Water Resour Manage (2014) 28:2293–2314
DOI 10.1007/s11269-014-0616-0

Predicting Water Level Fluctuations in Lake Michigan-Huron Using Wavelet-Expert System Methods

Abdüsselam Altunkaynak

Received: 12 November 2013 / Accepted: 30 March 2014 /
Published online: 20 April 2014

Abstract Understanding and forecasting water level fluctuations in Lake Michigan-Huron is important for a variety of water resource management operations such as flood control, local water supply management, shoreline maintenance, ecosystem sustainability, recreation, and economic development. In this study, wavelet transform, fuzzy logic and multilayer perceptron techniques are combined to obtain new approaches for forecasting lake level fluctuation. The wavelet approach is used to decompose water level time series into its spectral bands. Predictive models have been developed as stand-alone fuzzy logic, stand-alone multilayer perceptron combined wavelet-fuzzy and combined wavelet-multilayer perceptron models in order to forecast the water level fluctuations. The models were tested to predict the current water level (at t monthly time step) and lead times including $t+3$, $t+6$, $t+9$ and $t+12$ time steps from the water levels at two previous time steps ($t-2$ and $t-1$). In this study, the historic water level data was obtained from Lake Michigan-Huron for the period between 1855 and 2006. For the model development, monthly water level data was divided into two groups. The training group consists of the data for the first 101 years (from 1855 to 1955) with 1212 data points, which were, then, used to predict the water levels for remaining 51 years (from 1956 to 2006). The results reveal that all the four models can predict the water levels quite accurately. In comparison, the combined wavelet-fuzzy logic and combined wavelet-multilayer perceptron models outperformed the stand-alone fuzzy and multilayer perceptron models for lead times of 1, 3, 6, 9 and 12 months. This comparison was performed based on the root mean squared error (RMSE), the coefficient of efficiency (CE), the mean absolute deviation (MAD) and the skill score (SS) between observed data and prediction results.

Keywords Lake Michigan-Huron · Wavelet · Water level fluctuations · Wavelet-Fuzzy logic · Wavelet-multilayer perceptron

1 Introduction

Lakes Michigan, Huron, Superior, Erie and Ontario, which are collectively known as the Great Lakes, constitute the largest freshwater resource in the United States. Among them, lakes

A. Altunkaynak (✉)
Faculty of Civil Engineering, Hydraulics Division, Istanbul Technical University, Maslak 34469 Istanbul, Turkey
e-mail: altunkay@itu.edu.tr

Michigan and Huron are interconnected by a deep channel through the Straits of Mackinac, which are considered to behave hydraulically as one lake, and they are referred as Lake Michigan-Huron (Bishop 1990; Brinkman 2000). These two lakes are individually found to exhibit very similar patterns in terms of water level variability.

Water level changes in the Lake Michigan-Huron represent a change in water availability or stored water volume. These changes occur on several timescales. The seasonal variability in the water level reflects the annual hydrologic cycle in the Great Lakes Basin. Water level fluctuations in Lake Michigan-Huron also occur on longer timescales such as interannual, decadal and higher. These fluctuations reflect the influence of large-scale climatic processes through their effect on precipitation and river flow in the Great Lakes Basin and surrounding areas (Changnon 1987, 2004; Thompson and Baedke 1997; Baedke and Thompson 2000; Argyilan and Forman 2003; Assel et al. 2004; Booth et al. 2002; Ghanbari and Bravo 2008; Harnahan et al. 2009; Hartmann 1990; Polderman and Pryor 2004).

Water level fluctuations can directly affect the hydrology and ecology within the lake and the surrounding watersheds (Coops et al. 2003; Leira and Cantonati 2008; Webb 2008). High water levels can degrade reservoir operations, affect the lakeside plant and animal communities and result in shoreline erosion (Meadows et al. 1997). High levels of water may also boost the amount of nutrients from runoff and flooded lakeshore soils. In addition, fluctuating water levels may lead to erosion of the lakebed by exposing new surfaces. Erosion may occur at low, average and high water levels. High water levels can intensify the erosion process. On the other hand, low water levels may adversely affect water supply in the area, and cause stressful conditions for fish and other aquatic species. Wide variations in lake level result in the formation of wetlands surrounding the lake area. Lake level changes may also influence a variety of human activities including commercial shipping and recreational boating. In addition, water level variations can serve as a proxy for climate change in the Great Lakes region and adjoining areas.

Forecasting the water levels in Lake Michigan-Huron is useful for a variety of water resources management operations including flood control, local water supply management, shoreline maintenance, ecosystem sustainability, recreation, and economic development. Only a few researchers have studied on predicting water levels in the Great Lakes (Cohn and Robinson 1976; DeCooke and Meregian 1967; Irvine and Eberthardt 1992; Slivitzky and Mathier 1993; Lofgren et al. 2002). Almost all of these studies were performed using conventional parametric time series modeling techniques. Several studies on wavelet models have been published in the literature (Makarynskyy et al. 2004; Lee et al. 2007; Zhang et al. 2010). Recently, Cengiz (2011) applied continuous wavelet transforms and global spectrums in order to determine periodic structures and water level variations with time scale. It was found that the periodic components vary between 1-year (annual cycle) and 43-year scales in the Great Lakes water levels. In addition, major lake level periodicities are generally determined as an annual cycle. To my best knowledge, wavelet-expert system methods have not been used for predicting lake level changes in the Great Lakes Basin. The purpose of this work was, therefore, to develop predictive models for water level changes in Lake Michigan-Huron using wavelet-expert system methods such as wavelet-fuzzy logic and wavelet-multilayer perceptron. Fuzzy logic and multilayer perceptron models were also used to assess the efficiency of the wavelet-fuzzy and wavelet-multilayer perceptron methods developed here.

2 Study Area and Data

A geographical map showing the locations of the Great Lakes is presented in Fig. 1. The Great Lakes watershed can be divided into a southern lowland region and a northern upland region.

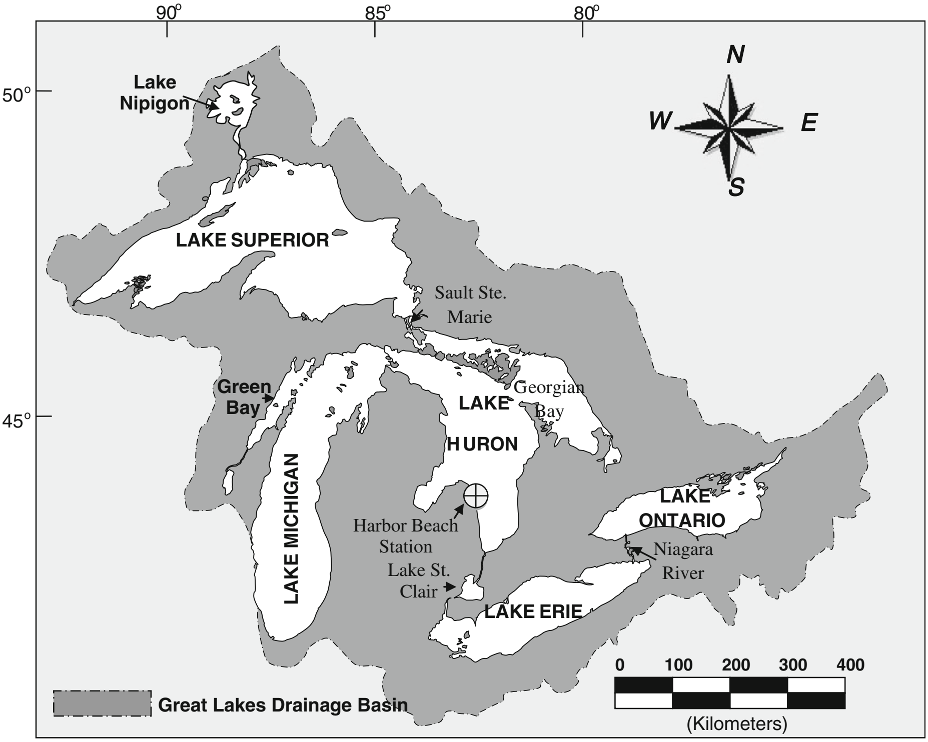


Fig. 1 Geographical map showing the locations of the Great Lakes

The lowland region includes the Erie and Michigan basins and the most of the Huron and Ontario basins. The upland region contains the most of the Superior basin and parts of the Ontario basin (Larson and Schaetzl 2001). The Great Lakes Basin, including the international section of the St. Lawrence River above Cromwall, Ontario, and Massena, New York, covers about 402 km^2 (Neff and Nicholas 2005). It includes parts of eight states and one province: Minnesota, Wisconsin, Illinois, Indiana, Michigan, Ohio, Pennsylvania, New York, and Ontario. About 59 % of the basin is located in the United States, and about 41 % is in Canada. The basin is about 1125 km long from north to south, and about 1450 km long from the west to the outlet of Lake Ontario at Cromwell and Massena in the east. The population of the basin is about 33 million. Major commerce and industries in the basin include manufacturing, tourism and agriculture (Wilcox et al. 2007).

Lake Erie is the shallowest part of the Great Lakes with an average depth of 19 m, followed by Lakes Huron (60 m), Michigan (85 m), Ontario (87 m), and Superior (148 m). As mentioned in Section 1, the lakes Michigan and Huron are considered to behave hydraulically as one lake, and are referred to as Lake Michigan-Huron. Water levels in Lake Michigan-Huron have been accurately monitored since the middle of the nineteenth century. Monthly historical data of water levels in Lake Michigan-Huron was obtained from http://www.glerl.noaa.gov/data/now/wlevels/dbd/GLWLDDataDownloads2.html for the period between 1855 and 2006. Also, the water levels data was measured at Harbor Beach Station as depicted in Fig. 1.

