Proceedings of the 2000 IEEE
International Conference on Robotics & Automation
San Francisco, CA • April 2000

Computing 3-D Optimal Form-Closure Grasps*

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Abstract

In this paper, we address the problem of computing optimal fingertip locations yielding $n-$finger form-closure grasps of polyhedral objects. Given grasping points of m fingers not in form-closure, we show that the calculation of optimal grip points for the $n-m$ fingers can be viewed as a non-linear programming problem. We propose a sufficient and necessary condition for the $n-m$ fingers to achieve a n-finger form-closure grasp with the m fingers given. Based on this condition, we derive a set of non-linear constraints for the non-linear programming problem. Furthurmore, we give a performance index serving as an objective function which measures the distance between the center of mass of the grasped object and the contact points to be determined. In addition, the algorithm proposed can be applied to the frictionless grasp. We have implemented the algorithm and verified its efficiency by two examples.

1 Introduction

In recent years, there has been a growing interest towards multifingered robot hand due to its flexibility and dexterity. The stability of a grasp is one of the fundemental issues concerning multifingered grasps. A number of previous papers have focused on the discussion of grasp stability. The stability of a multifingered hand grasp is characterized by form-closure and force-closure[1][10] under which arbitrary forces and torques exerted on the grasped object can be balanced by the contact forces applied by the fingers. Nguyen[7] proposed an algorithm for constructing force-closure grasps based on the shape of the grasped object. Ponce and Faverjon[8] computed stable grasps of polygonal objects using a projection algorithm based on linear programming and variable elimination among linear constraints. Recently, Liu [3] showed that the non-form-closure region for n-finger planar grasp of polygonal objects consists of two convex polytopes in the parameter space representing grasping points. Only a few literature touched the topic of computing 3-D form-closure grasps due to complicated geometry and high dimension of the grasp space. Ponce[9] further extended his algorithm to compute stable grasps of polyhedral objects. However, they only give a detailed algorithm and results for 3-D concurrent grasps, which seldom occurs in practical cases. Recently, the authors[4] developed an efficient algorithm for computing all grasping points of one finger to achieve a 3-D form-closure grasp with the other $n-1$ fingers given.

In order to simplify the synthesis of 3-D form-closure grasps, searching for the optimal location of contact points on objects with a known geometry is a practical way. However, many previous works tended to discuss optimal force distribution problem and only a limited number of papers have addressed the problem of searching optimal fingertip positions. Ponce[9] used linear optimization within the valid configuration space regions to compute the maximal object regions where fingers can be positioned independently while ensuring force-closure. Omata[11] has suggested algorithms for determining the optimal location of grip points for the 3-D grasp of a polyhedral object when the external force exerted on the body is known and fixed. Mangialardi[12] proposed an optimization criterion which minimized the grasp forces required to balance any external force acting on the object and thus determined the optimal grip points.

In this paper, we propose a sufficient and necessary condition for $n-$finger 3-D form-closure grasp and formulate it as a set of non-linear constraints. A grasp quality index is chosen by centering as well as possible the center of mass of the grasped object in the geometry formed by the contact points. Furthermore, the

*This work is supported in part by the Hong Kong Research Grant Council under grants CUHK4151/97E and CUHK4166/98E. Email: yhliu@mae.cuhk.edu.hk

algorithm proposed in the paper extends our previous one[3] by calculating the $n-m$ optimal fingertip positions ensuring a $n-$finger form-closure grasp with the m fingers positions given. Moreover, we also showed that the alogrithm can be applied to the frictionless grasp. Finally, we have implemented the proposed algorithm and verified its use and efficiency by two examples.

2 Form-Closure Grasp and Problem Definition

Hard-finger contact model is adopted and Coulomb friction with friction coefficient μ exists between the fingertips and the object. To avoid the slippage of the finger i on the surface of the object, the applied finger force f_i must remain in the *friction cone* (Figure 1) at all times. Note that the friction cone imposes a non-linear constraint on f_i. To simplify the problem,

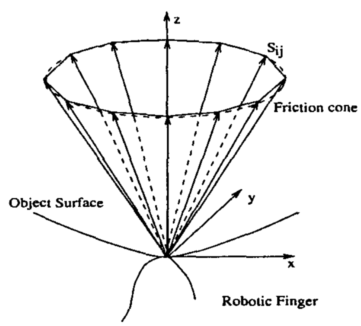

Figure 1: The friction cone at a grasping point.

