Econometrics II - Assignment 3

Uncensored sloths

23 Jan 2022

Question 1

(i) Compute for each of the colors the treatment effect.

The average treatment effects of each color can be estimated by taking the difference between the average outcome of the treatment group and the average outcome of the control group. Hence, the average treatment effect of purple is 2, the average treatment effect of blue is 5 and the average treatment effect of green is 1.

(ii) Compute the average treatment effect in the full population.

To compute the average treatment, we weigh the average treatments per color with their proportion within the data:

```
0.5*2+0.25*5+0.25*1
```

[1] 2.5

(iii) Compute the average treatment effect on the treated.

To compute the average treatment effect on the treated, we weigh the average effect of all colors (treated and control) with the distribution of the treated group and take the difference between the treatment and control group:

```
(100*9+75*13+25*10)/200-(100*7+75*8+25*9)/200
```

[1] 3

(iv) Give an example where the average treatment effect on the treated would be more useful to consider than the average treatment effect, and explain why.

In general, it makes sense to differentiate between the average treatment effect and the average treatment of effect of the treated when the assignment of treatment is not fully random as the ATE and ATET will not be equal as a result.

The average treatment effect on the treated is more applicable when we want to now the effect of a treatment on the treated group. For example, when we conduct a cost-benefit analysis of a treatment, we are interested to know whether the effect is sufficient given the costs such as in job support program. In this case, we want to know whether the support measures helped the participants to gain a job. Another situation is the effect of medication where we are interested to know whether a certain medicine helped the participants or not.

However, why do even have to differentiate between ATE and ATET? The reason is because these effects differ unless we have a fully random assignment of the treatment. But in reality we often have selection biases, because a certain type of people tends to participate in the treatment. If we invite unemployed individuals to participate in a program that is supposed to help them to get a job, it may happen that mostly only motivated individuals will decide to participate for instance.

Question 2

2 correct

3 gender

```
# Load data
data <- read.csv("assignment3.csv")</pre>
# dummy for important: 0 for sport, 1 for school
data$important[data$important == "sport"] <- 0</pre>
data$important[data$important == "school"] <- 1</pre>
# dummy for better: 0 for language, 1 for math
data$better[data$better == "Math"] <- 1</pre>
data$better[data$better == "Language"] <- 0</pre>
# dummy for preferred hand: 0 for right, 1 for left
data$preferredhand[data$preferredhand == "Left"] <- 1</pre>
data$preferredhand[data$preferredhand == "Right"] <- 0</pre>
data$treatment [data$treatment == "Nothing"] <- 0</pre>
data$treatment [data$treatment == "Candybar"] <- 2</pre>
data$treatment [data$treatment == "Fruit"] <- 1</pre>
data$important[data$important == ""] <- NA</pre>
data %<>%
      mutate_each(funs(if(is.character(.)) as.integer(.) else .))
## Warning: `funs()` was deprecated in dplyr 0.8.0.
## Please use a list of either functions or lambdas:
##
##
     # Simple named list:
     list(mean = mean, median = median)
##
##
     # Auto named with `tibble::lst()`:
##
##
     tibble::1st(mean, median)
##
##
     # Using lambdas
     list(~ mean(., trim = .2), ~ median(., na.rm = TRUE))
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
## Warning: `mutate_each_()` was deprecated in dplyr 0.7.0.
## Please use `across()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
 (i) Compute the fraction of pupils in all three groups (control, fruit, and candy bar) that have the number
     correct and that are expected to lie. Show within a table whether pupil characteristics are balanced
     over the treatment groups.
balance <- balance_table(data[, !names(data) %in% "id"], "treatment")</pre>
balance
## # A tibble: 9 x 6
   variables1
                   Media_control1 Media_trat1 Media_trat2 p_value1 p_value2
     <chr>>
                                                                          <dbl>
##
                             <dbl>
                                          <dbl>
                                                       <dbl>
                                                                <dbl>
## 1 better
                             0.511
                                          0.625
                                                       0.471 0.173
                                                                        0.648
```

