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The Process of Search Planning: Current Approaches and Continuing Problems

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This paper discusses an approach to search planning in which the planner blends subjective and objective methods and produces plans that are a compromise between theoretical optimality and operational feasibility.

AS ORIGINALLY CONCEIVED, operations research sought to apply scientific principles to operational problems. In the United States, the first application of operations research was to naval operations during World War II. The report by Morse and Kimball [1946] documents many of the techniques developed by the U.S. Navy's operations research group during World War II. The analysis of search operations was the subject of an intensive effort by this group and resulted in a separate publication, "Search and Screening," by B. O. Koopman [1946], who wrote an updated and expanded version of this report (Koopman [1980]).

Search operations are still a large part of naval operations. Maritime search and rescue operations are a major responsibility of the U.S. Coast Guard. The U.S. Air Force has responsibility for coordinating land search and rescue for downed aircraft and for finding and tracking space satellites. The techniques of search theory have been applied to fishing problems (Paloheimo [1971], Mangel and Clark [1982]), police patrol analysis (Larson [1972]), oil exploration (Allais [1957], Cozzolino [1972], Harbaugh et al. [1977]), and medical screening (Gorry et al. [1973]). Searches are planned and executed every day.

The purpose of this article is to present an approach to search planning. We propose to do this by stepping the reader through the tasks that comprise the planning and analysis of a search problem. In the process, we will state principles that provide guidance for accomplishing these tasks and then illustrate these principles with examples.

The reader will see that our approach encompasses both objective and subjective methods and that it provides for compromises between theoretically optimal plans and operationally feasible ones. In this approach,

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the analysis provides the framework on which to build the search recommendations.

The search planning techniques discussed in this article build upon the work of Koopman [1946], but represent a considerable development beyond that work, particularly in the use of computers in search planning. These techniques have been developed over the course of many years of search operations including the 1966 search for the H-bomb lost near Palomares, Spain, the search for the lost U.S. nuclear submarine *Scorpion* in 1968 (Richardson and Stone [1971]), the development of a computer assisted search planning system (CASP) for the U.S. Coast Guard (Richardson and Discenza [1980]) and the clearance of unexploded ordnance from the Suez Canal in 1974 (Richardson et al. [1975]), as well as numerous submarine search operations for the U.S. Navy. As a result of the *Scorpion* search, a manual for the operations analysis of ocean bottom search was prepared for the U.S. Navy by Richardson et al. [1971].

1. OUTLINE OF OPERATIONS RESEARCH TASKS

The first task of the analyst is to identify the type of search problem. Is the goal detection, surveillance, clearance, or exploration? In order to be specific, we shall consider a detection search. This is the most common type of search, and the one that we know most about.

The analyst's tasks are as follows:

1. Compute a prior distribution on target location (and motion),
2. Obtain a good estimate of sensor capabilities,
3. Determine a detection function,
4. Develop a search plan and estimate its success probability,
5. Update for search feedback, and
6. Estimate search effectiveness.

2. PRIOR TARGET LOCATION DISTRIBUTION

Obtaining the prior distribution requires both subjective and objective assessments. First, clues and possible explanations for the target's location are grouped into scenarios, each of which is logically self-consistent and provides a probabilistic description of the target's location. The search planner must quantify the uncertainties in each scenario. Sometimes this can be done by using statistical data or by knowing the design of the system providing the locating information. For example, if a position at sea is reported by LORAN navigation, then examining the inaccuracies in the LORAN system at that location would determine an error distribution for the reported position. If this cannot be done objectively, then it must be done subjectively by talking to people familiar with the system involved and eliciting their opinions. An example of the latter might occur when a fishing boat without navigation equipment is in distress and reports its best estimated position. One could ask other fisherman who operate in the same area how they determine their position. Are there landmarks or buoys that are used for navigation? How well can they estimate their position? On the basis of all this information, the search planner would estimate the positional uncertainty and incorporate it into a probability distribution for target location.

When there are multiple scenarios, one must assign probabilities or credences to each scenario. Since each scenario corresponds to a probability distribution, the resulting target location distribution is a mixture, weighted by the scenario probabilities, of the scenarios. The scenario probabilities are usually determined subjectively, and the Coast Guard recommends that the scenarios and weights be determined by a group and not by a single person (Richardson and Discenza). They suggest that the group members should consider all possible scenarios, and they emphasize that none should be discarded entirely. After all the scenarios have been listed, each member of the group should assign his subjective probability weights independently of the other members of the group. The resulting weights are then compared and the final weights determined by averaging or by consensus.

As the reader can see, the process of devising a probability distribution quickly goes beyond a straight scientific or analytic method. Instead, it is a blend of objective and quantified subjective inputs. One cannot say the result is right or wrong in the traditional scientific sense. However, the resulting distribution does provide the foundation for rational planning based on the search planner's best understanding, both subjective and objective, of the search problem. Moreover, there is no good substitute for this combined subjective/objective approach. To leave out the subjective information is to throw away valuable information because there is no unique or "scientific" way to quantify it.

An example will help to illustrate this point. A ship in distress reports its position and a search is begun. A small plane hearing about the search reports having seen this same vessel 100 miles away from its reported position 1 hour after the distress report. The 100-mile difference cannot be accounted for by the ship's movement or the nominal navigation error of either the ship or aircraft. Did the aircraft see the ship in question or another one? Did the ship report its position correctly? Was its navigation gear malfunctioning? Obviously, it would be unwise to discard either piece of information (i.e., scenario). Instead, the search planner must make a subjective estimate of the relative reliability of the two positions and form weighted scenarios accordingly. It is important to realize that discarding one of the pieces of information is in effect making the subjective judgment that its weight is zero and the other is one.

Example

We now present an example of how one produces a probability distribution for a scenario.

Consider a case where a ship in distress reports its estimated position as 30°N, 130°W. By checking the *National Search and Rescue Manual* (U.S. Coast Guard), one finds that the navigational system of the ship has a circular probable error (CPE) of 10 miles. This corresponds to a circular normal error distribution with standard deviation $\sigma = 8.5$ miles in any direction. Thus, the initial position distribution is circular normal with mean at the reported position.

Suppose the ship is believed adrift, and we receive no further position reports. By querying the U.S. Coast Guard's historical wind and current files, one can obtain the mean sea current and wind direction for the location of the ship along with distributions of the variation about the mean. The motion of the ship is a combination of its drift due to the total ocean current (geostrophic plus wind current) and its leeway, the motion due to the wind and the sail effect of the ship's freeboard. Suppose that, to a reasonable approximation, both the leeway and total current have a two-dimensional normal distribution. (This is often, but not always, true.) The resulting velocity is the sum of two normally distributed random variables and is therefore normal. Suppose the resulting distribution has

Mean speed 2 knots with standard deviation 1 knot.
Mean course 180° with standard deviation 20°.

To obtain the probability distribution of the target's location at time T after the time of the reported position, we model each component of the target's velocity as a stationary Gauss-Markov process with autocorrelation $\rho(t) = e^{-\lambda t}$ for $t \geqq 0$. This process is also called the Ornstein-Uhlenbeck process (see Feller [1971]). The autocorrelation reflects the fact that ocean currents change over time, but these changes are not completely independent of the past. To complete the description of the process, we must specify λ, the parameter that controls the rate at which ocean velocities decouple or become independent. We choose $\lambda = ¼$ hours. The stochastic process describing the target position is obtained by integrating the random velocities of the Ornstein-Uhlenbeck process. The resulting process is called the Integrated Ornstein-Ohlenbeck process or the IOU process. One can use the method described in Richardson [1979] to calculate the probability distribution of this process and the target's position at time T.

