

Econometrics II - Assignment 4

Uncensored sloths

30 Jan 2022

Question 1

- a) Use the Wald estimator to compute the causal effect of a prison sentence on the probability of being arrested later.

Since treatment participation $D_i = 1$ (i.e. a prison sentence) is likely related to unobserved characteristics, instrumentation using the random assignment to either judge Z_i is necessary. Since the probability of being sentenced to prison is higher for judge Jones, assignment to Jones when $Z_i = 1$ is treated as the assignment to the treatment hereinafter. Hence, the Wald estimator reads as:

$$\hat{\delta}_{Wald} = \frac{E[Y|Z=1] - E[Y|Z=0]}{E[D|Z=1] - E[D|Z=0]} = \frac{(70\% \cdot 40\% + 30\% \cdot 60\%) - (40\% \cdot 20\% + 60\% \cdot 50\%)}{70\% - 40\%} = 26.6\%$$

```
((0.7*0.4+0.3*0.6)-(0.4*0.2+0.6*0.5))/(0.7-0.4)
```

```
## [1] 0.2666667
```

- b) What is the interpretation of the estimated effect? And for which fraction of the population does this causal effect hold?

Hence, the Wald estimator states that a prison sentence increases the probability of being arrested later by 26.6%. However, assuming monotonicity this causal effect only holds for compliers, i.e. people which would be sentenced to prison by Jones, but not by Smith. By the monotonicity assumption, i.e. ruling out the existence of defiers, which would not be sentenced to prison by Jones, but would be by Smith, the fraction of compliers in the population can be estimated by the (thus identifiable) fraction of compliers in the sample.

Assuming away defiers, we know that all 40% of the people sentenced to the treatment by Smith, i.e. for which $D(0) = 1$ are always takers. Moreover, we know that under the non-existence of defiers, all 30% of the people not sentenced to the treatment by Jones, i.e. for which $D(1) = 0$, are never takers. Hence, assuming that cases are truly randomly assigned to the judges, this implies that there must be $100\% - 40\% - 30\% = 30\%$ compliers in the sample and thus that the causal effect identified by the Wald estimator holds for (approximately) 30% of the population.

There is serious doubt, however, whether the assumption of monotonicity holds in the application at hand. After all, it seems anything but far fetched to assume that there are some individuals which are sentenced to prison by Smith, but not by Jones (conditional on some characteristics), even though Jones is on average more likely to convict people to prison. If this is the case, it becomes very difficult to interpret the Wald estimator as it becomes biased by the presence of defiers (Angrist et al, 1996). One could, however, still consider alternative assumptions to validate the interpretation of the Wald estimator outlined above. First, one could argue along the lines of Angrist et al. (1996) that the treatment effect for defiers and compliers is identical. Second, one could impose the complier-defier assumption brought forward by De Chaisemartin (2017) and assume that there exists some group of compliers which has the same size and average treatment effect as the defiers such that the bias brought along by defiers vanishes. However, if these assumptions do not reasonably hold, the interpretation of the Wald estimator is rather inconclusive.

c) Explain what an always taker is in this setting and which fraction of the population are always takers?

People that are sentenced to prison by both judges should be considered as always takers in this setting. Hence, for Smith they make up 40% of the sample once we assume away defiers since compliers and never takers are never sentenced to prison by Smith. For Jones, they are part of the 70% together with the compliers as only the never takers are never sentenced to prison with Jones as a judge under the monotonicity assumption. Consequently, always takers constitute 40% of the sample/population assuming monotonicity.

Question 2

a) Perform a power calculation for the number of students that the teacher should include in the field experiment.