0.362

0.293

0.412 0.662

0.392 0.829

0.872

0.168

0.398

0.277

```
## 4 grade
                              3.03
                                           3.5
                                                       3.10
                                                                0.0299
                                                                        0.807
## 5 important
                                           0.845
                                                                         0.00873
                              0.870
                                                       0.667
                                                                0.679
## 6 oftenexpelled
                              0.138
                                           0.109
                                                       0.137
                                                                0.600
                                                                         0.986
## 7 preferredhand
                                                       0.157
                                                                0.754
                                                                         0.768
                              0.138
                                           0.121
## 8 siblings
                              0.989
                                           0.914
                                                       0.980
                                                                0.0549
                                                                        0.689
## 9 youngestchild
                                                       0.431
                                                                0.892
                              0.351
                                           0.362
                                                                         0.351
```

First, we compute the balancing table. For all three groups, we can see that the average of correct numbers is above $\frac{1}{6}$ which is why we use the given formula to calculate the fraction of pupils that lie:

```
fraction_nothing <- (balance[2, 2]-(1/6))/(1- (1/6))
fraction_fruit <- (balance[2, 3]-(1/6))/(1- (1/6))
fraction_candybar <-(balance[2, 4]-(1/6))/(1- (1/6))
fractions <- cbind(fraction_nothing, fraction_fruit, fraction_candybar)
fractions</pre>
```

```
## Media_control1 Media_trat1 Media_trat2
## 1 0.2774194 0.2344828 0.2941176
```

As the results show, around 27.74% of the pupils in the control group have a right number and are expected to lie. In the treatment group that receives a fruit it is around 23.45% of the pupils and in the treatment group that receives candy it is around 29.41%.

```
# dummy treatment_fruit: 1 for fruit, 0 for the others
data$treatment_fruit [data$treatment == 0] <- 0
data$treatment_fruit [data$treatment == 2] <- 0
data$treatment_fruit [data$treatment == 1] <- 1
# dummy treatment_candy: 1 for candy, 0 for the others
data$treatment_candy [data$treatment == 0] <- 0
data$treatment_candy [data$treatment == 2] <- 1
data$treatment_candy [data$treatment == 1] <- 0</pre>
```

Looking at the balancing table, we can see that the not all pupil characteristics are balanced over the treatment groups. For gender, the preferred hand, being the youngest child, being often expelled and the prioritization of either school or sport, we have no significant imbalances among the control and treatment groups. However, in case of the grades, we observe that pupils of the treatment group that receives fruits are on average in a significantly higher grade (we assume that grade is the school year, not the mark) than the other pupils of the other groups. Further, children of the treatment group that receives candies rate sport more important on average at a significant level. Last but not least, having siblings also is unbalanced for the fruit treatment group as on average less pupils have a sibling in this treatment group. However, this imbalance is only significant at 10%.

Hence, we would like to point out that the experiment could have a potential non-random assignment of the treatment. While it appears to be a coincidence that in one treatment group pupils prioritize sport more on average and in the other treatment group, pupils are more often single children on average, there could be a potential non-random assignment based on the grades as the teachers or researchers may assigned the children to the treatment groups based on the grade they attend. However, as we do not have any information about the selection process we cannot make any further inferences on that matter.

(ii) Use the linear probability model to regress the dummy variable for having the correct number on the dice on the assignment to the three treatment groups. Do you detect significant lying? Show how robust you results are to including additional covariates.

```
## Loading required namespace: car
```

\$lm_robust

```
##
                      Estimate Std. Error
                                             t value
                                                         Pr(>|t|)
                                                                    CI Lower
                    0.39784946 0.05102911 7.7965194 3.485201e-13 0.2972223
## (Intercept)
## treatment fruit -0.03578050 0.08158532 -0.4385654 6.614522e-01 -0.1966632
## treatment_candy 0.01391524 0.08630331 0.1612365 8.720707e-01 -0.1562711
##
                    CI Upper DF
## (Intercept)
                   0.4984767 199
## treatment fruit 0.1251022 199
## treatment_candy 0.1841016 199
##
## $1h
##
                       Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
                                                                         0.3315 199
## (Intercept) = 0.167
                         0.2308
                                   0.05103
                                             4.524 1.043e-05
                                                               0.1302
```

First, we estimate an OLS regression including the treatments as the only regressors. Note that we estimate robust standard errors as a binary outcome variable always involves heteroskedasticity. As the results show, the intercept is significant at 1%. Hence, we can reject the null hypothesis that the intercept equals $\frac{1}{6}$ and conclude that the children of the control group lie despite getting no treatment. As the estimations of the treatments are not significant, we can conclude that they do not behave substantially different from the control group.

