Introduction to Python

- Python is a high-level, interpreted, object-oriented, and general-purpose programming language.
- It emphasizes code readability and allows developers to write programs with fewer lines compared to other languages.

Python Data Types

Data types define the kind of values a variable can hold.

(a) **Numeric Types**

- int → Whole numbers (e.g., 10, -5)
- float → Decimal numbers (e.g., 3.14, -0.75)
- complex → Complex numbers (e.g., `3+5j`)

(b) Text Type

- str → Sequence of characters enclosed in quotes ("Hello" or 'Python')

(c) Boolean Type

- bool → Represents True or False

(d) Sequence Types

- list → Ordered, mutable collection ([1, 2, 3])
- tuple → Ordered, immutable collection ((1, 2, 3))
- range → Sequence of numbers (range(0,10))

(e) Set Types

- set → Unordered collection of unique values ({1, 2, 3})

- frozenset → Immutable version of a set

(f) Mapping Type

- **dict** → Key-value pairs ({"name": "Alice", "age": 25})

Python Operators

Operators are symbols used to perform operations on variables and values.

(a) Arithmetic Operators

- + (Addition)
- - (Subtraction)
- * (Multiplication)
- / (Division)
- % (Modulus – remainder)
- // (Floor division – integer result)
- ** (Exponentiation – power)

(b) Comparison (Relational) Operators

- == (Equal to)
- != (Not equal to)
- > (Greater than)
- < (Less than)
- >= (Greater than or equal to)
- <= (Less than or equal to)

(c) Logical Operators

- and (True if both are True)
- or (True if at least one is True)
- not (Negates condition)

(d) Assignment Operators

- = (Assign value)
- +=, -=, *=, /= (Update value)
- //=, %=, **= (Compound assignments)

(e) Identity Operators

- is → Returns True if both variables reference the same object
- is not → Returns True if they are different objects

(f) Membership Operators

- in → Checks if a value exists in a sequence
- not in → Checks if a value does not exist in a sequence

(g) Bitwise Operators

- & (AND)
- | (OR)
- ^ (XOR)
- ~ (NOT – complement)
- << (Left shift)
- >> (Right shift)

Python Conditional Statements

- Conditional statements are used to make decisions in a program.
- They allow the program to execute certain blocks of code only when a specific condition is True.
- In Python, conditions are usually Boolean expressions (evaluate to True or False).

Types of Conditional Statements:

(a) if statement

- Executes a block of code only if the condition is True.
- If the condition is False, the block is skipped.

(b) if-else statement

- Provides an alternative block of code if the condition is False.

(c) if-elif-else statement

- Used when there are multiple conditions to check.
- elif (else if) allows testing several conditions sequentially.

(d) Nested if statement

- An if statement inside another if.
- Used when decisions depend on multiple levels of conditions.

Iterative Statements

1. for Loop

- Used when we know how many times we want to iterate.
- Works directly with sequences (like list, tuple, string, range).

Syntax : for variable in sequence:

2. while Loop

- Used when we don't know the exact number of iterations in advance.
- Runs until the condition becomes False.

Syntax: while condition:

Loop Control Statements

Used inside loops to control flow:

- break → exits loop completely
- continue → skips current iteration and goes to next
- pass → does nothing (placeholder)

Math Notations & Statistics

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Math Notations

population average notation - μ

standard deviation - σ

sample mean - $\bar{x}$

Variance $= (S.D)^2$

σ^2 - variance

σ - S.D

n - no. of elements

$$\sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$SD = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$

