POLITECNICO
MILANO 1863

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1
Introduction

The aim of our project is to understand how an assigned satellite can move from a point of an initial orbit to another one of a final orbit. In order to work on that, the coordinates of the initial position and velocity were given to us as well as the semi-major axis a, inclination i, eccentricity e, longitude of the ascending node Ω, the orbital argument of periapsis ω of the final orbit and the true anomaly θ of the final point.
We analysed a first strategy that involves the creation of three different auxiliary orbits in order to reach the final point and after that we calculated the amount of velocity and time that are necessary to complete the mission. This strategy, however, is not the only one available and possible; in fact, we also studied a second strategy and a bi-elliptical one to compare the velocity and time data: all of this was done to understand which one give us a better trade-off. The report that we did is divided in three parts: the first part is dedicated to the analysis of the first orbit, then the final one is described and finally the possible transfers trajectory too.

2
Initial orbit characterisation

2.1 Initial orbit parameters

Starting from the position and the velocity of the spacecraft in its initial point, we found the orbital parameters and the true anomaly of the initial point by using the function *car2kep*.

$\mathbf{r}\,[km]$			$\mathbf{v}\left[\frac{km}{s}\right]$		
r_x	r_y	r_z	v_x	v_y	v_z
-4990.62	-6741.97	1226.64	3.967	-4.069	-3.824

a $[km]$	e $[-]$	i $[rad]$	$\Omega[rad]$	$\omega[rad]$	$\theta[rad]$
8458.84	0.050758	0.6243	1.13798	1.22691	1.66454
		(35.771°)	(65.201°)	(70.297°)	(95.371°)

2.2 Results discussion

In order to better define the orbit we calculated some parameters, as follows:

Orbital period	$T=2\pi\sqrt{\frac{a^3}{\mu}}$	$2.15h$
Orbital energy	$E=\frac{1}{2}v^2-\frac{\mu}{r}$	$-23.56\frac{km^2}{s^2}$
Semi-latus rectum	$p=a(1-e^2)$	$8437.05km$
Apogee radius	$r_a=\frac{p}{1-e}$	$8888.20km$
Perigee radius	$r_p=\frac{p}{1+e}$	$8029.49km$
Apogee velocity	$v_a=\sqrt{\frac{\mu}{p}}(1-e)$	$6.5245\frac{km}{s}$
Perigee velocity	$v_p=\sqrt{\frac{\mu}{p}}(1+e)$	$7.2223\frac{km}{s}$
Specific angular momentum (vector)	$\mathbf{h}=\mathbf{r}x\mathbf{v}$	$[30772.5;-14218.0;47052.2]\frac{km^2}{s}$
Orbital node (versor)	$\mathbf{n}=\frac{\mathbf{K}x\mathbf{h}}{\lvert\mathbf{K}x\mathbf{h}\rvert}$	$[0.4194;0.9078;0]$

where $\mathbf{K}$ is the versor [0;0;1].
As we can see the specific angular momentum is directed as the positive z-axis so we expected an anticlockwise rotation of our spacecraft around the Earth; this characteristic can be also noted since the inclination of the initial orbit is minor than 90°. Moreover, the energy is negative, as expected for an elliptical orbit.

2.3 Graphical representation of the initial orbit

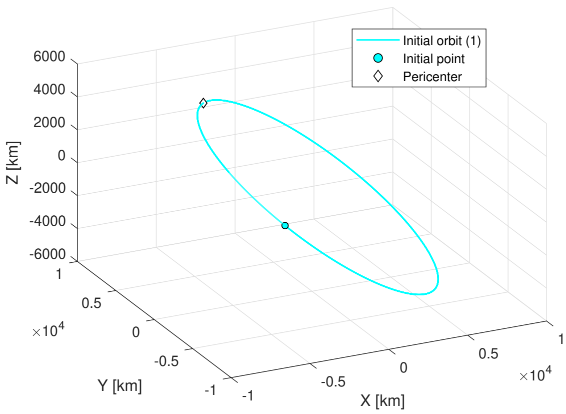


Figure 1: Initial orbit

3
Final orbit characterisation

3.1 Final orbit characterisation

For the final orbit we were provided with the six orbital parameters:

a $[km]$	e $[-]$	i $[rad]$	$\Omega[rad]$	$\omega[rad]$	$\theta[rad]$
13360	0.2302	1.103	1.464	0.6745	2.933
		(63.197°)	(83.881°)	(38.646°)	(168.049°)

