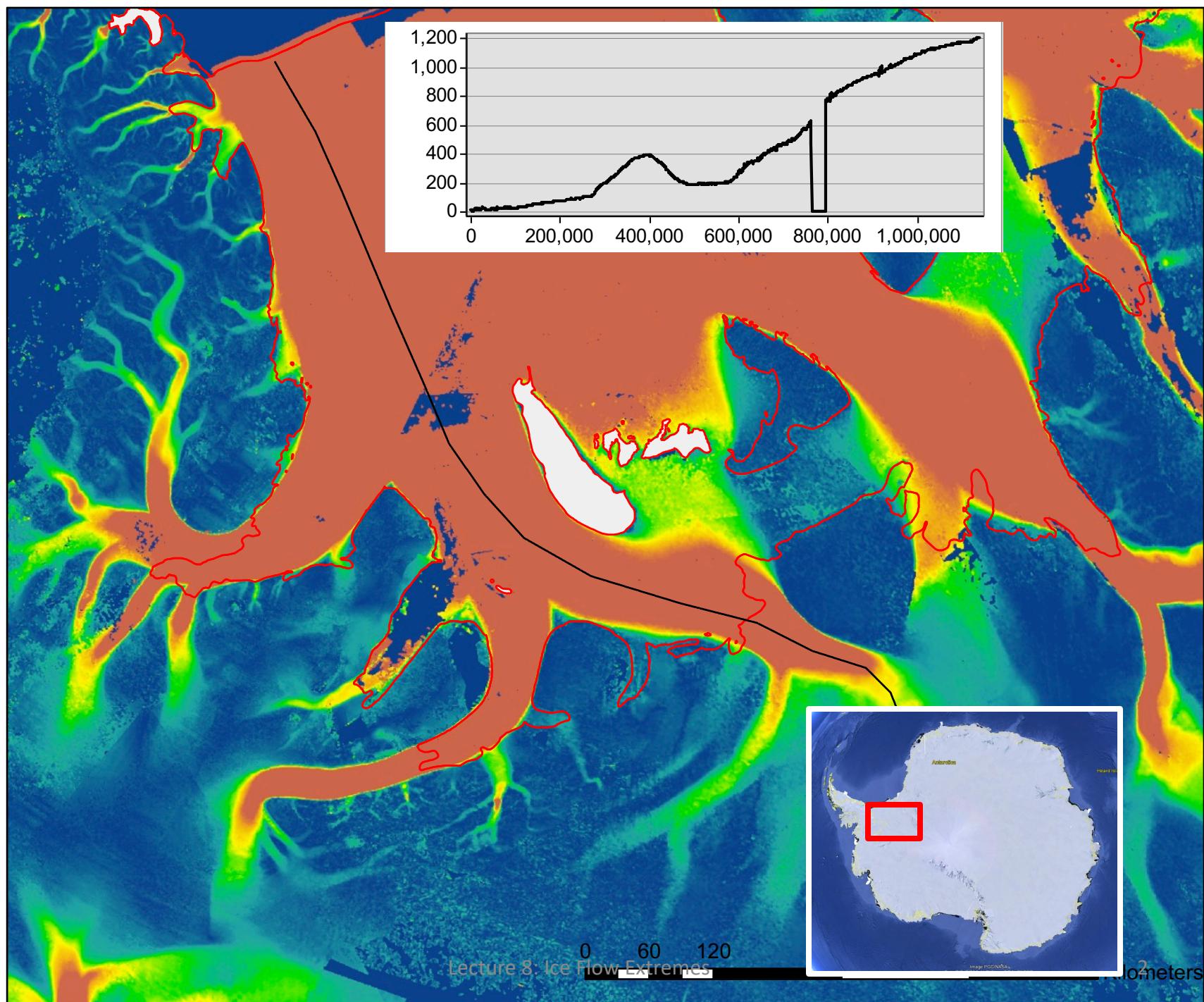
# Lecture 8: Ice-flow extremes

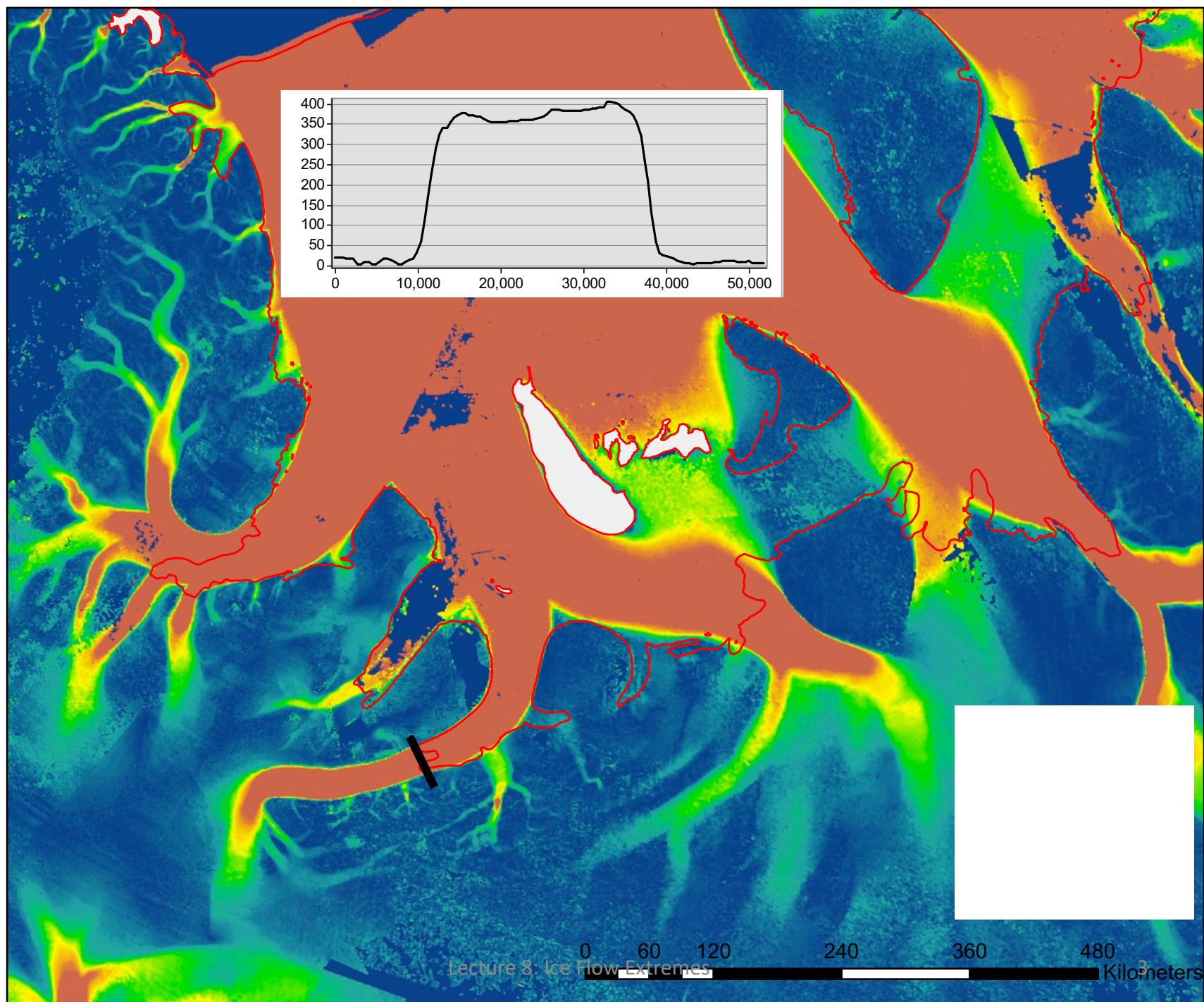
**Ice divides --- very slow and complex!**