When we test the significance using the data directly (we test whether the mean is significantly from $\frac{1}{6}$), we get similar results. Hence, in every group, there are pupils who lie:

```
t.test(subset(data, treatment == 0)$correct, y = NULL,
       alternative = c("greater"),
       mu = 1/6, paired = FALSE, var.equal = FALSE,
       conf.level = 0.95)
##
##
   One Sample t-test
##
## data: subset(data, treatment == 0)$correct
## t = 4.5304, df = 92, p-value = 8.82e-06
## alternative hypothesis: true mean is greater than 0.1666667
## 95 percent confidence interval:
## 0.3130602
## sample estimates:
## mean of x
## 0.3978495
t.test(subset(data, treatment == 1)$correct, y = NULL,
       alternative = c("greater"),
       mu = 1/6, paired = FALSE, var.equal = FALSE,
       conf.level = 0.95)
##
   One Sample t-test
##
##
## data: subset(data, treatment == 1)$correct
## t = 3.0696, df = 57, p-value = 0.00164
## alternative hypothesis: true mean is greater than 0.1666667
## 95 percent confidence interval:
  0.2556329
## sample estimates:
## mean of x
```

0.362069

When we include all available characeteristics of the pupils, we see that our intercept becomes insignificant. However, the estimation of the intercept only differs by around 0.047 to the previous estimation. For the estimations of the treatments, we have different estimates than in the previous estimation (the estimates are almost twice as high as before). Therefore, our results are not robust when we include all covariates.

0.4117647

```
lh_robust(correct ~ treatment_fruit + treatment_candy + gender + grade + important + better + preferre
    "(Intercept) = 0.167")
```

```
## $lm_robust
##
                       Estimate Std. Error
                                               t value
                                                         Pr(>|t|)
                                                                     CI Lower
## (Intercept)
                    0.350956461 0.22829469 1.53729577 0.12593873 -0.09945539
## treatment_fruit -0.068746569 0.08307589 -0.82751530 0.40901699 -0.23265036
## treatment_candy 0.047802878 0.09056749 0.52781498 0.59826368 -0.13088139
## gender
                   -0.178369458 0.07555158 -2.36089644 0.01927699 -0.32742823
## grade
                   -0.019189028 0.02570181 -0.74660217 0.45625663 -0.06989717
                   0.117115149 0.08670234 1.35077268 0.17842707 -0.05394341
## important
## better
                  -0.002941433 0.07225331 -0.04071001 0.96757122 -0.14549292
## preferredhand
                  -0.025107805 0.10485004 -0.23946396 0.81101238 -0.23197069
## siblings
                   0.065931914\ 0.17839137\ 0.36959140\ 0.71211197\ -0.28602364
## youngestchild
                   0.049070585 0.07825385 0.62706925 0.53139115 -0.10531961
                  -0.117725353 0.10403655 -1.13157687 0.25928499 -0.32298328
## oftenexpelled
                      CI Upper DF
## (Intercept)
                    0.80136831 184
## treatment_fruit 0.09515722 184
## treatment_candy 0.22648715 184
## gender
                   -0.02931068 184
## grade
                   0.03151912 184
## important
                   0.28817371 184
## better
                    0.13961006 184
## preferredhand
                   0.18175508 184
## siblings
                    0.41788747 184
## youngestchild
                   0.20346078 184
## oftenexpelled
                   0.08753257 184
##
## $1h
##
                       Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
## (Intercept) = 0.167
                                    0.2283 0.8058 0.4214 -0.2665
                          0.184
```