The final position and velocity are:

r $[km]$			**v** $\left[\frac{km}{s}\right]$		
r_x	r_y	r_z	v_x	v_y	v_z
1733.919	-14858.595	6547.685	1.9786	1.5123	-3.5752

this results were found by using the function kep2car, which is described in the section below.

3.2 Results discussion

We can define some relevant parameters also for the final orbit:

Orbital period	$T=2\pi\sqrt{\frac{a^3}{\mu}}$	$4.27h$
Orbital energy	$E=\frac{1}{2}v^2-\frac{\mu}{r}$	$-14.92\frac{km^2}{s^2}$
Semi-latus rectum	$p=a(1-e^2)$	$12652.03km$
Apogee radius	$r_a=\frac{p}{1-e}$	$16435.47km$
Perigee radius	$r_p=\frac{p}{1+e}$	$10284.53km$
Apogee velocity	$v_a=\sqrt{\frac{\mu}{p}}(1-e)$	$4.3208\frac{km}{s}$
Perigee velocity	$v_p=\sqrt{\frac{\mu}{p}}(1+e)$	$6.9050\frac{km}{s}$
Specific angular momentum (vector)	$\mathbf{h}=\mathbf{r}x\mathbf{v}$	$[63024.1;-6756.5;32022.0]\frac{km^2}{s}$
Orbital node (versor)	$\mathbf{n}=\frac{\mathbf{K}x\mathbf{h}}{\lvert\mathbf{K}x\mathbf{h}\rvert}$	$[0.1076;0.9943;0]$

where **K** is the versor [0;0;1].
As for the initial orbit a positive direction of the specific angular momentum respected to the z-axis determine an anticlockwise rotation of our spacecraft and the energy is negative.

3.3 Graphical representation of the final orbit

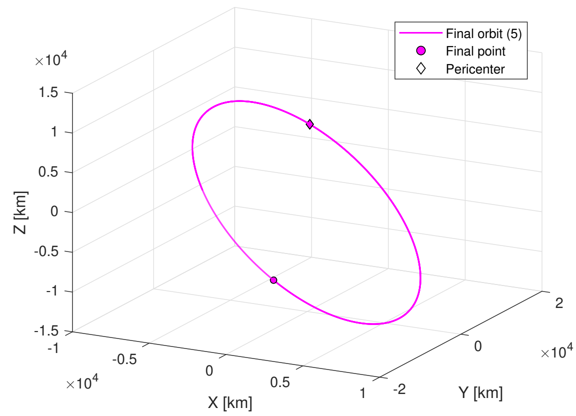


Figure 2: Final orbit

4
Transfer trajectory definition and analysis

To achieve the final position and velocity we had to change all the orbital parameters. Thus we had to change the shape of the orbit, the orbital plane and moreover to correct the argument of the periapsis.
In order to reach the final point we studied three different strategies, which are:

1. Change of plane, change of ω, change of shape and size $\rightarrow$ four cases
2. Circularization, change of plane, change of shape and size, circularization $\rightarrow$ four cases
3. Bi-elliptical strategy (through the circularization of the initial and final orbits) $\rightarrow$ four cases