3 Methodology

3.1 Expert System Methods

Expert system consists of a family of methods that imitate human intelligence with the goal to create tools with human-like capabilities such as learning, reasoning and decision making. These methods are based on fuzzy logic, multilayer perceptron and genetic algorithms etc. (Zadeh 1994; Haykin 1994; Sales et al. 1994; Goldberg 1989; See and Openshaw 1999). Advances in soft computing have led to the development of new methods for solving complex nonlinear systems which could not be addressed by traditional methods. Expert system techniques are being increasingly used for modeling and forecasting nonlinear time series in diverse applications.

3.2 Fuzzy Logic Model

Recently, fuzzy inference systems based on fuzzy logic set theory have been applied to many areas ranging from sociology to hydrology (Altunkaynak 2010; Altunkaynak and Şen 2007; Altunkaynak et al. 2005; Chang and Chang 2006; Alvisi et al. 2006; Bardossy and Duckstein 1995; Bogardi et al. 2003; Chang et al. 2005; Hatiboglu et al. 2010; Katambara and Ndiritu 2009; Nayak et al. 2004; Ozelkan and Duckstein 2001; Ozger 2009; Sen and Altunkaynak 2006; Tsoukalas and Uhrig 1997; Uyumaz et al. 2006; Xiong et al. 2001). The main idea behind fuzzy logic is using soft borders rather than using crisp borders. In classical set theory, an element is either a member of a set or not. In contrast, a fuzzy set contains elements with varying degrees of membership ranging between 0 and 1. This feature provides a very flexible tool for fuzzy inference system. A fuzzy inference system establishes a link between the inputs and outputs. For the present study on Lake Michigan-Huron, a fuzzy inference system was used with two inputs and one output. The system configuration is shown in Fig. 2. The fuzzy inference system uses a number of components such as membership functions, logical operations, and IF-THEN rules. The membership functions are used to determine the weights of the fuzzy IF-THEN rules, and these rules are employed to connect the inputs to outputs. In this study, the Takagi-Sugeno architecture with these inputs, outputs and fuzzy rules were used as shown in Fig. 3. There are mainly two different inference systems which are proposed by Mamdani (1974) and Takagi and Sugeno (1985). The Mamdani approach can process verbal data (Altunkaynak 2010; Sen and Altunkaynak 2009; Hatiboglu et al. 2010). It can be very useful where numerical data is absent. On the other hand, the Takagi-Sugeno approach is very

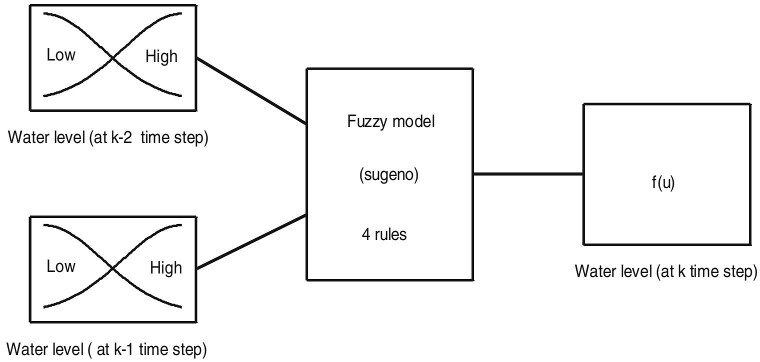


Fig. 2 Configuration of the fuzzy inference system

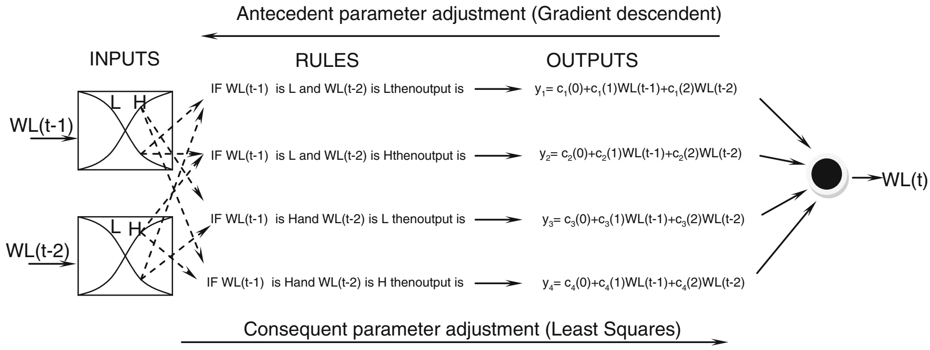


Fig. 3 Takagi-Sugeno architecture demonstrating inputs and outputs of system

effective when a large amount of numerical data exists. We use the Takagi-Sugeno approach (Takagi and Sugeno 1985). A sample fuzzy rule is shown as:

$$R_r : \mathrm{IF}(\mathrm{I}_1 \text{ is } S(1), \mathrm{I}_2 \text{ is } S(2), \ldots., \mathrm{I}_m \text{is } S(m), \quad \mathrm{THEN}\ y_r = f_r(\mathrm{I}_1, \quad \mathrm{I}_2, \ldots, \mathrm{I}_m) \quad (1)$$

where $S(i)$ is a fuzzy set of the input variable I_j in the *r-th* IF-THEN rule, m the number of input variables, f_r (.) a function of the m input variables, and y_r is the output of the *r-th* IF-THEN inference rule R_r. Application of the Takagi-Sugeno fuzzy inference system involves the following steps: (a) fuzzification of inputs by membership functions which hold information on the elements of the set and their membership degrees to the set, (b) determining the weights of each rule using a fuzzy operator, namely, the min (minimum) or prod (product) operator, (c) shaping the consequent part after determining the weights (between 0 and 1) of each rule from the previous step, (d) aggregation of all outputs obtained from the shaped consequent part, and (e) defuzzification to obtain a single value from the aggregated outputs. It is possible to express the general algorithm of the Takagi-Sugeno fuzzy inference system as follows. Assume that there are R_r (r =1, 2,…, n) rules in the fuzzy IF-THEN rules system.

i) For each application of R_r, the output y_r is determined as a function f_r in the consequent part.

$$y_r = f_r(I_1, \quad I_2, \quad I_3\ldots\ldots, I_m) = c_r(0) + c_r(1)I_1 + \ldots + c_r(m)I_m \quad (2)$$

ii) Weights of the fuzzy rule are calculated as

$$w_r = \left(d_1^r \wedge d_2^r \wedge d_3^r \ldots\ldots, d_k^r\right) x R^r \quad (3)$$

Here, d_1^r, d_2^r, ... d_k^r indicate the membership degrees obtained from input variables for the r-th rule. Occurrence probability of the rule is R^r, which is considered 1 due to its simplicity, and the character $\wedge$ stands for min operations.

iii) The desired final output, O, is calculated from the weighted average of all y_r with the rule weights w_r.

$$O = \frac{\sum_{r=1}^{n} w_r y_r}{\sum_{r=1}^{n} w_r} \quad (4)$$

3.3 Multilayer Perceptron Model

The Artificial Neural Networks (ANNs), essentially a powerful black-box model, have a flexible mathematical structure that is capable of identifying complex non-linear relationships between inputs and outputs without predefined knowledge of the underlying physical processes involved in the transformation (Seckin et al. 2013). An Artificial Neural Network (ANN) is a computationally efficient alternative for addressing a problem (Chen et al. 2013).

The concept of the artificial neural network (ANN) was introduced in 1958 by Rosenblatt in the form of a single-layer feed forward network called perceptron. Following this, the multilayer perceptron (MLP) consisting of three or more layers was developed, but these ANN applications could not be used effectively to analyze complex systems until Rumelhart et al. (1986) proposed the back propagation algorithm as a means of training the network. Since then, ANNs have enjoyed their popularity in many applications including hydrological processes (Abrahart et al. 2004; Altunkaynak 2007; Altunkaynak and Strom 2009; Alvisi et al. 2006; ASCE Committee 2000; Bardossy et al. 2006; Campolo et al. 1999; Dawson and Wilby 2001; Daliakopoulos et al. 2005; Govindaraju and Rao 2000; Hartmann et al. 2007; Jain and Kumar 2007; Krishna et al. 2008; Lliadis and Maris 2007; Maier and Dandy 2000; May and Sivakumar 2009; Mutlu et al. 2008; Nayak et al. 2006; Panagoulis 2006; Ramirez et al. 2005; Riad et al. 2004; Thimuralaiah and Deo 2000; Mohanty et al. 2010). According to Desalegn and Babel (2011), ANNs have become quite popular in the analysis of non-linear and non-stationary hydrologic data. During the past decade, there have been further interests in using ANNs in conjunction with other expert system techniques such as fuzzy logic and genetic algorithms (Altunkaynak 2009, 2013; Firat and Gungor 2008; Katambara and Ndiritu 2009; Wu and Chau 2006; Nayak et al. 2004). Desalegn and Babel (2011) used ANN to develop a model for a long-term streamflow forecasting at a stream gauging station in the Awash River Basin, Ethiopia. There are a vast number of additional reports on ANNs and their applications in the literature (Samarsinghe 2007). For the present purpose, the characteristic features of a three-layer perceptron, which is one of the simplest and the most commonly used multilayer perceptron architecture, is briefly described and this was also used here.