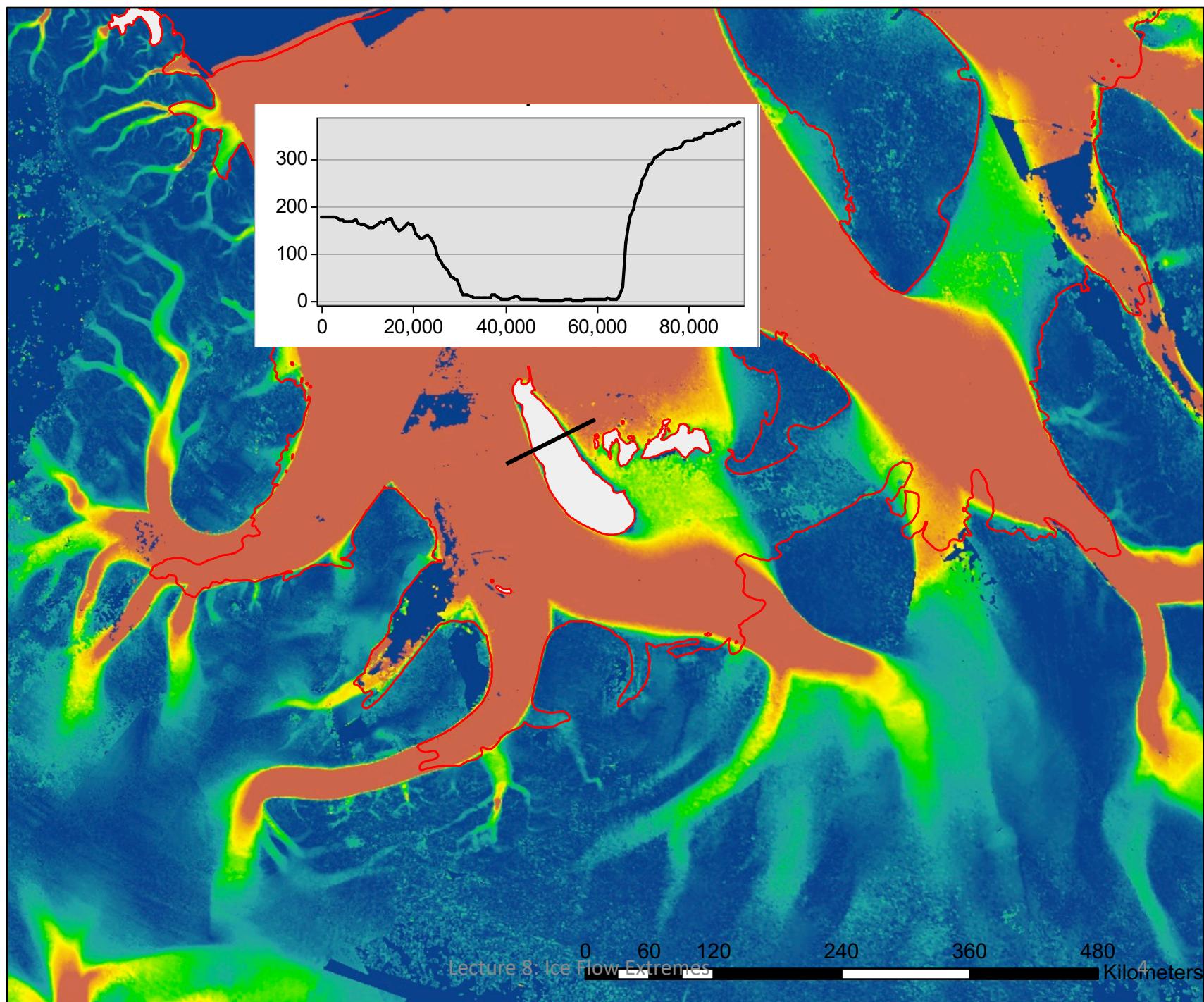


**Grounding lines --- usually fast and complex!**

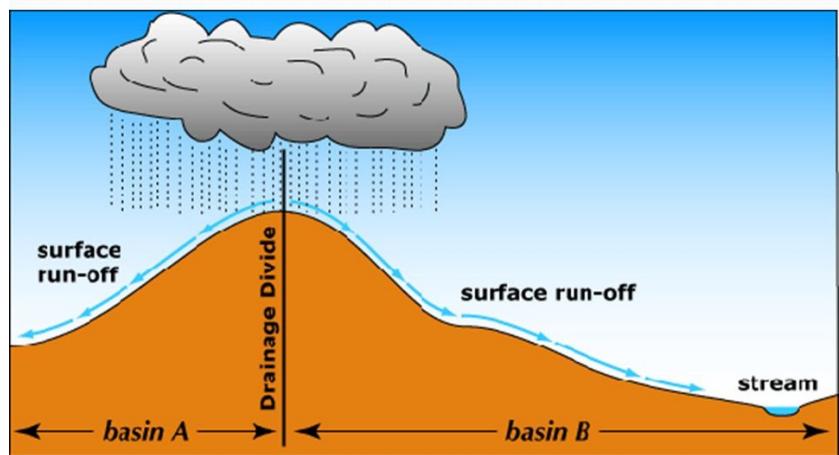






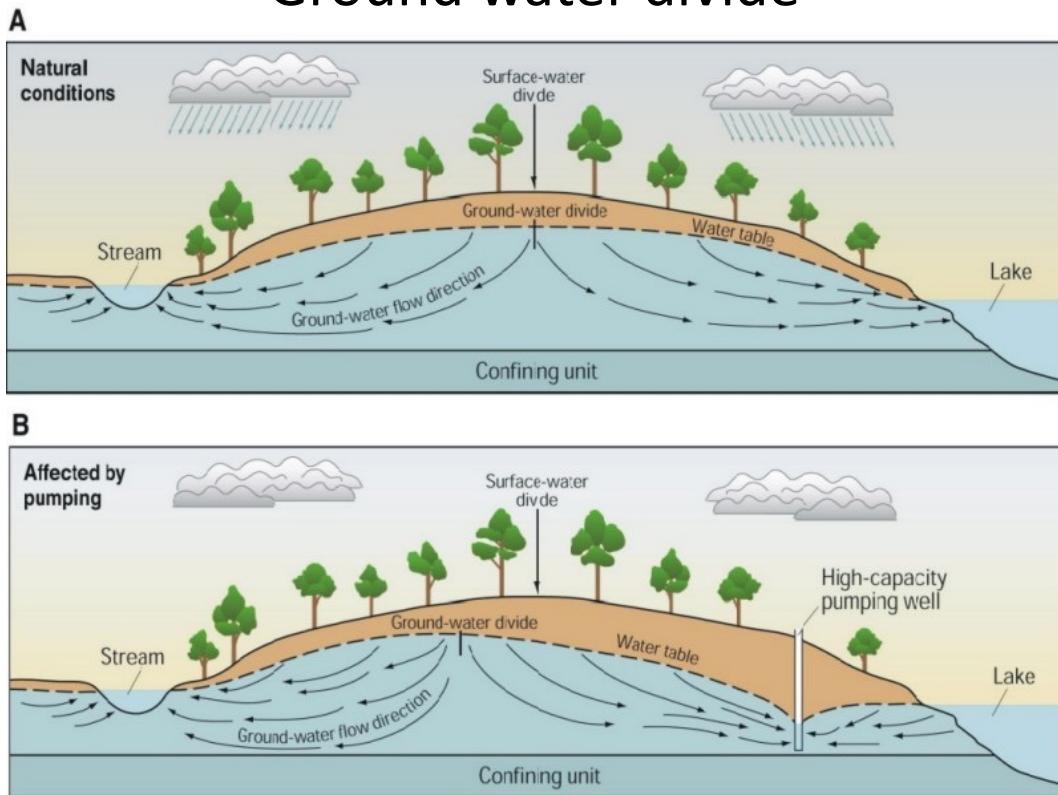


## Surface water divide



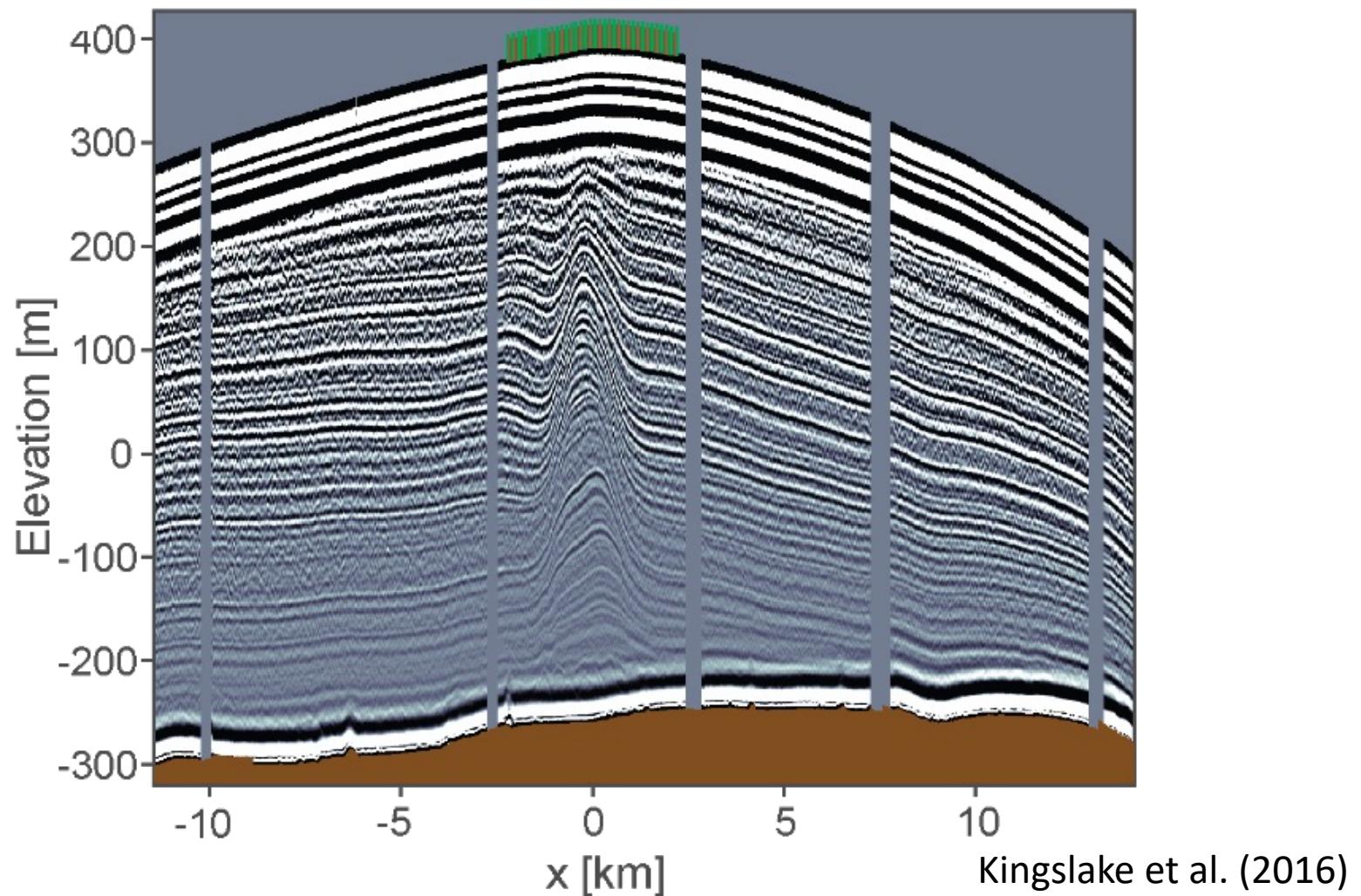
Cornelia Gibbs: Rivers an Introduction. <http://slideplayer.com>

## Ground water divide

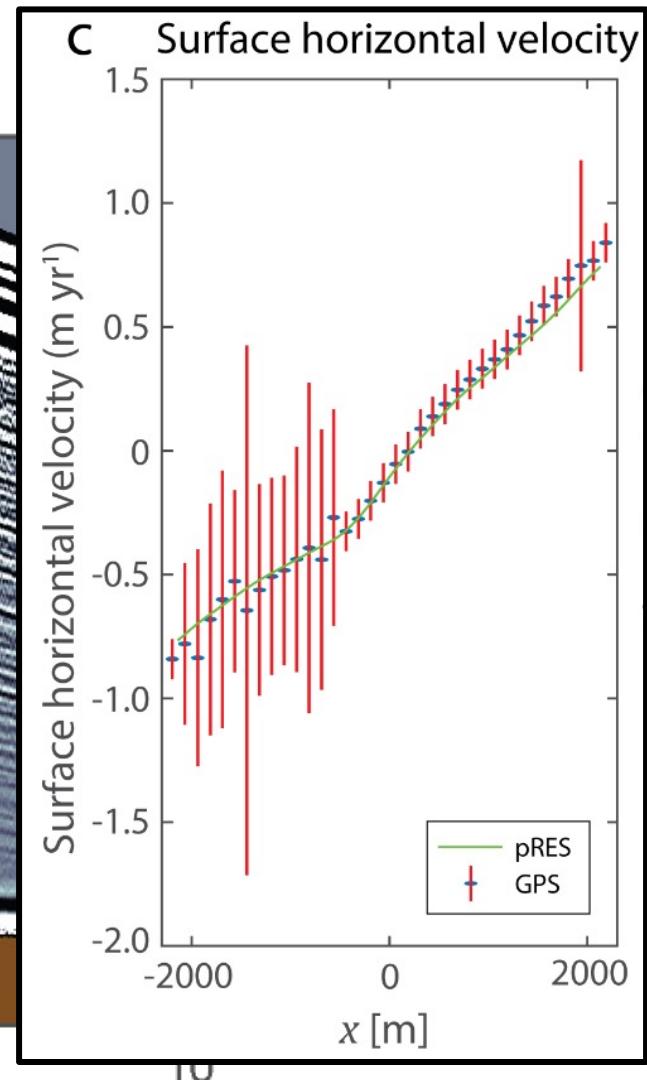
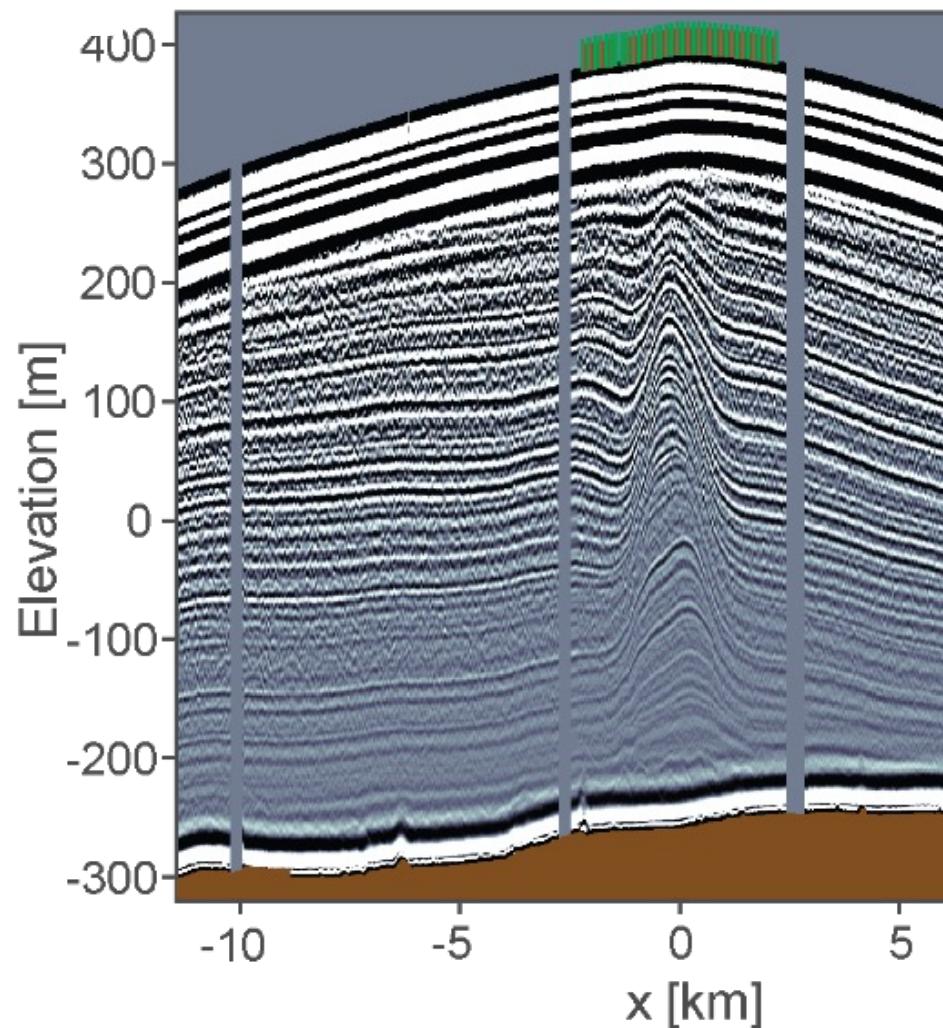


