Mandatry Assignment 2

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1 Introduction

For this assignment I cooperate with Elsa Cecconello and Karl Jacobsen. In this assignment we are asked to solved the wave equation numerically. The equation have the following form:

$$u_{tt} - \partial_x \left(q(x) u_x \right) = f(x,t) \tag{1}$$

where u_{tt} is short notation for the partial derivative in time and u_x the derivative with respect to coordinate, $q(x)$ is just the wave velocity squared and the $f(x,t)$ is the source term. In the assignment we are given the function for the $q(x)$ (the different functions for a) and b) part) and need to estimate the source term ourselves using the symbolic python. We have a Neumann condition for the problem at $u_x at x = 0, L$.

2 Exercise 13

Exercise 13 a) and b)

In part a) we have a $q(x)$ in a polynomial form.

$$q(x) = 1 + (x - \frac{L}{2})^4 \tag{2}$$

In part b) we have $q(x)$ being the periodical function.

$$q(x) = 1 + \cos(\frac{\pi x}{L}) \tag{3}$$

Here we are given the exact solution and need to find the source term, so that our numerical solution is close to exact one. The exact solution is the same for both a) and b) part. Fist we start discretizing the equation. Here we have a problem on the boundaries and in order to fix it we introduce some points that are not included in the discretization domain. So-called ghost points at $[-\Delta x, 0]$ and $[L, \Delta x]$. In order to do this we need to adjust our array for the q and u so it fit the new scheme. Before we have $Nx + 1$ point and now we move to $Nx + 3$. Now the only thing that is left is to introduce the values for this ghost points. And here we are using different approximations for the q function at the boundaries in a) and b) parts. In the a) part we have a ghost points for the q:

$$q_{i-1} = 2q_i - q_{i+1}, for \quad i = 0, i = Nx + 2 \tag{4}$$

In part b) we have another approximation for the ghost points

$$q_{i-1} = q_{i+1}, for \quad i = 0, i = Nx + 2 \tag{5}$$

For the u function we have in both parts the same approximation for the ghost points

$$u_{i-1} = u_{i+1}, for \quad i = 0, i = Nx + 2 \tag{6}$$

The solution we have is very close to the exact one. The convergence rate are in the range from 1 to 2, but when the number of grid points increase we have something wrong with the convergence. Might by some problems caused by the round off errors.