As we have imbalances among the treatment groups regarding grades, the prioritization and siblings, there might be difference between the treatment groups regarding their tendencies to lie which is why we also estimate a model including all these covariates. Again, our results are not robust as the intercept is lower than before and not significant. The estimations for the treatment effects also differ from the original estimation. Note, however, that neither of the included covariates is significant. We also estimated models where we included only on the three covariates, as well as paired combinations. However, we never could observe any significant effect which is why we would conclude that the imbalances do not affect the lying behavior among the treatment groups.

```
lh_robust(correct ~ treatment_fruit + treatment_candy + grade + important + siblings, data = data, lin
"(Intercept) = 0.167")
```

```
## $lm_robust
##
                      Estimate Std. Error
                                              t value Pr(>|t|)
                                                                   CI Lower
## (Intercept)
                    0.28699928 0.21040927
                                           1.3640049 0.1741464 -0.12798409
## treatment_fruit -0.01962170 0.08258157 -0.2376039 0.8124391 -0.18249465
## treatment candy 0.03289945 0.08859484
                                           0.3713473 0.7107841 -0.14183327
  grade
                   -0.02768932 0.02492212 -1.1110335 0.2679290 -0.07684241
  important
                    0.11646221 0.08551163
                                           1.3619459 0.1747943 -0.05218959
##
  siblings
                    0.10200722 0.17819195 0.5724570 0.5676752 -0.24943498
##
##
                     CI Upper DF
## (Intercept)
                   0.70198266 194
## treatment_fruit 0.14325125 194
## treatment_candy 0.20763218 194
## grade
                   0.02146378 194
## important
                   0.28511402 194
## siblings
                   0.45344943 194
##
## $1h
##
                       Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
                           0.12
                                                               -0.295
## (Intercept) = 0.167
                                    0.2104
                                           0.5703
                                                      0.5691
                                                                         0.535 194
```

Finally, we also estimated a model including gender. Again our estimates differ again from the original model, especially the estimate for treatment with candy. However, different from other estimations, the intercept is significant again. Moreover gender is significant at 10%, providing us with evidence that boys tend to lie more than girls.

```
## $lm_robust
##
                      Estimate Std. Error
                                                          Pr(>|t|)
                                              t value
                                                                     CI Lower
## (Intercept)
                    0.43521568 0.05551478
                                           7.8396364 2.728733e-13
                                                                    0.3257396
## treatment_fruit -0.03397165 0.08070092 -0.4209574 6.742427e-01 -0.1931153
## treatment candy
                   0.02896318 0.08658251 0.3345153 7.383448e-01 -0.1417790
## gender
                   -0.13365608 0.07206023 -1.8547828 6.511407e-02 -0.2757601
##
                      CI Upper DF
## (Intercept)
                   0.544691792 198
  treatment_fruit 0.125171978 198
## treatment candy 0.199705401 198
  gender
                   0.008447958 198
##
##
## $1h
##
                       Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
## (Intercept) = 0.167
                         0.2682
                                   0.05551
                                              4.831 2.711e-06
                                                                0.1587
                                                                         0.3777 198
```

Generally, we tried different estimations with different combinations of covariates. However, except for gender no covariate was significant and additionally led to models with higher standard errors.

(iii) The researcher is interested in testing if boys and girls behave differently. Write down and estimate a model that can test if boys lie more than girls and whether boys and girls respond differently to incentives (e.g. treatment).

We already saw in previous results that gender has a weakly significant effect, providing us with evidence that boys tend to lie more than girls. To account for potentially different responses among the treatment groups, we also include interaction effects between gender and the respective treatments.

```
lh_robust(correct ~ treatment_fruit + treatment_candy + gender*treatment_fruit + gender*treatment_candy
"(Intercept) = 0.167")
```

```
## $lm_robust
##
                               Estimate Std. Error
                                                         t value
                                                                     Pr(>|t|)
                           0.4477611940 0.06120892 7.315293126 6.423232e-12
## (Intercept)
## treatment_fruit
                          -0.0331270477 0.09906739 -0.334389006 7.384436e-01
## treatment_candy
                          -0.0284063553 0.10891775 -0.260805555 7.945163e-01
                          -0.1785304248 0.10777919 -1.656446111 9.923149e-02
## gender
## treatment_fruit:gender -0.0008096039 0.17008776 -0.004759919 9.962070e-01
## treatment_candy:gender 0.1591755861 0.17990130 0.884793962 3.773519e-01
##
                            CI Lower
                                       CI Upper DF
## (Intercept)
                           0.3270486 0.56847383 196
## treatment_fruit
                          -0.2285019 0.16224785 196
## treatment_candy
                          -0.2432075 0.18639484 196
## gender
                          -0.3910862 0.03402536 196
## treatment_fruit:gender -0.3362467 0.33462748 196
## treatment candy:gender -0.1956152 0.51396636 196
##
## $1h
##
                       Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
## (Intercept) = 0.167
                         0.2808
                                   0.06121
                                             4.587 8.012e-06
                                                                  0.16
                                                                         0.4015 196
```