4.1 Different strategies and method of solution

Our project reports an analyses of the transfer between an initial and a final point. In order to do it, we have to change the parameters that characterize our orbit that are a, e, i, Ω and ω. These are the functions we defined in order to reach our goal:

changeOrbitalPlane: starting from the initial orbit, with this function we changed its inclination by finding the new argument of the periapsis and the true anomaly where the transfer takes place. This orbit has the same shape of the initial orbit and the same plane of the final one, through a single impulse.
Our data concern the situation where the sign of $\Delta\Omega$ and Δi are both positive; by using the following formulas we found the true anomaly of the intersection point, where *1* stands for the old orbit while *2* for the new one:

$$\alpha = \arccos(\cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos \Delta\Omega) \qquad u = \arctan\frac{\sin u}{\cos u}$$

$$\cos u_2 = \frac{\cos i_1 - \cos\alpha \cos i_2}{\sin\alpha \sin i_2} \qquad \sin u_2 = \frac{\sin i_1 \sin\Delta\Omega}{\sin\alpha}$$

$$\cos u_1 = \frac{-\cos i_2 + \cos\alpha \cos i_1}{\sin\alpha \sin i_1} \qquad \sin u_1 = \frac{\sin i_2 \sin\Delta\Omega}{\sin\alpha}$$

$$\theta_2 = u_1 - \omega_1 = \theta_1 \qquad \omega_2 = u_2 - u_1 + \omega_1$$

changePericenterAn: this function is used to understand how the orbit is orientated compared with the final one; in order to do that we have to find its ω and the θ of the intersection point.
The analysis starts by using the eccentricity vectors which identify the pericenter. In fact the scalar product between the eccentricity vectors of the studied orbit and the final one was calculated: if their product has the same sign then they are orientated in the same way.
Then we reach a new orbit by using the following formulas, since in our case their scalar product is positive.

The two points of intersection (relatively *a, b*) between the old (*1*) and the new (*2*) orbits are:

$$\omega_2 = \omega_{final} \rightarrow \Delta\omega = \omega_2 - \omega_1$$

$$\theta_{1,a} = \frac{\Delta\omega}{2} \qquad \theta_{2,a} = 2\pi - \frac{\Delta\omega}{2}$$

$$\theta_{1,b} = \pi + \frac{\Delta\omega}{2} \qquad \theta_{2,b} = \pi - \frac{\Delta\omega}{2}$$

And the velocity is

$$\Delta v_2 = 2\sqrt{\frac{\mu}{\pi}} e_2 \sin\frac{\Delta\omega}{2}$$

This new orbit has the same shape and size of the old one and is aligned with the final orbit.

changeOrbitShape: with this function we have to study the shape of the transfer orbit that connects the old orbit with the new one. Since we have two coaxial orbits we have to use a bitangent transfer that consists in two manoeuvres. In fact, we have to find a velocity (Δv_1) that let the satellite go from the old orbit to the transfer arch and then a velocity (Δv_2) that completes the transfer by letting the satellite to go from the transfer arch to the new orbit.
We consider two cases: the first one happens when ω of the old orbit is the same of the final one, and the other when it is equal to $\omega_{final} + \pi$. Since our data lead us to the first case, this final transfer can start from the apocenter of the old orbit and end in the pericenter of the new orbit and vice versa for the first case we have considered.
The orbital parameters that identify the transfer orbit derive from the studied case, while the impulses are calculated as

$$\Delta v_1 = \sqrt{2\mu\left(\frac{1}{r_{a,t}} - \frac{1}{2a_t}\right)} - \sqrt{2\mu\left(\frac{1}{r_{a,old}} - \frac{1}{2a_{old}}\right)}$$

$$\Delta v_2 = \sqrt{2\mu\left(\frac{1}{r_{a,final}} - \frac{1}{2a_{final}}\right)} - \sqrt{2\mu\left(\frac{1}{r_{a,t}} - \frac{1}{2a_t}\right)}$$

$$\Delta v_{total} = |\Delta v_1| + |\Delta v_2|$$

in these formulas the *t* stands for the transfer orbit and *r* for the apocenter or the pericenter radius considered in each case.

