Orbital Mechanics
FINAL ASSIGNMENT 1

Interplanetary Explorer Mission

Group ID: 32

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1 Notation

In this project the following notation has been used:
Celestial bodies:

♁	Earth
♂	Mars
♀	Venus
♄	Saturn
☾	Moon

Orbital parameters:

a	Semi-major axis	[km]	Ω	Right ascension of the ascending node	[rad]
e	Eccentricity	[−]	ω	Argument of the perigee	[rad]
i	Inclination	[rad]	θ	True anomaly	[rad]

Parameters:

J_2	Coefficient that expresses of the Earth	[−]	m	Number of Earth revolutions	[−]
$\mathbf{R}$	Position vector	[km]	n	Mean motion of the spacecraft	$[\frac{rad}{s}]$
λ	Longitude	[rad]	ω_E	Earth rotational velocity	$[\frac{rad}{s}]$
k	Number of spacecraft revolutions	[−]	M_0	Mean anomaly	[rad]
f	Frequency	[μHz]			

Notation:

FB	Fly-by	GEO	Geostationary orbit
SOI	Sphere of influence	GT	Ground track
ARR	Arrival	SYN	Synodic
DEP	Departure	INF	Infinite
TOF	Time of flight	CO	Cut-off
DOFs	Degrees of freedom	GA	Powered Gravity Assist
S/C	Spacecraft	UN	Unperturbed
ATM	Atmosphere	P	Pericenter
REP	Repetition of the GT	S	Sampling

2 Abstract

The PoliMi Space Agency is carrying out a feasibility study for a potential Interplanetary Explorer Mission visiting three different planets in the Solar System.
The aim of this project is therefore to design the optimum transfers between the first planet (Saturn ♄) and the last (Venus ♀), by performing a powered gravity assist at the intermediate planet (Mars ♂), where the quantity to optimize is the Δv_{TOT} of the overall transfer.
The earliest departure and the latest arrival dates to reach Venus from Saturn are fixed as:

Earliest	Latest
2020.01.01	2060.01.01

Table 5: Earliest and latest allowable dates

Since the problem is undetermined and the Δv of each transfer repeats evenly with time, as described in the following sections, the best solution has been found as a trade off among the total time of the transfers (Δt_{TOT}), Δv_{TOT} and the time of the departure. Different methods are hence analysed to reach the optimum.

3 Preliminary Analysis

3.1 First considerations

The project is based on the method of patched conics; it inspects two Lambert transfers and an intermediate gravity assist fly-by.
When the S/C approaches at a certain distance the planet where the fly-by takes place, the reference system has to be changed. Thus, the overall mission is in a heliocentric frame, but the fly-by, since as the S/C enters into the sphere of influence of Mars, the new reference system has this planet as the main mass.
Also, the planetary departures and insertions will not be examined, so the initial heliocentric orbit is esteemed as the one of the departure planet, as well as the final heliocentric orbit with respect to the arrival planet. Thus the radius of the departure and arrival points coincide with the ones of the planets.

To analyse the figure of merit of the mission (Δv_{TOT}) one needs to consider that here each transfer is characterised by the time of departure and the time of arrival (or the time of flight of the transfer itself, which in particular is the case of the Lambert's problem).
Indeed, the degrees of freedom that will be employed are:

- the departure time (t_{DEP}) from Saturn
- the time of flight between Saturn and Mars (which corresponds to the first transfer): $\text{TOF}_{♄♂}$
- the time of flight between Mars and Venus (last transfer): $\text{TOF}_{♂♀}$

Also, the time of flight of the fly-by is not considered since it is quite small with respect to the others (as it can be seen in the results in the following sections).

3.2 Constraints

Regarding the design of the mission, one of the main constraints is related to the minimum allowable altitude during the fly-by. Since MARS Odyssey[1] theoretically will reach the planet performing an aerobraking[2] with a minimum altitude at the perigee of 120 kilometers, in this project h_{ATM} will be fixed as 150 km (in order to consider a proper and "safe" difference of 20%).