USGS

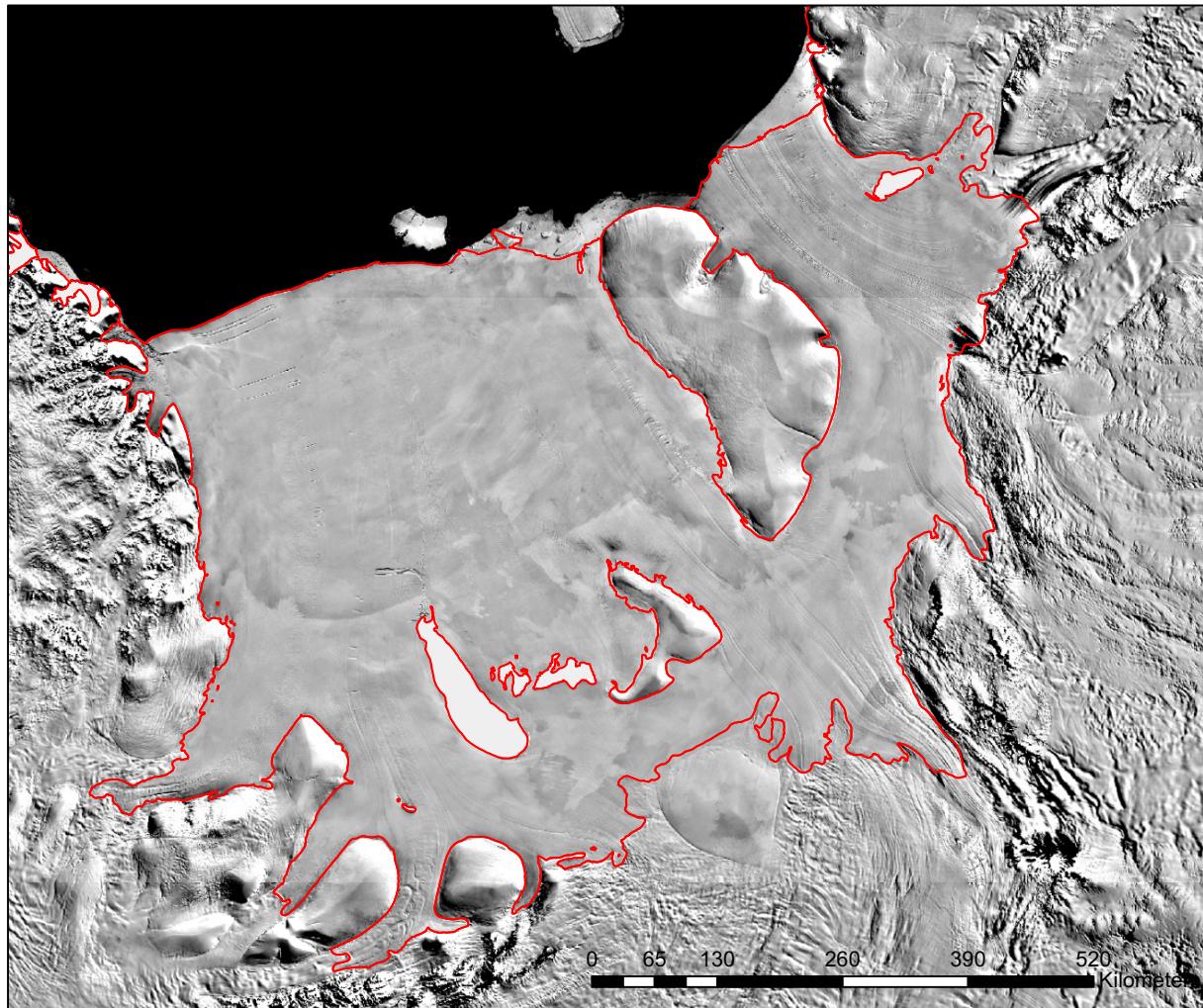
# Ice divide



# Ice divides

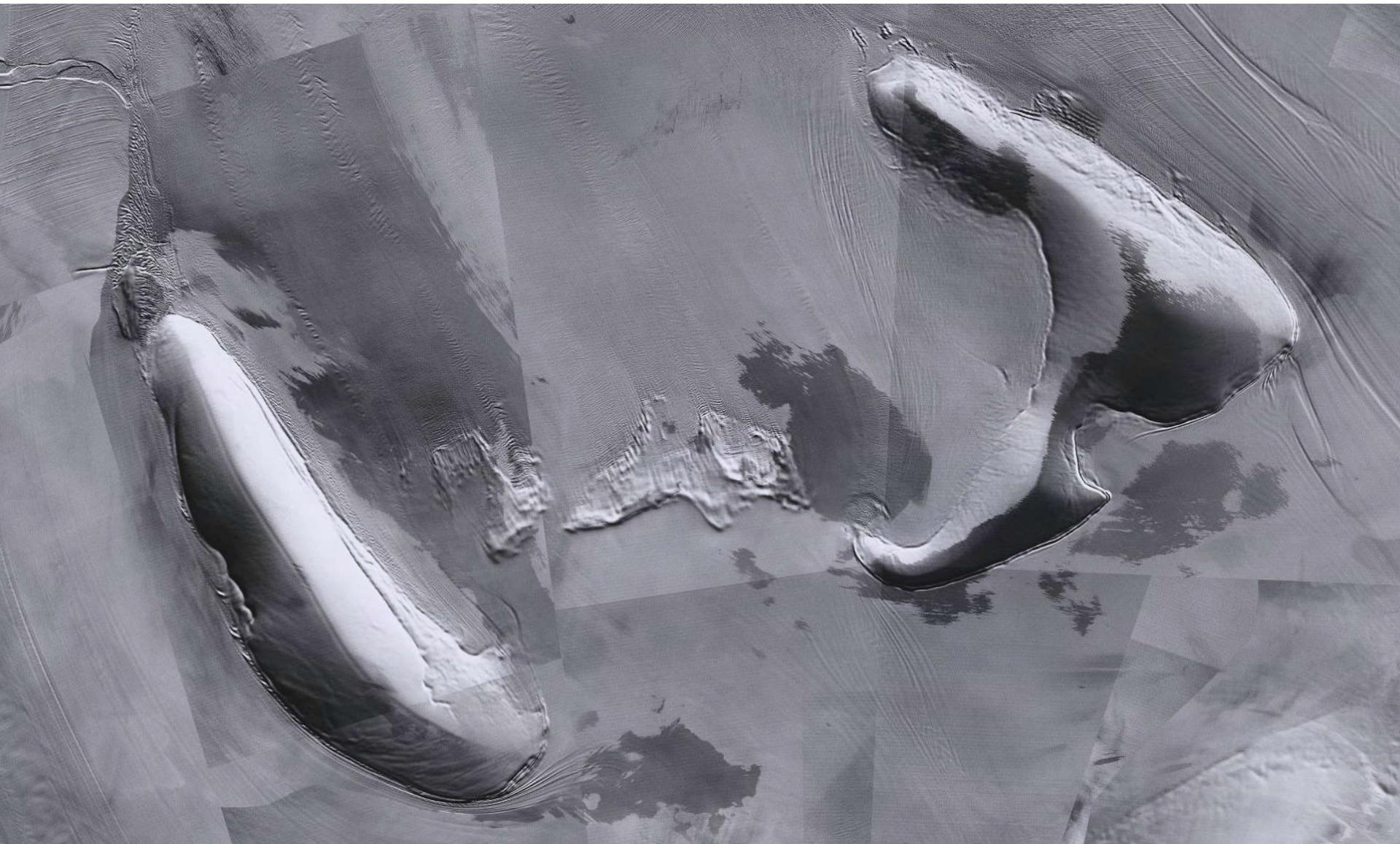


# Ice divides from satellite imagery



MODIS mosaic of Antarctica (MOA); Scambos et al. (2005)

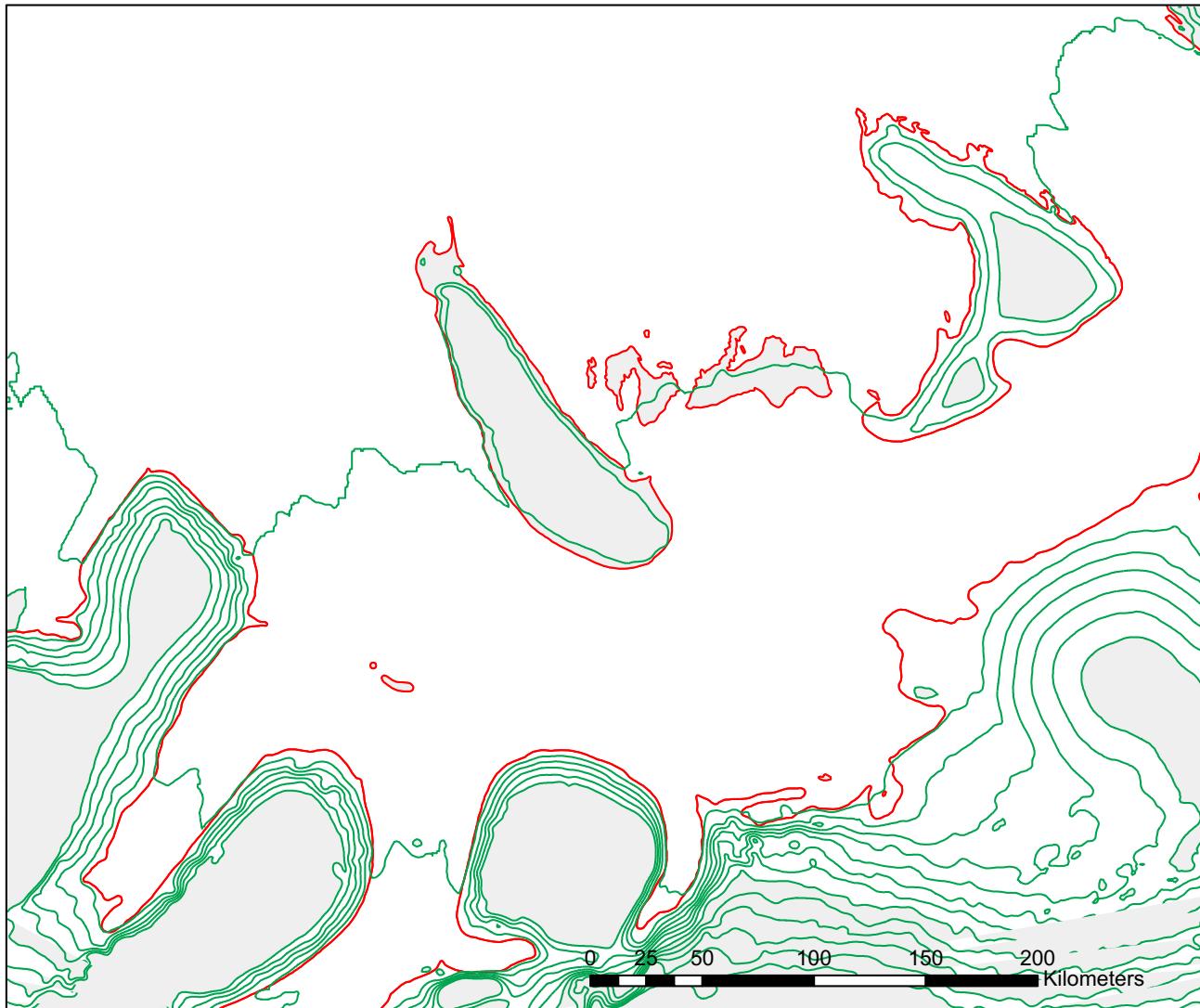
# Ice divides from satellite imagery



MODIS mosaic of Antarctica (MOA); Scambos et al. (2005)