Suppose the search is to begin 10 hours after the time of the last reported position. Figure 1 represents the target's distribution at the time of its last reported position and Figure 2 shows the distribution at the time the search begins. For pictorial purposes these distributions are represented by ellipses centered at the mean of the distribution and oriented along the major and minor axes of the distribution. These ellipses are equal probability density contours that contain 86% of the probability mass. The numbers 1 through 9 indicate relative probability

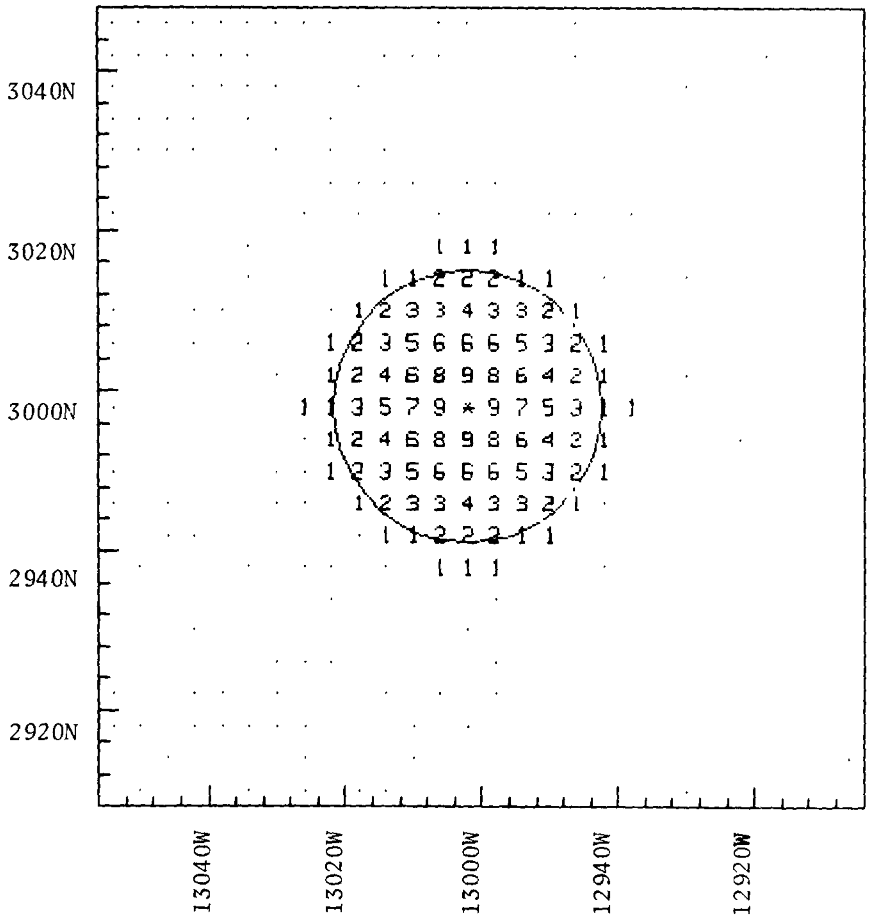

Figure 1. Target location map at time of initial distress.

densities. The asterisk (*) indicates the cell with the highest density. In many search operations, one displays the target location distribution by specifying a grid of cells and computing the probability of the target being in each cell of the grid.

This example demonstrates the basic features of developing a probability distribution for one scenario, namely

(i) Specifying the target's initial distribution,
(ii) Specifying a model of target motion, and
(iii) Updating the location distribution to the time of the search.

Monte-Carlo Methods

In the previous example, we obtained the target distribution by purely analytical methods. Distributions for more complicated problems are

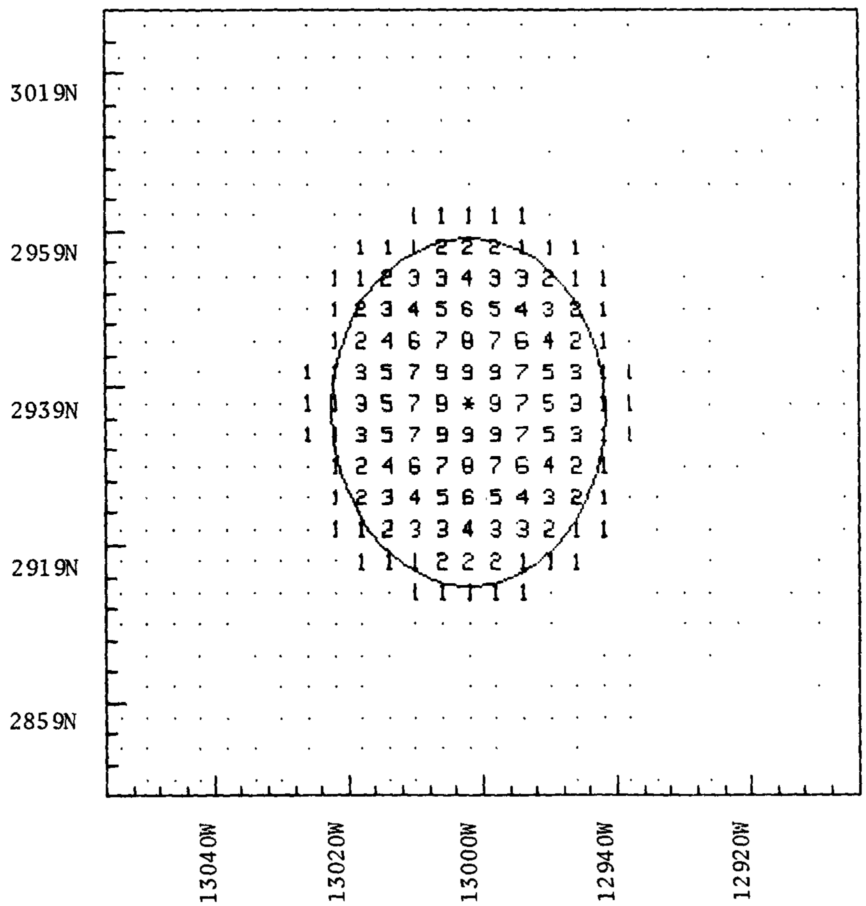

Figure 2. Target location map at start of search ($T = 10$ hours).

usually derived by Monte Carlo methods exemplified by those employed in the Coast Guard CASP system (Richardson and Discenza).

The CASP system develops probability distributions using a multi-scenario approach. The scenarios can be chosen from three scenario types. Within each type, the scenario is determined by specifying the required parameter values for that scenario. The scenarios can be weighted and combined to produce the final probability map.

Systems like CASP typically generate a large number (e.g., 10,000) of target points. The target distribution is represented by J points, and the jth target point is a vector

$$(X_j, Y_j, U_j, V_j, T_j, w_j, n_j)$$

where

X_j = latitude of jth point,
Y_j = longitude of the jth point,
U_j = north/south velocity of the jth point,
V_j = east/west velocity of jth point,
T_j = time associated with position (X_j, Y_j) and velocity (U_j, V_j),
w_j = weight or probability that the jth point represents the actual target position,
n_j = scenario from which jth point is drawn.