[1] https://mars.nasa.gov/odyssey/mission/timeline/mtaerobraking/
[2] spaceflight technique wherein an orbiting spacecraft brushes against the top of a planetary atmosphere

Furthermore, there are other constraints related to the definition of the time windows. In fact, in order to locate the optimum Δv one needs to choose time branches long enough to capture all the possible relative positions between departure and arrival points. However, since the time lapse that has to be considered for the overall transfer is quite large ($\sim$ 40 years) to be analysed with numerical methods, prior considerations about the different time of flights have to be taken into account to reduce each branch.

The period of each orbit can be evaluated through the ephemerides of the planets and moreover they can be used to evaluate the synodic periods as follows:

$$T_{\text{SYN}}{}^{♄♂} = \frac{T_{♄} T_{♂}}{|T_{♄} - T_{♂}|} = 733.7012\,\text{days} \qquad T_{\text{SYN}}{}^{♂♀} = \frac{T_{♀} T_{♂}}{|T_{♀} - T_{♂}|} = 333.9220\,\text{days} \tag{3.1}$$

The following restrictions related to the specified DOFs are be examined:

- t_{DEP}: assuming the orbit of Saturn approximately circular ($e_{♄} = 0.05548$), the window considered for the departure is function of the orbital period of the planet itself ($T_{♄} = 29.535$years); the branch also starts from the earliest allowable date (day_0).
- **TOF**$_{♄♂}$ and **TOF**$_{♂♀}$: the TOF to analyse the Lambert's transfers depend on the synodic periods. Indeed, from the pork-chop plots [Fig. 3.1] one can notice a repetition among the Δv. Thus for the first and last transfers proper multiples of the synodic periods are examined.

The time windows that have been examined in this analysis are:

DEP_{min}	DEP_{max}	$\text{TOF}_{♄♂\,min}$	$\text{TOF}_{♄♂\,max}$	$\text{TOF}_{♂♀\,min}$	$\text{TOF}_{♂♀\,max}$
day_0	$\text{day}_0 + T_{♄}$	$0.1\,T_{\text{SYN}}{}^{♄♂}$	$3.41\,T_{\text{SYN}}{}^{♄♂}$	$0.1\,T_{\text{SYN}}{}^{♂♀}$	$1.29\,T_{\text{SYN}}{}^{♂♀}$

Table 6: Time windows

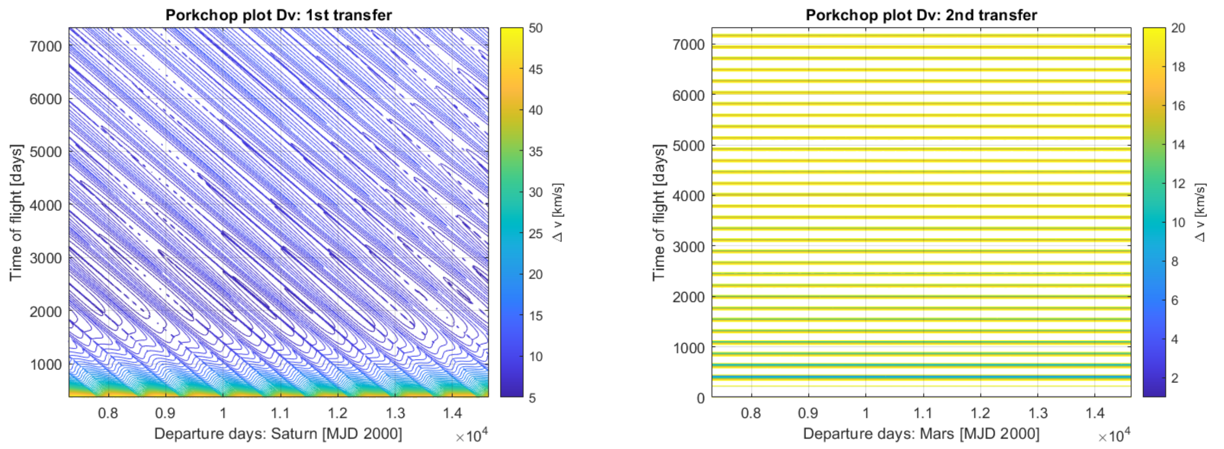
where day_0 is the earliest allowable departure date.

