Домашнее задание 4 (часть)

Задача 2

а) $ye^{xy} = f(x,y)$

$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$z - be^{ab} = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$f_x(x,y) = y \cdot (e^{xy})' = y \cdot (xy)' \cdot e^{1 \cdot (xy)} =$

$= y \cdot y \cdot e^{xy} = y^2 \cdot e^{xy}$

$f_y(x,y) = y' \cdot (e^{xy}) + y \cdot (e^{xy})' =$

$1 \cdot e^{xy} + y \cdot (xy)' \cdot e^{1 \cdot (xy)} =$

$e^{xy} + xy \cdot e^{xy}$

$z - be^{ab} = b^2 \cdot e^{ab} \cdot (x-a) + (e^{ab} + ab \cdot e^{ab}) \cdot (y-b)$

в) $(0{,}1; 1{,}1) \approx (0,1)$

$f(x,y) \approx f(0,1) + 1^2 \cdot e^0 \cdot (x-0) + (e^0 + 0 \cdot e^0)(y-1)$

$1e^0 = 1$

$1 + 1 \cdot 1 \cdot x + 1(y-1) \approx x + 1 + y - 1 \approx$

$\approx x + y \approx f(0,1;\ 1.1) \approx 1$

Упражнение 3

$f(x,y) = \sin(x+y) - \cos(x^2)$

в точке $(0,0)$

$f(x,y) = f(0,0) + f_x'(0,0)(x-0) + f_y'(0,0)(y-0) +$

$+ \frac{1}{2!}(f_{xx}''(0,0)\cdot(x-0)^2 + 2f_{xy}''(0,0)(x-0)\cdot(y-0) +$

$+ f_{yy}''(0,0)(y-0)^2) + R_2$

1) $f(0,0) = \sin(0+0) - \cos(0^2) = \sin 0 -$

$\cos 0 = 0 - 1 = (-1)$

2) $f_x'(x,y) = (x+y)' \cdot \sin'(x+y) - (x^2)' \cdot \cos'(x^2) =$

$= 1 \cdot \cos(x+y) + 2x \cdot \sin x^2$

$f_x'(0,0) = \cos(0+0) + 2 \cdot 0 \cdot \sin^2 0 = 1$

3) $f_y'(x,y) = (x+y)' \cdot \sin(x+y) = 1 \cdot \cos(x+y)$

$f_y'(0,0) = \cos 0 = 1$

$f_{xx}(x,y) = (x+y)' \cdot \cos'(x+y) + 2 \cdot \sin x^2 + 2x \cdot (x^2) \cdot \sin'(x^2) = -\sin(x+y) + 2\sin x^2 + 4x^2 \cdot \cos x^2$

$f_{xx}(0,0) = -\sin 0 + 2\sin 0 + 2 \cdot 0 \cdot 2 \cdot 6 \cdot \cos 0^2 = 0$

5) $f_{xy}(x,y) = (x+y)' \cdot \cos(x+y) = 1 \cdot (-\sin(x+y))$

$f_{xy}(0,0) = -\sin 0 = 0$

6) $f_{yy}(x,y) = (x+y)' \cdot \cos(x+y) - 1 \cdot (\sin(x+y))$

$f_{yy}(0,0) = -\sin = 0$

$f(x,y) = (-1) + 1x + 1 \cdot y + \frac{1}{2} \cdot 0 \cdot x^2 + 2 \cdot 0 \cdot xy + 6 \cdot y^2 + R_2 = (-1) + x + y + R_2$