Each point represents a possible target position. If there are N scenarios, and the nth scenario has probability q_n where $\sum_{n=1}^{N} q_n = 1$, then $q_n J$ points are drawn for scenario n. The initial position of these points is drawn in a Monte Carlo fashion from the initial target location distribution for scenario n. When the positions of these points are updated for motion, their velocities are chosen from the target motion distribution for scenario n. Initially, $w_j = 1/J$ for $j = 1, \cdots, J$. The weights are modified for unsuccessful search in the manner described in Section 6 (see Richardson and Discenza).

To obtain a probability map at time T, the methods move the target points to their positions at time T and modify their weights to account for unsuccessful search. A grid of cells is chosen. For each cell the weights of the points falling in that cell are summed to obtain the probability that the target is in that cell.

Examples of probability maps generated in this fashion may be found in Richardson and Stone and Richardson and Discenza.

Problem Areas

One of the greatest difficulties in generating prior probability maps is the lack of systematic, proven techniques for eliciting subjective inputs for search scenarios. Several researchers (Savage [1971], Lindley et al. [1979], Spetzler and Staël von Holstein [1975], Morris [1977] and Winkler [1981]) have studied methods for eliciting subjective probabilities. However, this work has not been applied to search problems. In addition to obtaining subjective probabilities, we also have the problem of obtaining subjective estimates of uncertainties, times, and other quantitative information needed to form scenarios. For example, do fishermen tend to overestimate or underestimate their navigational accuracy, and if so, by how much?

Many computer search planning systems develop probability maps by Monte Carlo simulation. There is evidence (Weisinger [1978]) that smoothing to reduce Monte Carlo sampling noise would improve the search planning advice based on the maps, but no consistent and satisfactory method of smoothing has been developed so far.

There are numerous problems concerning the modeling of target motion even for common search problems such as the Coast Guard maritime search and rescue problem. One of the major obstacles is in obtaining a model for the variation of ocean currents in space and time. The model used to generate the above example is essentially that of a Gaussian random field with a stationary autocorrelation function. However, this model has not been tested due to a sparsity of data. We do not have good estimates of parameters, such as the relaxation rate λ, nor do we even know if this is a reasonable model. We face a similar problem in developing motion models for submarines for naval searches. The answer to these problems is to collect and analyze the data necessary to develop and test the models; this topic has not, however, generated much research funding.

3. ESTIMATE SENSOR CAPABILITIES

The probability map identifies the location of the high probability cells. A natural first reaction is to search in the high probability cell. This is often (but not always) reasonable. Even if it is a good strategy to start searching the high probability cell, a search plan must determine how long to search in this cell, which cells to search next, and for how long. Ideally, the chosen plan should maximize the detection probability within the limits of the search resources. In order to develop a search plan, we must first have an estimate of our sensor capabilities.

For search problems, we characterize the sensor by its lateral range function. Following Koopman [1946], we consider a search sensor that approaches a stationary target at a constant speed along a long straight path, as shown in Figure 3. We assume this path begins well beyond the maximum detection range of the target and continues past the target to a point well beyond the maximum detection range. We define the lateral range of the sensor as its range r at the point of closest approach to the target and take this to be a signed quantity, with negative lateral ranges indicating that the target is to the left of the sensor track. Let

$$\alpha(r) = \text{probability of detecting the target when the sensor's path has lateral range } r.$$

Then α is the *lateral range* function of the sensor. The *sweep width* W of the sensor is defined to be the area under the lateral range function, i.e.,

$$W = \int_{-\infty}^{\infty} \alpha(r)dr.$$

Note that α is not a probability density, so W may be greater or less than 1. In addition, the lateral range function and the sweep width usually

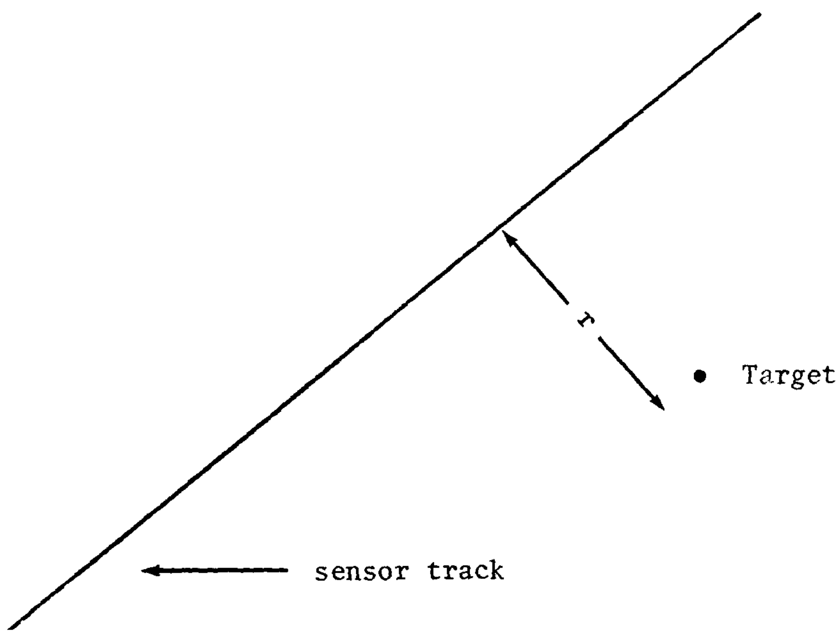

Figure 3. Lateral range.

depend on the relative speed of the target and sensor. Figure 4 shows some typical lateral range functions. The definite range law is the simplest and is sometimes a reasonable approximation for visual searches in which a camera is towed over the ocean bottom. For this law, the probability of detection is 1 out to lateral range R and is 0 beyond; the sweep width $W = 2R$.

The lateral range function, and even more so the sweep width, represents the sensor's capabilities rather crudely. However, in an actual search operation one is lucky to have even this much information about the detection capabilities of the search sensor under the actual operational conditions. When this information is lacking, it is important to test the sensor on site against a test target similar to the search target to determine the lateral range function.