Quartiles

25% - Q_1

50% - Q_2

75% - Q_3

1, 3, 5 | 8, 10, 12 | 15, 17, 19 | 20, 26,

Q_1 Q_2 Q_3

50% Q_2 - median

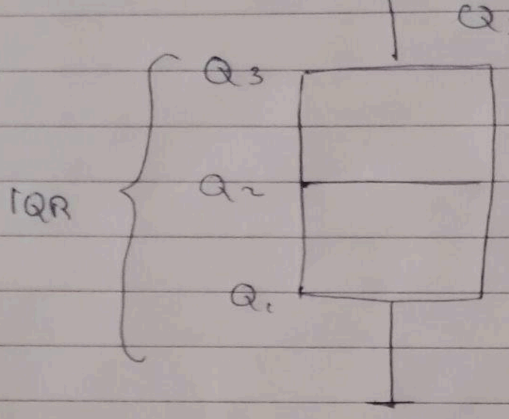
IQR - Inter Quartile Range

$IQR = Q_3 - Q_1$

Box plot

upper limit

$Q_3 + 1.5 * IQR$



lower limit = $Q_1 - IQR * 1.5$

Covariance - $Cov(x,y)$

$$\frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Corelation

$$\frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

Matrix, Eigen values, Eigen vectors, u-v decomposition

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Matrix

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} r \times c \Rightarrow m \times n$$

Rectangle matrix

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}_{2 \times 2} \Rightarrow \text{square matrix}$$

$$J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{diagonal matrix}$$

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{Identity matrix}$$

Matrix addition

Same order and addition is possible

Eg:- $3 \times 1 + 3 \times 1$ matrix

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$$D = \begin{bmatrix} 6 & 3 \\ 9 & 4 \\ 0 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 11 & 31 \end{bmatrix}$$

$$D + C = \begin{bmatrix} 7 & 5 \\ 12 & 8 \\ 11 & 38 \end{bmatrix}$$

<u>multiplication</u>

single value vech multiplication possible
$\downarrow$
scalar

1st matrix	2nd matrix
Eg :- $\underset{r}{3} \times \underset{c}{2}$	$\underset{r}{2} \times \underset{c}{4}$

$\Downarrow$
Same

Answer will be 3×4

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$$M = \begin{bmatrix} -5 & -1 \\ -6 & 0 \\ 11 & 24 \end{bmatrix}_{3\times 2} \qquad A = \begin{bmatrix} 15 & 6 & 4 \\ 1 & 2 & 3 \end{bmatrix}_{2\times 3}$$

$$M \times A = \begin{bmatrix} (-5\times15)+(-1\times1) & (-5\times6)+(-1\times2) & (-5\times4)+(-1\times3) \\ (-6\times15)+(0\times1) & (-6\times6)+(0\times2) & (-6\times4)+(0\times3) \\ (11\times15)+(24\times1) & (11\times6)+(24\times2) & (11\times4)+(24\times3) \end{bmatrix}$$

$$= \begin{bmatrix} -76 & -32 & -23 \\ -90 & -36 & -24 \\ 189 & 114 & 116 \end{bmatrix}$$

$$A \times M = \begin{bmatrix} 15 & 6 & 4 \\ 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} -5 & -1 \\ -6 & 0 \\ 11 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} (15\times-5)+(6\times-6)+(4\times11) & (15\times-1)+(6\times0)+(4\times24) \\ (1\times-5)+(2\times-6)+(3\times11) & (1\times-1)+(2\times0)+3\times24 \end{bmatrix}$$

$$= \begin{bmatrix} -67 & 18 \\ 16 & 71 \end{bmatrix}_{2\times 2}$$

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Transpose

$\Rightarrow$ $F = \begin{pmatrix} -67 & 81 \\ 16 & 71 \end{pmatrix}$

$F^T = \begin{bmatrix} -67 & 16 \\ 81 & 71 \end{bmatrix}$

T ⇒ Transpose

$\Rightarrow$ $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 7 & 8 & 9 & 10 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 7 \\ 2 & 8 \\ 3 & 9 \\ 4 & 10 \end{bmatrix}$

$\Rightarrow$ $M = M^T \Rightarrow$ symmetric

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Determinants

possible on square matrix

$$\Rightarrow \quad M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad\qquad M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|M| = (ad - bc)$$

$$|M| = 1 \times 4 - 2 \times 3$$
$$= 4 - 6 = \underline{\underline{-2}}$$

$$\Rightarrow \quad M = \begin{bmatrix} 10 & 12 \\ 9 & 7 \end{bmatrix}$$

$$\begin{array}{r} 12 \\ 9 \\ \hline 108 \end{array}$$

$$|M| = 10 \times 7 - 12 \times 9$$
$$70 - 108 = \underline{\underline{-38}}$$

Inverse

$$M \times M^{-1} = \text{Identity Matrix } [I]$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \cdot v = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (1 \times 2) + 0 \times 0 \\ (0 \times 1) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$A \cdot u = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{matrix} 2 \times 0 + 0 \times 1 \\ 0 \times 0 + 3 \times 1 \end{matrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A \cdot w = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 0 \times 1 \\ 0 \times 1 + 3 \times 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