The architecture of a fully-connected three-layer perceptron is depicted in Fig. 4. It consists of an input layer, a hidden layer and an output layer. There are m input neurons, n neurons in the hidden layer, and t neurons in the output layer. Each neuron from the input layer is connected by a weight to each neuron in the hidden layer; similarly, the neurons in the hidden layer are connected to each neuron in the output layer. The input and output signals of the network are designated by I_1, I_2.....I_m, and O_1, O_2.....O_t, respectively. The output signals forming the hidden neurons are denoted by h_1, h_2.....h_n. The bias for the jth-th hidden neuron is α_j, and θ_k is the bias for the kth neuron in the output layer. The weight of the connection from an input neuron i to a hidden neuron j is denoted by a_{ij}, whereas b_{jk} represents the weight of the connection from a hidden neuron j to a neuron k in the output layer. The connection weights in this network can be determined by training the network. The input to the jth neuron in the hidden layer is the weighted sum of the input signals (I_1, I_2.....I_m) and the bias. The output form of this hidden neuron is given by:

$$h_j = f\left(\alpha_j + \sum_{i=1}^{m} a_{ij}\ I_i\right), \tag{5}$$

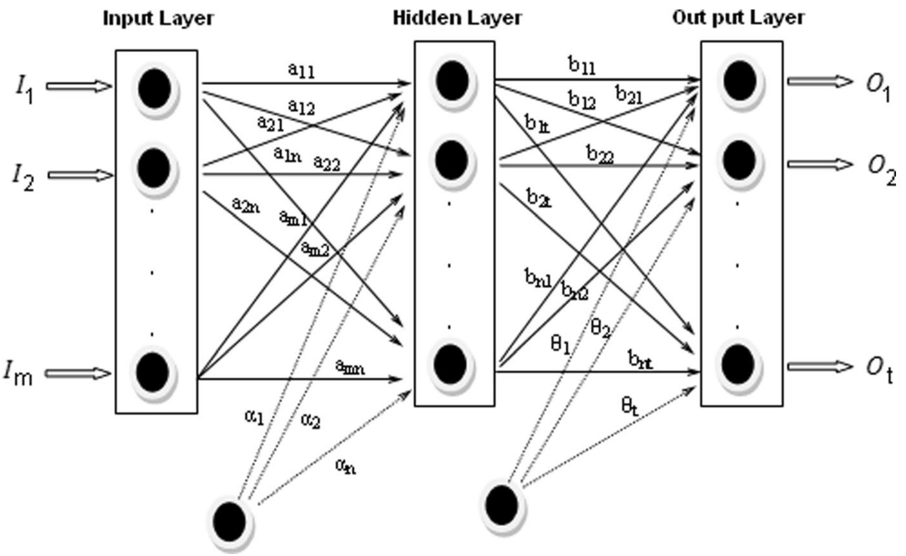


Fig. 4 Configuration of a three-layer perceptron with *m* nodes in the input layer, *n* nodes in the hidden layer, and *p* nodes in the output layer

where f (.) is an activation function. Similarly, the output from the *k*th neuron in the output layer is given by:

$$O_k = g\left(\theta_k + \sum_{j=1}^{n} b_{jk} \; h_j\right). \tag{6}$$

Here, g(.) is an activation function for the output layer. The activation function for the hidden and output layers can be chosen from a variety of functions such as a linear function, step function, sigmoid or tangent sigmoid (tansig) function. The multilayer perceptron can be trained by several approaches using observed data to determine the optimum weights. For this purpose, we used the back propagation training algorithm as given by Rumelhart et al. (1986).

The number of neurons in a hidden layer improves the skill of the network to model complex systems. However, it does not necessarily mean that the higher the number of neurons, the more capable the model is. Excessive number of neurons may cause over-learning in most cases and degrade the performance of the network (Altunkaynak 2007). In this study, tansig and pureline functions were employed in the hidden and output layers, respectively, because they give better prediction results compared to other activation functions.

3.4 Wavelet Transform

A wavelet is an irregular and asymmetric waveform which constitutes the hub of the wavelet transform. Currently, wavelet transformation is most important in terms of time and frequency analysis. According to specific purposes, wavelet transforms can be classified as continuous wavelet transforms, discrete wavelet transforms, and complex wavelet transforms etc. As opposed to Fourier series, wavelet analysis separates a series into shifted and scaled part of the mother wavelet. The information about time and frequency is provided by the transformed signal. Fourier transformation includes information with short time transformation. Wavelet transformation includes short time and also indicates the resolution in time at higher analysis frequencies of the main function (Torrence and Compo 1998). The time series can be divided into its sub-series using both time and frequency domains by wavelet transform. The wavelet transform facilitates determining the occurrence time of the low and high frequency information. According to Cobaner (2013), the wavelet transform is capable of computing separately for different segments of the time-domain signal at different frequencies and is the important derivative of the Fourier transform. In other words, it decomposes the signal into a mutually orthogonal set of wavelets. This is a technique that can be applied to all kind of time series.

A special wavelet function ψ(t), which is called mother wavelet, is determined before initiating any application. Daughter wavelets are generated by using mother wavelet. New generated wavelets are translated and scaled versions of the mother wavelet. The function can be applied for both wavelet decomposition and composition transforms (Torrence and Compo 1998). A wavelet function ψ(a,b) is given as:

$$\Psi(a,b) = b^{-1/2}\Psi\left(\frac{t-a}{b}\right) \tag{7}$$

where, t is time, and a and b are position and scale parameters, respectively. The continuous wavelet transform (CWT) was applied in this study. Continuous time series can be divided into wavelets using CWT, which has the ability to superpose time and frequency representation of signals. Figure 5 shows the power of the wavelet transform for the water level time series data. Power is calculated by taking the square of absolute value of W(a,b). The squared absolute value gives information on the relative power at a certain scale.

The CWT of time series $f(t)$ is defined as follows:

$$W(a,b) = b^{-1/2}\int_{-\infty}^{+\infty} f(t)\Psi^{*}\left(\frac{t-a}{b}\right)dt \tag{8}$$

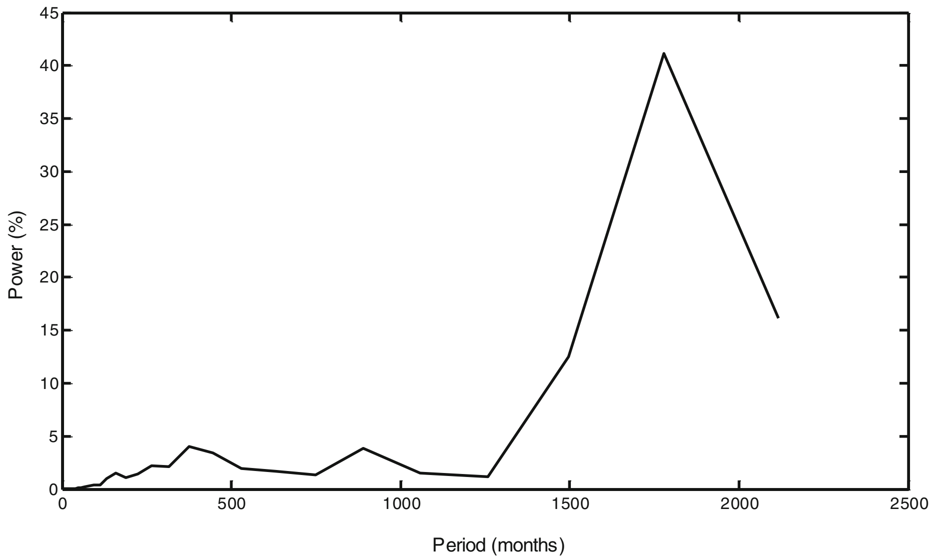


Fig. 5 Average wavelet spectra for water level series

where * is the complex conjugate. The Morlet wavelet function was used for the application. Morlet wavelet is defined as the product of a complex exponential wave and a Gaussian envelope. Information in detail about decomposition procedures can be found in the study by Torrence and Compo (1998).

3.5 Wavelet Band Decomposition

Selection of bands having significant power is the most important step for wavelet modeling. To date, no decomposition rule that divides time series into several bands has been developed. Using the average wavelet spectra for band separation constitutes the main approach. No band selection algorithm has been developed in the literature so far. Significant spectral bands can be found from average wavelet spectra. Considering the power distribution, bands that contain power peaks can be selected as spectral bands in the study. Significant spectral bands that are detected from average wavelet spectra are then inversely transformed to obtain new subseries. Six bands are used and the band intervals are 0–9, 10–15, 16–20, 21–29, 30–32 and 33–41 which corresponds to bands-1,2,3,4,5 and 6, respectively. The water level series are separated into six sub-bands using average wavelet spectra as shown in Fig. 5. Significant power at specific intervals in the average wavelet spectra can be observed from this figure. Each of the intervals includes at least one peak power determined as sub-bands of the variable concerned. It is not possible to see the specific information directly from the time series. However, the characteristic of time series such as trend, periodicity, noise etc. can be seen by using the sub-bands of the time series. Each sub-band holds a property that belongs to the observed data. For instance, the lower level band which shows the variation at high frequencies gives insight about the noisy part of the data. Higher level band which represents the low frequencies holds information about long term cycles.