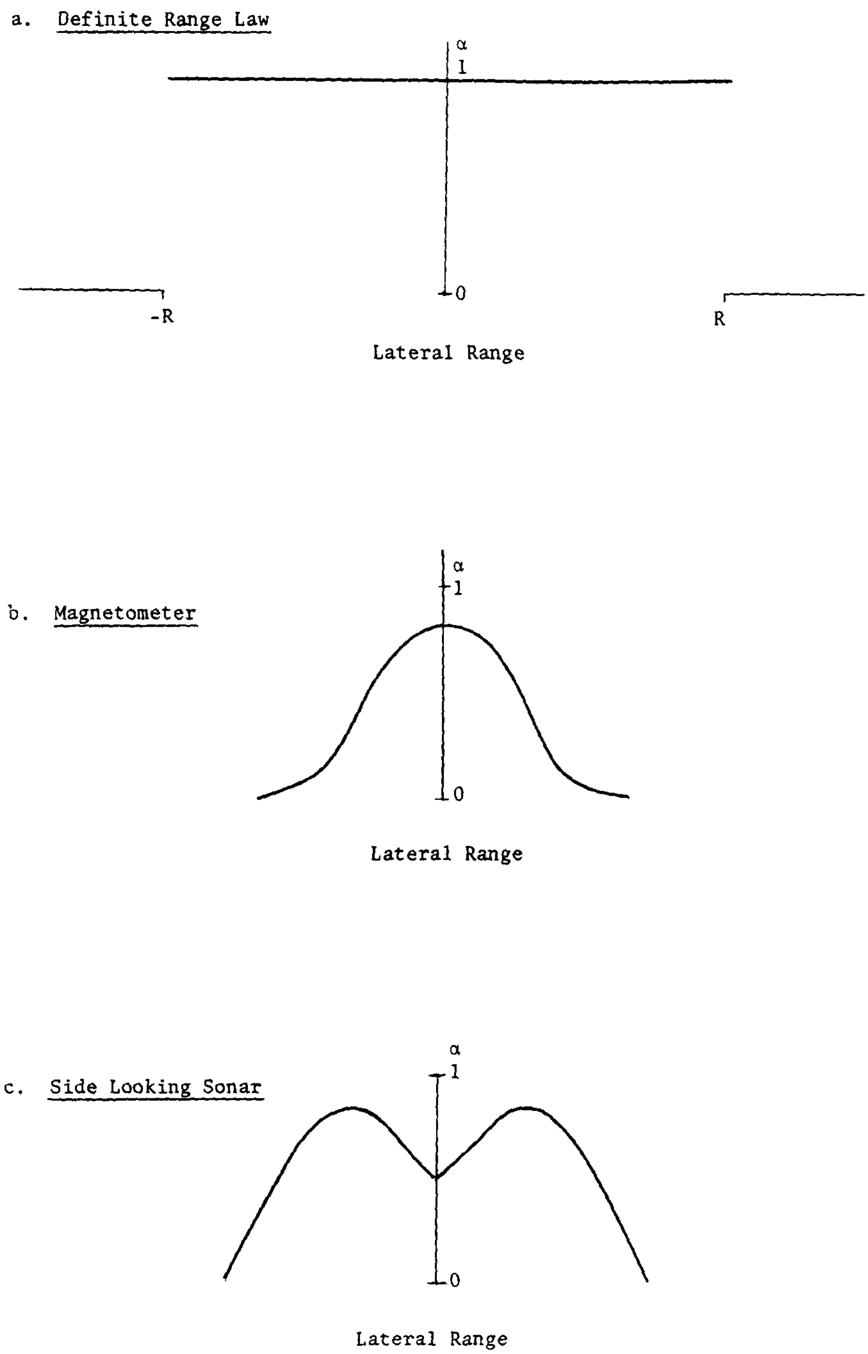

Figure 4. Lateral range functions.

The general characteristics of the sensor are usually enough to determine the shape of the lateral range function. For example, using a magnetometer (and averaging over all possible orientations of the target), we would expect to obtain a lateral range function that is symmetric about $r = 0$ and that is monotonically decreasing as the absolute value of the lateral range increases. The monotone regression techniques of Barlow et al. [1972] could be used when estimating the lateral range function for this sensor. For lateral range functions that increase and then decrease as the absolute value of the lateral range increases, a modification of the pooled adjacent violators algorithm described in Barlow et al. is available. Richardson et al. [1975] used this modification during the 1974 Suez Canal clearance to estimate side-looking sonar lateral range functions of the type shown in Figure 4(c).

Often search sensors are not tested. In these instances, the search planner must rely on the design specifications of the sensor to estimate the lateral range function and should be aware, however, that these estimates are usually optimistic. In fact, Koopman ([1980], p. 21) says that experience in World War II showed that, in an operational situation, systems typically performed at 60–70% of their design capability.

Problem Areas

The detection models used for search sensors do not account for false alarms. Work has been done on the problem of false targets, i.e., objects that yield essentially the same sensor response as the target; but very little is known about incorporating operator controlled false alarm rates into the detection models used for search. Shaw and Kinchla [1981] have suggested an approach to this problem for visual detection.

The search community needs a catalog of lateral range functions obtained by testing sensors in a variety of search situations. In the event of a search problem requiring quick response, the planner could choose the lateral range function that was obtained under the situation closest to that of the search and thereby obtain a more realistic estimate than would be obtained by relying on design specifications or subjective judgments alone.

The best methods now available for estimating lateral range functions, e.g., monotone regression, do not yield confidence intervals for the estimates.

4. DETERMINE DETECTION FUNCTION

Having characterized the sensor's capabilities by its lateral range function, we still require a detection function, namely, a function that relates search effort applied in an area to the probability of detecting the target given it is in that area.

More specifically, suppose that the target is stationary, the search region is divided into J cells, and effort is measured in terms of time spent in each cell. We require knowledge of the *detection function*, b, which is defined by

$b(j, t)$ = probability of detecting the target given that the target is in cell j, and t hours of search have taken place in cell j for $j = 1, \cdots, J$ and $t \geqq 0$.

Knowing the detection function allows us to calculate the probability of detection resulting from any search allocation. For example, let $p(j) =$ probability the target is in cell j for $j = 1, \cdots, J$, where $\sum_{j=1}^{J} p(j) = 1$. If t_j hours are spent searching in cell j, for $j = 1, \cdots, J$, the total probability of detecting the target is

$$\sum_{j=1}^{J} p(j) b(j, t_j).$$

Suppose the sensor searches at speed v and has sweep width W_j in cell j which has area A_j for $j = 1, \cdots, J$. The sweep width of a sensor often varies over the search region because of varying environmental conditions. In the case of visual searches, there may be fog in one part of the search region and clear weather in another. A classic and very useful detection function is the exponential one. For this function

$$b(j, t) = 1 - \exp(-W_j vt/A_j) \quad \text{for} \quad t \geqq 0 \quad \text{and} \quad j = 1, \cdots, J.$$

The typical derivations of the exponential detection function (called the random search formula by Koopman [1946]) assume that search effort is randomly distributed over the search area according to a uniform distribution and that each small increment of search effort is positioned independently of the past search effort. While almost no search situations satisfy these assumptions, the exponential detection function has proven very useful in search planning. One reason is its computational convenience. Another is that it provides a reasonable and conservative estimate for the detection function for a wide class of searches.

To support the latter claim, let us consider a sensor with a definite range law and sweep width, W. Suppose we are searching for a stationary target whose position distribution is uniform over a rectangular region of area A. In this case there is one cell, so we shall suppress the variable j in the detection function b. Since $p(1) = 1$, $b(t) =$ total probability of detection by time t. The intended search tracks are parallel and spaced a uniform distance S apart as shown in Figure 5. If we choose $S = W$, search at speed v, and there is no placement error in the tracks, then the probability of detection as a function of time will be given by the straight line in Figure 6. However, it is usually not possible to position the tracks precisely.

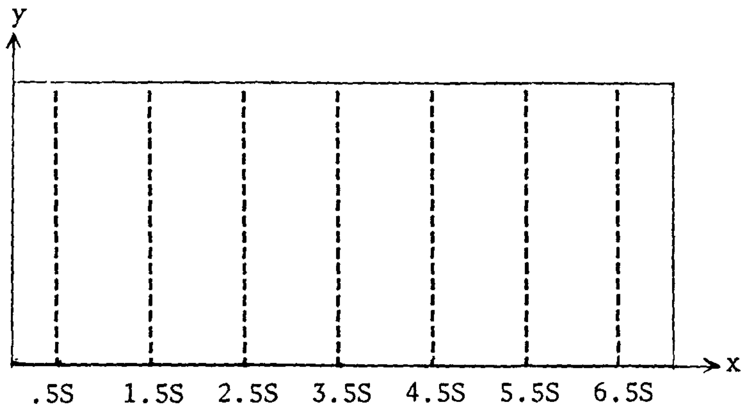

Figure 5. Nominal track placement for parallel path search.