3.3 Triple loop grid search

A first approach to determine the minimum Δv_{TOT} is based on a triple loop grid search; since the degrees of freedom are three one can implement an algorithm in order to consider all the possibilities of t_{DEP}, $\text{TOF}_{♄♂}$ and $\text{TOF}_{♂♀}$.
The advantage of this method is that it permits to analyse each value that the DOFs can assume and to build up a 3D matrix over the three degrees of freedom using a nested triple loop[3]. However, at the same time it requires a high computational cost and its precision is directly proportional not only to how large the time windows are, but also to how many values are considered within them.

A useful tool to analyse an interplanetary mission is the porkchop plot of the transfer, that links the time spans selected with the cost of every possibility.
Hence, by spacing the time vectors of the DOFs of few days each and using larger time windows, one obtains the following porkchop plots that link the departure date and the time of flight of each transfer[4].

[3]see MATLAB function "Preliminary_analysis_basic"
[4]see MATLAB functions "Analysis_windows_dp_da", "Analysis_windows_dp_TOF", "PCP_dp_da" and "PCP_dp_TOF"

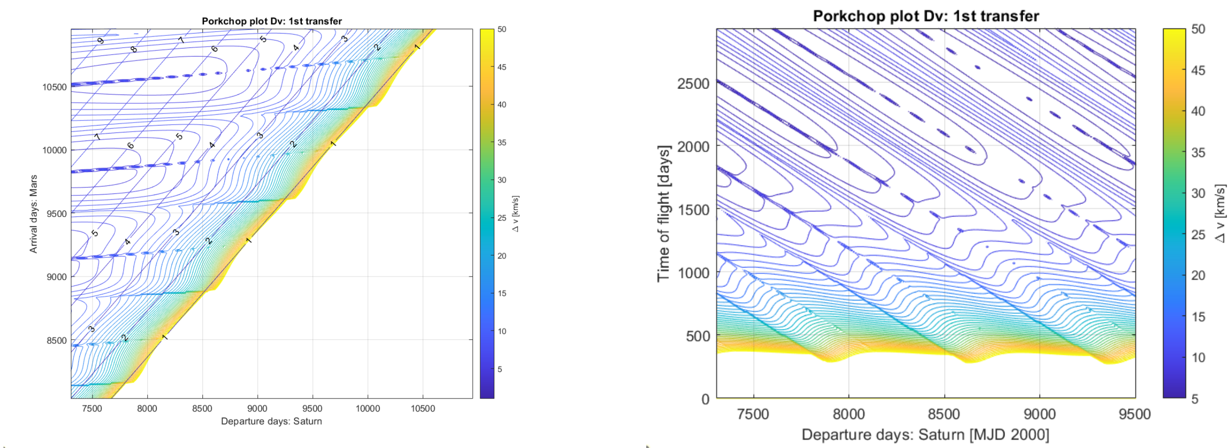


(a) First transfer

(b) Second transfer

Figure 3.1: Porkchop plots using wide time windows

From Fig. 3.1 one can notice that if for the first transfer a repetition can be spotted, for the second one the time window has to be slightly shrunk.
Hence, to underline the repetition among them, the porkchop plots have been rerun in order to obtain smaller ranges (Fig. 3.2, 3.3).
One obtains the following figures that link the departure date, time of flight of each transfer and the arrival date[5].

