Quantitative Risk Management

Assignment 1: Value-at-Risk and Expected Shortfall

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Abstract

In this report we focus on Value-at-Risk (VaR) and Expected Shortfall (ES) models. Our portfolio contains 7 risk factors and with initial investment amount being 1 million USD. By simulating different horizons for the VaR and ES using five models, we can understand how to allocate the capitals. We also do backtest and stress testing on all models to evaluate their fitness. We then give our recommendation of the models in the conclusion.

Contents

1 Portfolio and Data Description

In this assignment we create a long-only portfolio, which contains five assets - S&P 500 (GSPC), Nikkei 225 (N225), Bitcoin (BTCUSD=X), Invesco DB Commodity Index Tracking Fund (DBC), and German 1-year bond. We assume daily rebalancing and constant relative weight of 0.20 for all assets with initial investment amount of 1,000,000 USD. The Bitcoin as well as the exchange-traded funds S&P and DBC are denominated in USD, the German Bond is denominated in EUR and Nikkei 225 is denominated in JPY. We consider the following risk factors: the returns of the assets, the exchange rates EUR/USD and JPY/USD and the interest rate of the bond. We use historical data for the period from 31-03-2011 to 31-03-2020, considering that the latest observations reflect the recent health covid-19 crisis. The data sources are Yahoo! finance for the stocks and exchange rates and Investing.com for the bond yield. We synchronize the data by considering the days for which there are observations on all risk factors, leading to a dataset with 2019 observations. The mean for all factors is close to zero. Black et al. [1] and Bryne [2] suggest that data can be considered normal if the skewness and kurtosis are in the ranges (-2;2) and (-7;7), respectively. According to these measures, we conclude that the individual risk factors do not exhibit significant skewness, except for the Bitcoin. The kurtosis is postive for all risk factors and significant for the Bitcoin, S&P and the overall portfolio return, which is a sign of higher peaks and fatter tails. We set a criteria to exclude the date which the returns of Bitcoin are high than 50% or lower than -50%, these dates are 18-11-2013 (run on the bitcoin market after a flash crash a few weeks before [5]), 20-02-2014 (Tokyo-based exchange Mt.Gox has frozen withdrawals and caused turmoil [4]) and 26-02-2014 (first statement of the Mt. Gox management after the scandal [3]). The descriptive statistics for the risk factors on the final dataset of 2109 observations are exhibited in Table 8 (available in Appendix A) and their daily return changes dynamics are in Figure 7 (availabel in Appendix B).

2 Value-at-Risk and Expected Shortfall models

We simulated Value-at-Risk (VaR) and Expected Shortfall (ES) for our portfolio using five different methods. For each method, both 97.5% and 99% confidence intervals for the time horizon of 1 trading day are estimated.

2.1 Variance-covariance method

In this section, we first use variance-covariance method with two different distributions. We estimate VaR and ES by assuming both a multivariate-normal and multivariate-Student-t distribution for our risk factors. The calculation of VaR and ES is given below in Equation 1, where h denotes the risk horizon and α denotes the confidence level. Furthermore, μ is the mean for all the risk factors, σ the standard deviation, and q the critical value referring to the (1-α) quantile of the distribution. ES is the expected loss given that the loss exceeds VaR, L refers to the loss of the portfolio.

$$\begin{aligned} VaR_{h,\alpha} &= \mu_h + q_{1-\alpha}\sigma_h \\ ES_{h,\alpha} &= \mathbf{E}[L | L > VaR_{h,\alpha}] \end{aligned} \tag{1}$$

While both above equations hold for both distributions, the σ value calculation is different. For normal distribution, it is the simple square root of the sample variance. Regarding Student-t distribution, the critical value is the *t*-critical value and σ needs to be calibrated to account for the distributional properties of the Student-t. Therefore, the degree of freedom (ν) need to be take into consideration when calculating σ. Namely, the standard deviation becomes $\sigma = \sqrt{s^2 \frac{\nu-2}{\nu}}$ where s^2 is the sample variance.

To assess the validity of both distribution assumptions, we presented the histograms and Q-Q plots of our portfolio overall returns. The results are shown in Figure 1. The histogram on the top-left panel shows that the distribution has higher peaks and heavier tails than that of a normal distribution. These results are consolidated with the Q-Q plot for the normal distribution in the top-right pane. We can see quite clearly that our data does have heavy-tails when using the normal distribution. As from the Q-Q plot we observe that for Student-t distribution, the majority of the observations lie on the 45° line, which shows the relationship between the ordered values and the theoretical quantiles, which is in line with the histogram pattern. This means that the assumption of student-t distribution which has thinner tails is more realistic for our portfolio. We also performed normality check on the individual risk factors before and after the exclusion of the three Bitcoin outliers. We observe that if we do not exclude the extreme observations, the Bitcoin returns violate the student-t assumption.