Suppose that the search tracks are indeed parallel to the y-axis in Figure 5, but that the ith track has an x-coordinate that is normally distributed with mean $(2i - 1)S/2$ and standard deviation σ. This error in track placement results in unintended overlaps and gaps in sensor coverage. Figure 7 shows the probability of detection for one complete coverage of the rectangle as a function of σ/W. As σ/W becomes large (but not so large that a sizeable amount of search effort falls outside the rectangle), the detection probability falls to $1 - e^{-1}$ (see Reber [1956]). This is the probability that one would obtain from the exponential detection function. To see this, we observe that if the detection function, b, for this search is exponential, then

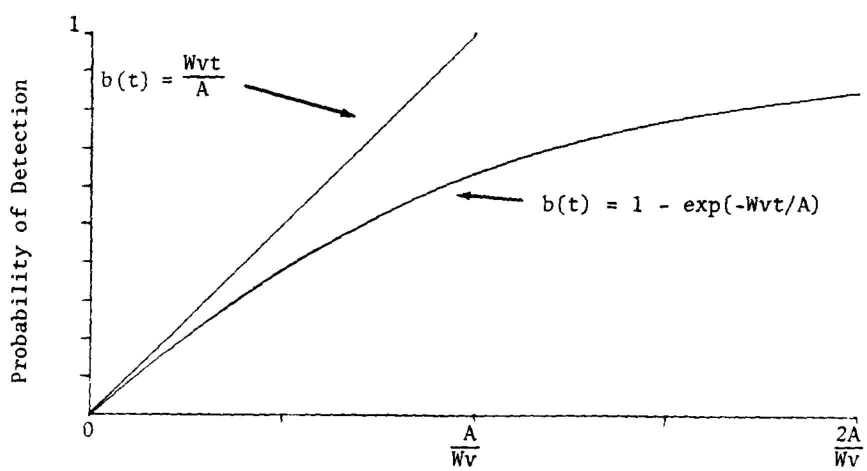

Figure 6. Detection probability for search.

$$b(t) = 1 - e^{-Wvt/A} \quad \text{for} \quad t \geqq 0. \tag{1}$$

It requires time $t = A/Wv$ to perform the above parallel path search. Evaluating (1) at this time, we obtain $1 - e^{-1}$ for the detection probability.

In many searches navigational error is large, and the exponential function is a good approximation to the actual detection function. For most searches in which one tries to spread effort uniformly over a rectangle, the actual detection function will lie somewhere between the straight and the exponential curve shown in Figure 6. Koopman [1946]

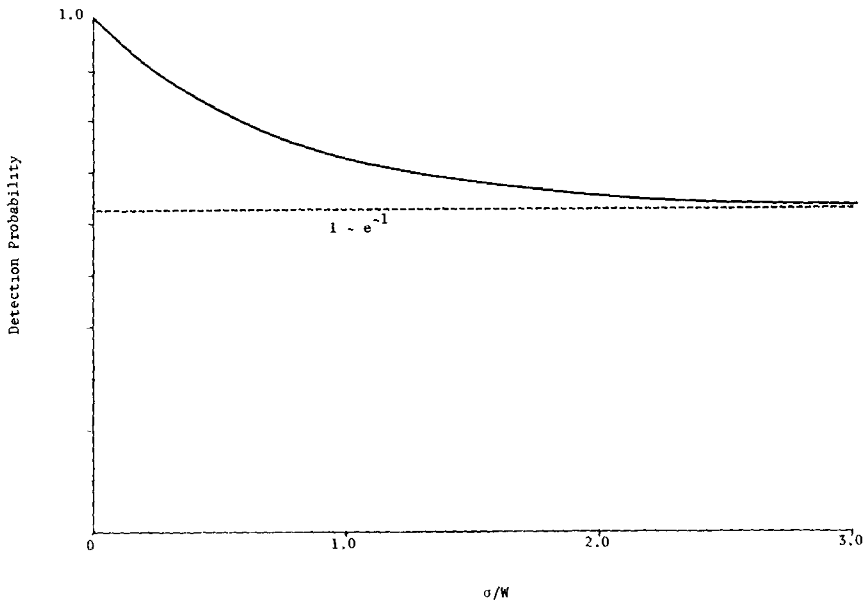

Figure 7. Detection probability for parallel path search.

derives the detection function for a sensor with an inverse cube lateral function (see Koopman [1946] or Stone [1975]). This detection function is appropriate for certain visual searches, and it also falls between the two curves in Figure 6.

When the sweep width (i.e., lateral range function) is uncertain, the exponential detection function is no longer good approximation. Even if the detection capability of the sensor is known, the sweep width can be uncertain because the condition of the target may be uncertain. For example, the searcher may not know whether the target is in one piece or many, or whether it is on top of the ocean bottom or buried beneath it. In this case one must specify a distribution on the sweep width, just as one has specified a distribution on target location.

Assume that the detection function b is determined by the sensor sweep width which is constant, but unknown, over the search region. Since the detection function does not depend on the cell j, we will again suppress the variable j. Suppose there are m possible values of sweep width, $\{\omega_i, i = 1 \cdots, m\}$ and that

$$\beta_i = \Pr\{\text{sweep width} = \omega_i\} \quad \text{for} \quad i = 1, \cdots, m,$$

where $\sum_{i=1}^{m} \beta_i = 1$. Let $B(t, \omega_i)$ be the detection function given that the sweep width has value ω_i. Define

$$\bar{b}(t) = \sum_{i=1}^{m} \beta_i B(t, \omega_i) \quad \text{for} \quad t \geqq 0.$$

Stone (Section 2.3) shows that one may plan the search as though $\bar{b}$ were the actual detection function. Richardson and Belkin [1972] give examples of the penalty, in terms of mean time to detection, that one may incur by not accounting for uncertainty in sweep width.

Problem Areas

Only for parallel path search have we been able to proceed rigorously from the lateral range function to the detection function. The random search formula of Koopman [1946] is derived under very unrealistic search motion assumptions. Nevertheless, we feel on the basis of the example discussed in this section that the random search formula gives a reasonable lower bound on the effectiveness of a search that attempts to spread effort uniformly over the search region.

On the other hand, if one searches randomly (e.g., follows a search path determined by a random walk), the detection probability may be lower than that given by the random search formula. To see this, let us return to the search for a stationary target whose distribution is uniform over the rectangle of Figure 5. Suppose the time available for search is $t = A/Wv$ and σ/W is large. If we attempt to perform the parallel path search shown in Figure 5, our distribution of search effort will (on the average) be uniform over the rectangle. Since σ/W is large, the exponential detection function holds, and the probability of detection computed from Equation 1 is $1 - e^{-1}$. Suppose, on the other hand, the searcher starts at the center of the rectangle, and for each time increment Δt, makes an independent draw from a uniform distribution over $[0°, 360°]$ to determine his direction θ. The searcher then travels in this direction for time Δt and draws another random direction and repeats the process. If Δt is small, the distribution of the searcher's position at time t after the start of the search will quickly converge to a normal one with mean at the center of the rectangle (Feller, Theorem 2, p. 260). As a result, the search effort density will be nonuniform with the highest density at the center. This nonuniform distribution is not optimal and will produce a detection probability lower than $1 - e^{-1}$, which results from the optimal (i.e., uniform) effort distribution.

In the above discussion, we have focused on a particular random walk, but the result holds for any two-dimensional random walk whose increments have mean (0, 0), provided the time step Δt of the walk is small.