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Eigen values & vectors

$AV = \lambda \cdot V$

eigenvalue eigen vector

$(A - \lambda I) V = 0$

$|(A - \lambda I)| = 0$

$$\left| \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{vmatrix} (1-\lambda) & 4 \\ 3 & (2-\lambda) \end{vmatrix} = 0$$

$= (1-\lambda)(2-\lambda) - 12 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 12 = 0$

$\lambda^2 - 3\lambda - 10 = 0$

$\lambda = 5, -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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$\lambda = 5$

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-4x_1 + 4x_2 = 0$, $3x_1 + -3x_2 = 0$

$\lambda = -2$

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$3y_1 + 4y_2 = 0$

$3y_1 + 4y_2 = 0$

$3y_1 = -4y_2$

$y_1 = \frac{-4}{3} y_2$

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$\Rightarrow$ $O = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $M = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$OM = \begin{bmatrix} 0\times 2 + 1\times 0 \\ 1\times 2 + 0\times 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$\Rightarrow$ $O = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $M = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$O \cdot M = \begin{bmatrix} 0\times 1 + 1\times 3 \\ 1\times 1 + 0\times 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$|(O - \lambda I)| = 0$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$

$= \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = \lambda^2 - 1 = 0$

$\lambda^2 = 1, \quad \lambda = \sqrt{1}, \quad \lambda = 1$

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$\lambda = 1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 + x_2 = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = -1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$$

$y_1 + y_2 = 0$

$y_1 + y_2 = 0$

$y_1 = -y_2$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Introduction, Conditional Probability, Bayes Theorem

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pillers of DSA

Linear algebra
probability & statistics
Calculus
computer science

probability

event - get an outcome from sample space.

probability = chance of Happen event

$$\text{probability} = \frac{\text{event count}}{\text{sample space count}}$$

Event
- Independent
- Dependent

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$\Rightarrow P(C) = \frac{15}{32}$ $\quad P(T) = \frac{17}{32}$

$P(M) = \frac{2}{32}$ $\quad P(N) = \frac{30}{32}$

	M	N	
C	$\frac{30}{32 \cdot 32}$	$\frac{450}{(32)^2}$	$\frac{480}{32 \times 32}$
T	$\frac{34}{(32)^2}$	$\frac{17 \times 30}{32^2}$	$\frac{17 \times \cancel{32}}{32 \times \cancel{32}} = P(T)$
	$\frac{64}{32 \times 32}$	$\frac{960}{32 \times 32}$	

Joint probability - Indepedent events

$P(C, M) = P(C) \cdot P(M)$

$P(C, N) = P(C) \cdot P(N)$

$P(T, M) = P(T) \cdot P(M)$

$P(T, N) = P(T) \cdot P(N)$

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$P(S) = \frac{9}{32}$ $P(w) = \frac{23}{32}$

$P(G) = \frac{15}{32}$ $P(B) = \frac{17}{32}$

$P(S, G) = P(S) \cdot P(G)$
$P(S, B) = P(S) \cdot P(G)$
$P(w, G) = P(w) \cdot P(G)$
$P(w, B) = P(w) \cdot P(B)$

	B	G	
S	$\frac{9 \times 17}{32 \times 32} = \frac{153}{32^2}$	$\frac{9 \times 15}{32 \times 32} = \frac{135}{32^2}$	$\frac{288}{32 \times 32} = \frac{9}{32}$
w	$\frac{23 \times 17}{32 \times 32} = \frac{391}{32^2}$	$\frac{23 \times 15}{32 \times 32} = \frac{345}{32^2}$	$\frac{736}{32 \times 32} = \frac{23}{32}$
	$\frac{544}{32 \times 32} = \frac{17}{32}$	$\frac{480}{32 \times 32} = \frac{15}{32}$	

Conditional Probability

- Conditional probability is the probability of event A occurring given that event B has already occurred.