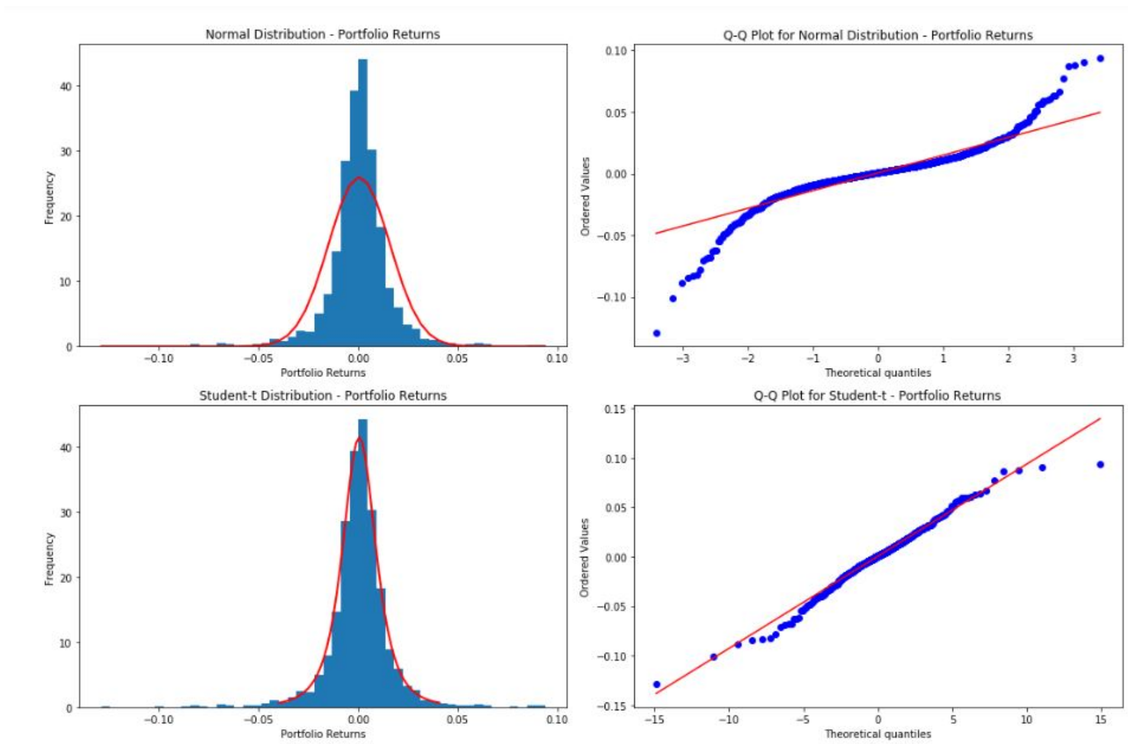

Figure 1. Portfolio Distribution

The plots show the fit of the portfolio returns to normal and student t distributions ($\nu = 3$) via histograms (left) and Q-Q plots (right). The graphs show that the data has higher peaks and heavier tails than the standatd normal distribution. Student t distribution which has thinner tail and nearly straight 45 degree line is a better fit for the portfolio data.

2.2 Historical simulation method

Next, we use historical simulation method to estimate VaR and ES. For this approach, we need to first calculate the portfolio return by multiplying the risk factors' log-returns with corresponding weights and sum them up where each risk factor has weight of 0.2. All the portfolio returns are multiplied by -1 to express the losses in positive terms. The porfolio losses are then sorted in magnitude from the largest to the smallest and the α percentile can be retrieved. i.e. if n (number of days) = 1000 and α = 0.99, the historical VaR is the (1 - 0.99) 1000 = 10 th largest loss. ES is calculated the same as in Equation 1.

2.3 Filtered Historical Simulation method with EWMA

The next model we apply the filtered historical simulation (FHS) method using exponentially weighted moving average (EWMA) for estimating the variance. EWMA model is calculated as Equation 2:

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)X_t^2 \tag{2}$$

where we updated the variance of the risk factors, σ^2 , by a factor of the previous period's variance determined by the constant λ and the previous period's squared return in the risk factor, X^2, multiplied by (1-λ). In order to apply FHS, we use the first 100 days for initiating the variance and use the next 500 days as the filtered period to estimated the next 1-day horizon VaR. The variances of these 600 days are estimated by EWMA. The residuals (Z) of the 500 days are obtained by those daily returns (R) divided by their standard deviation (σ). The 1-day horizon returns are calculated by multiplying the above 500 residuals by the new σ. The next is getting the desired percentile after sorting the returns as the way in historical simulation method. This process is done for each risk factor, then multiply with the weights and sum them together. As EWMA can capture the previous volatility of the assets (equation 2) and FHS focuses on the 500 most recent days and their distribution, this combined approach can have a good estimate of the VaR and ES values.