5. DEVELOP A SEARCH PLAN

Returning to the example in Section 2, we have a bivariate normal distribution for target location at the beginning of the search as shown in Figure 2. Suppose that we have 3 hours of search effort available and that we are searching visually in an aircraft flying at 125 knots. By checking the sweep width tables of the *National Search and Rescue Manual* (U.S. Coast Guard [1973]), we find that the visual sweep width is 4 nautical miles (nm) for the type of ship involved and the meteorological conditions in the search area. The total effort E, measured in swept area, for this search is

$$E = Wvt = (4 \text{ nm})\ (125 \text{ knots})\ (3 \text{ hours}) = 1500\ (\text{nm})^2.$$

To be conservative, we shall assume that we are using an exponential detection function as given in (1).

This is a moving target problem, and an optimal search plan for it could be calculated using the algorithms described in Brown [1980], Stone et al. [1978], or Discenza and Stone [1981]. However, in order to retain the simplicity of the discussion, we will treat this problem as a stationary target problem by calculating the target's distribution at the middle of the aircraft's search period and planning the search as though it were a stationary target problem with that distribution. This procedure is a common method for treating searches in practice. The ellipse in Figure 8 represents the target distribution at the middle of the aircraft's search period, $T = 11.5$ hours after the distress call.

Using the work of Koopman [1946], one can calculate the optimal distribution of search effort density. The parabola in Figure 9 shows a cross section of this density function. It is very difficult to implement the optimal plan in this case because the aircraft cannot distribute its search effort this finely. This difficulty has motivated the search for simpler plans. For example, the CASP development work investigated rectangle plans that allocated search uniformly over a rectangular region.

Rectangle Plans

For a bivariate normal distribution, consider the class of rectangles that are centered at the mean of the distribution and have their sides parallel to the principal axes of the distribution. Let σ_1 and σ_2 be the standard deviation of the target distribution along the first and second axes, and let L_1 and L_2 be the lengths of the sides of the rectangle parallel to the first and second axes. We restrict attention to rectangles for which $L_1 = 2K\sigma_1$ and $L_2 = 2K\sigma_2$ for some constant K. Figure 10 illustrates the family of rectangles under consideration. It is a one parameter family, with each value of the size factor, K, defining a rectangle for $K \geqq 0$.

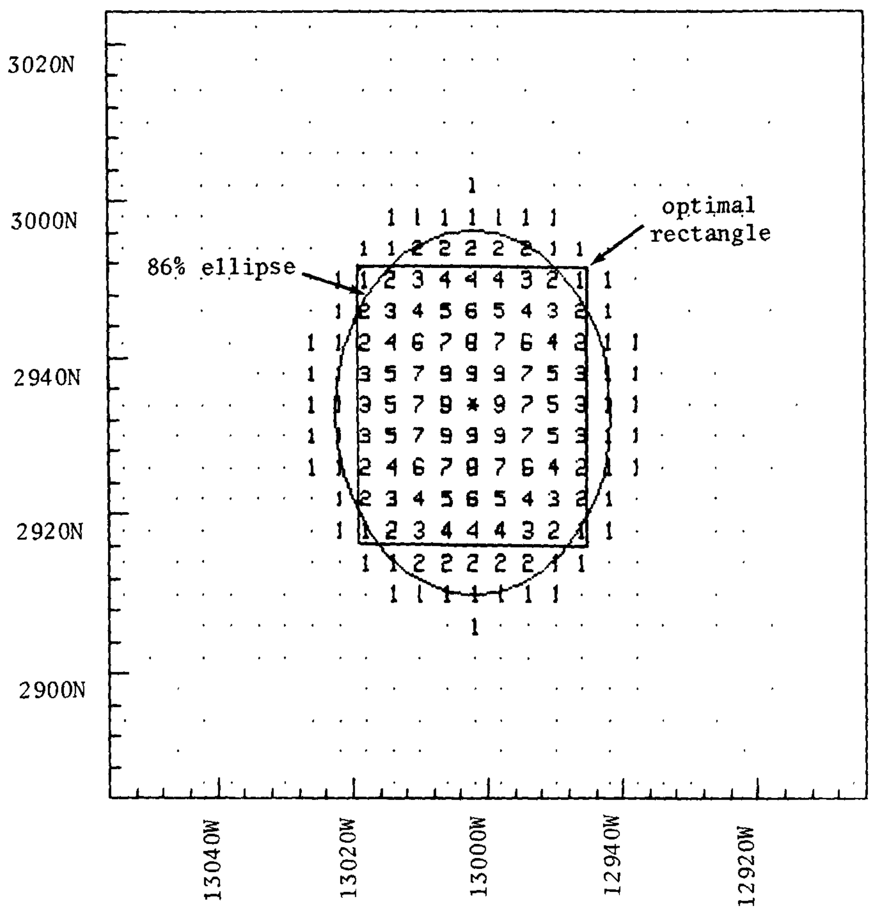

Figure 8. Target distribution at mid-search time ($T = 11.5$ hours).

Let Φ be the cumulative distribution function of a normal random variable with mean 0 and standard deviation 1. The probability of the target being in the rectangle with parameter K is $[\Phi(K) - \Phi(-K)]^2$. If we allocate effort $E = Wvt$, which is spread uniformly over the rectangle, the probability of detecting the target, given it is in the rectangle, is $1 - \exp(-E/4K^2\sigma_1\sigma_2)$. The unconditional probability of detection is

$$P_D(K) = [\Phi(K) - \Phi(-K)]^2[1 - \exp(-(E/(4K^2\sigma_1\sigma_2)))].$$

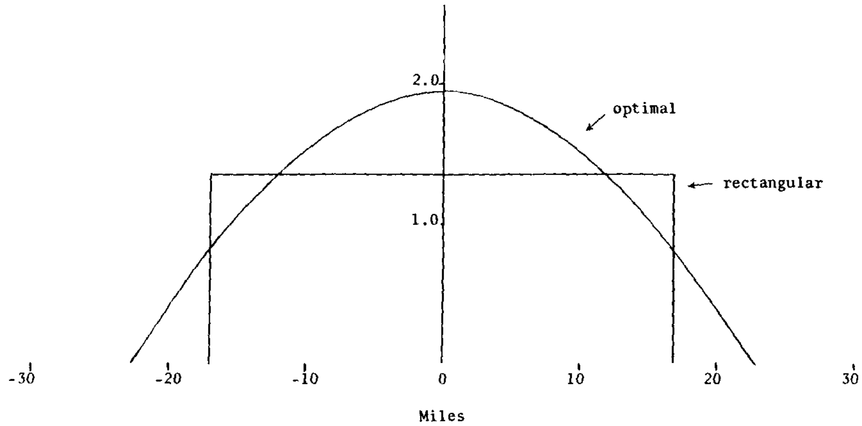

Figure 9. Cross section of optimal and rectangular search density.

As K increases, the first factor increases and the second decreases. Thus there is a K^* that maximizes P_D. An inspection of the equation shows that this K^* depends only on $E/\sigma_1\sigma_2$, the normalized search effort.

The graph of K^*, as well as the associated detection probability $P_D(K^*)$, as a function of normalized effort can be computed numerically and are shown in Figure 11, adapted from Richardson and Discenza. Using it, one may quickly and easily plan a rectangle search and obtain an estimate of its effectiveness.

The resulting rectangle plan seems reasonable, but how good an approximation to the optimal plan is it? To answer this question, one can perform a calculation similar to the one leading to Equation 2.2.21 of

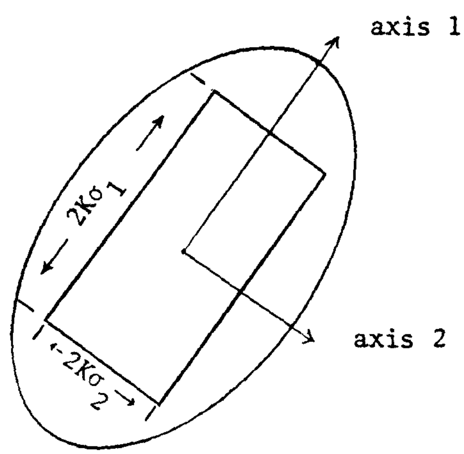

Figure 10. Family of rectangles.