The results show that gender is still weakly significant and negative, therefore boys tend to lie more than girls. However, there are no significant interaction effects between gender and the type of treatment. Therefore, it does not depend on the incentive whether boys lie more than girls on average but on the gender.

(iv) Estimate your preferred model specification (including pupil characteristics if this improves your model) and write down your main conclusions.

```
## $lm_robust
##
                      Estimate Std. Error
                                             t value
                                                          Pr(>|t|)
                                                                     CI Lower
## (Intercept)
                    0.43521568 0.05551478 7.8396364 2.728733e-13
                                                                   0.3257396
## treatment fruit -0.03397165 0.08070092 -0.4209574 6.742427e-01 -0.1931153
## treatment_candy 0.02896318 0.08658251 0.3345153 7.383448e-01 -0.1417790
                   -0.13365608 0.07206023 -1.8547828 6.511407e-02 -0.2757601
## gender
##
                      CI Upper DF
## (Intercept)
                   0.544691792 198
## treatment_fruit 0.125171978 198
## treatment_candy 0.199705401 198
## gender
                   0.008447958 198
##
## $1h
```

```
## Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
## (Intercept) = 0.167 0.2682 0.05551 4.831 2.711e-06 0.1587 0.3777 198
```

Following the discussion above, the preferred model includes gender as the only covariate due to its weak significance, but does not include interaction effects between gender and treatments which were found to be anything but significant. Estimation results of this relatively simple model are printed above and suggest that there are no significant treatment effects, while for all control/treatment groups there is clear evidence for lying (i.e. significant upward deviation in the share of pupils stating to have guessed correctly from 1/6). Moreover, the estimation yields very weak evidence for the proportion of pupils who lie to be lower for girls. However, since the experiment's power does not seem to accommodate the size of this effect only reasons for the former result are discussed in some detail.

One can think of many reasons why the experiment yields evidence for lying across all control/groups but none for significant treatment effects which ultimately result in the potential treatment effect to be so small that it would not be accommodated by the experiment's power, even if one would assume that it exists. First, it seems unclear whether the researcher explicitly told children in the control group that they would not receive any reward for indicating "correct" or whether nothing was communicated at all potentially leaving pupils in the control group with the belief that they would still receive some sort of reward. Thus, by this effective substitution effect the observed difference in the outcome variable might have been vanished. Second, one could assume that children simply like lying to the research (e.g. due his poor randomization practice) even without any economic incentivization. Third, it is likely that the experiment induced some sort of competitive environment among pupils (especially if conducted at a school), such that even non-incentivized students lied so that the would belong the "winners" of the game even though it would not come with any benefit other than the intrinsic ones of belonging to this group. This effect may have been aggravated by the incentivization of other treated pupils so that some form of spill-over effect comprimised the experiment's validity.

(v) Given the sample size and the estimates you have obtained above, what would be the minimum detectable effect size of this experiment?

To calculate the minimum detectable effect size (MDE) the customary power level q of 80% and significance level α of 5% are applied. Moreover, the model is estimated once again using the function 'rlm' to robustly estimate the residual standard error σ . Based on these parameters, the MDE is calculated using the equation below where all t-statistics have degrees of freedom equal to n-k, where n is the number of total observations and k then number of regressors including the intercept. Moreover, p equals the treatment density over both treatments so that the thus obtained MDE applies to both treatment effects studied.