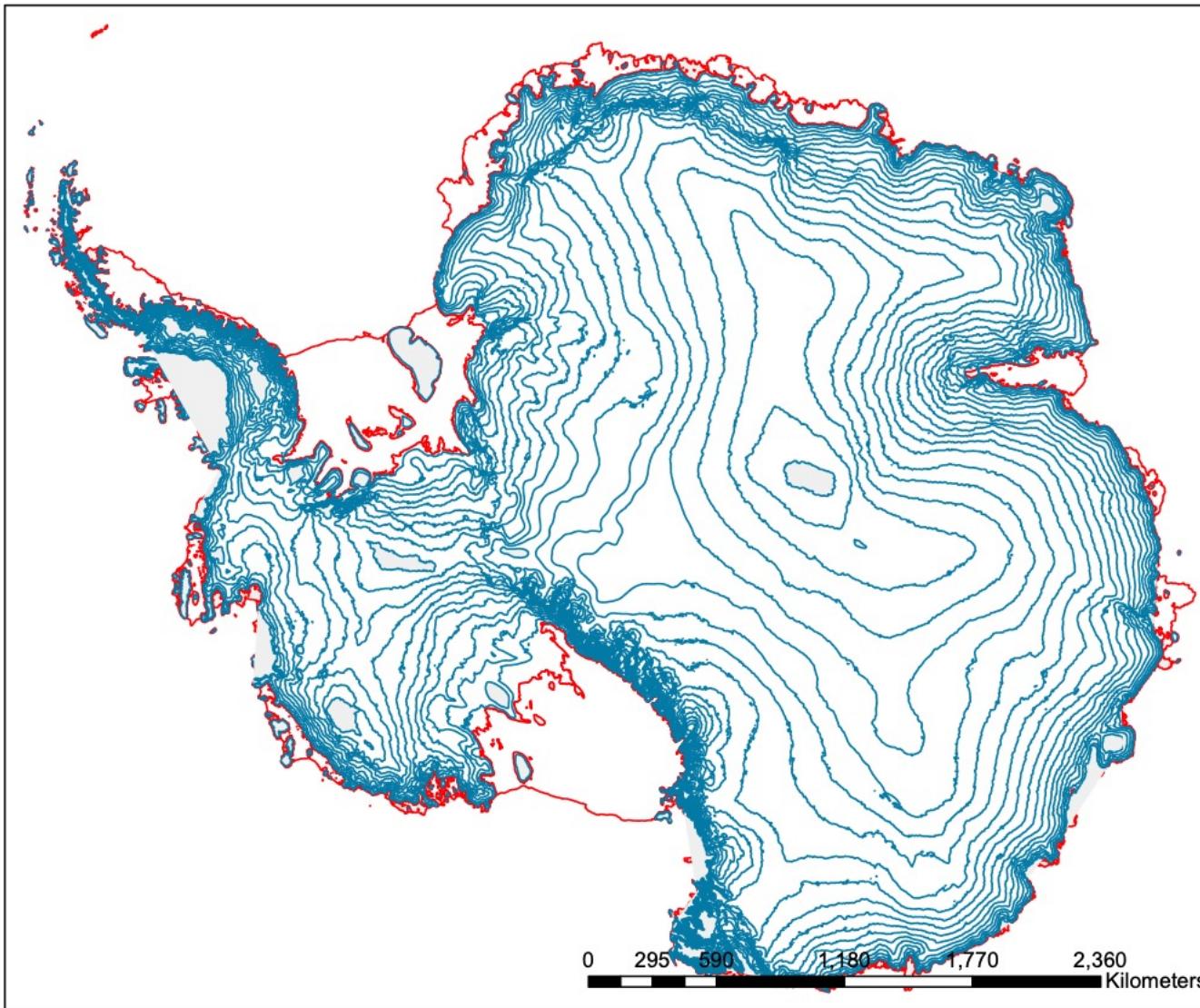
Lecture 8: Ice Flow Extremes

# Ice divides from a digital elevation model



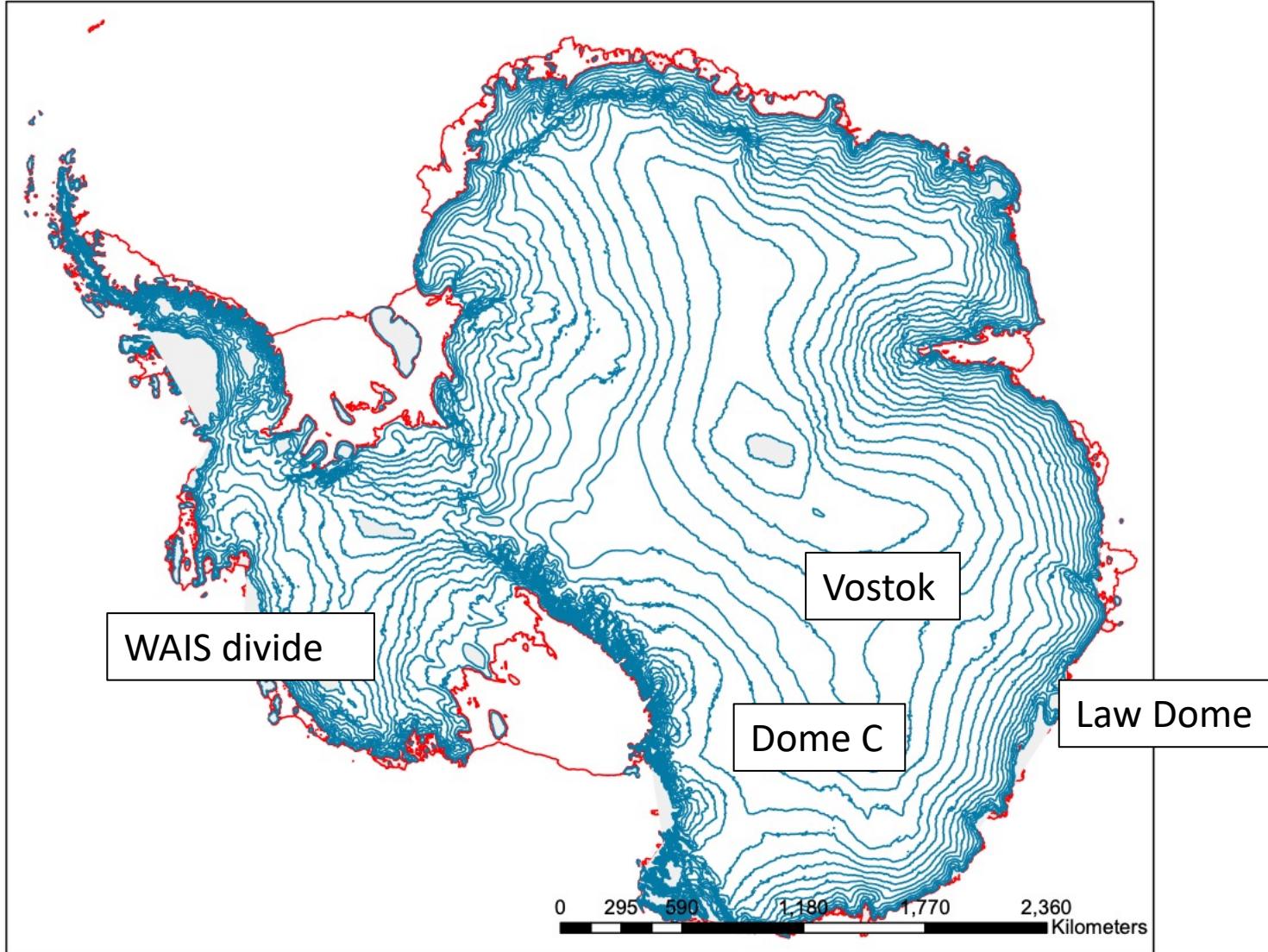
Fretwell et al. (2013)

# Ice divides from a digital elevation model



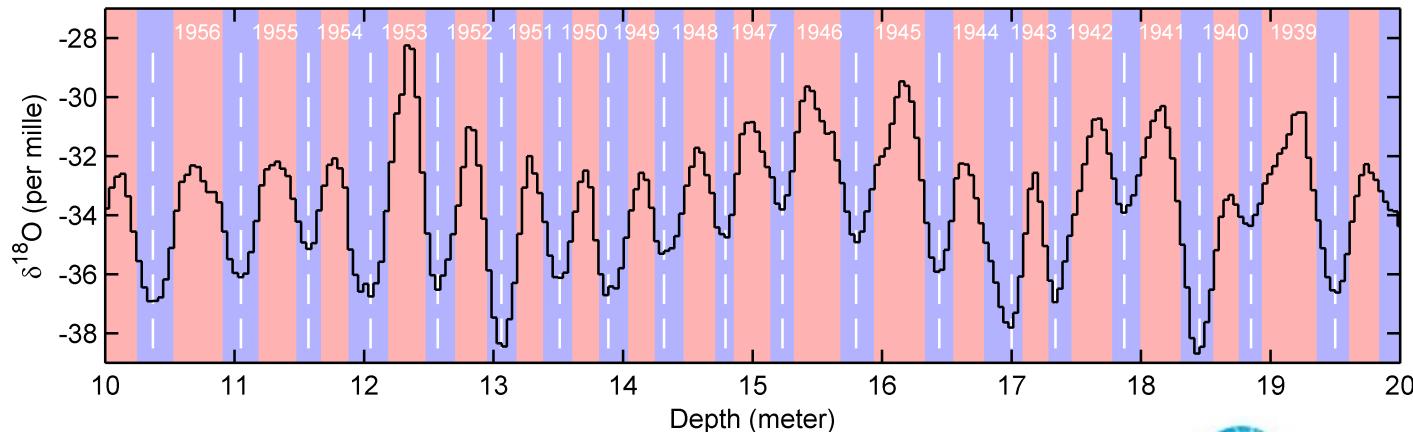
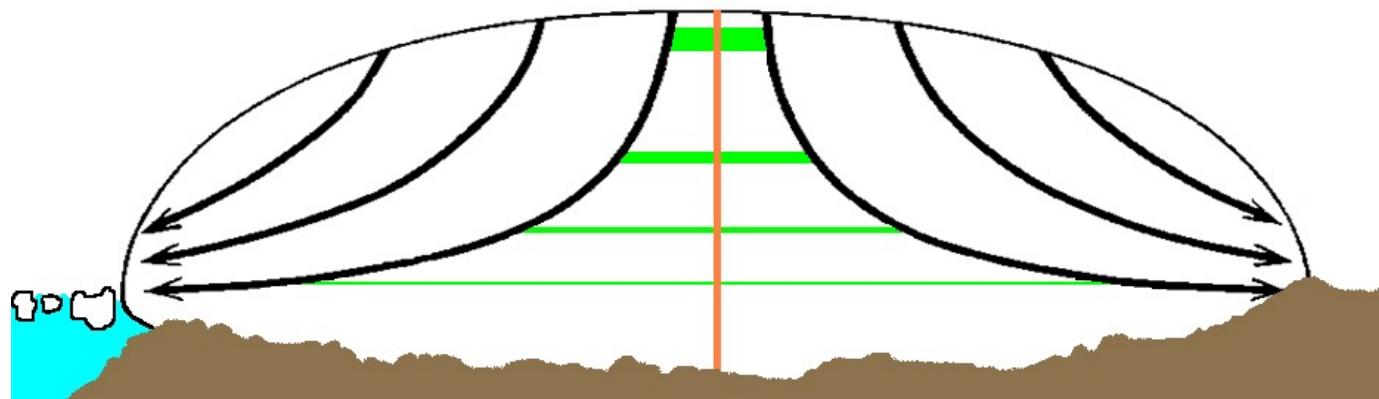
Fretwell et al. (2013)

# Ice divides, ideal for ice cores

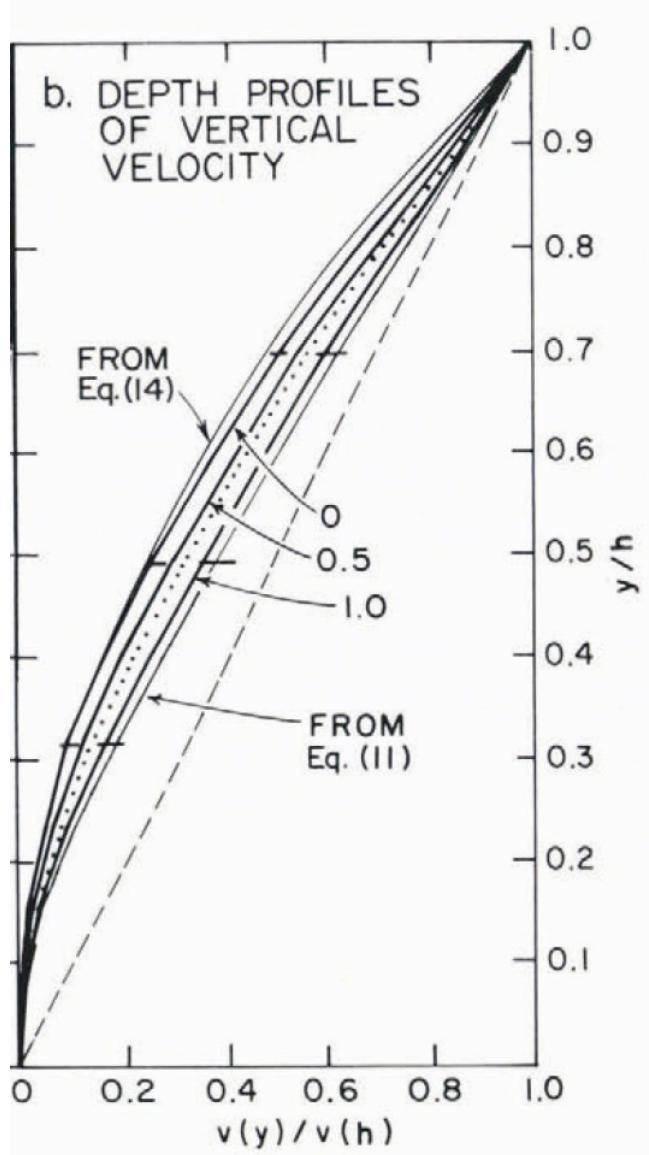


Fretwell et al. (2013)

# Ice divides, ideal for ice cores



# Modelling the vertical flow at ice divides



$$d_{xx} = \frac{\partial u}{\partial x}, \quad d_{yy} = \frac{\partial v}{\partial y},$$

$$\epsilon = \left( \frac{d_{xx}^2}{2} + \frac{d_{yy}^2}{2} + d_{xy}^2 \right)^{1/2}.$$

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \frac{\partial p}{\partial x} = 0,$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \frac{\partial p}{\partial y} - \rho g = 0$$

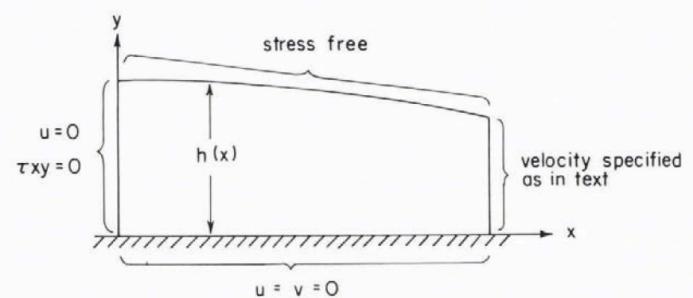
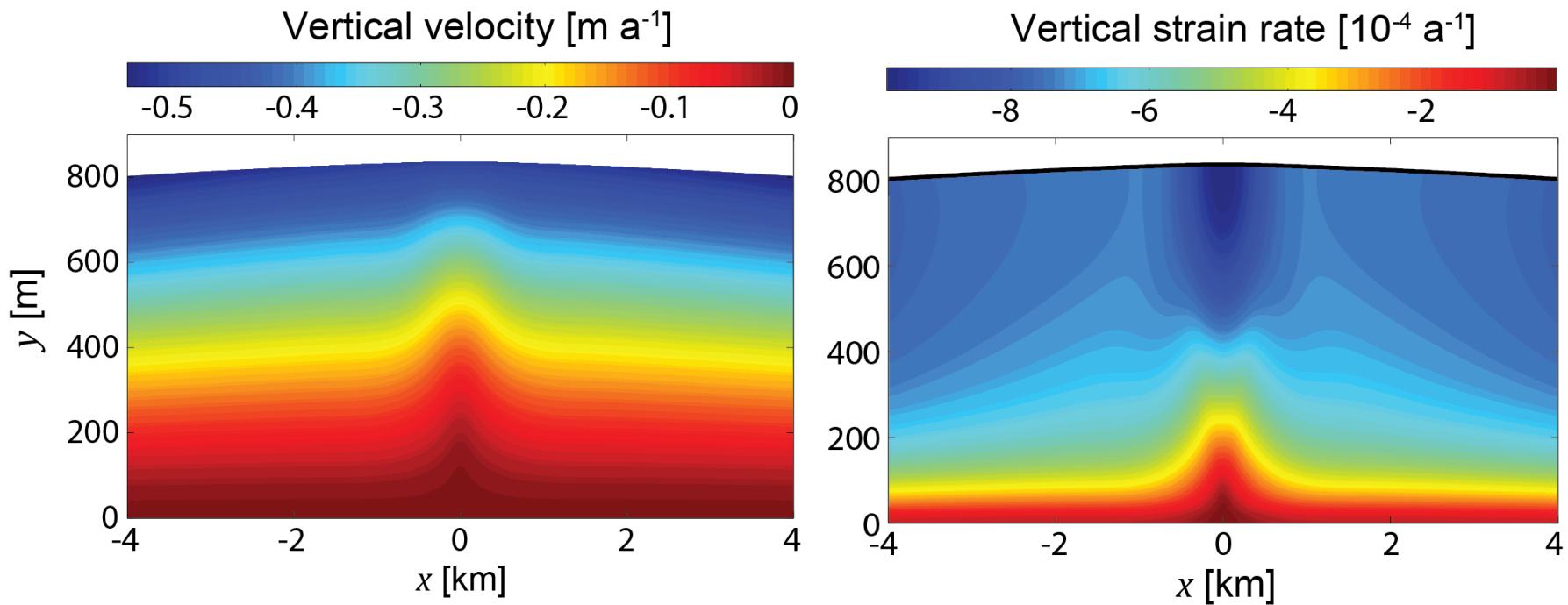


Fig. 1. Idealized cross-section of two-dimensional symmetric ice divide.