Formula:

$P(A \mid B) = P(A \cap B) / P(B)$, where $P(B) > 0$

Baye's Theorem | Baye's rule

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$P(A)$ = Prior
$P(B|A)$ = likelihood
$P(A|B)$ = Posterior
$P(B)$ = evidence

P-Test

- What it is: Not an actual test like t or z; instead, it's the probability value (p-value) used to decide whether to reject the null hypothesis.
- Interpretation:
  - If $p \leq \alpha$ (significance level, e.g., 0.05) → reject H0H_0H0 → result is statistically significant.
  - If $p > \alpha$ → fail to reject H0H_0H0.
- Use: Applies to *any hypothesis test*

T-Test

- What it is: Statistical test for comparing means when the population variance is unknown and/or sample size is small (<30).
- Types:
  - One-sample t-test → compare sample mean vs. population mean.
  - Independent two-sample t-test → compare means of two independent groups.
  - Paired t-test → compare means of the same group before & after treatment.

Z-Test

- What it is: Hypothesis test for comparing means or proportions when the population variance (σ2\sigma^2σ2) is known or when sample size is large ($n > 30$).
- Applications:

  - Comparing a sample mean with a known population mean.
  - Comparing two population proportions.

F-Distribution

- What it is: A probability distribution used mainly for testing ratios of variances.
- Common Use:

  - ANOVA (Analysis of Variance): To test if 3 or more group means are significantly different.

  - Compare variability between groups vs. within groups.

CALCULUS

Functions & Variables in Data Science

What is a Function?

- A function is a mathematical rule that maps input(s) (independent variables) to an output (dependent variable).
- Notation: $f(x)=yf(x) = yf(x)=y$.
- In programming, a function is a reusable block of code that takes input(s), performs operations, and returns output.

Independent vs Dependent Variables:

- Independent variable (x): The input that we can control or observe.
- Dependent variable (y): The output that depends on the independent variable(s)

Types of Graphs & Their Interpretations

Common Graphs in ML:

- Linear ($y = mx + c$): Models straight-line relationships.
- Quadratic ($y = ax^2 + bx + c$): Parabolic trends (e.g., errors vs iterations).
- Exponential ($y = a \cdot e^x$): Rapid growth/decay (used in population, loss decay).
- Sigmoid ($1 / (1+e^{-x})$): Used as activation function in neural nets.
- Logarithmic ($y = \log(x)$): Slow growth, used in scaling features.

ML Context:

- Activation functions (sigmoid, ReLU, tanh) decide neuron outputs.
- Loss curves show training performance.

Limits, Continuity & Chain Rule

Limits:

- Limit describes the value a function approaches as input approaches a point.
  - Example: $\lim_{x \to 0} \sin x / x = 1$

Continuity:

- A function is continuous at point *a* if:
  - f(a) exists.
  - Lim $x \to a$ f(x) exists.
  - Both are equal.

Chain Rule :

- If y = f(g(x)), then derivative:
  $dy/dx = f'(g(x)) \cdot g'(x)$

Derivatives & Chain Rule Applications

Rules:

- Power Rule: $d(x^n)/dx = n \cdot x^{n-1}$
- Sum Rule: $d(u+v)/dx = u' + v'$
- Product Rule: $d(uv)/dx = u'v + uv'$

Application in ML (Backpropagation):

- Neural networks use chain rule to propagate errors backward (gradient computation).