we linearize the friction cone by a polyhedral convex cone generated by P vectors. Thus the grasping force f_i can be represented as

$$f_i = \sum_{j=1}^{P} \alpha_{ij}\vec{s}_{ij}, \quad \alpha_{ij} \geq 0 \tag{1}$$

where $\vec{s}_{ij}$ represents the j-th edge vector of the polyhedral convex cone. Coefficients α_{ij} are non-negative constants. The force and torque, corresponding to the grasping force f_i, applied at the center of mass of the object is given by

$$\underline{w}_i = \begin{pmatrix} f_i \\ \tau_i \end{pmatrix} = \begin{pmatrix} f_i \\ r_i \times f_i \end{pmatrix} \tag{2}$$

where r_i denotes the position vector of the i-th grasping point w.r.t. the object coordinate frame origined at the center of mass. The combination $\underline{w}_i$ of the force f_i and moment τ_i is called *wrench.* Substituting eq. (1) into eq. (2), we get

$$\underline{w}_i = \sum_{j=1}^{P} \alpha_{ij} w_{ij} \tag{3}$$

where

$$w_{ij} = \begin{pmatrix} \vec{s}_{ij} \\ r_i \times \vec{s}_{ij} \end{pmatrix}$$

w_{ij} is called *primitive contact wrenches* of the fingers. The net wrench applied at the object by the fingers is

$$\underline{w}_{net} = \sum_{i=1}^{n}\sum_{j=1}^{P} \alpha_{ij} w_{ij} = W\alpha \tag{4}$$

where W and α are given by

$$W = (w_{11}, w_{12}, ..., w_{1P}, ..., w_{n1}, w_{n2}, ..., w_{nP})$$

$$\alpha = (\alpha_{11}, \alpha_{12}, ..., \alpha_{1P}, ..., \alpha_{n1}, \alpha_{n2}, ..., \alpha_{nP})^T$$

W is a $6 \times nP$ matrix called *wrench matrix* and its column vectors are the primitive contact wrenches. For convenience, we use w_i with a single subscript i, instead of w_{ij}, to denote the $i-$th column vector of grasp matrix W, and use α_i to represent the $i-th$ component of vector α later.

Definition 1 *Suppose that a n-finger frictional grasp is given. For any external wrench $\underline{w}_{ext}$ applied to the object, if it is always possible to find an α with $\alpha_i \geq 0$ such that*

$$W\alpha = -\underline{w}_{ext},$$

the grasp is said to be form-closure.

It is well known that a form-closure grasp is equivalent to that the convex hull of the primitive contact wrenches w_i contains the neighborhood of the origin of the R^6 wrench space.
Constructing a form-closure grasp is of great significance in the sense that all the external disturbance (force and moment) can be properly balanced. Since two surface coordinates are essential to represent grasping point of a finger on the surface of an object, the problem of computing a form-closure grasp, in general, is to find a solution in R^{2n}. As it is not realistic to search in such a high dimensional space, we address the following simpler problem:

Problem 1 *Suppose that grasping points $g_i (i = 1, 2, ..., m)$ of m fingers on the object have been specified. Assume that the m-finger grasp $\{g_1, g_2, ..., g_m\}$ is* *not form-closure. Find the optimal grasping points of the other $n-m$ fingers on the object such that the n-finger grasp $\{g_1, g_2, ..., g_n\}$ achieves form-closure under a performance index.*

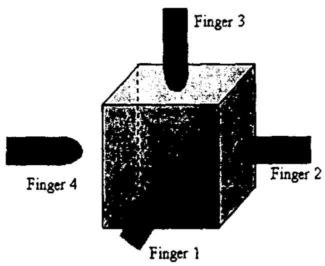

Figure 2: The problem definition.

For example, the three fingers in Figure 2 do not form a form-closure grasp. Our aim is to find an optimal grip point on the object for the fourth finger to achieve a form-closure grasp with the other three ones.

3 Representing a Grasping Point

The surface of a polyhedral object is not smooth at its edges and vertices, so different coordinates must be introduced to represent the grasping point on different faces. To represent the grasping point of finger i on a face, a local coordinate frame $\{\lambda_1^i, \lambda_2^i, \lambda_3^i\}$ is attached to the face (Figure 3). The origin of the coordinate frame is located at one vertex and the λ_3^i axis is parallel to the normal of the face. The other axes are defined according to the right-hand rule. The grasping point is represented by a local coordinate $(\lambda_1^i, \lambda_2^i)$. The coordinates of the grasping point g_i w.r.t. the object frame are calculated by

$$g_i = o_\lambda^i + R_\lambda^i \begin{pmatrix} \lambda_1^i \\ \lambda_2^i \\ 0 \end{pmatrix}$$

where o_λ^i and R_λ^i denote the origin and the rotation matrix of the local frame $\{\lambda_1^i, \lambda_2^i, \lambda_3^i\}$ w.r.t. the object frame respectively. The components of the vector g_i are all affine in λ_1^i and λ_2^i.