note: in this function we also took into consideration the case where the old orbit is bigger than the final one. In this case we also studied the secant transfer.

changeOrbitalPlaneBI we initialized the function starting from the orbital parameters of the initial and final orbits and also with the true anomalies of the main points of this transfer. They are: the initial, the final, the points where we circularized the two orbits (that are the initial and the final one) and the θ that represents the angle of the transfer from the first circularized orbit to the first elliptical transfer orbit.
In order to calculate the velocity (Δv) and the time (Δt) of the total transfer from the initial to the final orbits, we firstly setted the radius of the two circular orbits as the pericenter or the apocenter radius basing on the point where the circularization takes place.
Then, we set the pericenter radius of the two elliptical orbits equal to the radius of the two circularized orbits respectively. In addition to this, we required the apocenter radius as the pericenter one multiplied for *n* times, where *n* changes between 1 and 5.
Moreover, in order to evaluate the Δv of the elliptical transfer (from the first elliptical transfer orbit 1 to the second one 2) we used the theorem of the cosines and it is

$$\Delta\Omega = \Omega_2 - \Omega_1 \qquad \Delta i = i_2 - i_1$$

$$\alpha = \arccos(\cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos\Delta\Omega)$$

$$v_1 = \sqrt{\frac{\mu}{p}}(1-e) \qquad v_2 = \sqrt{\frac{\mu}{p}}(1-e)$$

$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos\alpha}$$

where α is the angle between the two orbital planes.

car2kep function to find the orbital parameters from the vectors of the velocity and the radius.

kep2car function to find the radius and the velocity from the orbital parameters.

timeOfFlight this function was used to calculate the necessary time to complete the manoeuvre from the initial point to the final one considered in each case.

4.2 Results discussion of the transfer orbits

4.2.1 FIRST STRATEGY

First case:

- *From (1) to (T2)*: we selected the point that minimizes both the Δv and the Δt
  $\rightarrow$ changeOrbitalPlane;
- *From (T2) to (T3)*: to change ω the Δv in the two points is the same, hence we selected the point the minimizes the Δt
  $\rightarrow$ changePericenterAn;
- *From (T3) to (T5)*: the case selected has been the transfer from the apocenter of (T3) to the pericenter of (T5)
  $\rightarrow$ changeOrbitShape.

Time	$\Delta v[\frac{km}{s}]$
$20798.12s = 5.77h$	4.925

Second case

- *From (1) to (T2)*: we selected the furthest point and not the one that minimizes both the Δt and the Δv
  $\rightarrow$ changeOrbitalPlane;
- *From (T2) to (T3)*: to change ω the Δv in the two points is the same, hence we selected the point the minimizes the Δt
  $\rightarrow$ changePericenterAn;
- *From (T3) to (T5)*: the case selected has been the transfer from the pericenter of (T3) to the apocenter of (T5)
  $\rightarrow$ changeOrbitShape.

Time	$\Delta v[\frac{km}{s}]$
$34498.159s = 9.58h$	5.1757

Third case:

- *From (1) to (T2)*: we selected the furthest point and not the one that minimizes both the Δt and the Δv
  $\rightarrow$ changeOrbitalPlane;
- *From (T2) to (T3)*: to change ω the Δv in the two points is the same, hence we selected the point that minimizes the Δt
  $\rightarrow$ changePericenterAn;
- *From (T3) to (T5)*: the case selected has been the transfer from the apocenter of (T3) to the pericenter of (T5)
  $\rightarrow$ changeOrbitShape.

Time	$\Delta v[\frac{km}{s}]$
$28623.666s = 7.95h$	4.4615

Forth case:

- *From (1) to (T2)*: we selected the point that minimizes both Δv and Δt
  $\rightarrow$ changeOrbitalPlane;
- *From (T2) to (T3)*: to change ω the Δv in the two points is the same, hence we selected the point the minimizes the Δt
  $\rightarrow$ changePericenterAn;
- *From (T3) to (T5)*: the case selected has been the transfer from the pericenter of (T3) to the apocenter of (T5)
  $\rightarrow$ changeOrbitShape.