2.4 Constant Conditional Correlation method with GARCH(1,1)

The last method is the generalized auto-regressive conditional heteroskadisticity (GARCH) model with constant coditional correlation (CCC). GARCH model is calculated as Equation 3:

$$\sigma_{t+1}^2 = \omega + \alpha X_t^2 + \beta\sigma_t^2 \tag{3}$$

where the updating of the variance relies on an intercept ω plus the previous period's squared returns in the risk factors and the previous period's returns and variance weighted by α and β , respectively, where α and β are restricted such that $\alpha + \beta < 1$. ω can be seen as the long term variance where it can be calculated by $\frac{\omega}{1-\alpha-\beta}$. These three parameters are estimated with the maximum likelihood method. We can calculate the forecasted variance for each risk factors and update the variance-covariance matrix where the initial correlation matrix is obtained by using a certain period (1 year or 2 year) correlations and keep it the same.

The VaR and ES results for each of the approaches are summarized in Table 1, where we can see that for both confidence levels FHS-EWMA VaR is the highest because it take into accounts for both the most recent days residuals distribution and put heavy weights on the previous day variances and returns.

We also estimated the VaR and ES with respect to the different lookback period. For this, we calculate historical simulation and FHS-EWMA VaR with lookback periods of 1, 5, and 8 years. As can be seen in Table 9 on page 12, the risk measures change substantially when varying the lookback period. For historical simulation, VaR and ES increase as more volatile time periods are included. Though only have values for 5 and 8 yeares [1], FHS-EWMA also has high VaR and ES when including the more volatile period. When varying the degrees of freedom parameter for the Student-t approach from 3 to 6, VaR changes slightly while ES are almost the same, as can be seen in Table 10 on page 12. The corresponding Q-Q plots for the portfolio and all risk factors can be found in the appendix on page 13.

[1] FHS requires at least 600 days data for forecasting VaR and ES, which in 1 year simulation cases the data amout is not enough

Method	$\mathrm{VaR}_{1,0.975}$	$\mathrm{ES}_{1,0.975}$	$\mathrm{VaR}_{1,0.99}$	$\mathrm{ES}_{1,0.99}$
Normal	2.96	4.81	3.52	5.58
	29,600	48,100	35,200	55,800
Student t	2.77	4.42	3.97	5.95
	27,800	44,200	39,700	59,500
HS	3.12	5.08	4.69	6.9
	31,200	50,800	46,900	69,000
EWMA	12.25	17.81	15.98	23.29
	122,500	178,100	159,800	232,900
GARCH	5.05	7.58	6.00	8.31
	50,500	75,800	60,000	84,100

Table 1. VaR, ES estimates for the whole sample

The VaR and ES estimates are calculated using 2109 observations between 31-03-2011 and 31-03-2020. The initial portfolio investment is 1,000,000 USD at 31-03-2011.The results are shown in relative (percentage) and absolute (USD) terms for each of the five evaluation methods. The USD values are rounded to the nearest hundred. The results are evaluated at 97.5% and 99% confidence levels. $\nu = 3$.

We evaluated the 5- and 10-day empirical VaR of our portfolio for each method except for FHS-EWMA [2] and compare them to the scaled VaR using the square-root-of-time rule. Our results are presented in Table 2. From these results we can find out that using the scaling values underestimate the VaR for each method. Yet for 10-day estimates, GARCH overestimates the VaR. We see that as the time horizon increases, the accuracy of VaR using the square-root-of-time rule decreases as we can see from the larger degree of error in the 10-day VaR estimate compared to the actual $\mathrm{VaR}_{10,\alpha}$. This is mostly because of the corona virus which causes a highly volatile markets. Using the scaled estimates cannot capture the change throughout the period.

To understand the influence of the stress period, we carried out a sensitivity analysis which compares the VaR trend with or without the stress period (corona virus crisis in 2020). For the non-stressed estimate, we exclude the most recent year. From figure 2 we can observe that when excluding the stress period, VaR values are lower than the one including it. This difference is the largest when close to the stress period. Overall the long period VaR values are similar since the effect of stress period is flattened throughout the period.

[2] The same reason

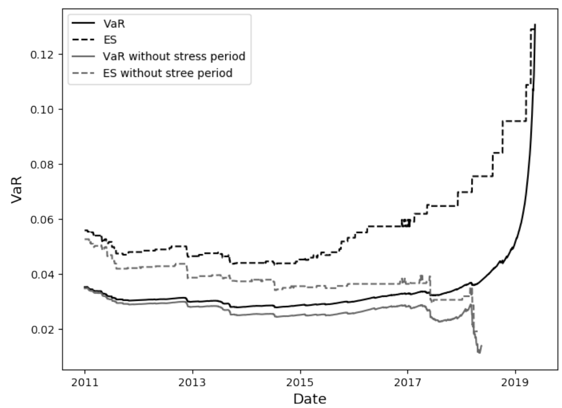

Figure 2. Sensitivity analysis

Sensitivity analysis of VaR regarding the stress period using variance-covariance method with normal distribution. The corona virus crisis is the selected stress period. The dataset includes 2109 observations from 03-31-2011 to 30-03-2020. The results are evaluated at 99% confidence levels.