At the grasping point, another frame $\{x_i, y_i, z_i\}$ is introduced to represent the grasping force. Slice the friction cone by $z_i = 1$. The grasping force f_i w.r.t. the local frame can be represented as

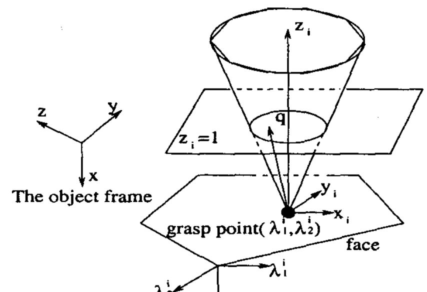

Figure 3: The representation of a grasping point on an object face.

$$f_i = \beta_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \tag{5}$$

where β_i is a non-negative constant. To guarantee that the grasping force f_i is inside the linearized friction cone,

$$B_i \begin{pmatrix} x_i \\ y_i \end{pmatrix} \leq b_i \tag{6}$$

where B_i is a $P \times 2$ constant matrix and b_i is a $P \times 1$ vector determined by the linearized friction cone. The grasping force f_i w.r.t. the object frame is given by

$$f_i = \beta_i R_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

where R_i is the rotation matrix of the local frame. The resultant wrench on the object by the grasping force f_i is

$$\underline{w}_i = \beta_i \underbrace{\begin{pmatrix} R_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \\ g_i(\lambda_1^i, \lambda_2^i) \times (R_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}) \end{pmatrix}}_{q_i} \tag{7}$$

where vector q_i is called the ***primitive grasping wrench*** of the finger i.

4 Searching for Optimal Fingertip Positions

First, we address the case when $m = n - 1$, i.e. only the grasping point of one finger is to be determined.

4.1 A Sufficient and Necessary Condition for Form-Closure Grasps

Note that we assume the grasp of the $n-1$ fingers is not form-closure, so the convex hull of their primitive contact wrenches does not contain the origin. Construct the convex cone $Cone(W, O)$ of the primitive contact wrenches W and the origin O.

Assumption 1 *Assume that such a 6-D convex cone can always be constructed.*

Such a 6-D cone cannot be found when all the primitive contact wrenches belong to a 5-D hyperplane. We call the case a singular one, which often occurs when only two fingertip positions are given or at least two fingers grasp in the same face of the polyhedral object. Till now our approach is only applicable when at least three fingertip positions are given. Note that the convex cone $Cone(W, O)$ is different from the convex hull. The convex cone is represented as

$$c_j^T x \geq 0, \quad j = 1, 2, ..., K$$

where K denotes the number of faces of the convex cone. c_j are constant 6×1 vector.

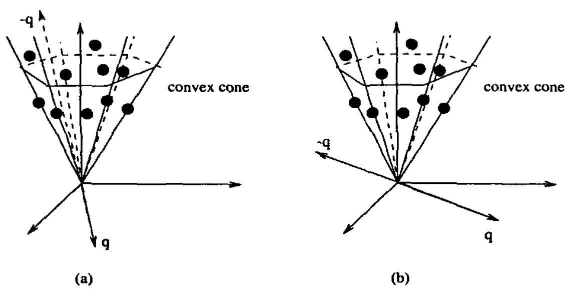

Figure 4: (a) A vector $-q_n$ intersecting with the interior region of $Cone(W, O)$, and (b) a vector $-q_n$ not intersecting the interior region.