Time	$\Delta v[\frac{km}{s}]$
$34415.05s = 9.56h$	4.878

4.2.2 SECOND STRATEGY

The second strategy started from the circularization of the first and the last elliptical orbits (that take place in the pericenter or in the apocenter). After this, we changed the orbital plane with the function *ChangeOrbitalPlane*, obtaining a new circular orbit; this transfer orbit lays on the same plane of the final one. In order to reach the final point, between this obtained transfer orbit and the last circular one, we did a bitangent transfer.
Hence, to reach the final orbit we have four different cases, depending by the points where the circularization of the first and the last orbits take place.
As shown in the table below, the best result for the Δt is given by the pericenter-pericenter option, while for Δv is given by the apocenter-pericenter option.

- *From (1) to (C1)*: the first orbit can be circularised in two different ways (from the apocenter or the pericenter), reported in the table;
- *From (C1) to (C2)*: to change only the orbital plane → changeOrbitalPlane;
- *From (C2) to (C3)*: a bitangent orbit is used from (C2) to (C3) → changeOrbitShape;
- *From (C3) to (5)*: the final orbit is reached in its apocenter or pericenter.

The table provides the results of four different cases:

Manoeuvre in (1)	**Manoeuvre in (5)**	**Time** $[h]$	$\Delta v[\frac{km}{s}]$
Pericenter	Apocenter	15.04	6.563
Apocenter	Pericenter	8.52	4.864
Apocenter	Apocenter	15.55	6.050
Pericenter	Pericenter	8.02	5.399

4.2.3 THIRD STRATEGY: bi-elliptical transfer

The third strategy begins with the circularization of the first orbit (in the pericenter or in the apocenter); then a transfer elliptical orbit (T1) has been used and it lays in the same plane of the first orbit.
The following transfer point between (T1) and the next elliptical orbit (T2) coincides with the apocenter of both orbits. (T2) lays in the same plane of the final one and in order to change the inclination of (1) we used the function *ChangeOrbitalPlaneBI*. Then (T2) makes another π angle in order to reach the final circular orbit. In fact, as the first one, even the final orbit has been circularized (in the pericenter or in the apocenter).
To study this strategy, we set the ratio between the apocenter and the pericenter radius of the first elliptical transfer orbit (T1) as different values. Then we fixed it to analyse all transfers, by calculating Δv and Δt.
We consider the ratio between $r_{a,T1}$ and $r_{p,T1}$ of the transfer orbit (T1) as:

$r_{a,T1}/r_{p,T1}$	$\Delta v(T1)\rightarrow(T2)[\frac{km}{s}]$	$\Delta v[\frac{km}{s}]$	**Time** $[h]$
5	0.985	5.457	15.79
4	1.160	5.314	12.89
3	1.493	5.103	10.26
2	2.102	4.772	7.94
1	3.613	4.695	5.97

In all cases reported above we assumed that the circularization of the first elliptical orbit (T1) takes place in the apocenter and the circularization of the last orbit in the pericenter. This strategy was selected because it permits the best trade-off between Δv and Δt.
Instead if the main target of the mission is to minimize the Δv, the case from apocenter to apocenter (respectively of the first and last orbits) has to be taken into consideration, even if it requires an enormous Δt.
The following table reports all the possible combinations concerning the case where the ratio between the radius is 3; the case apocenter-pericenter is not reported because previously studied:

Manoeuvre in (1)	**Manoeuvre in (5)**	$\Delta v(T1)\rightarrow(T2)[\frac{km}{s}]$	$\Delta v[\frac{km}{s}]$	**Time** $[h]$
Pericenter	Apocenter	1.889	4.698	13.54
Apocenter	Apocenter	1.731	4.565	16.35
Pericenter	Pericenter	1.614	5.198	11.83

4.3 Graphical representation of the orbits

The following graphs represent the transfer orbits of the first strategy.