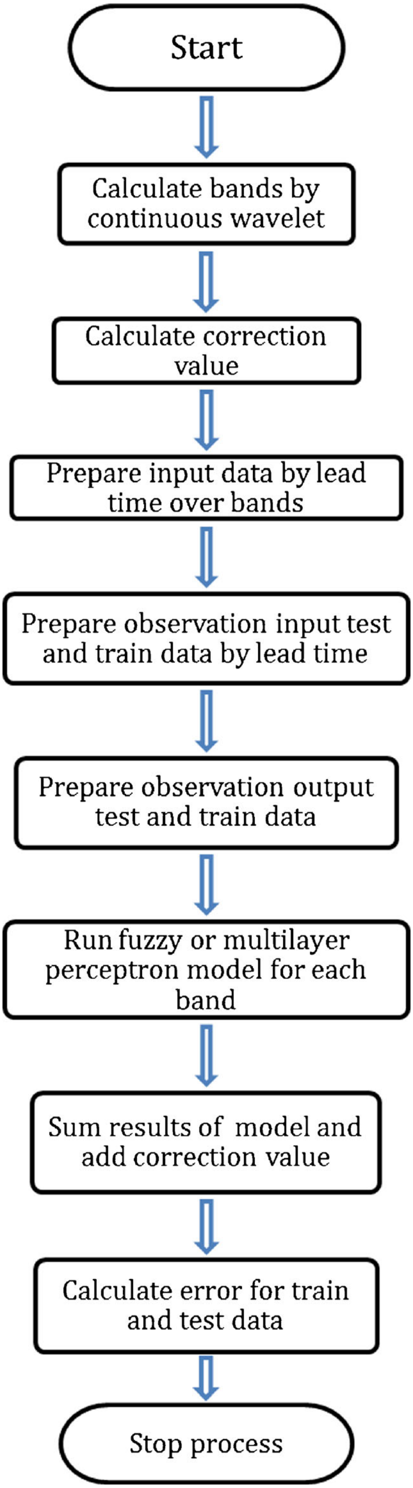
3.6 Wavelet Combination Models

Initial processing of data is useful for solving prediction problems. It has been demonstrated that prediction performance is augmented when a preprocessing technique is used. In this study, continuous wavelet transform (CWT) was used in order to process the water level time series data. Many applications on wavelet combination models have been studied in the literature (Kim and Valdes 2003; Kucuk et al. 2009; Webster and Hoyos 2004; Nourani et al. 2009; Ozger 2010; Shiri and Kisi 2010; Shiri et al. 2011). Monthly water level time series was decomposed into sub-signals (components) using continuous wavelet transform. Two combination models, wavelet-multilayer perceptron (W-MP) and wavelet-fuzzy logic methods (W-FL) are used. The main purpose of establishing a combination model is to separate the series into its subseries and subsequently to predict each subseries using an application method.

An application of wavelet transform was performed in order to acquire different subseries; and then, predictions were undertaken using multilayer perceptron and fuzzy logic by using the decomposed sub-series as inputs. The steps followed in this approach are summarized as: (1) The significant spectral bands from average wavelet spectra were assessed and the subseries were selected, (2) multilayer perceptron and fuzzy logic were used to model each subseries, individually. (3) Finally, concerned water level time series were obtained after reconstruction of the predicted subseries. Inverse continuous wavelet transform can be used for reconstruction. The flow chart of proposed models is presented in Fig. 6.

The purpose of making W-MP and W-FL combination is to predict water level for the next months including t, t+3, t+6, t+9 and t+12 time steps from two previous records. Future lake

Fig. 6 Flow chart of models



levels are considered as output of the model and previous lake levels as inputs for this model. The water level series are separated into six different spectral bands using wavelet transform. The water level series are separated into six sub-series as shown in Fig. 7(a–f). The water level sub-series are plotted on the probability paper as indicated in Fig. 8(a–f). Chi-square (χ^2) test was used to check if the distribution fits to normal distribution. It is found that subseries of water level fit not to normal distribution for 5 % significance level except Band-3 (Fig. 8c). This result can also be understood from Fig. 7 visually.

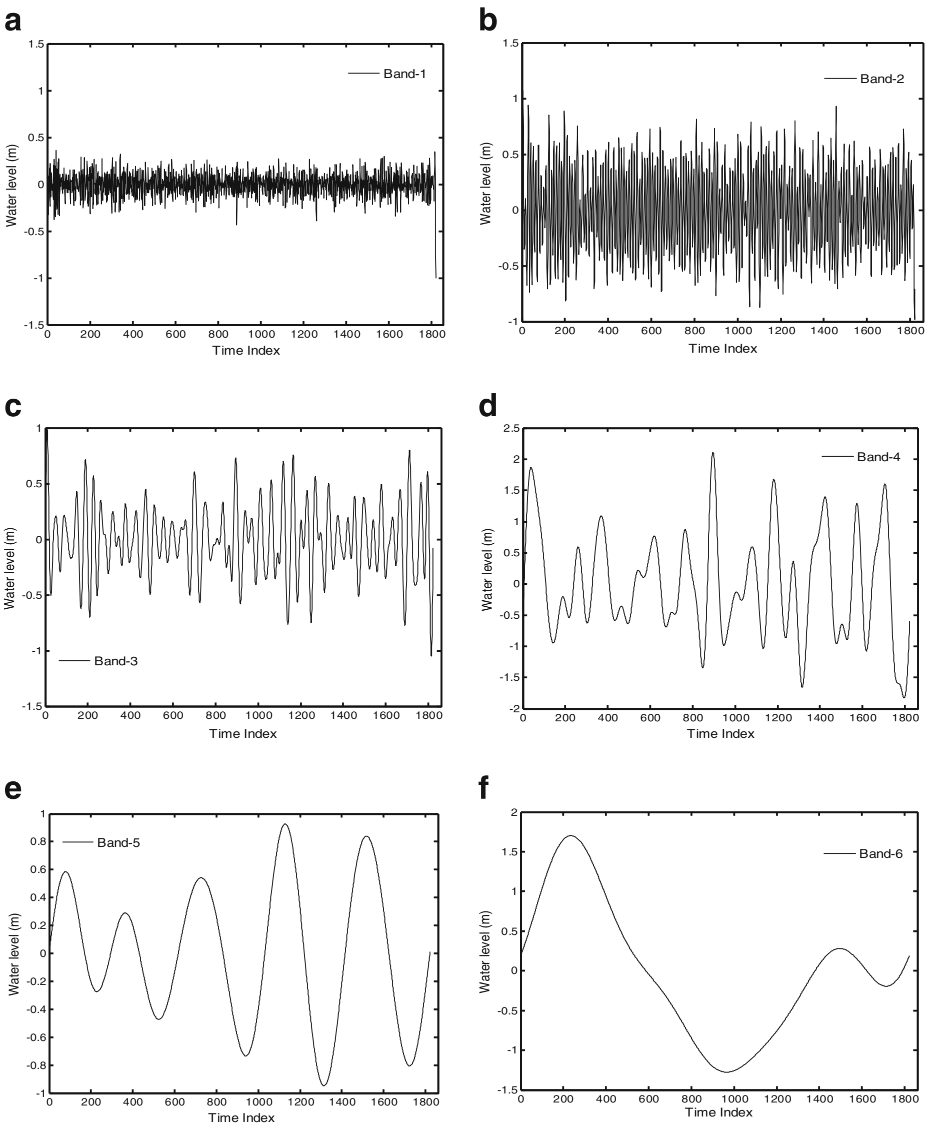


Fig. 7 The sub-series of the water level. **a** Band-1 (0–9), **b** Band-2 (10–15), **c** Band-3 (16–20), **d** Band-4 (21–29), **e** Band-5 (30–32)