Theorem 1 *Assume that the grasp of the $n-1$ fingers is not form-closure. The n-finger grasp $\{g_1, g_2, ..., g_n\}$ is form-closure if and only if there exists (x_n, y_n) such that*

(1) x_n and y_n satisfy the inequalities (6)

(2) the ray $-q_n$, opposite to the primitive grasping wrench q_n, intersects with interior region of the convex cone $Cone(W, O)$, (Figure 4) i.e.

$$c_j^T \begin{pmatrix} R_n \begin{pmatrix} x_n \\ y_n \\ 1 \end{pmatrix} \\ (O_\lambda^n + R_\lambda^n \begin{pmatrix} \lambda_1^n \\ \lambda_2^n \\ 0 \end{pmatrix}) \times (R_n \begin{pmatrix} x_n \\ y_n \\ 1 \end{pmatrix}) \end{pmatrix} \leq 0 \qquad j = 1, 2, ..., K \quad (8)$$

Note that c_j, R_n, R_λ^n and O_λ^n are constant vectors or matrices. From Theorem 1, we directly have

Proposition 1 *On the face of the object, the optimal grip point g_n that result in a $n-$finger form-closure grasp is a set of parameter $(\lambda_1^n, \lambda_2^n)$ that guarantee the existence of (x_n, y_n) satisfying both inequalities (6) and (8) under a performance index.*

4.2 Formulation of Non-Linear Constraints

The constraint inequalities (6) and (8) contain four variables x_n, y_n, λ_1^n and λ_2^n. Note that the constraints from condition (1) in Theorem 1 are linear ones while the constraints from condition (2) involve the multiplication of two variables, which indicates that those constraints are nonlinear. Suppose the face F_n on which the fingertip position will be determined is bounded by l edges, the parameters $(\lambda_1^n, \lambda_2^n)$ must also satisfy l linear constraints $a_j\lambda_1^n + b_j\lambda_2^n + c_j \leq 0$, with $j = 1, 2, ..., l$, which ensures the grip point on the surface of the object. a_j, b_j and c_j are constants defined by edge j.

Define the following vector z:

$$\begin{aligned} z_1 &= x_n \\ z_2 &= y_n \\ z_3 &= \lambda_1^n \\ z_4 &= \lambda_2^n \end{aligned} \quad (9)$$

We can rewrite those inequalities as follows:

$$\begin{array}{ll} gn_i(z) <= 0 & i = 1, ..., K \\ gl_i(z) <= 0 & i = K+1, ..., K+P \\ a_j z_3 + b_j z_4 + c_j <= 0 & i = K+P+1, ..., K+P+l \end{array} \quad (10)$$

Thus we get a set of constraints including 4 variables, K non-linear constraints and $P + l$ linear constraints.

4.3 Determination of Performance Index

Note that many criterions can be chosen as the performance index for the optimal fingertip positioning problem. Here we introduce a performance index similar to that of Ponce[9] by measuring the L_2 distance between the center of mass $O(x_o, y_o, z_o)$ of the grasped object and the center of the contact points $C(x_c, y_c, z_c)$.

$$u = \|O - C\|^2 = (x_o - x_c)^2 + (y_o - y_c)^2 + (z_o - z_c)^2$$

In other words, we try to center the center of mass of the object as well as possible in the geometry formed by the contact points. This enables us to decrease the effect of gravitational and inertial forces during the motion of the robot and thus acheives a robust grasp. It should be noted that the optimization problem is a non-linear programming(NLP) problem with both linear and non-linear constraints.

$$\min_{x \in R^4} \quad u(z)$$

subject to:

$$gn_i(z) \le 0, \quad i = 1, ..., K$$
$$gl_i(z) \le 0 \quad i = K+1, ..., K+P$$
$$a_j z_3 + b_j z_4 + c_j <= 0 \quad i = K+P+1, ..., K+P+l$$

5 Extension to General Cases

In this section, we extend the results obtained to a general case, i.e. when $m < n-1$. The net wrench applied by the $n-m$ fingers can be represented by

$$\underline{w}_{net} = \sum_{i=m+1}^{n} \beta_i \begin{pmatrix} R_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \\ g_i(\lambda_1^i, \lambda_2^i) \times (R_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}) \end{pmatrix} \quad (11)$$

Theorem 2 *The n-finger grasp is form-closure if and only if there exist proper $\beta_i \ge 0$ and (x_i, y_i) for all $i = m+1, ..., n$ such that*

(1) (x_i, y_i) satisfy the equation (6);

(2) the ray $-\underline{w}_{net}$ intersects in interior region of the convex cone of the primitive contact wrenches of the m fingers.