Method	$\text{VaR}_{5,\alpha}(\%)$	$\text{VaR}_{1,\alpha}\sqrt{5}(\%)$	$\text{VaR}_{10,\alpha}(\%)$	$\text{VaR}_{1,\alpha}\sqrt{10}(\%)$
$\alpha = 99\%$				
Normal	8.87	7.87	11.31	11.13
Student-t	10.03	8.89	12.8	12.57
Historical	12.3	10.48	12.61	14.83
GARCH	14.23	14.03	11.66	19.84
$\alpha = 97.5\%$				
Normal	7.43	6.61	9.46	9.35
Student-t	6.95	6.19	8.84	8.76
Historical	8.93	6.99	10.05	9.89
GARCH	11.95	11.8	9.75	16.69

Table 2. Empirical VaR Comparison

The comparison between the VaR for time horizons and the 1-day VaR multiplied by the square root of the time horizon. The results are in relative terms and are denoted in percentages of the portfolio value. The VaR methods are indicated in the leftmost column: Variance-covariance method assuming a normal distribution; variance-covariance method assuming a Student-t distribution with ν = 3; historical simulation method; FHS assuming a EWMA model with λ = 0.94 and CCC using GARCH(1,1). The dataset includes 2109 observations from 03-31-2011 to 30-03-2020. The results are evaluated at 97.5% and 99% confidence levels.

3 Backtest

In this section, we present the results from the backtesting of our VaR system. We used a rolling 2-year lookback period and calculated $\text{VaR}_{1,0.99}$ for every day and each approach as outlined above. The result of the calculations can be seen in figure 3, which plots the daily VaR estimates as well as the changes in portfolio losses. Due to the fact that FHS method requires 600 extra days (100 initiating days + 500 days for the residuals), we started our rolling window from the 601th day. For the static methods (normal, student-t and historical methods) we re-evaluate every 1 year. For

GARCH method, we re-estimate the parameters (ω, α andβ) every 1 year for the computation cost reason.
From the figure we can see there is a huge difference between the static and dynamic methods where both FHS-EWMA and CCC-GARCH changes the values accordingly to the daily loss changes. In particular, FHS-EWMA has the highest VaR values and hardly violate the daily losses. Both normal and student-t variance-covariance methods change slowly and suffer from the violations the most. Figrure 4 shows that for the static method, they have relatively more violation clustering in the times of high volatility. This means that the probability of VaR values violating loss values increases if there was a violation in the previous period. As we expect the number of violations to be 1-α overall times, this clustering behavior is not desired. On the other hand, CCC-GARCH violations spread over the period and don't cluster during the high volatility times. FHS-EWMA is the most appropriate approach with regards to our portfolio as there is nearly no violation. The estimated VaR are higher during the high volatility times which is a good characteristic since the company can have better capital allocation weights during stress periods.

To investigate deeper if our above observations are correct, we counted the number of VaR violations per year and compared this to the expected number of violations. The expected number of violation is calculated by multiplying the significance level (1-α) with the respected trading days for each year . From the average discrepancy value in Table 3, we can see that historical simulation method has close discrepancy value to the one of CCC-GARCH, yet the violations per year of HS shows significant clustering. This result is in line with what we see from figure 4 that FHS-EWMA is the best fit methods for our portfolio. From Table 4 we can see that ES values vary from year to year while the highest value is in 2020 for all methods because of the corona virus.

	2013	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**Sum**	**Average**
Trading day	46	230	234	236	239	236	232	55	1508	Discrepancy
$\alpha = 99\%$										
Expected	0.46	2.3	2.34	2.36	2.39	2.36	2.32	0.55	15.08	
Normal	2	4	4	2	4	10	4	5	35	2.17
Student-t	2	2	4	1	2	8	2	4	25	1.46
Historical	0	1	2	1	1	3	3	4	15	0.50
FSH-EWMA	0	0	0	0	0	0	0	1	1	-0.77
CCC-GARCH	0	0	4	2	3	7	3	3	22	0.94
$\alpha = 97.5\%$										
Expected	1.15	5.75	5.85	5.9	5.975	5.9	5.8	1.375	37.7	
Normal	4	5	5	3	7	13	5	5	47	0.69
Student-t	4	7	6	3	8	16	6	6	56	0.96
Historical	4	5	5	1	7	14	5	5	46	0.68
FHS-EWMA	0	0	0	0	0	1	0	3	4	-0.71
CCC-GARCH	2	13	7	3	5	12	5	3	50	0.45

Table 3. VaR Violations and discrepancy between violation and expected numbers.

This table shows the VaR violations as well as the expected number of VaR exceedances per year and for the latter 1508 samples. All results are calculated for $\alpha = 99\%$ and 97.5%. In the second row, the number of trading days per year is shown. The average discrepancy between the realised and expected number of VaR violations are listed in the rightmost column. The VaR methods are indicated in the leftmost column: Variance-covariance method assuming a normal distribution; variance-covariance method assuming a Student-t distribution with $\nu = 3$; historical method; FHS-EWMA with $\lambda = 0.94$ and CCC-GARCH. The dataset includes 1508 observations from 18-10-2013 to 30-03-2020.