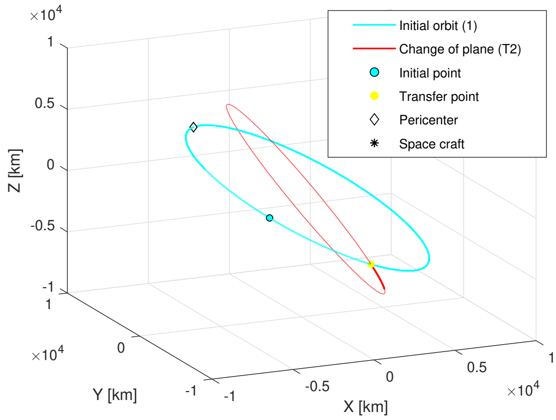


Figure 3: First trasfer orbit (T2)

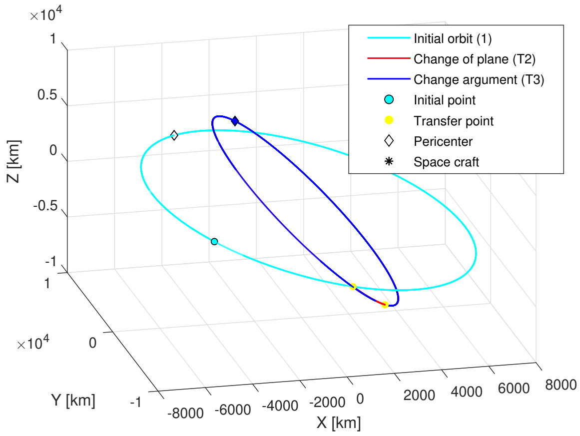


Figure 4: Change argument (T3)

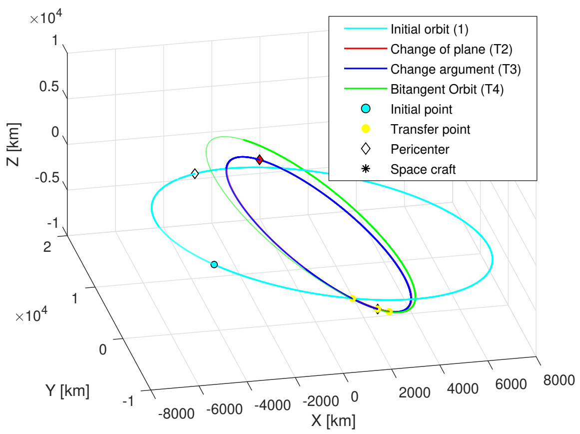


Figure 5: Bitangent orbit (T4)

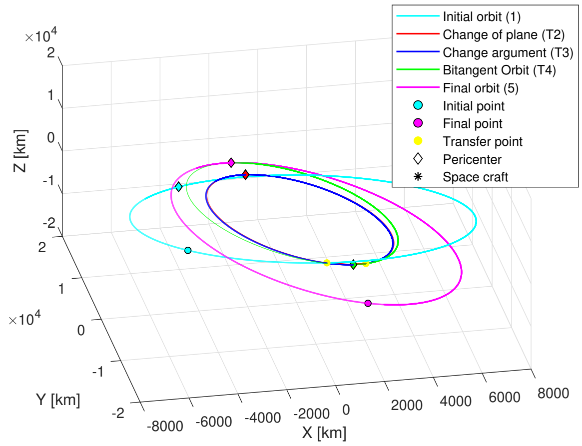


Figure 6: First strategy

The following graphs represent the second and the third strategy previously described.