Performing a power calculation, the number of students that need to be included in the field experiment can be calculated using the customary formula, where (i) $MDE = 0.1$ since the teacher thinks the passing rate is 10 percentage points lower for the treatment group, (ii) treatment intensity $p = 0.5$ by assumption since this minimizes the necessary sample size c.p., (iii) $\sigma = 0.5(1 - 0.5) = 0.25$ since the teacher assumes that 50% of students in the control group will pass the exam, and $t_{1-\alpha/2} = t_{0.975} = 1.960$ and $t_{1-q} = t_{0.3} = -0.524$ as specified by the teacher.

$$n = \left(\frac{t_{1-\alpha/2} - t_{1-q}}{MDE} \right)^2 \frac{\sigma}{p(1-p)}$$

```
MDE_target <- 0.1
t_q = -0.524
t_alpha <- 1.960
p = 0.5
sigma = (1-0.5)*0.5
n_target = ((t_alpha - t_q)/MDE_target)^2*((sigma)/(p*(1-p)))
ceiling(n_target)
```

```
## [1] 618
```

Hence, the teacher should include 618 students in the field experiment. The question whether the teacher should include any students in the experiment, i.e. whether to run it all, in light of ethical considerations is left to the interested reader.

b) The teacher assumes that 20% of the students randomized in the treatment group will actually have breakfast. How does this change the number of students required to participate in the field experiment?

Since the teacher does not expect full compliance, the number of necessary students should be computed as follows, where the compliance rate in the treatment group $r_t = 0.8$ since the teacher assumes that 20% of the students randomized in the treatment group will actually have breakfast and the treatment intensity in the control group $r_c = 0$ since the teacher assumes that all people in the control group will have breakfast.

$$n = \left(\frac{t_{1-\alpha/2} - t_{1-q}}{MDE(r_t - r_c)} \right)^2 \frac{\sigma}{p(1-p)}$$

```
rt <- 0.8
rc <- 0
n_compliance = ((t_alpha - t_q)/(MDE_target*(rt - rc)))^2*((sigma)/(p*(1-p)))
ceiling(n_compliance)
```

```
## [1] 965
```

As one would expect intuitively, non-compliance in the treatment group weakens the power of the experiment and thus increases the number of students that is required to participate to 965 (more than 50%) as not everyone complies with the treatment.

Question 3

```
# Load data
data <- read.csv("assignment4.csv")
```

- a) Compute for the children assigned to the control group the variance in flu incidence. If the researcher aims at reducing flu incidence by 0.05, how many children should participate in the randomized experiment.

```
q <- mean(data[data$treatgroup == 0, 'flu'])
sigma = (1-q)*q
sigma
```

```
## [1] 0.235434
```

The variance in flu incidence of the children assigned to the control group is approximately 0.2354.

As the researcher aims to reduce the flu incidence by 0.05, we set our MDE value to -0.05 . 80% of the children are supposed to get the flu shot which is why we set p to 0.8. The variance of the Bernoulli variable corresponds to the variance in flu incidence of the children assigned to the control group. Setting the power to 70% and the significance level to 5%, we get a required sample size of 3632 children.

```
MDE_target <- -0.05
t_q <- -0.524
t_alpha <- 1.960
p <- 0.8
n_flu = ((t_alpha - t_q)/MDE_target)^2*((sigma)/(p*(1-p)))
ceiling(n_flu)
```

```
## [1] 3632
```

- b) Compute which fraction of the children in the treatment group actually received a flu shot. What is the implication for the power analysis of the experiment?

```
rt <- mean(data[data$treatgroup == 1, 'flushot'])
rt
```

```
## [1] 0.6679552
```

Approximately, 66.8% of children in the treatment actually received a flu shot.

```
rc <- mean(data[data$treatgroup == 0, 'flushot'])
rc
```

```
## [1] 0
```

```
sigma = (1-q)*q
n_flu = ((t_alpha - t_q)/(MDE_target*(rt - rc)))^2*((sigma)/(p*(1-p)))
ceiling(n_flu)
```

```
## [1] 8140
```

Not every child that was supposed to get a flu shot, got a flu shot which is why we set the rate in the treatment group to $r_t \approx 0.668$. There are no children who were not supposed to get treatment but got a flu shot which is why the treatment intensity in the control group equals zero ($r_c = 0$). Including these

aspects into our calculation of the sample size, we find that the necessary sample size increases to 8140 children. Hence, if we do not increase the sample, this would decrease the power of the design considerably.