	2013	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**ES(T)**
Trading day	46	230	234	236	239	236	232	55	
$\alpha = 99\%$									
Normal	4.72	4.73	4.84	4.02	2.98	3.42	3.59	7.15	5.58
Student-t	4.72	5.5	4.84	4.64	3.53	3.66	4.19	8.2	5.95
Historical	N/A	6.3	5.47	4.64	4.05	4.82	3.82	8.2	6.9
FSH-EWMA	N/A	N/A	N/A	N/A	N/A	N/A	N/A	8.85	13.98
CCC-GARCH	N/A	3.25	3.97	3.55	3.05	3.14	3.76	8.63	8.31
$\alpha = 97.5\%$									
Normal	4.2	4.52	4.55	3.37	2.6	3.13	3.4	7.15	4.81
Student-t	4.2	4.11	4.27	3.67	2.51	2.91	3.15	6.34	4.42
Historical	4.2	4.52	4.55	4.64	2.6	3.05	3.4	7.15	5.08
FHS-EWMA	N/A	N/A	N/A	N/A	N/A	0.79	N/A	8.63	10.53
CCC-GARCH	4.21	2.5	3.93	3.5	2.6	2.65	3.4	8.63	7.58

Table 4. Average Expected Shortfall

This table shows the the average per year shortfalls and for the expected shortfalls at the end of the sample, at time T. All results are calculated for $\alpha = 99\%$ and 97.5%. In the second row, the number of trading days per year is shown. N/A indicates that VaR was not violated in that year and thus, there is no recorded ES. The values for ES are expressed in percentage terms. The methods are indicated in the leftmost column: Variance-covariance method assuming a normal distribution; variance-covariance method assuming a Student-t distribution with ν = 3; historical method; FHS-EWMA with λ = 0.94 and CCC-GARCH. The dataset includes 1508 observations from 18-10-2013 to 30-03-2020.

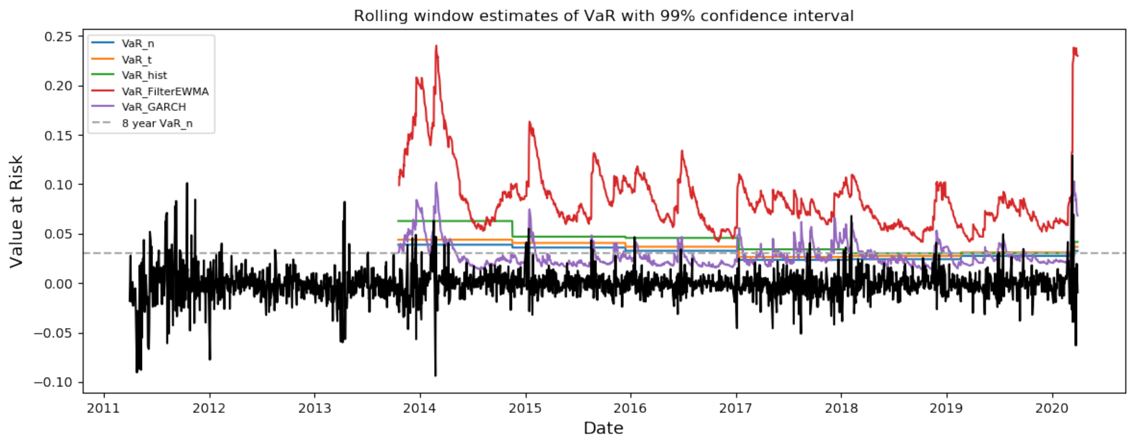

Figure 3. Rolling Window VaR of five methods

The figure shows the changes in portfolio losses as well as the rolling $\mathrm{VaR}_{1,0.99}$ estimates for every day. The VaR methods evaluated are those using the variance-covariance method assuming a normal and Student-t distribution with $\nu = 3$, historical simulation, FHS-EWMA ($\lambda = 0.94$) and CCC-GARCH. Furthermore, the $\mathrm{VaR}_{1,0.99}$ for the last day of the sample using a normal distribution and a 8-year lookback period is shown as the grey-dashed line.