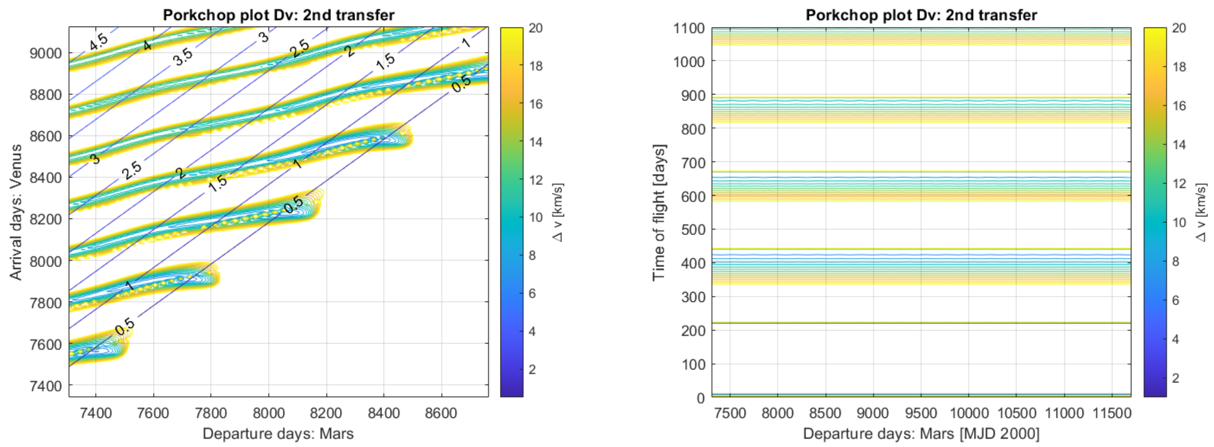


(a) Departure vs arrival

(b) Departure vs time of flight

Figure 3.2: First transfer: porkchop plots using smaller time windows

[5] see MATLAB functions "Analysis_windows_dp_da", "Analysis_windows_dp_TOF", "PCP_dp_da" and "PCP_dp_TOF"



(a) Departure vs arrival

(b) Departure vs time of flight

Figure 3.3: Second transfer: porkchop plots using smaller time windows

As it can be seen from Fig. 3.3, a smaller time span must be considered to go from Mars to Venus in order to avoid the regions where the cost of the transfer is too high. Indeed, one can notice that, as far as a larger window is analysed, the Δv is no longer acceptable.

4 Genetic Algorithm

Since a rough and preliminary analysis has been conducted to select proper time windows for the two Lambert's problems, a more refined strategy can now be used to explore the branches in a more efficient way.
The function that has been implemented is the Genetic Algorithm; however, even though this is an advanced strategy, the convergence to the optimum result is not guaranteed if a wide domain is examined. Therefore also in this case a loop cycle has been employed to find the best solution among different optimum results and the convergence to the best one then can be expected.

The Genetic Algorithm takes as input the three degrees of freedom previously mentioned. The first one, that is the departure time, covers the sidereal period of Saturn ($T_{♄} = 29.535$years), which has been analysed not only entirely but also dividing it into four subdomains; the other two instead thanks to the previous analysis have already been fixed (see Table 6).

The employed algorithm has been realized by making use of a function[6] which combines all the three transfers and gives as output the total cost of the overall manoeuvre and, finally, the latter is minimised by the Genetic Algorithm. As already stated, however, the algorithm provides different values even if repeated in the same time windows. Hence it has been rerun in the same time spans multiple times, in order to achieve to best result among a series of "minima". In the Appendix the main solutions obtained are reported.
Among these solutions, the lowest cost of the manoeuvre is achieved for the following departure, fly-by and arrival dates:

Departure	Fly-by	Arrival	Δv_{TOT}[km/s]
2045.07.07 19:57:58	2050.09.05 13:18:50	2050.11.15 10:52:26	18.5512

Table 7: Resulting dates

where a instantaneous fly-by has been considered.

[6] see MATLAB function "interTransfer"

If the time windows are enlarged other possible solutions can be considered. However the mission should not:

- overcome the maximum allowable arrival date;
- use a time of flight too wide.