$$MDE = (t_{1-q} - t_{1-\alpha/2})\sqrt{\frac{1}{p(1-p)}}\sqrt{\frac{\sigma^2}{n}}$$

```
model4_robust <- rlm(correct ~ treatment_fruit + treatment_candy + gender, data = data)

k=length(model4_robust$coefficients)
q=0.8
alpha=0.05

n <- length(data$treatment)
m <- length(data$treatment[!data$treatment == 0])

t_alpha <- qt(p=1-alpha/2, df=n-k, lower.tail=TRUE)
t_q <- qt(p=1-q, df=n-k, lower.tail=TRUE)
p = m/n

sigma= summary(model4_robust)$sigma</pre>
```

```
MDE = (t_alpha - t_q)*sqrt(1/(p*(1-p)))*sqrt(sigma^2/n)
MDE
```

[1] 0.2557005

Since this MDE would imply the fraction of pupils to lie to be more than 25 percentage points higher than in the control group, implying over 30% of treated pupils to lie, the original experiment cannot be characterized as particularly powerful and is thus unlikely to detect a significant effect, even if it may exist for some magnitude. In combination with the design issues discussed above, the power analysis gives a clear indication of why the experiment does not yield any significant results.

(vi) Initially, the researcher expected that no pupil would lie if there is nothing at stake (control group) and that a quarter of the student lie for a candy bar. How large should the sample size of the experiment have been in that case?

Since the researcher expects 25% of students in the candy bar treatment to lie while none do for the control group, so that $\frac{1}{6}$ of "controlled" students are expected to indicate correct, the implied MDE can be obtained as follows:

$$25\% = \frac{(1/6 + MDE_{target}) - 1/6}{1 - 1/6} \Rightarrow MDE_{target} = 25\% \cdot (1 - 1/6)$$

Based on MDE_{target} the necessary sample size can be computed using the following equation, where t-statistics have n - k degrees of freedom to avoid the necessity of a numerical solution method. Since the standardized t-distribution converges quite quickly in degrees of freedom and the necessary sample site is an integer value (rounded up), results should not be distorted in any way.

$$n_{target} = \left(\frac{t_{1-q} - t_{1-\alpha/2}}{MDE}\right)^2 \sqrt{\frac{\sigma^2}{n}}$$

```
MDE_target = 0.25*(1 - (1/6))
n_target = ((t_alpha - t_q)/MDE_target)^2*((sigma^2)/(p*(1-p)))
ceiling(n_target)
```

[1] 306

Since $MDE_{target} \approx 0.2083 < MDE \approx 0.2557$, the necessary sample size of 306 is bigger than in the original experiment.

(vii) Students assigned to the control group provide counterfactuals to both the fruit and the candy bar treatment. Show that by assigning more students to the control group than to each treatment group the same MDE can be achieved with fewer students in the experiment.

In terms of testing power, it is optimal to allocate 50% of the students into the control group and treat the other 50% (with either treatment). If one would anticipate an equal effect size across treatments an equal split between them, i.e. 25%-25%, would be appropriate. If there heterogeneity in effect sizes between treatments is assumed more nuanced power analysis techniques may be necessary to determine the optimal allocation of participants across treatments.

```
p2 \leftarrow 0.5
new_size = ((t_alpha - t_q)/MDE)^2 * (sigma^2)/(p2*(1-p2))
ceiling(new_size)
```

[1] 202

Since already close to 50% of the students are treated in the original experiment (53.69%), not much is to gain in the situation at hand. For ease of analysis, a plot of the relationship between the necessary sample

size and the treatment density is provided below. For less uniform distributions across control and treatment group the variance of the estimated treatment effect increases thus increasing the necessary sample size to accommodate the same power and MDE.

```
p1 <- seq.int(from = 0, to = 1, by=0.01)
n1 <- list()

for (i in 1:length(p1)){
    n1[i] = ((t_alpha - t_q)/MDE)^2 * (sigma^2)/(p1[i]*(1-p1[i]))
}

p1 <- as.list(p1)
sampsize = data.frame(unlist(p1),unlist(n1))
names(sampsize) <- c("p1", "n1")
sampsize[,2] <- as.numeric(sampsize[,2])
sampsize <- sampsize[c(-1,-101), ]

plot(sampsize$p1,sampsize$n1, type="1")
abline(h=length(data$treatment), col="red")</pre>
```