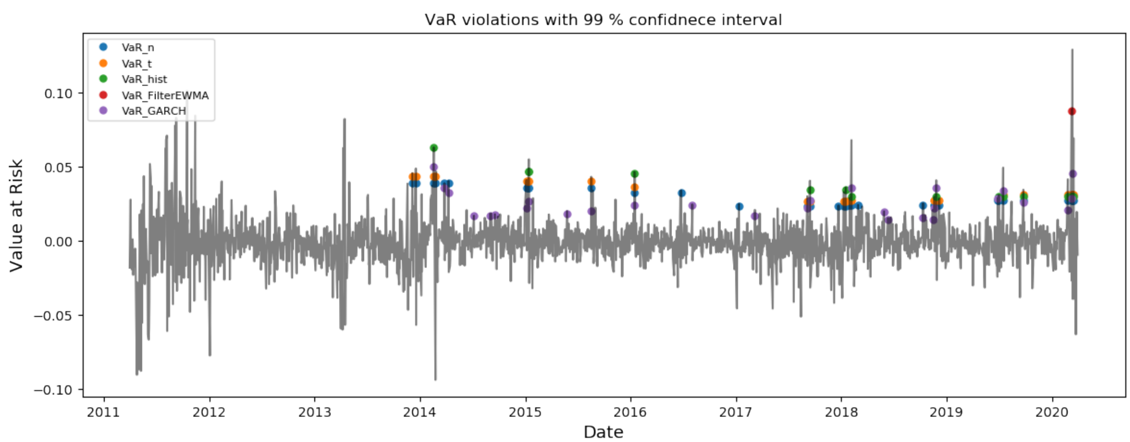

Figure 4. Rolling Window VaR Violations

The figure shows the changes in portfolio losses as well as the $\mathrm{VaR}_{1,0.99}$ violations. The VaR methods evaluated are those using the variance-covariance method assuming a normal and Student-t distribution with $\nu = 3$, historical simulation, FHS-EWMA ($\lambda = 0.94$) and CCC-GARCH.

4 Stress testing

In this section we discuss how extreme scenarios on the risk factors will affect the VaR estimates. The following stress scenarios are considered: increase or decrease of the equity and commodity indices by 20% and 40%, the foreign exchange rate by 10% and the interest rate pairs by 2% and 3%. We evaluate the shocks independently, meaning that we allow only one stress factor to affect the portfolio at a time. Given that the biggest exposure of our portfolio is to equity, it is expected that the equity shocks will have the biggest effect. Since we apply the stress to the current values and keep the remaining data the same, we also expect that using the whole history for the analysis will show results with less variability than using a shorter time period (having smaller number of observations allows for bigger weight, and therefore bigger impact of the stressed observations).

For FHS-EWMA (1-day, 99%) on the whole history we observe that that the stressed VaR (30.80%) is almost twice higher than the normal VaR (15.98%) when there is a -40% shock on equity and 1.5 times higher (23.95%) when there is a +40% shock on equity. The other equity, commodity and FX shocks produce VaR estimates in the range of 17.55%-19.97%. Interest rate shocks do not affect VaR significantly. When conducting the analysis on the smaller sample of 500 trading days, we notice that the shocks play an even more important role and even the interest rate stress becomes evident. Stressed VaR (42.05%) is almost three times higher than the normal VaR when equity is -40%. The rest of the results are in the range 22.39%-35.51% with the lowest values corresponding to interest rate shocks and the highest ones to equity and commodity shocks. This is in line with the initial expectations. We also note that the effect is reinforced by the currect covid health crisis. The results are shown in Table 5.

For comparison, the variance-covariance method with student-t distribution with $\nu = 3$ results exhibit some, yet quite small, variance on the whole sample when the equities are stressed. The results confirm the conclusion from FHS-EWMA stress test that equity +/-40% shock has most significant effect on the VaR on the 500 obs. sample. The interest rate and FX shocks do not affect the portfolio in that set-up. The numbers are presented in Table 6.

Table 5. FHS-EWMA method

	$\text{VaR}_{1,0.99}(\%)$	$\text{VaR}_{1,0.99}(\%)$
	2109 obs.	500 obs.
Equity		
+/- 20%	18.98/19.97	26.55/27.62
+/- 40%	23.95/30.80	33.51/42.05
Commodities		
+/- 20%	17.60/18.24	24.34/25.10
+/- 40%	19.78/22.47	26.96/30.17
Interest Rate		
+/- 2%	16.02/15.98	22.43/22.39
+/- 3%	16.05/15.99	22.46/22.40
FX		
+/- 10%	17.88/17.55	24.30/23.99

Table 6. Var-covar method, student-t distribution with $\nu = 3$

	$\text{VaR}_{1,0.99}(\%)$	
	2109 obs.	500 obs.
Equity		
+/- 20%	4.02/4.04	3.87/3.94
+/- 40%	4.14/4.33	4.36/5.10
Commodities		
+/- 20%	3.98/3.98	3.66/3.66
+/- 40%	3.99/4.01	3.73/3.81
Interest Rate		
+/- 2%	3.97/3.97	3.63/3.63
+/- 3%	3.97/3.97	3.63/3.63
FX		
+/- 10%	3.98/3.98	3.63/3.63

Table 7. Stress Testing Results

The stressed VaR and ES estimates are calculated using 2109 and 500 observations.The results are shown in relative (percentage) terms for FHS-EWMA. The results are evaluated for 1 day at 99% confidence levels.