Stone to compute the detection probability, P_D^*, for the optimal plan, namely

$$P_D^*(E) = 1 - (1 + H(E))\exp(-H(E))$$

where

$$H(E) = (E/(\pi\sigma_1\sigma_2))^{1/2}.$$

Upon comparing the detection probabilities, one finds that the probability for the optimal rectangle plan is always within 0.03 of the optimal

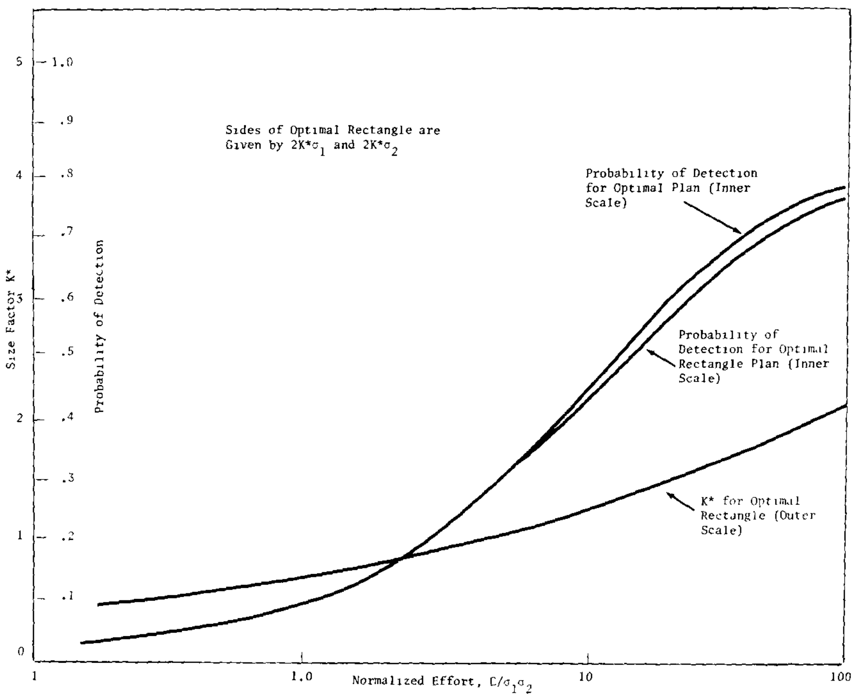

Figure 11. Optimal rectangle.

probability (see Figure 11). Although there may be better approximations, they cannot improve upon the optimal plan. Since we are already so close to optimal, there is no point in searching for better approximations. Thus, we feel confident in recommending a rectangle plan for this situation.

The above example illustrates the role of analysis in search planning. One might recommend a rectangle plan without doing any analysis, but analysis is required to determine the best rectangle plan and to quantify how well it approximates the optimal plan.

Figure 8 shows the optimal rectangle plan for the example in Section 2. The detection probability for this plan is 0.55, compared to a detection probability of 0.58 for the optimal plan.

This example considers a particularly simple situation. Suppose that the above search fails to detect the target. How should the next aircraft search? In order to maintain simplicity, one could require that the next search be rectangular. Engel [1981] has developed a method for finding such a rectangle, which depends only on the detection probability of the previous search effort.

Suppose one is faced with a non-normal probability map consisting of probabilities in cells, as is the case with the maps produced by CASP. For situations like this, the CASP system contains a program MULTI (Richardson and Discenza) that finds up to three disjoint rectangles with which to approximate the optimal search plan. Discenza [1980] has developed an alternate approach to this problem which allows for more than 3 rectangles.

With the exception of Engel [1981], who allows the inverse cube detection function mentioned earlier, the above approaches assume an exponential detection function. Nevertheless, there should be little difficulty in extending these methods to any regular detection function. A *regular detection function* is one whose derivative with respect to effort is continuous, positive, and strictly decreasing. Most detection functions are regular.

There is another simple and useful tool for planning searches when the target distribution is cellular. Using the method discussed on pages 50–51 of Stone, one can devise a computer program that computes the optimal distribution of search effort for stationary problems involving cellular distributions and regular detection functions. The optimal effort for each cell can be displayed as a map for the planner. The planner can use this map as a guide for developing an operationally feasible plan. When the detection function is exponential, the Charnes-Cooper algorithm described in Example 2.2.8 of Stone can be used to obtain the optimal search allocation.

When a stationary search problem involves false targets, optimal effort allocations can be found by using the results of Chapter 6 of Stone. However, like optimal plans without false contacts, these plans are usually too complicated to realize operationally. Chapter 7 of Stone describes an incremental approach that may be used to find simple approximations to optimal plans. These are usually not as easy to implement as rectangle searches.

Problem Areas

Finding simple, near-optimal search plans for moving target problems remains unsolved. Although Stone et al. made some progress on this problem, much remains to be done. Perhaps the multiple rectangle techniques of Discenza could be adapted to these problems.

At present there are no simple, near-optimal approximations to optimal search plans in the presence of false targets. Perhaps the rectangle search approach could be applied here as well.

6. UPDATE FOR SEARCH FEEDBACK

Most searches take place sequentially, or over a long enough period of time, so that the search planner can receive feedback from the search and adjust his plan accordingly.

In order to discuss feedback, we must distinguish between prior and posterior detection and target location probabilities. The prior detection probability is the probability that we calculate before the search takes place. After the search, the posterior detection probability is either zero or one, depending on whether the target was detected or not. The posterior target location probabilities account for all of the information gained during the search. If the target has been detected, then the posterior location probability is one in the cell where the target was detected and zero elsewhere. If the target was not detected, then the posterior location probabilities, or target map, are computed by the method described below.

Suppose that, in our earlier example, we have planned a second search if the first search fails to detect the target. The second search is also a rectangle search with the rectangle being determined by the prior success probability of the first search in the manner of Engel. In the middle of the first search, the aircraft develops engine problems and has to abandon the search halfway through the rectangle. By calculating the prior detection probability for this half search and using Engel's curves, one can adjust the second search to account for the partial first search.

Posterior Maps

In general, one accounts for search feedback by obtaining the records (e.g., the flight path) of the actual search and computing the posterior target location map given failure to detect. Suppose that the map is specified by a grid of J cells with (prior) probability $p(j)$ of the target being in the jth cell. Suppose the target is stationary, search effort is measured in units of time, and the detection function b is given by

$b(j, t)$ = probability of detecting the target, given that it is in cell j and t hours of search have taken place in cell j, for $t \geqq 0$ and $j = 1, \cdots, J$.

In this case the feedback from the search consists of the time t_j spent searching in the jth cell for $j = 1, \cdots, J$. The posterior target distribution, given failure to detect, is computed using Bayes' rule as

$$\tilde{p}(j) = p(j)(1 - b(j, t_j))/(\textstyle\sum_{k=1}^{J} p(k)(1 - b(k, t_k))) \quad \text{for} \quad j = 1, \cdots, J. \tag{2}$$

Having this posterior distribution, based on the actual search effort, the search planner can use the techniques of Section 5 to generate optimal or near-optimal plans for the next increment of effort, provided the detection function is exponential. If the detection function is not exponential, then the posterior detection function must be used in planning the next increment. That is, $b(j, \cdot)$ must be replaced by $\tilde{b}(j, \cdot)$ where

$$\tilde{b}(j, t) = (b(j, t + t_j) - b(j, t_j))/(1 - b(j, t_j)) \quad \text{for} \quad t \geqq 0, j = 1, \cdots, J. \tag{3}$$

Observe that for an exponential detection function, such as the one in Equation 1, $\tilde{b}(j, t) = b(j, t)$ for $t \geqq 0$.