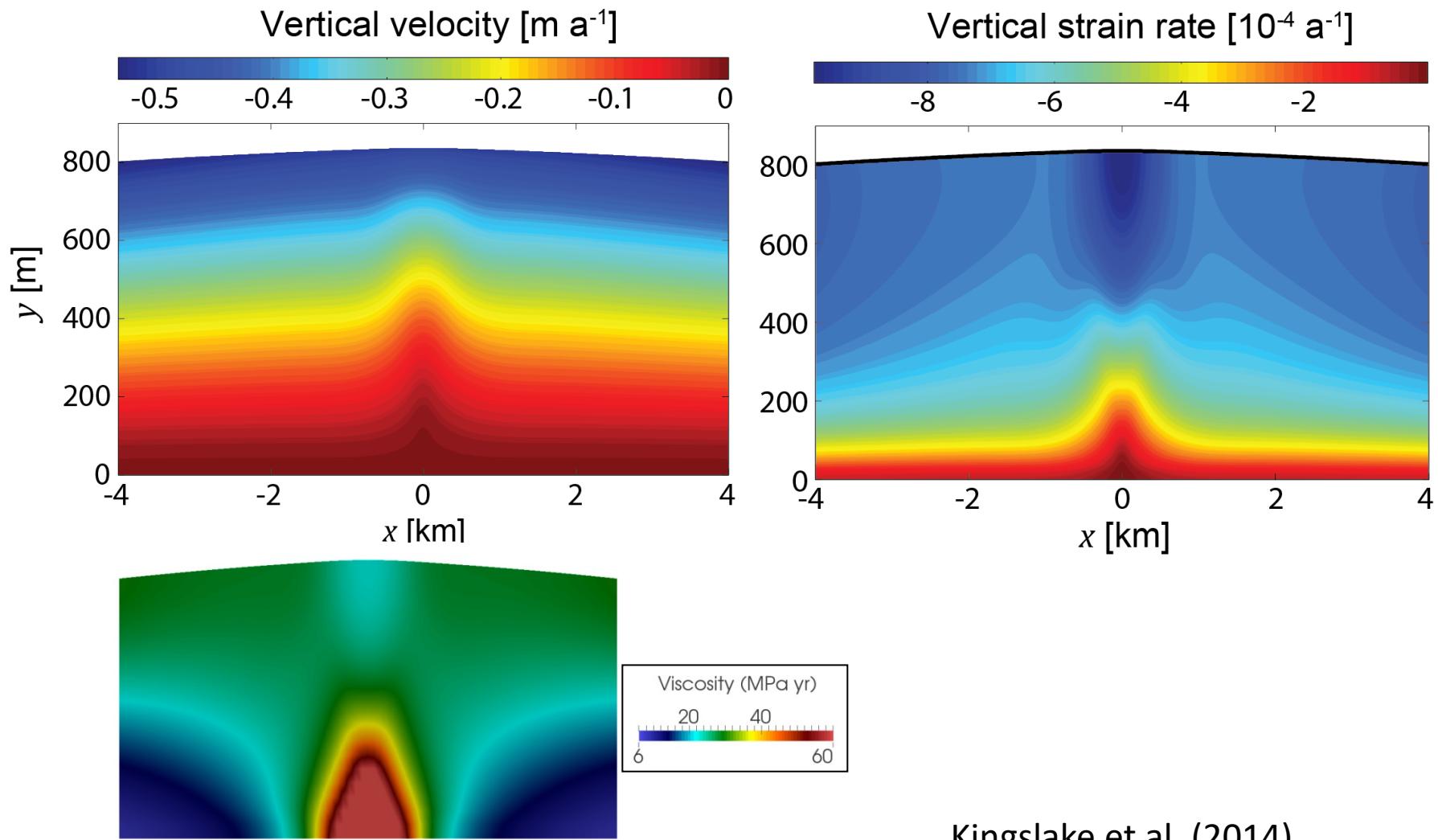
Raymond (1983)

# Modelling the vertical flow at ice divides



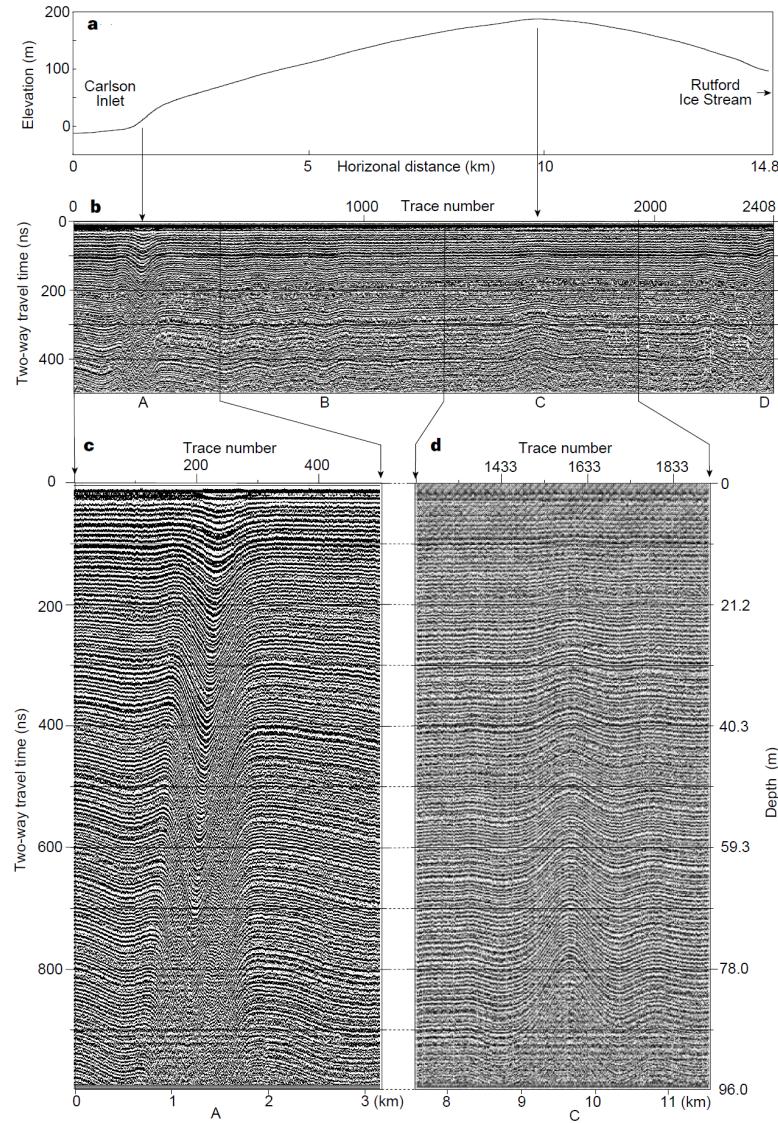
Kingslake et al. (2014)

# Modelling the vertical flow at ice divides



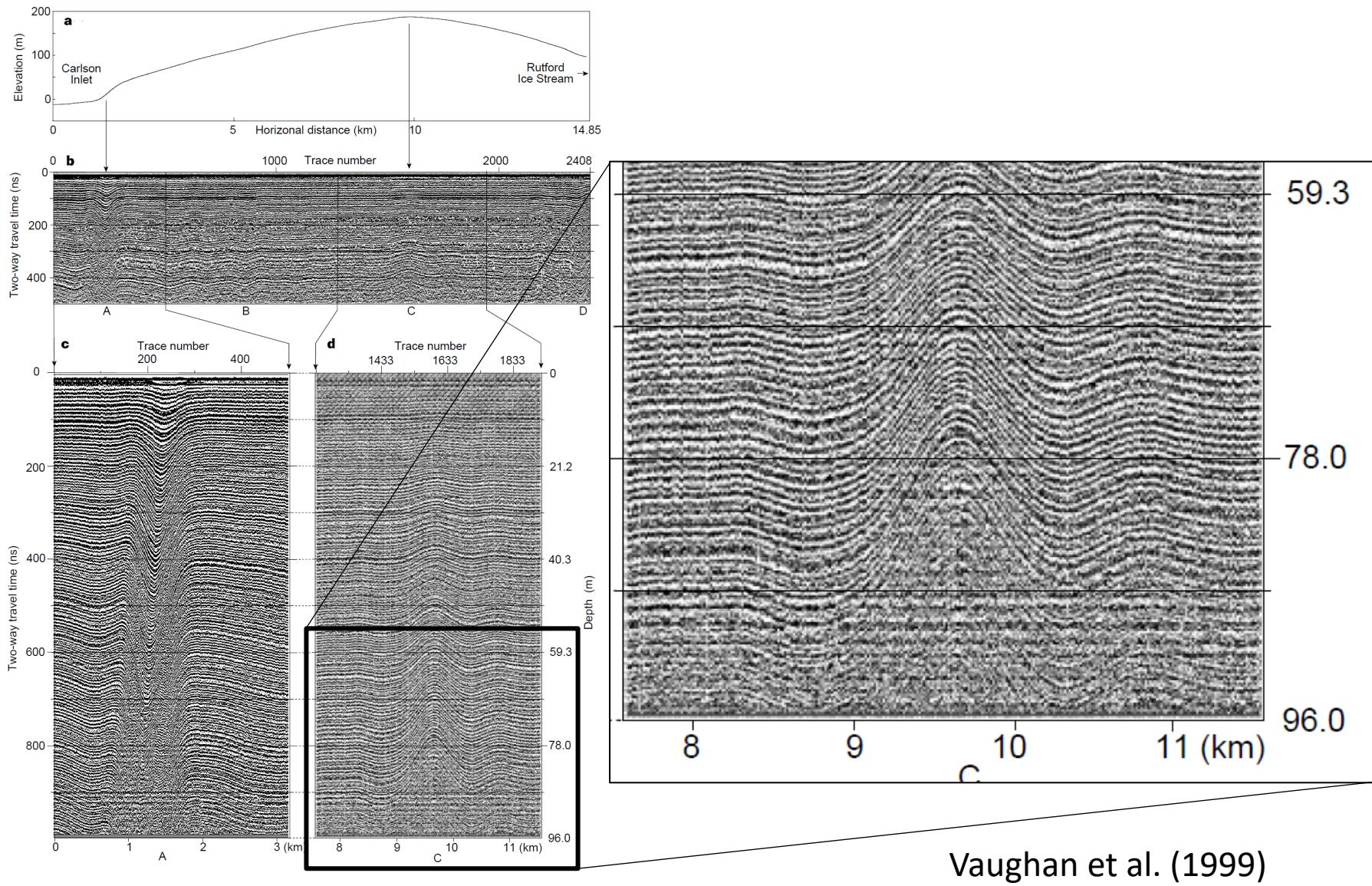
Kingslake et al. (2014)

# The first observed “Raymond Bumps”

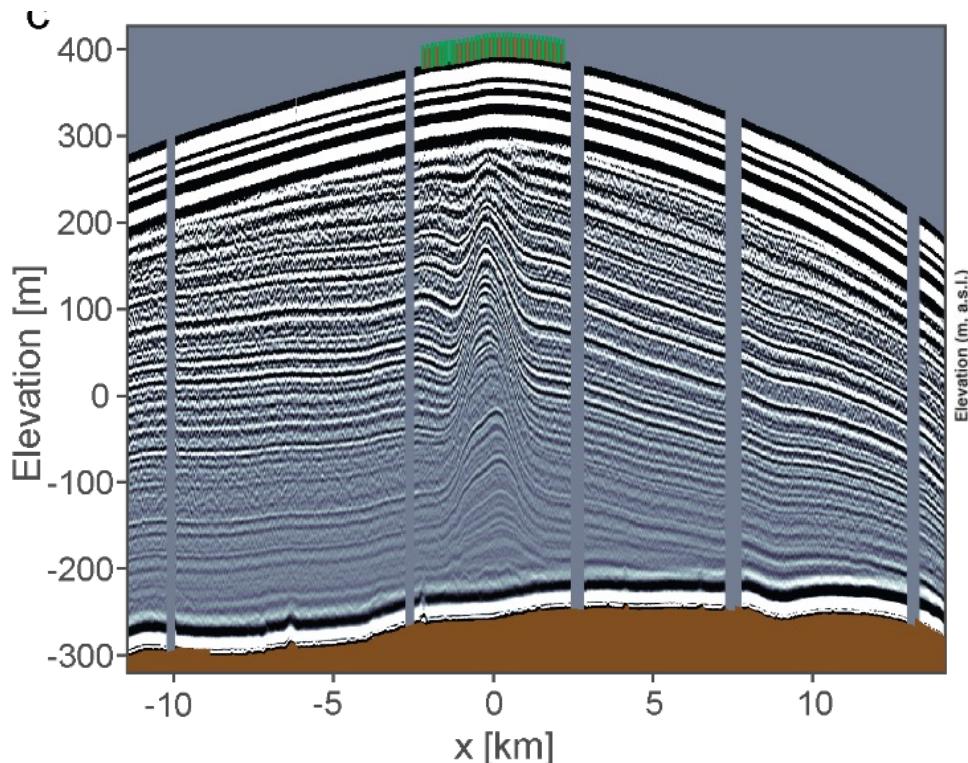


Vaughan et al. (1999)