5 Conclusion

In this report we studied and compared the strengths and weaknesses of 5 VaR and ES estimation methods for a portfolio of 5 assets. Based on the result in Table 1, FHS-EWMA amd CCC-GARCH can capture adequately volatility and market dynamics. Sensitivity analysis shows that the VaR and ES estimates of these two methods are nearly or over twice higher than the measures of the static methods when including the periods of the recent corona virus crisis. The sensitivity to shocks is further estimated through stress testing, which allows us to observe that for the FHS-EWMA and CCC-GARCH methods the movements in stock prices are better reflected in the stressed VaR estimates. Backtesting of observed over the predicted results show that the normal and student-t distribution have significantly higher number of violations than FHS-EWMA and CCC-GARCH. While the historical simulation might have lower discrepancy than CCC-GARCH, the clustering of violated days suggests that this method should be avoided in periods of high volatility. Although FHS-EWMA has the least violations and can change accordingly during the stress period, its high amount of VaR and ES may be a suboptimal approach in respect to the profitability of a financial institution, given the high requirement on capital reserves. Nevertheless, we advocate the use of dynamic approaches due to their general ability to better adapt to market volatility over static methods. However, we should note that they require more sophisticated implementation and have less stable measures. A specific recommendation we have is that the institution should implement a rolling system with a 1-day horizon and lookback equal to 1 year such that it can wholly capture extreme events. In addition, we recommend appropriate adjustment and margin of conservatism to be calculated and added to the VaR and ES estimates to correct for the data deficiencies associated to outlier treatment.

A Appendix A - Tables

	DE_1Y_BOND	BTC	DBC	GSPC	N225	FX_EUR	FX_YEN	Portfolio
mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
std	0.024	0.064	0.010	0.011	0.013	0.006	0.006	0.015
min	-0.166	-0.465	-0.099	-0.128	-0.083	-0.026	-0.040	-0.129
max	0.169	0.444	0.042	0.090	0.077	0.031	0.047	0.094
skew.	-0.336	2.988	-0.078	-1.218	-0.246	-0.047	-0.025	-0.503
kurt.	9.392	10.048	7.174	20.331	4.331	2.503	4.994	9.675

Table 8. Descriptive Statistics

The mean, standard deviation (sd), minimum value (min), maximum value (max), skewness (skew) and kurtosis (kurt) are listed for all seven risk factors and overall portfolio values. DE1YBOND: German 1-year bond,BTC: Bitcoin, FX_EUR: EUR/USD exchange rate, FX_YEN: YEN/USD exchange rate.

	VaR$_{hist}$	ES$_{hist}$	VaR$_{EWMA}$	ES$_{EWMA}$
1 year	59,500	95,600		
5 year	38,300	59,000	165,200	251,500
8 year	41,000	6,4000	159,100	234,500

Table 9. Historical simulation and FHS-EWMA VaR and ES for Different Lookback Periods

This table presents historical and FHS-EWMA VaR and ES evaluated with different lookback periods. The confidence level is 99%. The starting value of the portfolio is 1,000,000 USD at 31-03-2011. VaR and ES is expressed in absolute USD terms rounded to the nearest 100 Dolloars based on the value of the portfolio at the end of the sample.

	$\nu = 3$	$\nu = 4$	$\nu = 5$	$\nu = 6$
VaR	37900	40200	39500	38900
ES	59500	60100	59500	58400

Table 10. Student-t VaR and ES for Different Degrees for Degrees of Freedom from 3 to 6

This table presents Student-t distribution VaR and ES evaluated with different degree of freedom ($\nu = 3 - 6$). The confidence level is 99%. The starting value of the portfolio is 1,000,000 USD at 31-03-2011. VaR and ES is expressed in absolute USD terms rounded to the nearest 100 Dolloars based on the value of the portfolio at the end of the sample.

B Appendix B - Figures

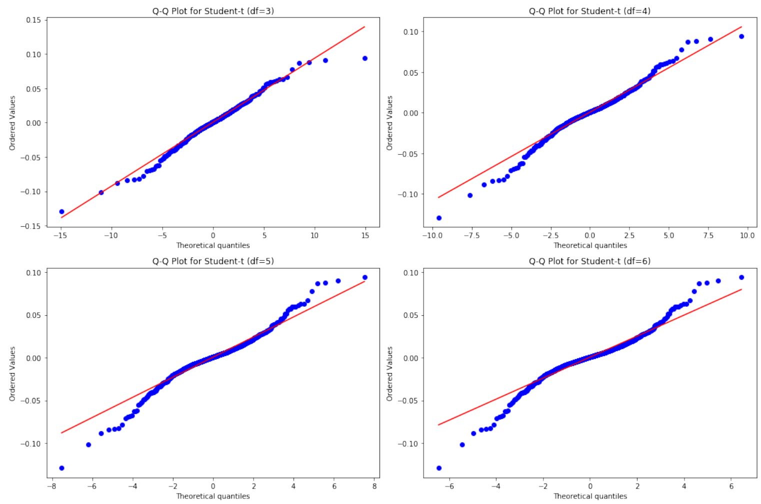

Figure 5. Student-t Distribution Q-Q Plots for Degrees of Freedom from 3 to 6

This figure shows the Q-Q-plots of the data using the Student-t distribution with different degrees of freedom (ν = 3-6). The dataset includes 2018 observations from 31-03-2011 to 30-03-2020.