- c) Make a table with summary statistics for (1) the control group, (2) the treated treatment group, and (3) the untreated treatment group. What do you conclude?

```
data$treatedT <- ifelse(c(data$treatgroup == 1 & data$flushot==1), 1, 0)
data$untreatedT <- ifelse(c(data$treatgroup == 1 & data$flushot==0), 1, 0)
data$treatment <- ifelse(data$treatedT== 1, 1, ifelse(data$untreatedT== 1, 2, 0))

balance <- balance_table(data[, !names(data) %in% c("treatgroup", "flushot", "treatedT", "untreatedT")])
balance
```

```
## # A tibble: 7 x 6
##   variables1 Media_control1 Media_trat1 Media_trat2 p_value1 p_value2
##   <chr>          <dbl>          <dbl>          <dbl>    <dbl>    <dbl>
## 1 agemother      26.1          26.6          24.9  1.22e-12 2.46e-52
## 2 educmother     12.3          12.5          11.8  5.86e- 6 1.31e-28
## 3 flu            0.621         0.401         0.675 1.54e-79 1.83e- 5
## 4 genderchild    0.508         0.503         0.501 6.80e- 1 6.22e- 1
## 5 housincome     2270.         2374.         2111.  1.45e- 5 4.90e-10
## 6 married        0.957         0.977         0.939 1.83e- 5 1.24e- 3
## 7 nationality     0.278         0.239         0.341 1.33e- 4 3.00e- 7
```

Except for the gender of the children, which is equally distributed among the control, the treated treatment and the untreated treatment groups, we have significant differences in terms of the age of the mother, the education of the mother, the household income, the proportion of marriages and the proportion of children with a migration background. The mothers in the untreated treatment group are younger than the mothers in the control group and the treated treatment group. Moreover, they have less years of education, the households have a lower income and a higher share of the mothers is not married. Last but not least, a higher share of the children is native in the untreated treatment group.

In the treated treatment group, on the other side, the mothers are older than in the control group. They have more years of education, a higher household income and higher share of the mothers is married. Further, a lower share of the children is native. The treated treatment group has a considerably lower share of children who got the flu while the untreated treatment group has a higher share than the control group.

Considering these evident differences between the control and the untreated treatment group, we can conclude that getting the flu shot is not the only aspect that impacts the probability of getting the flu as otherwise the untreated treatment group would not have significantly more flu infections on average than the control group. One reason why this group has more flu infections than the control group might be that the children have worse access to healthy diet as the mothers earn less on average and are more often single mothers (assuming that they raise their children alone as a result of being not married). This would have a negative effect on their immune system, making the children more prone to get the flu.

Further, we have to consider that whether a child receives the assigned treatment or not does not only relate to the treatment assignment but also to the age of the mother, the education of the mother, the household income, the marital status of the mother and the nationality of the child as there are considerable differences between the treated and untreated treatment group. However, to be able to make more inferences on that, we would have to do further estimations first.

- d) Estimate this model using OLS. Next, include subsequently the individual characteristics. What do you learn from these regressions?

```
model1_robust <- rlm(flu ~ flushot, data = subset(data, data$treatgroup == 1))
model2_robust <- rlm(flu ~ flushot + genderchild + nationality + agemother + educmother + married
                     + housincome, data = subset(data, data$treatgroup == 1))

stargazer(model1_robust, model2_robust)
```

% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu
 % Date and time: So, Jan 30, 2022 - 18:51:27

Table 1:

	<i>Dependent variable:</i>	
	flu	
	(1)	(2)
flushot	-0.274*** (0.010)	-0.164*** (0.010)
genderchild		0.016* (0.009)
nationality		0.092*** (0.010)
agemother		-0.048*** (0.002)
educmother		-0.030*** (0.003)
married		-0.024 (0.025)
housincome		0.00001 (0.00000)
Constant	0.675*** (0.008)	2.200*** (0.045)
Observations	10,089	10,089
Residual Std. Error	0.594 (df = 10087)	0.602 (df = 10081)
<i>Note:</i> *p<0.1; **p<0.05; ***p<0.01		

We estimate two OLS models with robust standard errors. The first model include whether a child received the flushot as its only regressor. The other model also includes individuals characteristics such as the gender and the migration background of the child as well as the age, the years of education, the marital status of the mother and the household income. As the researcher wants to focus only on those children randomized in the treatment group, we only use the data of the children who were assigned treatment.

In both model the flushot is estimated to decrease the probability of getting the flu significantly. However, the estimated impact is considerably lower when we include individual characteristics. Hence, our estimation is not robust. Further, the age of the mother and the education of the mother have a significant, negative effect while the child being native has a significant positive effect. Being a girl also has a significant positive effect but only at 10%. The F test shows us that the estimators are jointly significant.

As the estimations for the flushot differ considerably between the two models, we have to consider that the model specification may be wrong or that underlying assumptions do not hold (e.g. heteroskedasticity or endogeneity of the regressors).

```
linearHypothesis(model2_robust, c("genderchild=0", "nationality=0", "agemother=0", "educmother=0",
                                "married=0", "housincome=0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## genderchild = 0
## nationality = 0
## agemothere = 0
## educmother = 0
## married = 0
## housincome = 0
##
## Model 1: restricted model
## Model 2: flu ~ flushot + genderchild + nationality + agemothere + educmother +
##          married + housincome
##
##   Res.Df Df       F    Pr(>F)
## 1   10087
## 2   10081   6 254.08 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

e) Use 2SLS to estimate β_1 and check the robustness with respect to adding individual characteristics.

```
model3 <- feols(flu ~ 1 | flushot ~ treatgroup, data)
summary(model3)
```

```
## TSLS estimation, Dep. Var.: flu, Endo.: flushot, Instr.: treatgroup
## Second stage: Dep. Var.: flu
## Observations: 12,583
## Standard-errors: IID
##               Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)   0.620690   0.009705  63.9530 < 2.2e-16 ***
## fit_flushot -0.192779   0.016227 -11.8802 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.484649   Adj. R2: 0.059242
## F-test (1st stage), flushot: stat = 5,016.2, p < 2.2e-16 , on 1 and 12,581 DoF.
##               Wu-Hausman: stat =    17.1, p = 2.237e-5, on 1 and 12,580 DoF.
```

```
model4 <- feols(flu ~ genderchild + nationality + agemothere + educmother + married
                + housincome | flushot ~ treatgroup, data)
summary(model4)
```

```
## TSLS estimation, Dep. Var.: flu, Endo.: flushot, Instr.: treatgroup
## Second stage: Dep. Var.: flu
## Observations: 12,583
## Standard-errors: IID
##               Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)   2.15266853 0.04010533  53.675370 < 2.2e-16 ***
## fit_flushot -0.19827462 0.01508642 -13.142589 < 2.2e-16 ***
## genderchild   0.01342733 0.00804823   1.668358  0.09527 .
## nationality   0.09024013 0.00913374   9.879864 < 2.2e-16 ***
## agemothere   -0.04628152 0.00163287 -28.343613 < 2.2e-16 ***
## educmother   -0.02731668 0.00278890  -9.794798 < 2.2e-16 ***
```

```
## married      -0.02848121 0.02181911 -1.305333  0.19180
## housincome    0.00000357 0.00000423  0.844801  0.39824
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 0.451201 Adj. R2: 0.184224
## F-test (1st stage), flushot: stat = 5,400.9 , p < 2.2e-16 , on 1 and 12,575 DoF.
## Wu-Hausman: stat = 3.5763, p = 0.058633, on 1 and 12,574 DoF.
```

We estimate two models using the assignment to treatment as an instrumental variable for the regressor flushot. As in the previous subquestion, the first model includes flushot as its only regressors in the second stage regression and the second model includes the flushot and individual characteristics. In both models, we get an estimation of approximately -0.2 for the flushot. Further, in both models the estimation is significant at 0.1%. Hence, the estimation for the flushot is robust with respect to adding individual characteristics.

f) Estimate the first-stage regression using OLS. Are you afraid of a weak instruments problem?