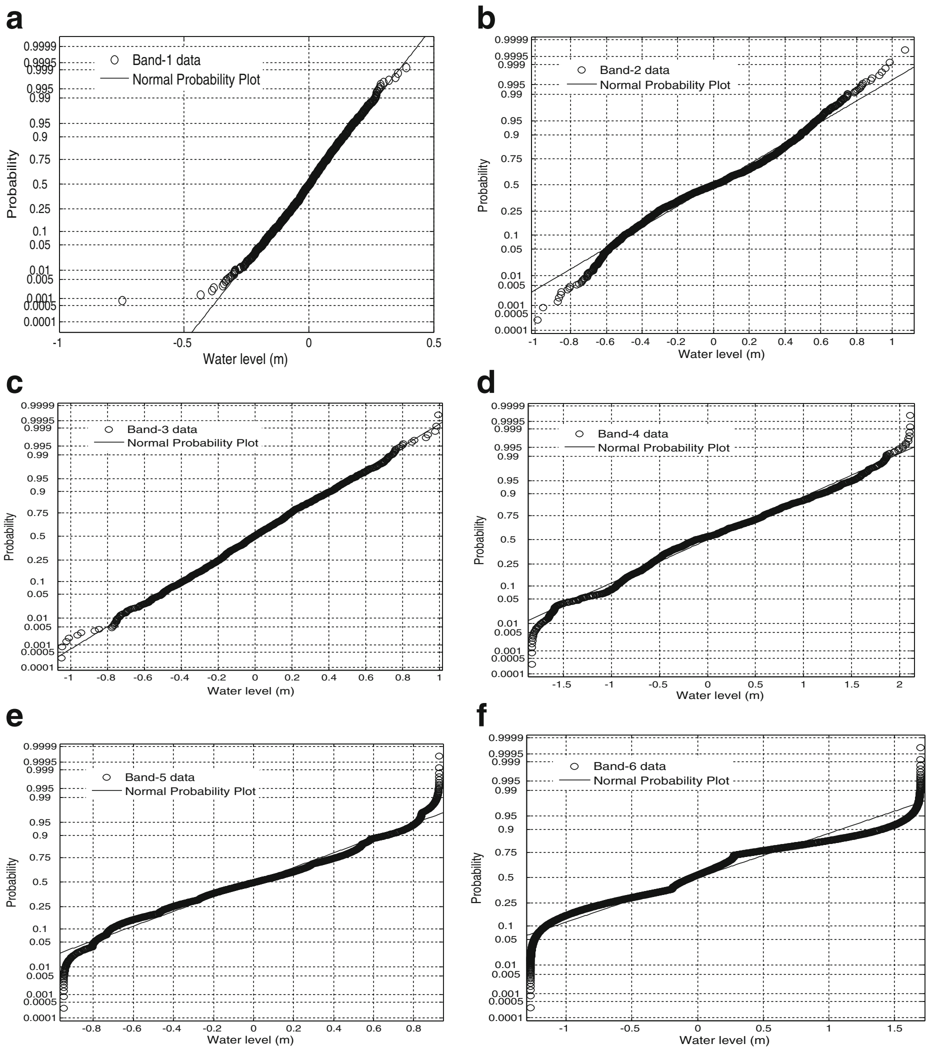


Fig. 8 Probability plots for the subseries of water level. **a** Band-1 (0–9), **b** Band-2 (10–15), **c** Band-3 (16–20), **d** Band-4 (21–29), **e** Band-5 (30–32)

Multilayer perceptron and fuzzy logic approaches were then used in order to make relationship between inputs and outputs. Fuzzy logic, multilayer perceptron, wavelet-fuzzy and wavelet-multilayer perceptron models were used to predict the water level fluctuations in Lake Michigan-Huron. These models were developed and tested to predict the current water level (at time step t) and lead times including $t+3$, $t+6$, $t+9$ and $t+12$ time steps from the water levels at two previous time steps ($t-2$ and $t-1$). In this analysis, the monthly historic water level data from January, 1855 through December, 2006 were used. The dataset was divided into two groups. The first dataset of 101 (1855–1955) years consisting of 1,212 data points were used for training (calibration), and the remaining 51 years (1956–2006) data consisting of 612 data points were used for prediction (verification). The water level series were separated into six different spectral bands as shown in Fig. 7(a–f). In addition, distribution plots of the

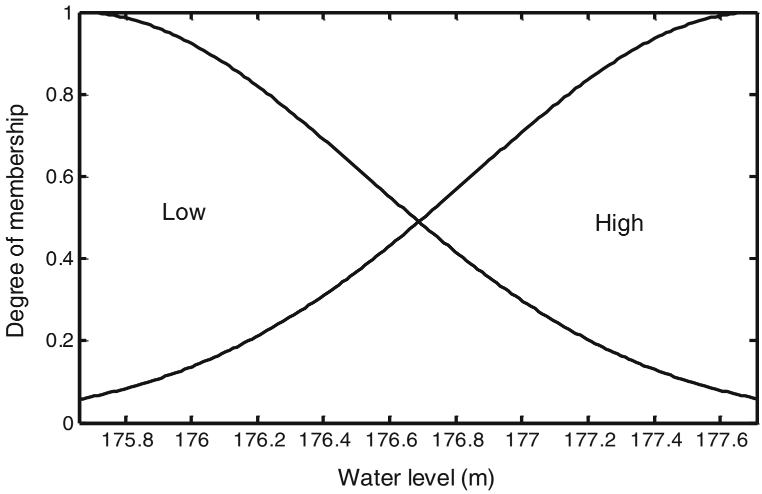


Fig. 9 Division of the input variables (water level at k-1 time step) into low and high fuzzy sets

bands are presented in Fig. 8(a–f). In the fuzzy logic formulation, the fuzzy input variables were divided into two fuzzy sets: Low (L) and High (H), as indicated in Fig. 9. Gaussien type membership function is used in the analysis. Two inputs and two fuzzy sets resulted in 2x2=4 fuzzy rules. For the Lake Michigan-Huron level data, these rules are described in Table 1. A sample of the overall fuzzy inference system is given in Table 2.

4 Results and Discussion

Development of a multilayer perceptron model requires selection of the appropriate number of neurons, and calibration of the weights using training data. Care should be taken in keeping the number of neurons in the hidden layer small in order to avoid over-learning by the network. Typically, anywhere between 3 and 7 neurons can be used effectively (Altunkaynak 2007). In this model, three neurons were used in the hidden layer. The functions, tansig and pureline, were used as activation functions for the hidden layer and output layer, respectively. The temporal variation of the Lake Michigan-Huron water levels is shown in Fig. 10. This is the monthly fluctuation that spans the period from 1855 to 2006.

Figures 10(a and b) depicts the time series plots of the observed water level data, training data, as well as the water levels predicted by the fuzzy logic, multilayer

Table 1 Fuzzy rule base

Rule no	Antecedent		Consequent
	WL(t−1)	WL(t−2)	Output, y
R1	Low	Low	y_1=−0.801* WL(t−1)+1.267* WL(t−2)+93.593
R2	Low	High	y_2=36.116* WL(t−1)−34.883* WL(t−2)−13.208
R3	High	Low	y_3=37.905* WL(t−1)−36.836* WL(t−2)−40.619
R4	High	High	y_4=−0.583* WL(t−1)+0.813* WL(t−2)+137.013

Table 2 A sample of the Takagi-Sugeno fuzzy inference system

Rules	Antecedent		Consequent, y	r_i production
R_1	Low 0.820 175.7	Low 0.637 175.7	176.08	0.820*0.637 = 0.522
R_2	Low 0.820 175.7	High 0.389 177.7	193.58	0.820*0.389 = 0.319
R_3	High 0.211 177.7	Low 0.637 175.7	136.69	0.211*0.637 = 0.135
R_4	High 0.211 177.7	High 0.389 177.7	177.78	0.211*0.389 = 0.082
	WL(t-1)=176.2 m	WL(t-2)=176.5 m		WL(t)=176.48 m

perceptron, wavelet-fuzzy, wavelet-multilayer perceptron models. Visual inspection of this figure indicates that there is a good agreement between the expert system methods and the wavelet-expert system model in predicting the water levels. Performance of these models was also assessed quantitatively using the root mean square error (RMSE), the coefficient of efficiency (CE) and the mean absolute deviation (MAD) measures, which are defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(WL_{pi}-WL_{oi}\right)^2} \tag{9}$$

$$CE = \left[1-\frac{\sum_{i=1}^{n}\left(WL_{pi}-WL_{oi}\right)^2}{\sum_{i=1}^{n}\left(WL_{oi}-WL_{a}\right)^2}\right] \tag{10}$$

$$MAD = \frac{1}{n}\sum_{i=1}^{n}\left|WL_{pi}-WL_{oi}\right| \tag{11}$$

Here, n is the total number of observations; WL_{pi}, WL_{oi}, WL_{ao} and WL_{pa} are predicted water level, observed data, average of the observed water levels, and average of predicted water levels respectively. For each model, the values of RMSE, CE and MAD were calculated using Eqs. (9), (10) and (11), respectively,

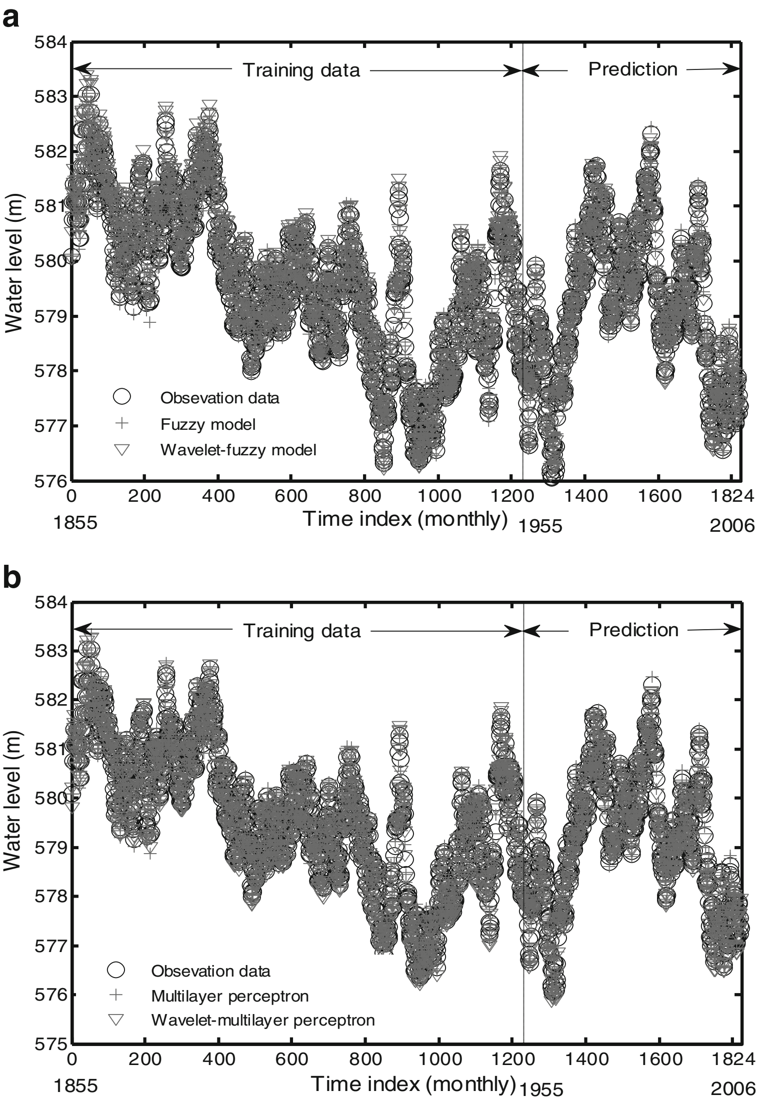


Fig. 10 Comparison of water levels predicted with observed data (**a**) fuzzy logic and wavelet-fuzzy (**b**) multilayer and wavelet-multilayer perceptron methods