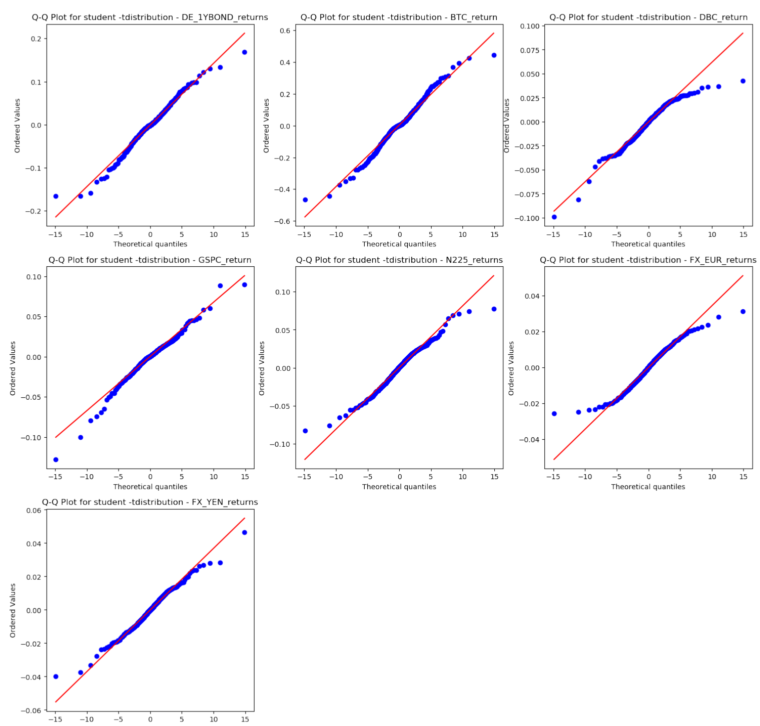

Figure 6. Student-t Distribution Q-Q Plots for Degrees of Freedom from 3 to 6

This figure shows the Q-Q-plots of the seven risk factors using the Student-t distribution with ν = 3. The dataset includes 2018 observations from 31-03-2011 to 30-03-2020.

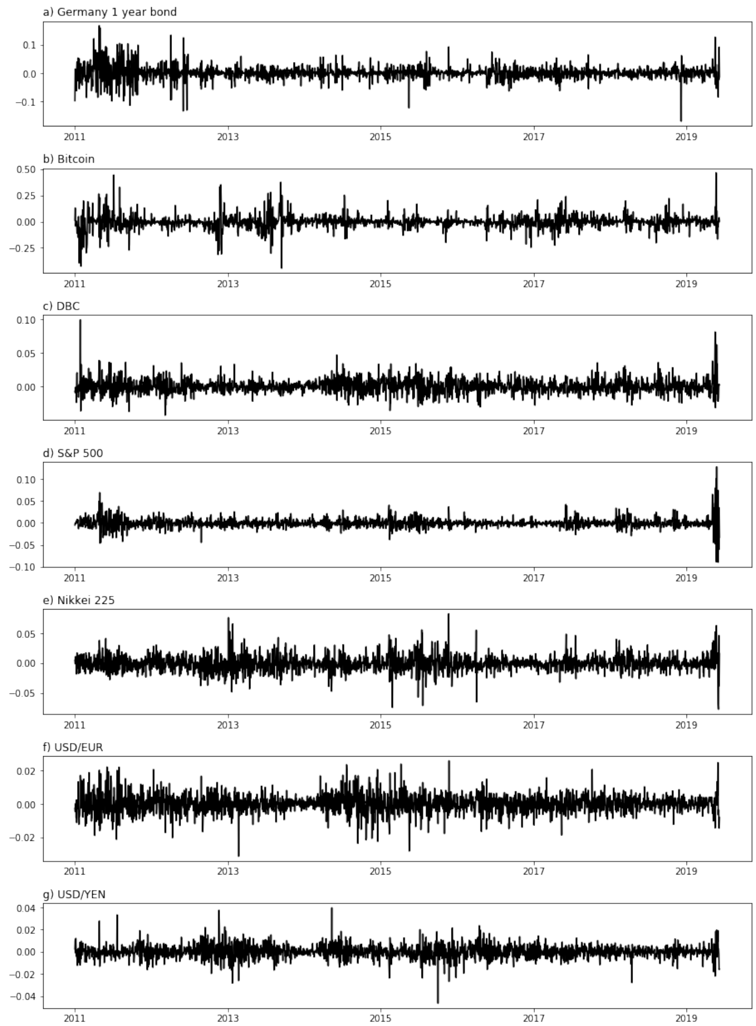

Figure 7. Risk Factor Changes

This figure shows the dynamic changes of all 7 portfolio risk factors over the whole period (2108 days) excluding the outliers.
a) Germany 1-year bond, b) Bitcoin, c) DBC, d) S&P 500, e) Nikkei 225, f) USD/EUR, g) USD/YEN

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