Indeed, another solution for example could have been:

Departure	Fly-by	Arrival	Δv_{TOT}[km/s]
2102.12.23	2109.11.29	2117.11.22	18.0352

Table 8: Resulting dates

but it is considered for the previous statements.

4.1 Heliocentric trajectories

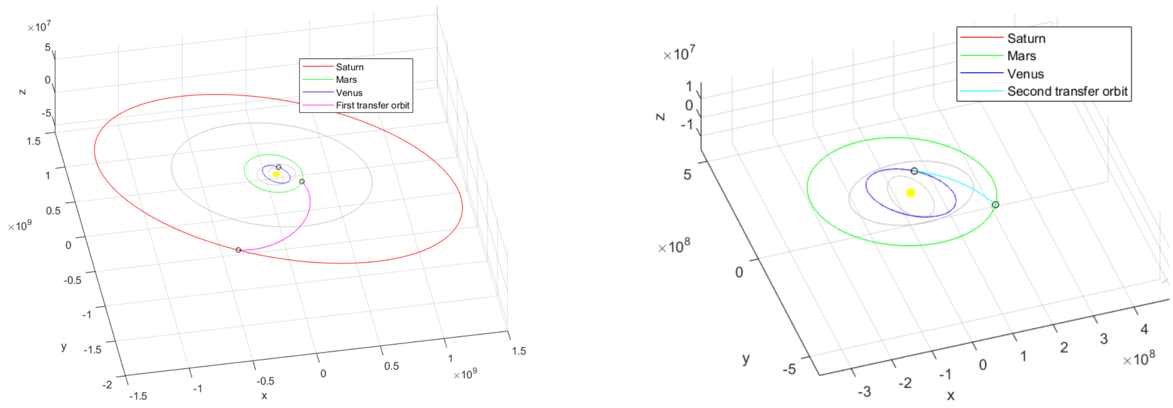
Forasmuch as a solution has been found, the characteristics and orbital parameters of the two heliocentric trajectories (from Saturn to Mars and from Mars to Venus) are given by:

	Departure	Arrival	a [km]	e [−]	i [rad]	Ω[rad]	ω[rad]
♄→♂	2045.07.07	2050.09.05	8.2043e+08	0.8636	0.0449	1.9577	5.4643
♂→♀	2050.09.05	2050.11.15	6.8702e+08	0.8431	0.0324	1.4344	6.0437

Table 9: Heliocentric trajectories

	Departure	Arrival	θ_{DEP}[rad]	θ_{ARR}[rad]
♄→♂	2045.07.07	2050.09.05	3.2186	4.7232
♂→♀	2050.09.05	2050.11.15	4.6667	0.0465