and they are listed in Table 3. Note that the RMSEs of the water level prediction for fuzzy logic, multilayer perceptron, wavelet-fuzzy and wavelet-multilayer perceptron models are 0.166 m, 0.165 m, 0.112 m and 0.118 m, respectively, for lead times of 1-month. Clearly, the RMSE values of the wavelet-fuzzy and wavelet-multilayer perceptron models are found to be very close to each other. However, the stand-alone fuzzy and multilayer perceptron models have higher RMSE values than the other two models. This indicates that prediction results of the fuzzy and multilayer perceptron models are worse than those of the two wavelet-expert system methods.

Table 3 Performance comparison of fuzzy logic, multilayer perceptron, wavelet-fuzzy and wavelet-multilayer perceptron models in terms of the root mean square error (RMSE) the coefficient of efficiency (CE) and the mean absolute deviation (MAD) for lead times including t, t+3, t+6, t+9 and t+12 time steps

Lead time	Criterion used	Fuzzy Logic	Multilayer perceptron	Wavelet-fuzzy	Wavelet-multilayer
Lag-1	RMSE (m)	0.166	0.165	0.112	0.118
	MAD (m)	0.132	0.131	0.085	0.093
	CE	0.984	0.984	0.993	0.992
Lag-3	RMSE (m)	0.494	0.494	0.139	0.136
	MAD (m)	0.399	0.399	0.111	0.109
	CE	0.861	0.860	0.989	0.989
Lag-6	RMSE (m)	0.775	0.773	0.205	0.199
	MAD (m)	0.631	0.631	0.167	0.158
	CE	0.656	0.658	0.976	0.977
Lag-9	RMSE (m)	0.744	0.744	0.251	0.246
	MAD (m)	0.617	0.617	0.204	0.196
	CE	0.683	0.683	0.964	0.965
Lag-12	RMSE (m)	0.746	0.748	0.350	0.323
	MAD (m)	0.613	0.614	0.274	0.263
	CE	0.681	0.680	0.930	0.940

As seen from Table 3, the CE values of fuzzy logic, multilayer perceptron, wavelet-fuzzy, wavelet-multilayer perceptron models are 0.984, 0.985, 0.993 and 0.992, respectively, for lead times of 1-month. Also, the MAD values for fuzzy logic, multilayer perceptron, wavelet-fuzzy, wavelet-multilayer perceptron models are 0.132, 0.131, 0.085 and 0.093, respectively. These statistical values indicate the presence of high consistency between the observed data and the predicted results by these four models. The CE values for fuzzy logic and multilayer perceptron models are almost the same; however, the CE values for the expert system models are less than those of the wavelet-expert system models. RMSE and CE of the models for all the lead times are tabulated in Table 3. As can be seen from Table 3, RMSE and CE values are close to each other for fuzzy and multilayer models, and those are close to each other for each lead time in wavelet-fuzzy and wavelet-multilayer perceptron models. As a result, it can be concluded that performances of wavelet-fuzzy and wavelet-multilayer perceptron models are better than the stand-alone fuzzy and multilayer models for all lead time predictions. Water levels predicted by fuzzy logic, multilayer perceptron, wavelet-fuzzy and wavelet-multilayer perceptron models as compared to the observed water levels were plotted on a 1:1 line and are presented in Fig. 11(a–d). The 1:1 line (perfect model line) is also shown as the 45° degree diagonal line in this figure. Figures 11(a) and (b) indicate the presence of a very good agreement between the observed data and the fuzzy and multilayer model predictions. Figures 11(c) and (d) reveal that there is an excellent match between the observed data and the results predicted by the wavelet-fuzzy logic and wavelet-multilayer perceptron models, respectively. On the other hand, from Figures 11(c) and (d), it is clear that the results of the wavelet-fuzzy and wavelet- multilayer perceptron models tend to predict the observed water levels more accurately as the scattered plots are closer the 1:1 line. In addition, the scatter plots for lead time prediction at t+12 time step are shown in Fig. 12 (a–d). It is clear that the scattered plots of wavelet-fuzzy logic

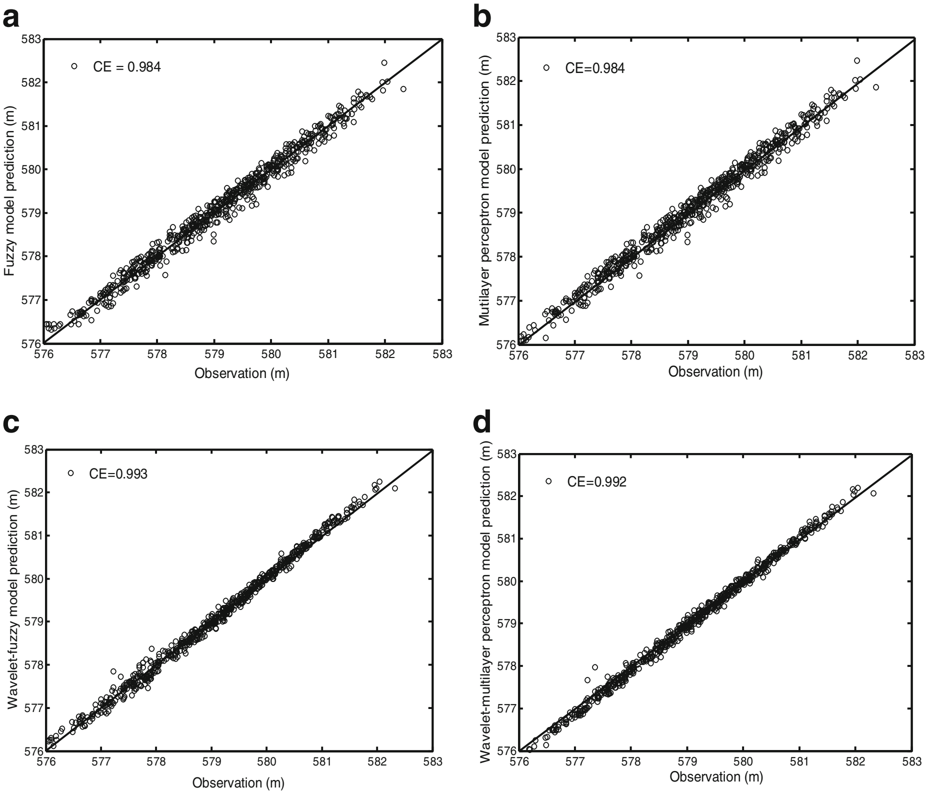


Fig. 11 Perfect model line for current water level at t time step: (**a**) fuzzy model prediction, (**b**) multilayer perceptron model prediction (**c**) wavelet-fuzzy model prediction, and (**d**) wavelet-multilayer perceptron model prediction

and wavelet-multilayer perceptron models are better than those of fuzzy and multilayer models.

To validate the results of the models developed in this study, the skill score (SS) parameters were also calculated for wavelet-fuzzy and wavelet-multilayer perceptron models. The SS parameter is defined as:

$$SS = 1 - \frac{MSE_P}{MSE_R} \tag{12}$$

where, MSE_P=mean square error of wavelet-fuzzy and wavelet-multilayer perceptron models; MSE_R=the mean square error of fuzzy and multilayer perceptron models. Here, in order to calculate SS for wavelet-fuzzy model, reference value is taken as the mean square error of fuzzy model. Similarly, for wavelet-multilayer model, the reference value is assumed to be the mean square error of multilayer perceptron model. The SS parameters are found to be 0.549 and 0.487 for wavelet-fuzzy and wavelet-multilayer perceptron models, respectively. These results are presented in Table 4 for lead times 1, 3, 6, 9 and 12 months. It is clearly observed, from this table, that wavelet-fuzzy logic and wavelet-multilayer perceptron models produce very close results to each other, taking the SS values in to consideration. Therefore, it can be generalized that wavelet-fuzzy and wavelet-multilayer perceptron models are found to have