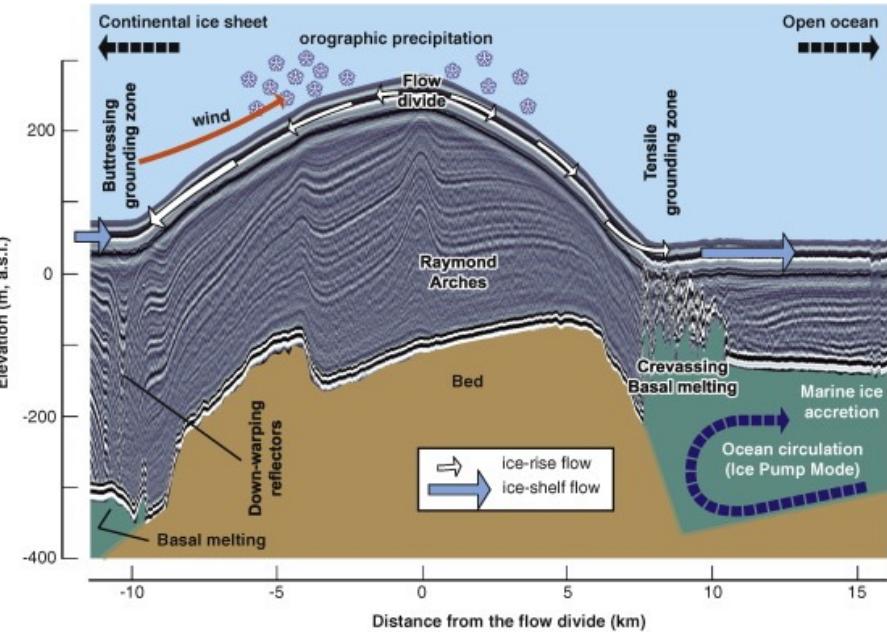
# The first observed “Raymond Bumps”



# Raymond Bumps have now been observed in many places.



Kingslake et al. (2016)



Matsuoka et al. (2015)

# The Raymond effect has even been measured!

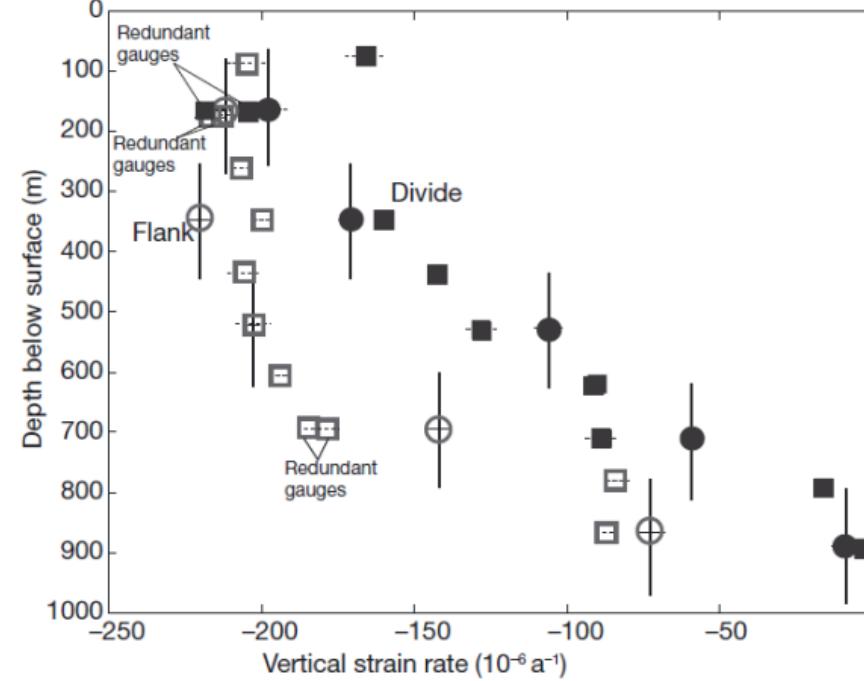
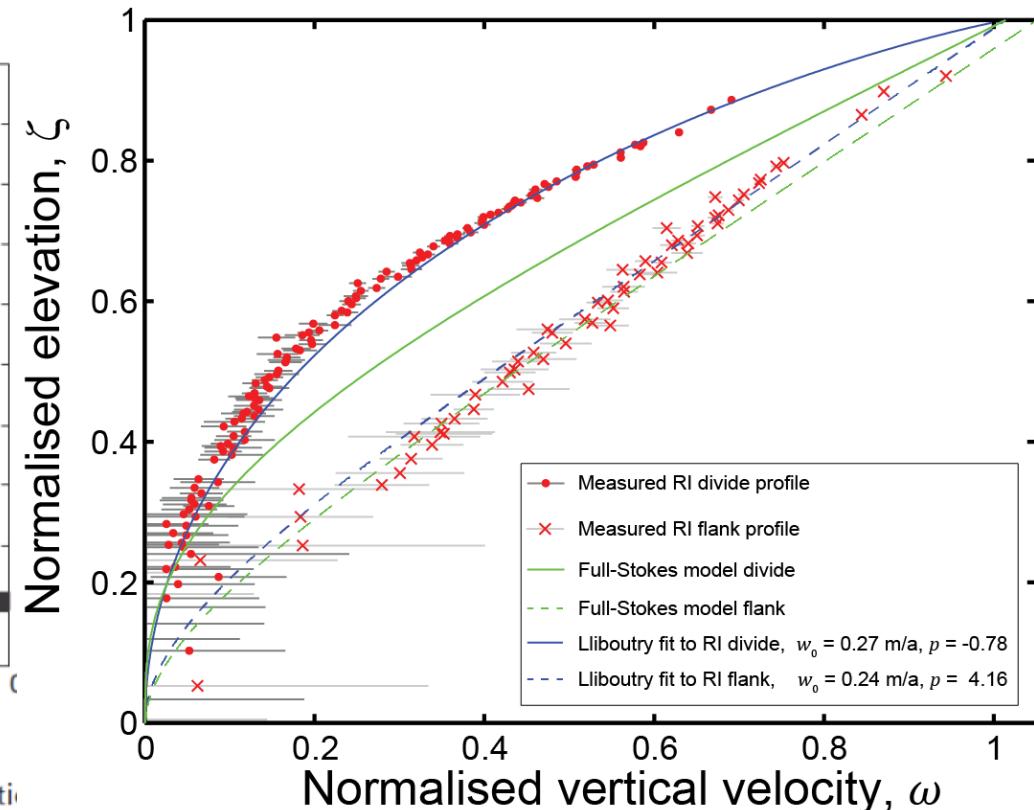


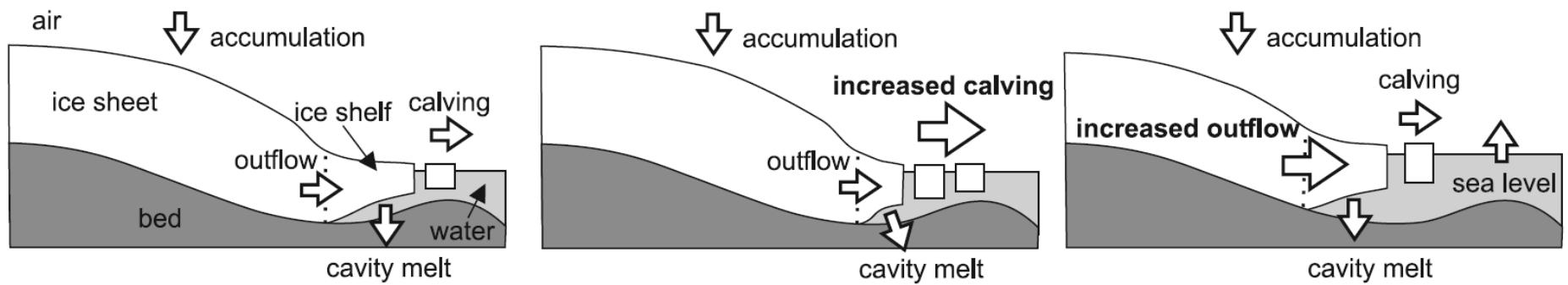
Fig. 4. Strain-rate data from the wire (squares) and fiber-optic (circles) instruments. The open symbols are flank-site measurements,

Pettit et al. (2011)



Kingslake et al. (2014)

# Flux at the grounding line controls mass loss in Antarctica



**Figure 1.** Main components in the mass balance of a marine ice sheet.

Schoof (2007)

# The marine ice sheet instability

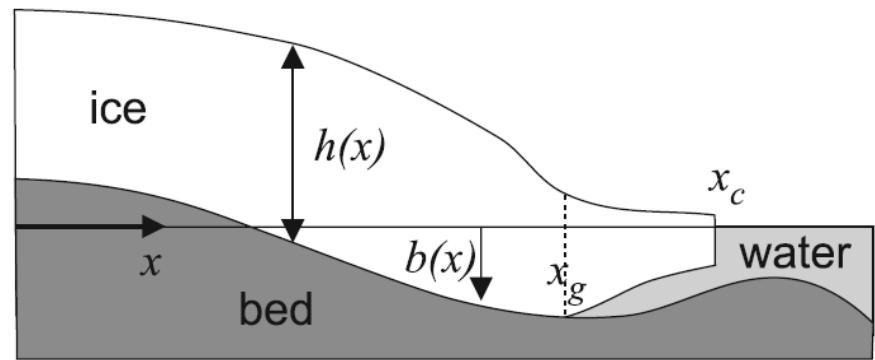
STABILITY OF THE JUNCTION OF AN ICE SHEET AND AN  
ICE SHELF

*By J. WEERTMAN*

- Proposed by Weertman
  - If a grounding line retreats into deeper water, flux will increase and cause further retreat (a positive feedback).
- Much debated.
- Eventually Schoof (2007) derived a ‘boundary layer’ model that has been widely accepted.

# Schoof's (2007) theory

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = b$$

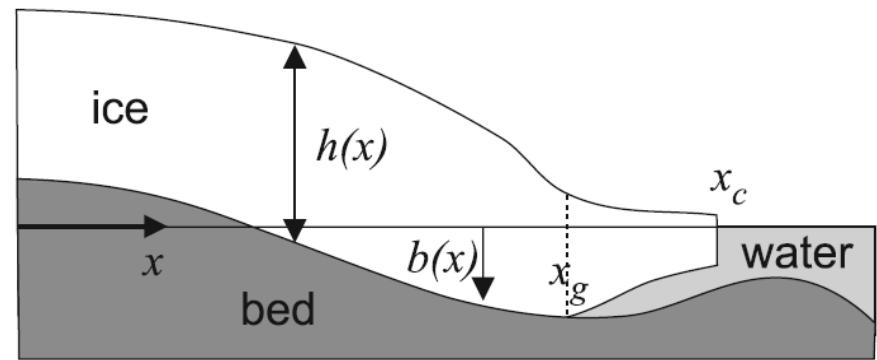


$$b(x) = - \left[ 729 - 2184.8 \times \left( \frac{x}{750 \text{ km}} \right)^2 + 1031.72 \times \left( \frac{x}{750 \text{ km}} \right)^4 - 151.72 \times \left( \frac{x}{750 \text{ km}} \right)^6 \right] \text{ m},$$

# Schoof's (2007) theory

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \dot{b}_i$$

Specific mass balance



Mass conservation

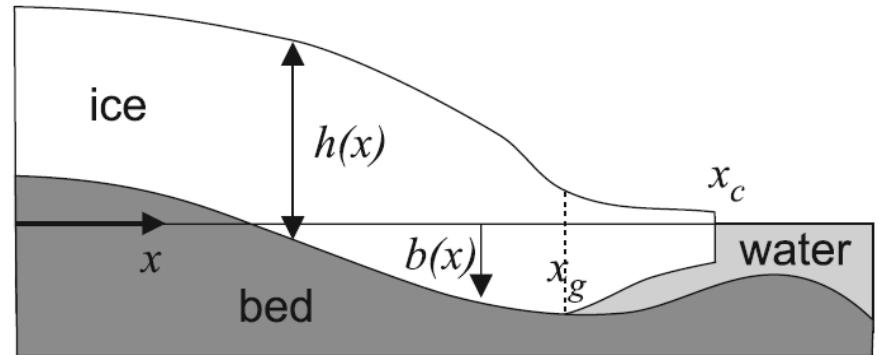
In steady state:  $q = \dot{b}_i x$

$$b(x) = - \left[ 729 - 2184.8 \times \left( \frac{x}{750 \text{ km}} \right)^2 + 1031.72 \times \left( \frac{x}{750 \text{ km}} \right)^4 - 151.72 \times \left( \frac{x}{750 \text{ km}} \right)^6 \right] \text{ m},$$

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$$b(x) = - \left[ 729 - 2184.8 \times \left( \frac{x}{750 \text{ km}} \right)^2 + 1031.72 \times \left( \frac{x}{750 \text{ km}} \right)^4 - 151.72 \times \left( \frac{x}{750 \text{ km}} \right)^6 \right] \text{ m}, \quad (10)$$

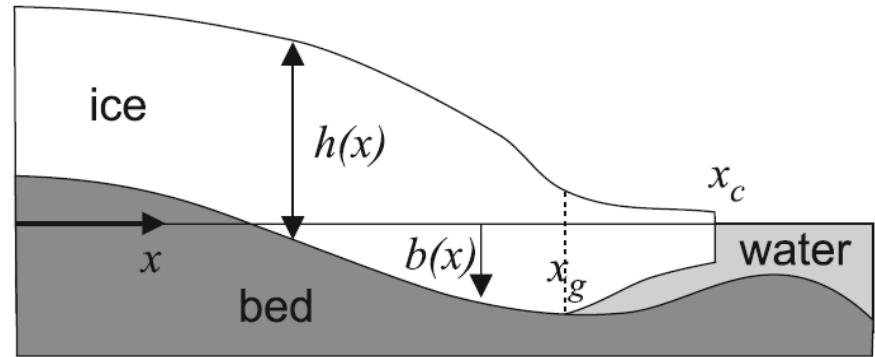
Flux at the grounding line:

$$q(x_g) = \left( \frac{\bar{A}(\rho_i g)^{n+1} (1 - \rho_i / \rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} [h(x_g)]^{\frac{m+n+3}{m+1}}$$

