Glaciology
EESCGU4220

Practical 3:
Solving equations with the finite-difference method

Imagine we have a model saying how ice thickness H varies in time.

$$\frac{dH}{dt} = \frac{(H_f - H)}{B}$$

H is ice thickness
T is time
H_f is final ice thickness
B is a constant

Solve numerically

Solve analytically and evaluate

Compare the results

How to quantify the comparison?
How does the comparison change with the resolution of the model?

Solve analytically under the initial condition of $H(t=0) = 0$

H is ice thickness
T is time
H_f is final ice thickness
B is a constant

(1) $$\frac{dH}{dt} = \frac{(H_f - H)}{B}$$

$$\boxed{H = H_f(1 - \exp\left(-\frac{t}{B}\right))}$$

To check this:

Sub. this back into (1)

$$\frac{dH}{dt} = H_f(\frac{1}{\mathrm{B}}\exp\left(-\frac{t}{B}\right))$$

$$\frac{dH}{dt} = \frac{(H_f - H)}{B}$$

Today:

1. Compute H directly from the analytical solution.

$$H = H_f(1 - \exp\left(-\frac{t}{B}\right))$$

2. Compute it numerically using the finite difference approximation:

Our model equation

$$\frac{dH}{dt} = \frac{(H_f - H)}{B}$$

Finite-difference approximation

$$\frac{dH}{dt} = \frac{H^{j+1} - H^{j}}{\Delta t}$$

?

3. Compare the two solutions, for example by plotting them on the same axes or computing the RMS mismatch. Then do this for many different values of Δt (a convergence test).