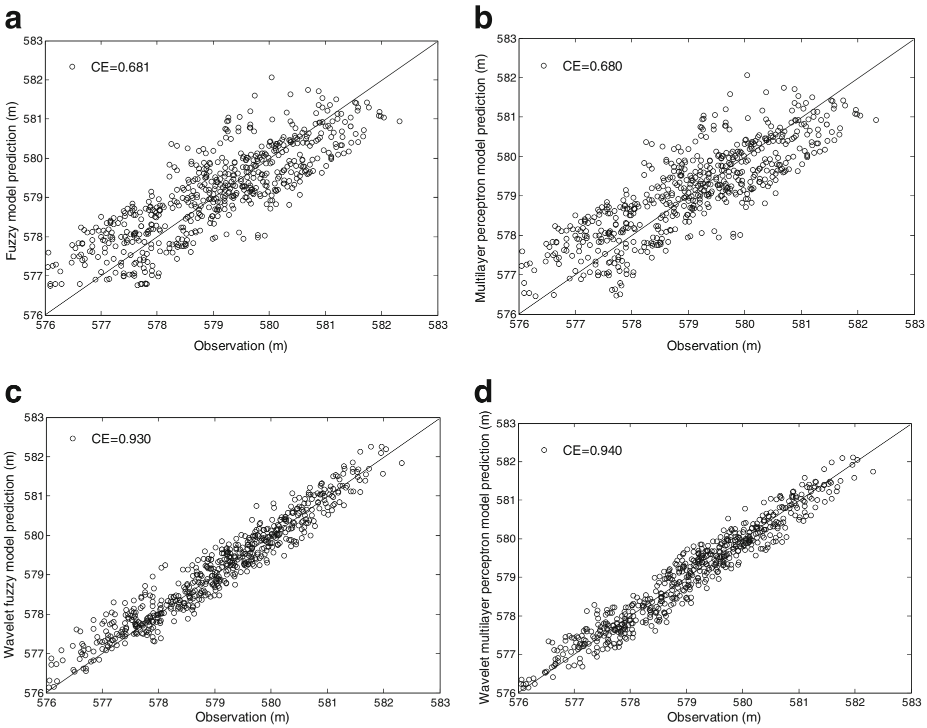


Fig. 12 Perfect model line for current water level at t+12 time step: (**a**) fuzzy model prediction, (**b**) multilayer perceptron model prediction (**c**) wavelet-fuzzy model prediction, and (**d**) wavelet-multilayer perceptron model prediction

better performance compared to fuzzy logic and multilayer perceptron models based on the statistical parameters.

In summary, the rule-set in the fuzzy logic model used here consists of four simple rules. it is noted that the fuzzy model can predict the water levels very accurately using these rules without depending on complicated mathematical operations and restrictive assumptions. Similarly, a three-layer feed forward multilayer perceptron with one hidden layer and three neurons in this hidden layer are found to give accurate predictions for the water levels. In comparison, the wavelet-fuzzy logic and wavelet-multilayer perceptron models outperform the fuzzy and multilayer perceptron models.

Table 4 Performance of wavelet-fuzzy and wavelet-multilayer perceptron models in terms of the skill score (SS) for lead times including t, t+3, t+6, t+9 and t+12 time steps

Lead time	Criterion used	Wavelet-fuzzy	Wavelet-multilayer
Lag-1	SS	0.549	0.487
Lag-3	SS	0.921	0.925
Lag-6	SS	0.930	0.933
Lag-9	SS	0.886	0.891
Lag-12	SS	0.780	0.813

5 Concluding Remarks

The wavelet technique is combined with fuzzy logic and multilayer perceptron to develop new predictive approaches. The water level series is decomposed to its spectral bands. Using fuzzy logic, multilayer perceptron, wavelet-fuzzy and wavelet-multilayer perceptron models, predictive models were developed in order to forecast water level fluctuations for Lake Michigan-Huron. Monthly historic water level data from the period 1855–2006 consisting of 1,824 data points was used for the study. In this study, the developed models are tested to predict current water level and lead time steps including 3, 6, 9 and 12 months from water levels at two previous months. For the model development, the observed data was divided into two groups. The training group consisted of the data for the first 101 years including 1,212 data points, which were used to predict the remaining 612 data points. All four models were found to predict the water levels quite accurately. Their performance was quantitatively assessed using the root mean square error, coefficient of efficiency, mean absolute deviation and skill score. Both wavelet-fuzzy logic and wavelet-multilayer perceptron models indicate a better prediction performance for all lead time steps compared to the fuzzy logic and multilayer perceptron models. One of the simplest multilayer perceptron models, the three-layer perceptron, which produced very accurate predictions for the water levels, has been employed.

References

Abrahart RJ, Kneale PE, See LM (2004) Neural networks for hydrological modeling. Balkema Publishers, London

Altunkaynak A (2007) Forecasting surface water level fluctuations of Lake Van by artificial neural networks. Water Resour Manag 21:399–408

Altunkaynak A (2009) Sediment load prediction by genetic algorithms. Adv Eng Softw 40:928–934

Altunkaynak A (2010) A predictive model for well loss using fuzzy logic approach. Hydrol Proced 24:2400–2404

Altunkaynak A (2013) Prediction of significant wave height using geno-multilayer perceptron. Ocean Eng 58: 144–153

Altunkaynak A, Şen Z (2007) Fuzzy logic model of water level fluctuations in Lake Van, Turkey. Theor Appl Climatol 90:227–233

Altunkaynak A, Strom KB (2009) A predictive model for reach scale morphology classification in mountain streams using multilayer perceptron methods. Water Resour Res 45, W12502. doi:10.1029/2009WR008055

Altunkaynak A, Özger M, Çakmakçı M (2005) Water consumption prediction of Istanbul city by using fuzzy logic approach. Water Resour Manag 19:641–654

Alvisi S, Marcellani G, Franchini K, Bardossy A (2006) Water level forecasting through fuzzy logic and artificial neural network approaches. Hydrol Earth Syst Sci 10:1–17

Argyilan EP, Forman SL (2003) Lake level response to seasonal climatic variability in the Lake Michigan-Huron system from 1920 to 1995. J Great Lakes Res 29:488–500

ASCE Task Committe on Application of Artificial Neural Networks in Hydrology (2000) Artificial neural networks in hydrology II. Hydrologic applications. J Hydrol Eng 5:124–137

Assel RA, Quinn FH, Sellinger CE (2004) Hydro-climatic factors in the recent record drop in Laurential Great Lakes water levels. Bull Am Meteorol Soc 85:1143–1151

Baedke SJ, Thompson TA (2000) A 4,700-year record of lake level and isostasy for Lake Michigan. J Great Lakes Res 26:416–426

Bardossy A, Duckstein L (1995) Fuzzy-rule based modeling with application to geophysical, biological and engineering systems. CRC Press, Boca Raton

Bardossy A, Mascellani G, Franchini M (2006) Fuzzy unit hydrograph. Water Resour Res 42, W02401

Bishop CT (1990) Historical variation of water levels in lakes Erie and Michigan-Huron. J Great Lakes Res 16: 405–425

Bogardi I, Bardossy A, Duckstein L, Pongracz R (2003) Fuzzy logic in hydrology and water resources. In: Demicco R, Klir G (eds) Fuzzy logic in geology. Elsevier, Amsterdam, pp 153–190

Booth RK, Jackson ST, Thompson TA (2002) Paleo-ecology of a northern Michigan lake and the relationship among climate, vegetation, and Great Lakes water levels. Quater Res 58:120–130

Brinkman WAR (2000) Causes of variability in monthly Great Lakes water supply and lake levels. Clim Res 15: 151–160
Campolo M, Soldati A, Andreussi P (1999) Forecasting river flow rate during low-flow periods using neural networks. Water Resour Res 35:1191–1197
Cengiz TM (2011) Periodic structures of great lakes levels using wavelet analysis. J Hyrol Hydromech 59(1):24–35
Chang FJ, Chang YT (2006) Adaptive neuro-fuzzy inference system for prediction of water level in reservoir. Adv Water Resour 29:1–10
Chang IC, Chang FJ, Tsai YH (2005) Fuzzy examplar-based inference system for flood forecasting. Water Resour Res 41:W02005
Changnon SA (1987) Climate fluctuations and record high levels of Lake Michigan. Bull Am Meteorol Soc 68: 1394–1402
Changnon SA (2004) Temporal behavior of levels of Great Lakes and climate variability. J Great Lakes Res 30: 184–200
Chen YW, Chang LC, Huang CW, Chu HJ (2013) Applying genetic algorithm to the conjunctive use of and subsurface water. Water Resour Manag 27:4731–4757
Cobaner M (2013) Reference evapotranspiration based on Class A pan evaporation via wavelet regression technique. Irrig Sci 31:119–134
Cohn BP, Robinson JE (1976) A forecast model for Great Lakes water levels. J Geol 84:455–465
Coops H, Beklioglu M, Crisman TL (2003) The role of water level fluctuations in lakes and wetlands—introduction. Hydrobiologia 539:189–204
Daliakopoulos IN, Coulibaly P, Tsanis IK (2005) Groundwater level forecasting using artificial neural netwrorks. J Hydrol 309:229–240
Dawson CW, Wilby RL (2001) Hydrological modeling using artificial neural networks. Prog Phys Geogr 25:80–108
DeCooke BG, Meregian E (1967) Forecasting the levels of the Great Lakes. Water Resour Res 3:397–403
Desalegn CE, Babel MS (2011) Application of ANN-based streamflow forecasting model for agricultural water management in the Awash River Basin, Ethiopia. Water Resour Manag 25:1759–1773
Firat M, Gungor M (2008) Hydrological time series modeling using neuro-fuzzy inference system. Hydrol Proced 22:2122–2132
Ghanbari RN, Bravo HR (2008) Coherence between atmospheric teleconnections, Great Lakes water levels and regional climate. Adv Water Resour 31:1284–1298
Goldberg DE (1989) Genetic algorithms in search optimization and machine learning. Addison-Wesley, Reading
Govindaraju RS, Rao AR (eds) (2000) ARTIFICIAl neural networks in hydrology. Kluwer Academic Publishers, London
Harnahan JL, Kravtsov SV, Roebber PJ (2009) Quasi-periodic decadal cycles in levels of lakes Michigan and Huron. J Great Lakes Res 35:30–35
Hartmann HC (1990) Climate chnage impacts on Laurential Great Lakes levels. Clim Change 17:49–67
Hartmann H, Becker S, King L (2007) Predicting summer rainfall in the Yangtze river basin using neural networks. Int J Climatol 28:925–936
Hatiboglu MA, Altunkaynak A, Özger M, Iplikcioglu AC, Coşar M, Turgut N (2010) A predictive tool by fuzzy logic for outcome of patients with intracranial aneurysm. Expert Syst Appl 37:1043–1049
Haykin S (1994) Neural networks: a comprehensive introduction. Prentice Hall, New Jersey
Irvine KA, Eberthardt AK (1992) Multiplicative seasonal ARIMA models for Lake Erie and Lake Ontario. Water Resour Bull 28:385–396
Jain A, Kumar AM (2007) Hybrid neural network models for hydrologic time series forecasting. Appl Soft Comput 7:585–592
Katambara Z, Ndiritu J (2009) A fuzzy inference system for modeling streamflow: Case of Letaba River. S Afr Phys Chem Earth 34:688–700
Kim T, Valdes JB (2003) Nonlinear model for drought forecasting based on a conjunction of wavelet transforms and neural networks. J Hydrol Eng 8(6):319–328
Krishna B, Rao YRS, Vijaya T (2008) Modeling groundwater levels in an urban aquifer using artificial neural networks. Hydrol Proced 22:1180–1188
Kucuk M, Kahya E, Cengiz TM, Karaca M (2009) North atlantic oscillation influences on Turkish lake levels. Hydrol Proced 23:893–906
Larson G, Schaetzl R (2001) Origin and evolution of the Great Lakes. J Great Lakes Res 27:518–546
Lee T-L, Makarynskyy O, Shao C-C (2007) A combined harmonic analysis-artificial neural network methodology for tidal predictions. J Coast Res 23(3):764–770
Leira M, Cantonati M (2008) Effects of water level fluctuations in lakes: an annotated bibliography. Dev Hydrobiol 613:171–184
Lliadis LS, Maris F (2007) An artificial neural network model for mountainous water-resources management: The case of Cyprus mountainous watersheds. Environ Model Softw 22:1066–1072