Incremental Plans

For stationary target problems with regular detection functions and no false targets, Stone (Chapter 3), shows that the searcher, who plans each increment of his search in an optimal fashion (using the posterior distributions in (2) and (3) at each stage), will obtain an optimal allocation of the total effort. For stationary target problems with regular detection functions, incremental plans are optimal. This means that the search planner need not consider the time horizon in planning stationary searches, but need plan only each increment optimally. This pleasant state of affairs does not hold for moving target problems. In those problems, the optimal allocation at time 1 and other times depends on the duration of the search. That is, incremental plans are not optimal for moving targets.

False Targets

In many searches there are objects, called false targets, whose detection characteristics are almost identical to that of the target. That is, the false targets have the same detection function as the real target. In underwater search, a large rock on the ocean bottom of roughly the same size and shape as the target may be almost indistinguishable from the target on a side-looking sonar trace. Suppose the distribution of these rocks is Poisson with parameter $\delta(j)$, in the jth cell, i.e.,

$$\Pr\{n \text{ false targets in cell } j\} = (\delta(j))^n e^{-\delta(j)}/n! \quad \text{for} \quad n = 0, 1, \cdots.$$

The Poisson distribution is often a reasonable model for random scatters. We assume that the parameters $\delta(j)$, $j = 1, \cdots, J$ are known or have been estimated from geological information about the search area.

Suppose that the search takes place in two separate phases, broad search and contact investigation. This is often the case in underwater searches. The broad search phase is performed with a sensor, such as sonar, that has a large sweep width against the target, but that might respond to a false target with a detection. Whenever a detection is made, it is called a contact. The second phase employs a separate sensor, such as a camera, that typically has a smaller sweep width and is capable of identification (i.e., distinguishing true from false targets). Entering the contact investigation phase requires that broad search be terminated.

Consider the case where one has searched for time t_j in cell j, $j = 1, \cdots, J$, and this search yields n target-like contacts in cells $j_1, \cdots, j_n$. If none of these contacts has been investigated, then one can use the results of Section 7 of Stone and Stanshine [1971] to compute the posterior target location distribution. Let

$$P = \textstyle\sum_{j=1}^{J} p(j)b(j, t_j). \tag{4}$$

This is the prior probability of detecting the target with search allocation $\{t_1, \cdots, t_J\}$. The posterior probability that the ith contact is the target is

$$\gamma_i = (p(j_i)/\delta(j_i))/(1 - P + \textstyle\sum_{k=1}^{n} p(j_k)/\delta(j_k)) \qquad i = 1, \cdots, n. \tag{5}$$

The posterior probability that the target is in the jth cell, either as a contact or undetected, becomes

$$\tilde{p}(j) = p(j)[1 - b(j, t_j)]/(1 - P + \textstyle\sum_{k=1}^{n} p(j_k)/\delta(j_k)) + \sum_{j_i = j} \gamma_i. \tag{6}$$

The second sum on the right hand side is simply the sum over all contacts that are in cell j. If there are no contacts in cell j, then this sum is zero.

Equations 5 and 6 provide guidance as to which contacts to investigate first and which cells to search next.

Problem Areas

Whenever contact investigation is conclusive and contributes little or no search effort outside the immediate vicinity of the contacts, one can account for the feedback from the contact investigation by simply removing the investigated contacts from the contact list. When contact investigation is inconclusive or when it contributes significant search effort, accounting for its effort is more complicated. In fact, at present we know of no satisfactory method for doing this.

There are additional difficulties involving false targets. For example, the false targets may be caused by moving objects such as fish, or there may be so many contacts in an area that it is difficult to count them accurately.

7. ESTIMATE SEARCH EFFECTIVENESS

In many search operations one is faced with the question of evaluating the thoroughness of the search effort. A simple measure is to note whether or not the target has been found. If one has found the target, this measure is fine, but if the target has not been found, this measure gives no clue as to the thoroughness and efficiency of the search, nor does it provide guidance on when to halt the search.

No False Targets

If one is searching with a sensor for which detection and identification of the target are simultaneous, then it is easy to construct a measure of search effectiveness. The prior probability P of detection given in Equation 4 serves very well. Often visual or camera searches fall into this category.

A high value for P, of say 0.9, implies that if a large number of identical searches were performed under the assumed conditions on target distribution and detection function, then the target would have been found by this time in 90% of those searches. Thus, either bad luck or a faulty assumption, and not poor planning, has led to the failure. The search has been performed intelligently given the best information at hand. Of course, this information might be wrong. Thus, when the measure P becomes high, we have a basis for concluding that we have exhausted the information on which we based the search, and if no new information becomes available, we should stop. Of course, the definition of high is subjective.

When there are false targets and contacts to be investigated, the problem of defining a measure of effectiveness becomes more complicated.

False Targets

During the 1966 search for the H-bomb lost off the coast of Spain near Palomares, H. R. Richardson introduced a measure called Search Effectiveness Probability (SEP) that applies when the nonvisual sensors are incapable of target identification. It is calculated as follows. Let

V_j = prior probability that the target would be detected by visual search given the target is in cell j.

D_j = prior probability that the target would be detected either visually or nonvisually given it is in cell j.

C_j = prior probability that if the target were one of the contacts in cell j, it would have been identified by the investigation effort in cell j. ($C_j = 1$ by convention if there are no contacts in cell j.)

Let $E_j = V_j + C_j(D_j - V_j)$ for $j = 1, \cdots, J$. Then

$$\text{SEP} = \sum_{j=1}^{J} P(j)E_j.$$

Note that when the search is purely visual, SEP = P.

SEP is a useful measure and has been applied on a number of searches, including the *Scorpion* search (Richardson and Stone) and the Suez Canal clearance (Richardson et al. [1975]). However, there are difficulties with it. Suppose that one is searching with a nonvisual sensor, e.g., a side-looking sonar that is not capable of target identification, and there have been no contacts. In this case $E_j = D_j$, the detection probability for the nonvisual sensor. As one applies more search effort to the cells, the probabilities, $D_j, j = 1, \cdots, J$, continue to rise. As long as one obtains no contacts, SEP also continues to rise. However, as soon as one obtains a contact, say in cell j, $E_j = 0$, and SEP drops! What should be good news for the search produces a degradation in the SEP performance measure.

Problem Areas

Although Richardson et al. [1971] investigated a number of alternative measures of SEP, no completely satisfactory alternative has been found. The problem remains to find a definition of SEP that has a natural intuitive meaning and that is monotonically increasing for a well-planned search.

8. CONCLUSION

In this paper we have discussed the process of search planning and identified a number of operations research problems associated with it. Through this discussion, we have shown how theory, practice, and judgment combine in solving search problems. Planning a search is not solely an analytical exercise. Since subjective judgments are crucial to good search planning, search will always, to some degree, be an art. Nevertheless, applying good analytical techniques will improve the art and provide better search planning. Applying analytical techniques to those parts of an operational problem that can benefit from analysis is the heart of practicing operations research.

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