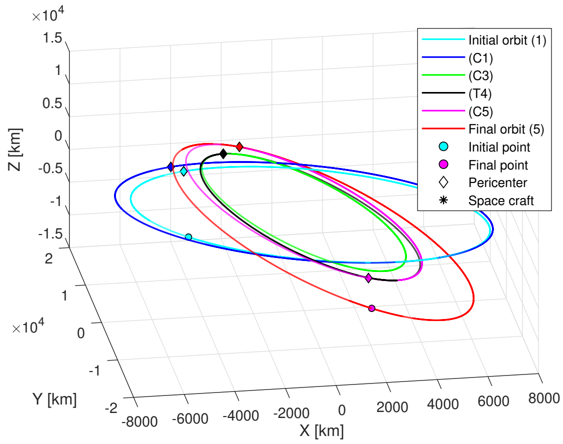


Figure 7: Second strategy

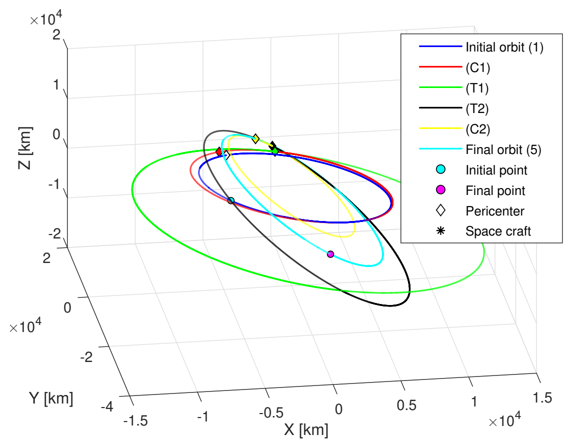
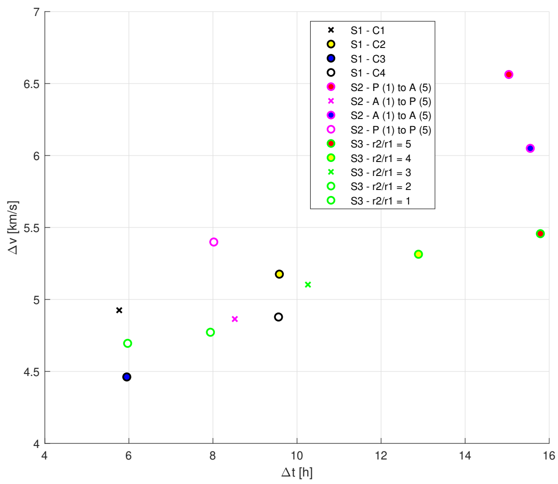


Figure 8: Third strategy

5
Conclusions

In this reports we examined three different kind of strategies, each with multiple manoeuvres taken in different points. For all strategies we selected the case that permits the best trade-off between Δt and Δv.
The first strategy, which consists in a change of plane, a change of argument and a change of shape, is certainly the one that lets us find the best trade-off between time and impulse needed (see case 1).
The second strategy thanks to the circularization of the initial and final orbits lets us do the transfer without a manoeuvre of change of argument, however that is payed with a major impulse and a major time.
In the end, the bi-elliptical transfer, despite the very low impulse needed in the change of plane, is not so convenient as expected, due to the low radius ratio and the low rotation angle between the two circular orbits.



6
Appendix

These tables report the transfer from the initial orbit to the last one. For each strategy we selected only one case, which is the one that represents the best trade-off between Δv and Δt.
The first column of the following tables represents the studied transfer between two orbits. For each orbit we described all its orbital parameters (a, e, i, Ω, ω); every line concerns one transfer and it describes the position of the same point (which is identified by θ) in the two orbits of the transfer.
Moreover, we also analysed the Δv and the Δt, which are respectively the impulse to apply to the first orbit to move to the next one the necessary time. The third column ($T/2$) describes the semi-period of each orbit.