# Schoof's (2007) theory

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \dot{b}_i$$

Specific mass balance



Mass conservation

In steady state:  $q = \dot{b}_i x$

$$b(x) = - \left[ 729 - 2184.8 \times \left( \frac{x}{750 \text{ km}} \right)^2 + 1031.72 \times \left( \frac{x}{750 \text{ km}} \right)^4 - 151.72 \times \left( \frac{x}{750 \text{ km}} \right)^6 \right] \text{ m},$$

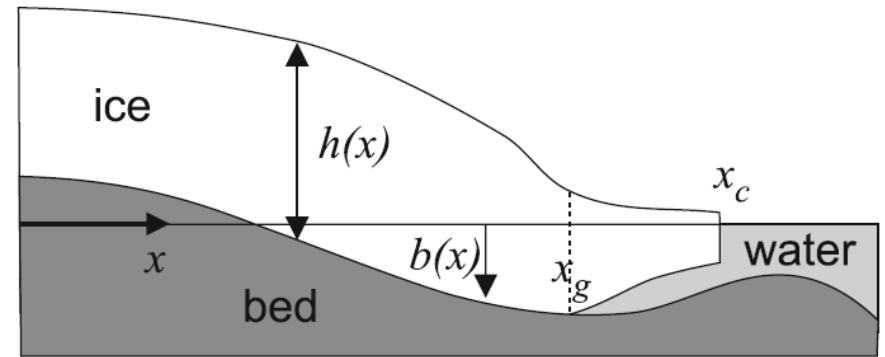
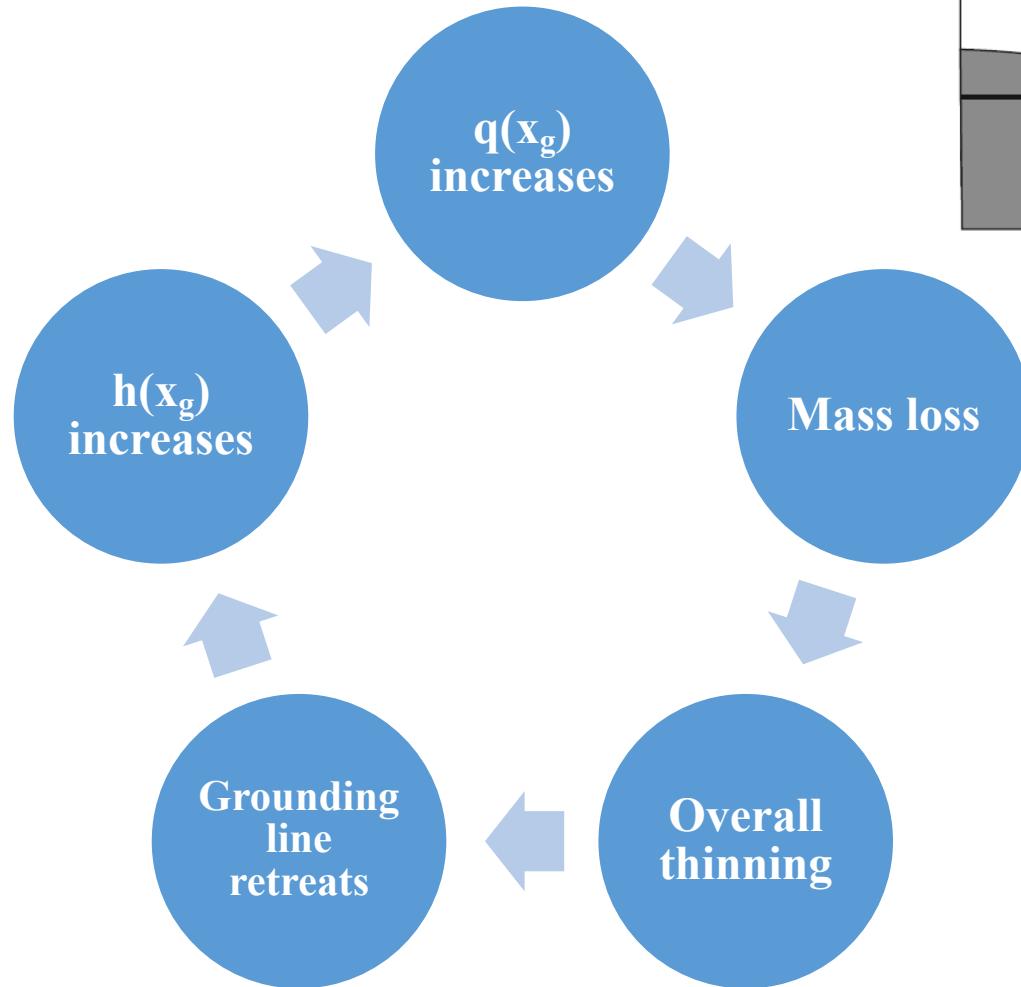
Flux at the grounding line:

$$q(x_g) \propto h(x_g)^{4.75}$$



This quantifies a key part of the marine ice sheet instability!

# The marine ice sheet instability

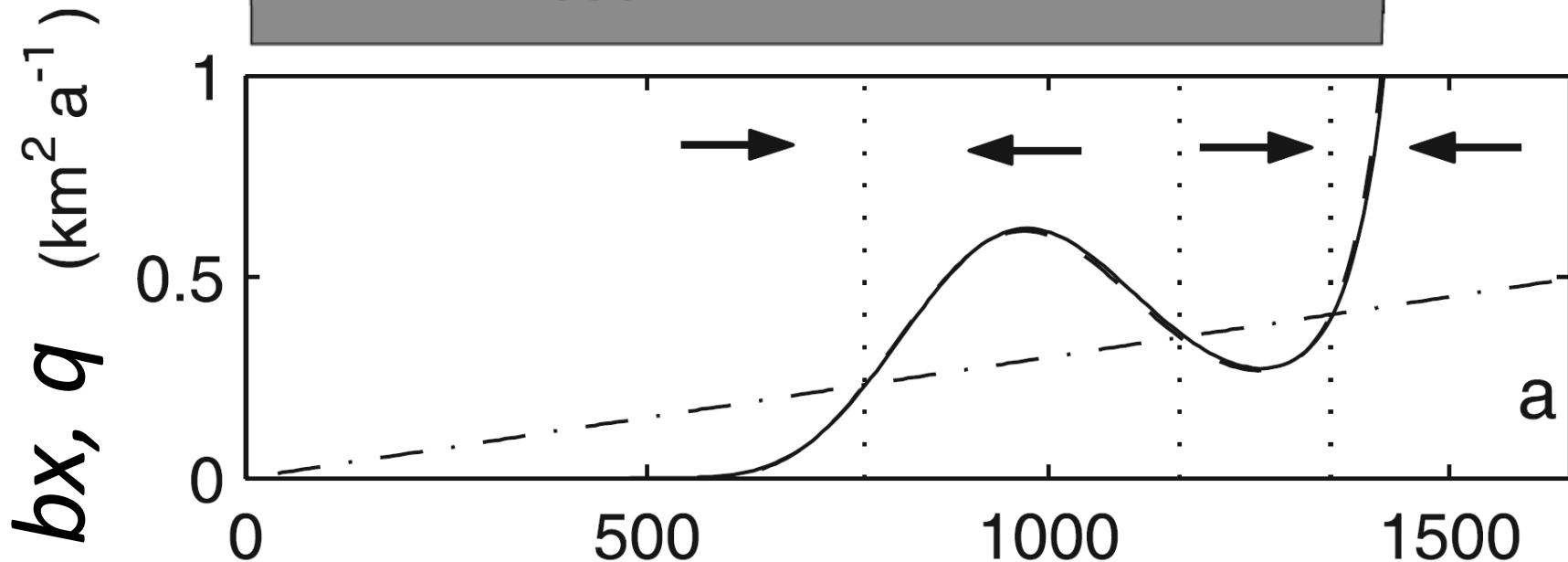
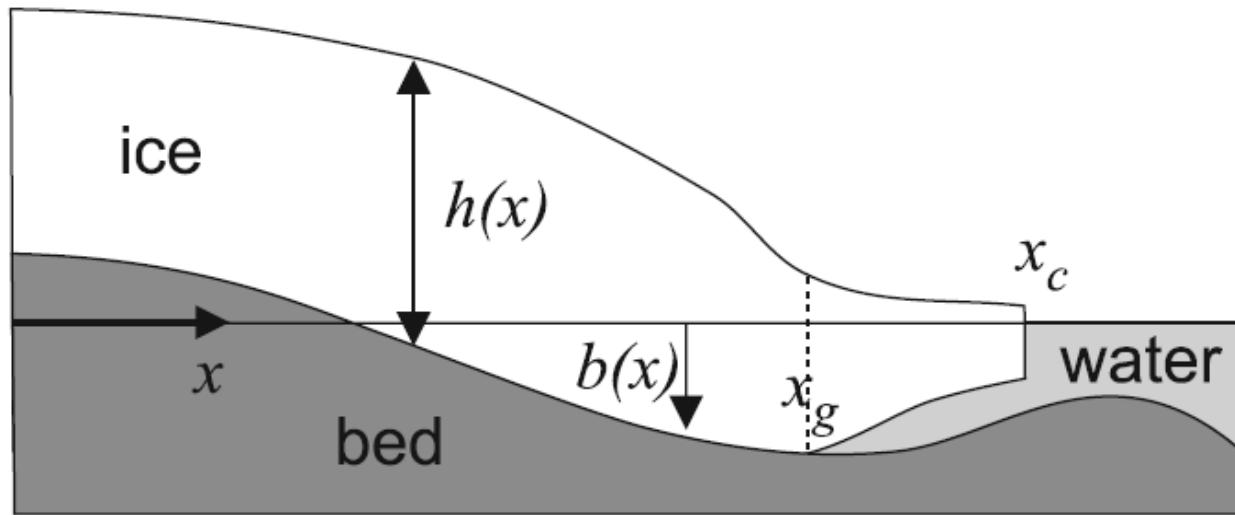