Lofgren BM, Quinn FH, Clites AH, Assel RA, Eberhardt AJ, Luukkonen CL (2002) Evaluation of potential impacts on Great Lakes water resources based on climate scenarios of two GCMs. J Great Lakes Res 28: 537–554
Maier HR, Dandy GC (2000) Neural networks for the prediction and forecasting of water resources variables: a review of modeling issues and applications. Environ Model Softw 15:101–124
Makarynskyy O, Makarynska D, Kuhn M, Featherstone WE (2004) Predicting sea level variations with artificial neural networks at Hillary Harbour, Western Australia. Estuar Coast Shelf Sci 61:351–360
Mamdani EH (1974) Application of fuzzy algorithm for control of simple dynamic plant. Proc IEEE 121:1585–1588
May DB, Sivakumar M (2009) Prediction of urban stormwater quality using artificial neural networks. Environ Model Softw 24:296–302
Meadows GA, Meadows LA, Wood WL, Hubert JM, Perlin M (1997) The relationship between Great Lakes water levels, wave energies and shoreline damage. Bull Am Meteorol Soc 78:675–683
Mohanty S, Jha MK, Kumar A, Sudheer KP (2010) Artificial neural network modeling for groundwater level forecasting in a river island of eastern India. Water Resour Manag 24:1845–1865
Mutlu E, Chaubey I, Hexmoor H, Bajwa SG (2008) Comparison of artificial neural network models for hydrologic applications in an agricultural watershed. Hydrol Proced 22:5097–5106
Nayak PC, Sudheer KP, Rangan DM, Ramasastri KS (2004) A neuro-fuzzy computing technique for modeling hydrological time series. J Hydrol 291:52–66
Nayak PC, Rao YRS, Sudheer KP (2006) Groundwater level forecasting in a shallow aquifer using artificial neural network approach. Water Resour Manag 20:77–90
Neff BP, Nicholas JR (2005) Uncertainty in the Great Lakes water balance. U.S. Geological Survey Scientific Investigations Report 2004–5100, 42 pp
Nourani V, Alami MT, Aminfar MH (2009) A combined neural-wavelet model for prediction of Ligvanchai watershed precipitation. Eng Appl Artif Intell 22:466–472
Ozelkan EC, Duckstein L (2001) Fuzzy conceptual rainfall-runoff models. J Hydrol 215:32–55
Ozger M (2009) Comparison of fuzzy inference systems for streamflow prediction. Hydrol Sci J 54:261–273
Ozger M (2010) Significant wave height forecasting using wavelet fuzzy logic approach. Ocean Eng 37(16): 1443–1451
Panagoulis D (2006) Artificial neural networks and high and low flows in various climate regimes. Hydrol Sci J 51:563–587
Polderman NJ, Pryor SC (2004) Linking synoptic-scale climate phenomena to lake-level variability in the Lake Michigan-Huron basin. J Great Lakes Res 30:419–434
Ramirez MC, Velho HF, Ferreira J (2005) Artificial neural network technique for rainfall forecasting applied to the Sao Paulo region. J Hydrol 301:146–162
Riad S, Manaia J, Bouchaou L, Naijar Y (2004) Predicting catchment flow in a semi-arid region via an artificial neural network technique. Hydrol Proced 18:2387–2393
Rosenblatt F (1958) The perceptron: a probabilistic model for information storage and organization in the brain. Psychol Rev 65:386–408
Rumelhart DE, Hinton GE, Williams RJ (1986) In: Rumelhart DE, McClelland JL (eds) Learning internal representations by error propagation. The MIT Press, London
Sales PRH, Pereira B, Vieira AM (1994) Linear procedures for time series analysis in hydrology. In: Hipel KW, McLeod AI, Panu US (eds) Stochastic and statistical methods in hydrology and environmental engineering. Springer, New York
Samarsinghe S (2007) Neural networks for applied sciences and engineering. Auerbach Publications, New York
Seckin N, Cobaner M, Yurtal R, Haktanir T (2013) Comparison of artificial neural network methods with L-moments for estimating flood flow at ungauged sites: the case of East Mediterranean River Basin, Turkey. Water Resour Manag 27:2103–2124
See L, Openshaw S (1999) Applying soft computing approaches to riverflow forecasting. Hydrol Sci J 44:763–778
Sen Z, Altunkaynak A (2006) A Comparative fuzzy logic approach to runoff coefficient and runoff estimation. Hydrol Proced 20:1993–2009
Sen Z, Altunkaynak A (2009) System modeling of drinking water consumption prediction. Expert Syst Appl 36: 11745–11752
Shiri J, Kisi O (2010) Short term and long term streamflow forecasting using a wavelet and neuro-fuzzy conjunction model. J Hydrol 394:486–493
Shiri J, Makarynskyy O, Kisi O, Dierickx W, Fakheri FA (2011) Prediction of short term operational water levels using an adaptive neuro-fuzzy inference system. J Water Port Coast Ocean Eng 137(6):344–354
Slivitzky M, Mathier L (1993) Climatic changes during the 20th century on the Laurentian Great Lakes and their impacts on hydrolgic regime. Proc. NATO Advanced Study Institute, Deauville
Takagi T, Sugeno M (1985) Fuzzy identification of systems and its application to modeling and control. IEEE Trans Syst Man Cybern 15:116–132

Thimuralaiah K, Deo MC (2000) Hydrologic forecasting using neural networks. J Hydrol Eng 5:180–189
Thompson TA, Baedke SJ (1997) Strand-plain evidence for late Holocene lake-level variations in Lake Michigan. Bull Geol Soc Am 109:666–682
Torrence C, Compo GP (1998) A practical guide to wavelet analysis. Bull Am Meteorol Soc 79:61–78
Tsoukalas LH, Uhrig RE (1997) Fuzzy and neural approaches in engineering. Wiley Interscience, New York
Uyumaz A, Altunkaynak A, Ozger M (2006) Fuzzy logic model for equilibrium scour downstream of a dam's vertical gate. J Hydraul Eng 132:1069–1075
Webb PW (2008) The impact of changes in water level and human development on forage fish assemlages in Great Lakes coastal marshes. J Great Lakes Res 34:615–630
Webster PJ, Hoyos CD (2004) Prediction of monsoon rainfall and river discharge on 15–30-day time scales. Bull Am Meteorol Soc 85(11):1745–1765
Wilcox DA, Thompson TA, Booth RK, Nicholas JR (2007) Water-level variability and water availability in the Great Lakes. U S Geol Surv Circ 1311:32p
Wu CL, Chau KW (2006) A flood forecasting neural network model with genetic algorithm. Int J Env Poll 28: 261–273
Xiong L, Shamseldin AY, O'Connor KM (2001) A non-linear combination of the forecasts of rainfall-runoff models by the first-order Takagi-Sugeno fuzzy system. J Hydrol 245:196–217
Zadeh LA (1994) Fuzzy logic, neural networks, and soft computing. Comm ACM 3:77–84
Zhang Q, Xu C, Chen Y (2010) Wavelet-based characterization of water level behaviors in the Pearl River estuary, China. Stoch Env Res Risk A 24:81–92