```
first_stage <- rlm(flushot ~ treatgroup + genderchild + nationality + agemother + educmother + married
                  + housincome, data = data)
stargazer(first_stage)
```

```
% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu
% Date and time: So, Jan 30, 2022 - 18:51:28
```

We estimate the first stage including the instrumental variable and all exogenous variables as regressors. The assignment to treatment increases the probability of getting a flushot. The same goes for the age, the marital status and the years of education of the mother, although their impact is considerably lower than in the case of the assignment to treatment. The household income also has a small, positive and significant effect, while the child being a native has a negative effect. Hence, we can conclude that children, whose mothers are younger, have less years of education and are single, were less likely to get the flushot. On the other hand, children that have a migration background were more likely to get a flushot.

We do not see an issue with weak instruments here, as the effect of being in the treatment group is significant at 0.01%. Including the estimation from the previous subquestion, we can observe that for both estimations we have a first-stage F statistics above 10. Hence, we have evidence that the assignment to the treatment group is not a weak instrument.

g) Explain why in this case the local average treatment effect is the same as the average treatment effect on the treated.

LATE is the local treatment effect of the compliers, so when $D(1) - D(0) = 1$. ATET is the effect on the treated, so when $D = 1$. D equals one in four cases. First, the individual was not assigned treatment but still got the treatment, so $D(0) = 1$. In this case the individual is either an always taker or a defier. If the individual was assigned treatment and got the treatment, so when $D(1) = 1$, then the individual is either an always taker or a complier.

$$D_i = 1 \begin{cases} D(0) = 1 & \text{if } i \text{ is Always Taker or Defier} \\ D(1) = 1 & \text{if } i \text{ is Always Taker or Complier} \end{cases}$$

Based on the data, we can exclude always takers and defiers completely because no one of the control group received a flushot despite not being assigned the treatment. Therefore, we have only compliers and never takers in our sample.

As ATET only considers individuals that got treatment but is not able to differentiate between defiers, compliers and always takers, ATET usually does not equal LATE. However, in our case, we were able to exclude the possibility of always takers and defiers in our sample. Hence, ATET only considers compliers here and therefore is the same as LATE.

Table 2:

	<i>Dependent variable:</i>
	flushot
treatgroup	0.691*** (0.010)
genderchild	0.0001 (0.008)
nationality	−0.089*** (0.009)
agemother	0.030*** (0.002)
educmother	0.013*** (0.003)
married	0.075*** (0.022)
housincome	0.00001*** (0.00000)
Constant	−1.015*** (0.041)
Observations	12,583
Residual Std. Error	0.451 (df = 12575)
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01

Note that the existence of never takers in our sample is not an issue because neither LATE nor ATET considers never takers.

Mathematically, this means that $D(1) - D(0) = D(1)$ as $D(0) = 0$. Hence, we can write:

$$LATE = E(Y_1^* - Y_0^* | D(1) - D(0) = 1) = E(Y_1^* - Y_0^* | D(1) = 1) = E(Y_1^* - Y_0^* | D = 1) = ATET$$

References

Angrist, J. D., Imbens, G. W., & Rubin, D. B. (1996). Identification of causal effects using instrumental variables. *Journal of the American statistical Association*, 91(434), 444-455. De Chaisemartin, C. (2017). Tolerating defiance? Local average treatment effects without monotonicity. *Quantitative Economics*, 8(2), 367-396.