Table 10: Heliocentric trajectories



(a) First transfer (b) Second transfer

Figure 4.1: Transfers

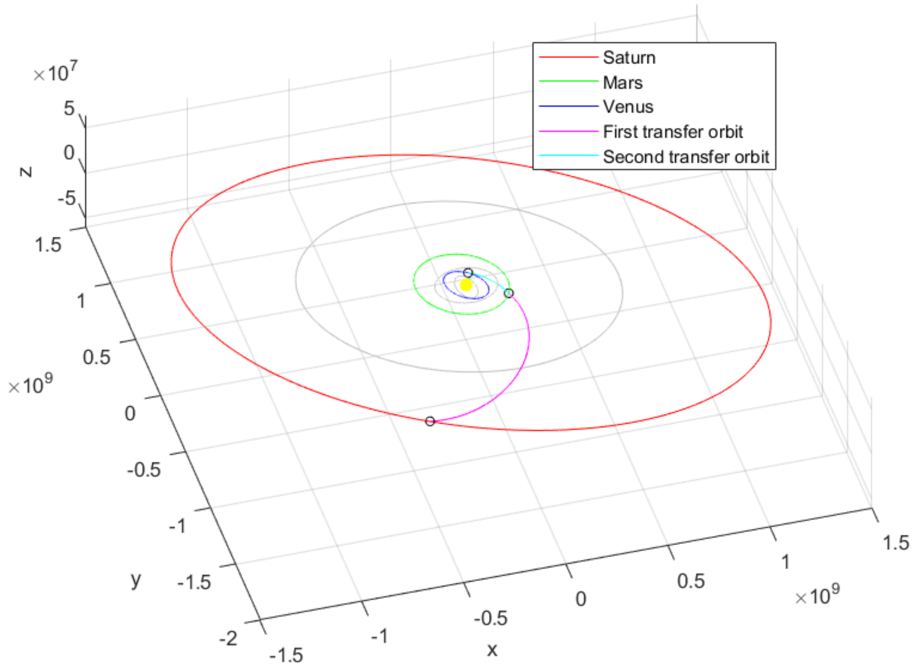


Figure 4.2: Overall mission

4.2 Gravity Assist Fly-by

By making use of the S/C velocities of the incoming and outgoing transfer arcs, the fly-by is fully determined and does not require any other degrees of freedom to be characterized. Also, one can notice that the excess velocities at the boundary of the hyperbolic arcs are not equal in norm, thus the fly-by has to be powered and a Δv has to be given at the pericenter of the two hyperbolic arcs.
The following table displays the relevant parameters of the incoming and outgoing arcs:

h_p[km]	r_p[km]	$e^-[-]$	$e^+[-]$	Δv_{inf}[km/s]	t_{FB}[h]
1155.4911	4545.3911	51.383657	51.383651	0.8481	14.6831

Table 11: Fly-by characterization

Where t_{FB} is the time spent by the S/C in the sphere of influence, hence the time spent performing the fly-by.

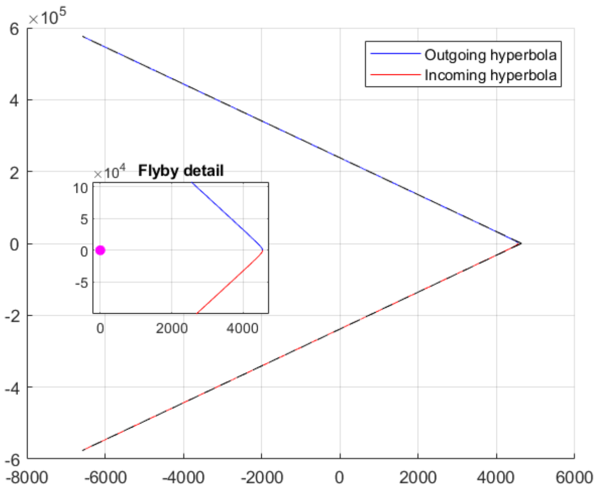


Figure 4.3: Fly-by

5 Final Considerations

In the following table are reported the selected costs of each manoeuvre in terms of Δv :

Δv_{DEP}[km/s]	$\Delta v_{P,GA}$[km/s]	Δv_{ARR}[km/s]	Δv_{TOT}[km/s]
5.9671	$1.2613e{-}06$	12.5841	18.5512

Table 12: Fly-by characterization

From the obtained results one can notice that, as expected, the cost required by the fly-by is really low with respect to the other two transfers; moreover the cost of the second interplanetary cruise is larger with respect to the first one.

As one can notice from Table 8, another way to obtain a lower Δv_{TOT} is by enlarging the time windows, in particular the one related to the departure and the time of flight of the first Lambert's transfer.
However, they exceed the maximum and minimum allowable dates previously fixed for this mission and the difference is not so relevant.

Another possible strategy for the mission in analysis could have been a single transfer arc from Saturn to Venus without performing a fly-by around Mars. However, this manoeuvre would have required too much propellant.
Indeed, in a real life mission a higher number of fly-bys would have been considered in the cruise from the first planet to last one in order to decrease the total cost of the mission. The higher the number of the fly-bys is, the lower is the Δv_{TOT}.