To satisfy condition (2) in this theorem,

$$c_j^T \underline{w}_{net} \le 0, \quad j = 1, 2, ..., K$$

The inequalities (6) can be changed to

$$B_i \begin{pmatrix} x_i \\ y_i \end{pmatrix} \le \beta_i b_i \quad i = m+1, ..., n \quad (12)$$

Note that the only difference between the general case and the case when $m = n-1$ is the increase of variables for NLP problem. Introduce the vector λ:

$$\lambda = (\lambda_1^{m+1}, \lambda_2^{m+1}, ..., \lambda_1^n, \lambda_2^n)^T$$

Define the following vector:

$$z = (x_i^{m+1}, y_i^{m+1}, ..., x_i^n, y_i^n, \lambda, \beta_{m+1}, ..., \beta_n)^T$$

Thus the non-linear constraints for the general case can be summerized as

$$g_i(z) \le 0 \quad (13)$$

6 Extension to Frictionless Grasps

The results can also be applied to the construction of frictionless grasps. In this case, the problem becomes much simpler because forces can only be applied in the normal direction and hence

$$x_i = 0, \quad y_i = 0 \quad (14)$$

Accordingly, the vector z becomes

$$z = (\lambda, \beta_{m+1}, ..., \beta_n)^T$$

In this case, the constraints can be made linear and the optimization problem becomes a quadratic programming one. Furthermore, if a linear objective function is chosen, we can have a linear programming problem, which can be solved efficiently.

7 Implementation

We have implemented the algorithm on a polyhedral object shown in Fig.5. Friction coefficient $\mu = 0.3$. The Qhull Algorithm available on the website http://www.geom.umn.edu/software/qhull, is used to calculate the convex cone. We adopt Sequential Quadratic Programming method to solve the non-linear programming problem.

Case I: Suppose the grip points of 3 fingers are given by $r_1 = (0.0, 2.0, 1.5)^T$, $r_2 = (0.0, 1.0, 2.0)^T$, $r_3 = (1.0, 1.0, 1.0)^T$. By the qualitative test algorithm[5], the three fingers fail to form a form-closure grasp. Our aim is to find an optimal grip point on the back face 5 such that the 4−finger grasp is form-closure(Fig.5). We give the initial values for $(x_5, y_5, \lambda_1^5, \lambda_2^5)$ as $(0.0, 0.0, 0.0, 0.0)$ and the final optimal solution obtained is $(-0.22, 0.19, 2.24, 1.80)$. The optimal grip point w.r.t the object frame is $(-1.0, -0.20, -0.24)$. The CPU time for calculating $6-D$ convex cone is 0.31 and for solving the nonlinear programming problem 0.54. Case II is about a 5−finger grasp of the object

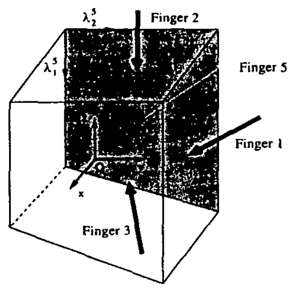

Figure 5: The optimal grip point for case I.

shown in Fig.6. Suppose the three fingertip positions known as $r_1 = (0.0, 2.0, 1.5)^T$, $r_2 = (0.3, 2.0, -0.5)^T$, $r_3 = (0.0, 0.0, 2.0)^T$, the aim is to find the optimal positions for finger 4 and 5 respectively. The initial value for $(x_n^4, y_n^4, \lambda_1^4, \lambda_2^4, x_n^5, y_n^5, \lambda_1^5, \lambda_2^5, \beta^4, \beta^5)$ is (0.0, 0.0, 2.0, 2.0, 0.0, 0.0, 2.0, 2.0, 0.0, 0.0). The optimal solution obtained is (-0.2, 0.0, 1.02, 3.27, -0.2, 0.0, 4.05, 0.1, 1.56, 0.1). From the result, the optimal location for finger 4 is (1.02, 3.27) and for finger 5 (4.05, 0.10) both viewed w.r.t. local frame. From the object frame they will be (1.0, 1.27, 0.98) and (-1.0, -1.9, -2.05). The CPU time for calculation of convex cone and the nonlinear programming problem are 0.20 and 6.67 respectively.

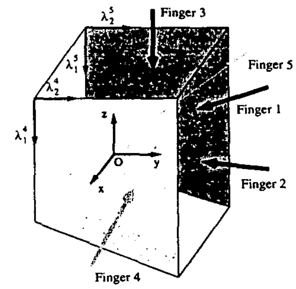

Figure 6: The optimal grip points for Case II.

8 Conclusions

This paper demonstrated that the problem of computing optimal fingertip positions for $n-m$ fingers yielding form-closure with other m grip points given can be transformed to a non-linear programming problem subject to a set of constraints. A performance index is given by considering the L_2 distance between the center of mass of the grasped object and the contact points to be determined. Finally the performance of the algorithm is verified by two examples. It is expected that the results can be applied to 3D grasp and regrasp planning.

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