First strategy, first case:

	$\Delta t[s]$	$T/2[s]$	$a[km]$	$e[-]$	$i[deg]$	$\Omega\,[deg]$	$\omega[deg]$	$\theta[deg]$	$\Delta v[km/s]$
Initial point			8458.84	0.050758	35.77	65.20	70.30	95.37	
$1 \rightarrow 2$	1090.00	3871.22	8458.84	0.050758	35.77	65.20	70.30	143.79	3.487
			8458.84	0.050758	63.20	83.88	57.74	143.79	
$2 \rightarrow 3$	628.10	3871.22	8458.84	0.050758	63.20	83.88	57.74	170.45	−0.116
			8458.84	0.050758	63.20	83.88	38.65	189.55	
$3 \rightarrow 4$	7515.51	3871.22	8458.84	0.050758	63.20	83.88	38.65	180	0.412
			9586.36	0.07283	63.20	83.88	218.65	0	
$4 \rightarrow 5$	4670.48	4670.48	9586.36	0.07283	63.20	83.88	218.65	180	0.910
			13360	0.2302	63.20	83.88	38.65	0	
Final point	6894.03	7684.06	13360	0.2302	63.20	83.88	38.65	168.05	

Second strategy, second case:

	$\Delta t[s]$	$T/2[s]$	$a[km]$	$e[-]$	$i[deg]$	$\Omega\ [deg]$	$\omega[deg]$	$\theta[deg]$	$\Delta v[km/s]$
Initial point			8458.84	0.050758	35.77	65.20	70.30	95.37	
$1 \rightarrow C1$	1945.02	3871.22	8458.84	0.050758	35.77	65.20	70.30	180	0.172
			8888.20	0	35.77	65.20	70.30	180	
$C1 \rightarrow C3$	7500.50	4169.67	8888.20	0	35.77	65.20	70.30	143.79	3.542
			8888.20	0	63.20	83.88	57.74	143.79	
$C3 \rightarrow 4$	5008.51	4169.67	8888.20	0	63.20	83.88	57.74	0	0.239
			9586.36	0.07283	63.20	83.88	57.74	0	
$4 \rightarrow C5$	4670.48	4670.48	9586.36	0.07283	63.20	83.88	57.74	180	0.231
			10284.53	0	63.20	83.88	38.65	218.20	
$C5 \rightarrow 5$	4639.24	5189.88	10284.53	0	63.20	83.88	38.65	0	0.679
			13360	0.2302	63.20	83.88	38.65	0	
Final point	6894.03	7684.06	13360	0.2302	63.20	83.88	38.65	168.05	

Third strategy: we selected the case where the ratio between the apocenter and pericenter radius is 3. Moreover, the transfer points coincide with the apocenter of the first orbit and the pericenter of the last one:

	$\Delta t[s]$	$T/2[s]$	$a[km]$	$e[-]$	$i[deg]$	$\Omega\ [deg]$	$\omega[deg]$	$\theta[deg]$	$\Delta v[km/s]$
Initial point			8458.84	0.050758	35.77	65.20	70.30	95.37	
$1 \rightarrow C1$	1945.02	3871.22	8458.84	0.050758	35.77	65.20	70.30	180	0.172
			8888.20	0	35.77	65.20	70.30	180	
$C1 \rightarrow T1$	3330.83	4169.67	8888.20	0	35.77	65.20	70.30	323.79	1.505
			17776.40	0.5000	35.77	65.20	34.08	0	
$T1 \rightarrow T2$	11793.60	11793.60	17776.40	0.5000	35.77	65.20	34.08	180	1.493
			18474.56	0.44331	63.20	83.88	21.53	180	
$T2 \rightarrow C2$	12495.17	12495.17	18474.56	0.44331	63.20	83.88	21.53	0	−1.254
			10284.53	0	63.20	83.88	38.65	342.89	
$C2 \rightarrow 2$	493.44	5189.88	10284.53	0	63.20	83.88	38.65	0	0.679
			13360	0.2302	63.20	83.88	38.65	0	
Final point	6894.03	7684.06	13360	0.2302	63.20	83.88	38.65	168.05	0