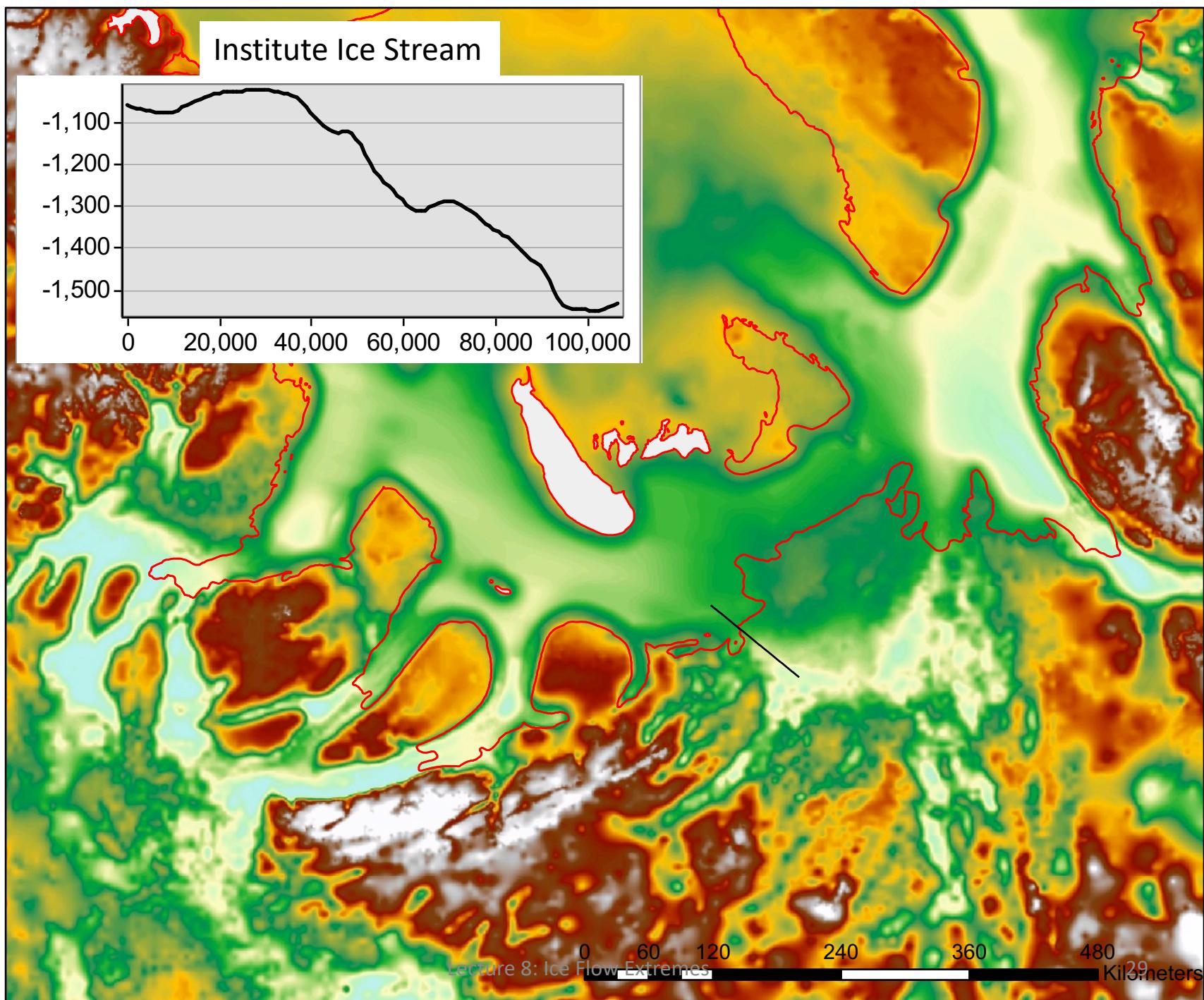
$$q(x_g) \propto h(x_g)^{4.75}$$

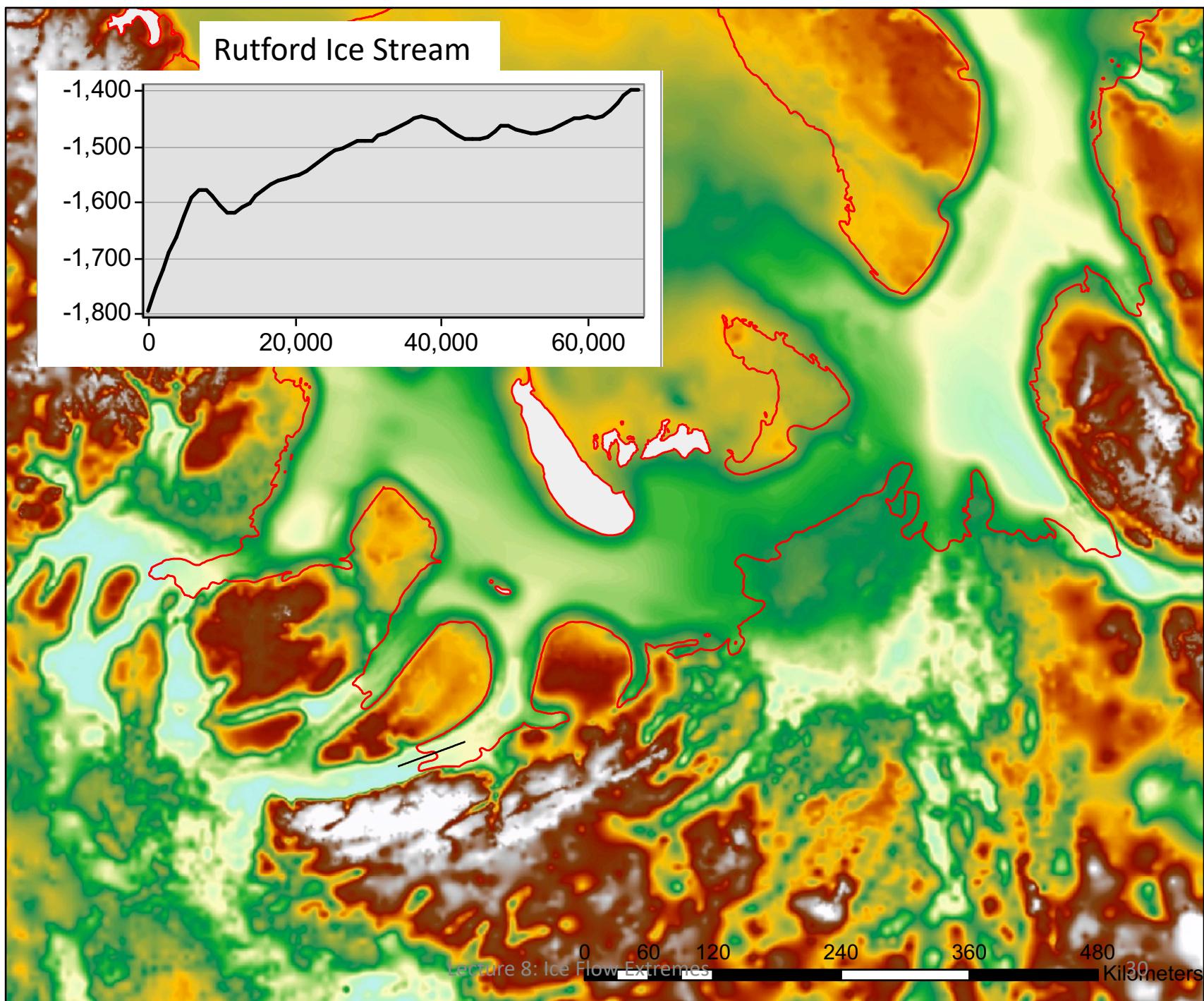
# Mass balance

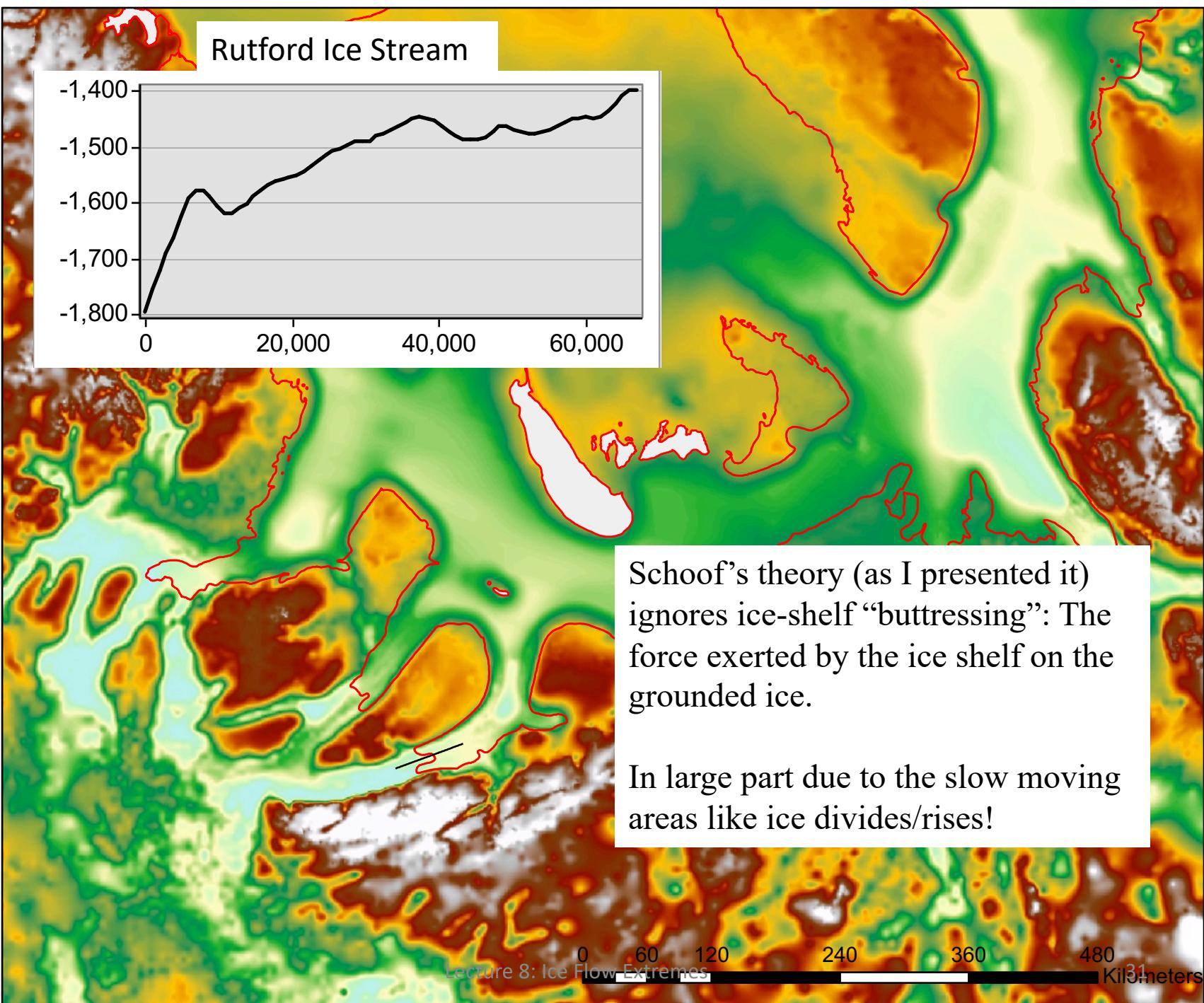
$$q(x_g) \propto h(x_g)^{4.75}$$

$$q = \dot{b}_i x$$



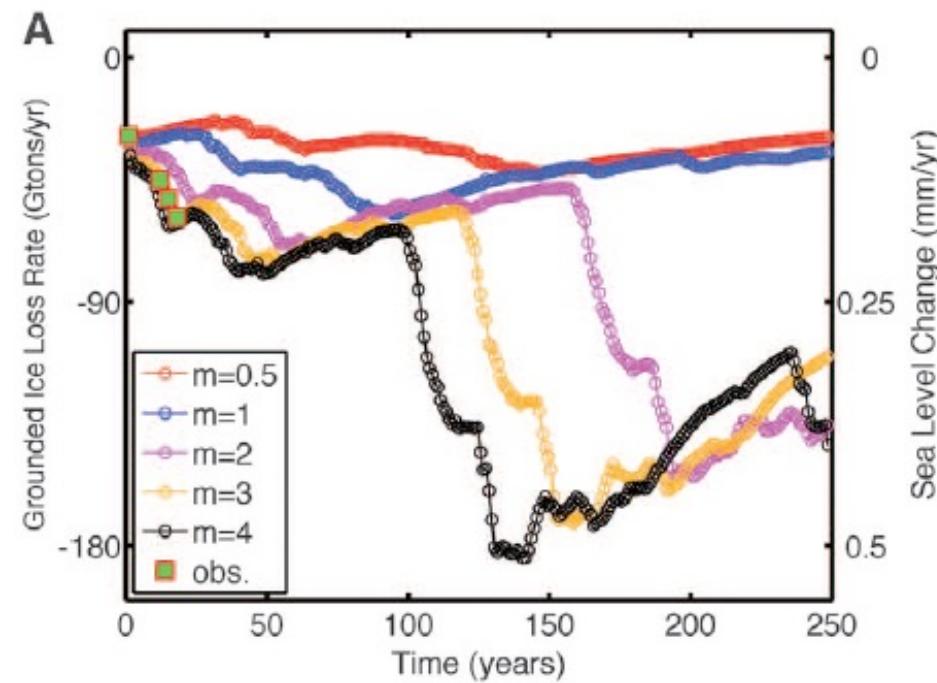
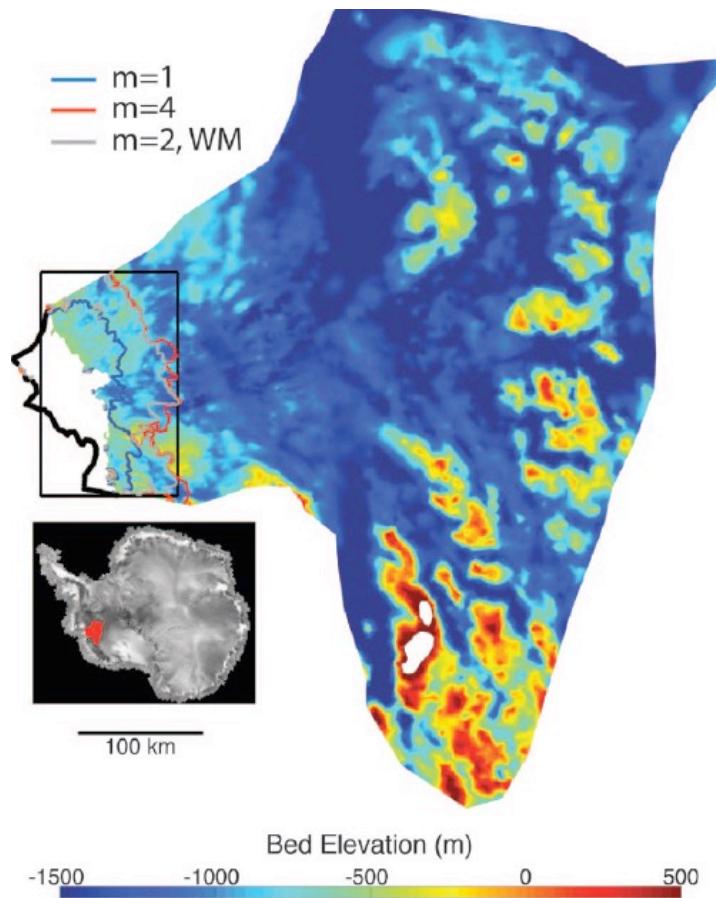






# Marine Ice Sheet Collapse Potentially Under Way for the Thwaites Glacier Basin, West Antarctica

Ian Joughin, Benjamin E. Smith, Brooke Medley



# Reading

Mercer 1978:

**West Antarctic ice sheet and CO<sub>2</sub> greenhouse effect: a threat of disaster**

<https://www.nature.com/articles/271321a0>

# Summary

- Ice divides are slow moving regions that form in the interior of ice sheets and on isolated domes (ice rises) near the coast.
- Nonlinear ice rheology causes deep ice to stagnate beneath divides and causes upwarping in isochrones (Raymond bumps).
- The grounding line separates grounded ice from floating ice.
- Flux through the grounding line partly controls Antarctic mass balance.
- Theory suggests that grounding lines migrate unstably (MISI).
- Ice-shelf buttressing stabilizes grounding lines.

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- Weertman, J., 1974. Stability of the junction of an ice sheet and an ice shelf. *Journal of Glaciology*, 13(67), pp.3-11.