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Limits

Eg:-1 $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$ approaches no. 4

$f(x) = \frac{2^2-4}{2-2} = \frac{0}{0}$ } we can't find the value

take no closer to 2

$x = 1.9$

$\frac{1.9^2-4}{1.9-2} = 3.9$

$x = 1.99$

$\frac{1.99^2-4}{1.99-2} = 3.99$

$x = 1.999$

$\frac{1.999^2-4}{1.999-2} = 3.999$

Eg:- $\lim_{x \to 3} x^2 + 7x + 3$

$= 3^2 + 7(3) + 3$

$= 9 + 21 + 3$

$= 30 + 3 = \underline{33}$

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$x=0$ is not defined

Eg:- $\lim\limits_{x \to 0} \frac{\sin x}{x}$

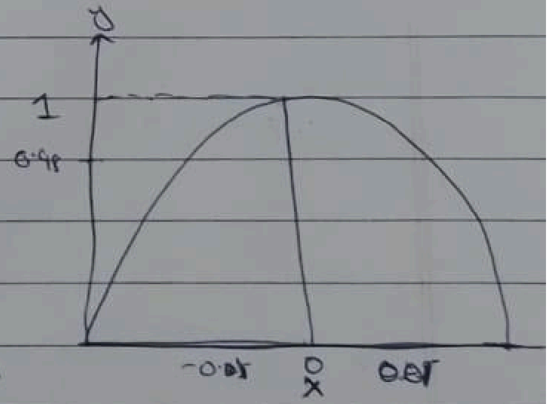
$\frac{\sin 0}{0} = 0$ undefined

$x = 0.1$, $\frac{\sin 0.1}{0.1} = 0.9983$

$x = 0.01$, $\frac{\sin(0.01)}{0.01} = 0.99998$

$x = -0.01$, $\frac{\sin(-0.01)}{-0.01} = 0.99998$

$\frac{\sin x}{x} = 1$



Eg:- $\frac{(\cancel{x-2})(x+2)}{\cancel{x-2}} = x+2$

$\lim\limits_{x \to 2} x+2 = 2+2 = 4$

Left hand limit & Right hand limit

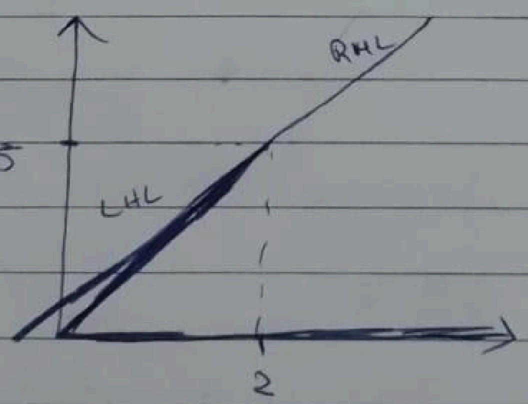
LHF approaches from left

Eg:- $f(x) = \begin{cases} 2x+1 & x<2 \\ x+3 & x>2 \end{cases}$

$$\lim_{x\to 2^-} (2x+1) = 2\times 2+1 = \underline{\underline{5}}$$

$$\lim_{x\to 2^+} (x+3) = 2+3 = \underline{\underline{5}}$$

$\boxed{LHL = RHL}$ limit exist



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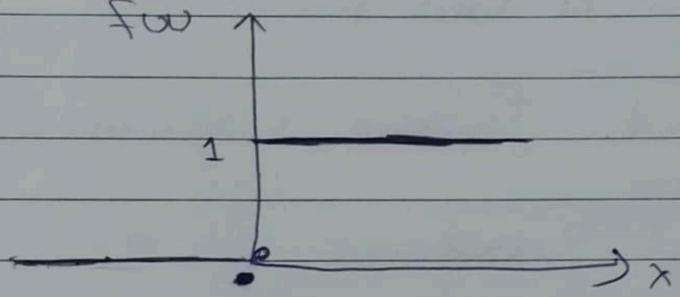
LHL ≠ RHL

Eg:-2 $f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$



$\lim_{x \to 0^-} 0 = \underline{\underline{0}}$

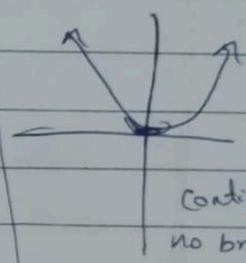
$\lim_{x \to 0^+} 1 = \underline{\underline{1}}$

$\boxed{\text{LHL} \neq \text{RHL}}$

does not exist
step function

continuity

- → continuous form of data
- → edayil no break points
- → polynomial functions continuous
- → rational discontinuous $x=0$



continuous
no breaks

Eg:-

$$f(x) = \begin{cases} 5x+3 & x<1 \\ x^2+4 & 1 \le x < 2 \\ x^3 & x \ge 2 \end{cases}$$

$5(1)+3 = 8$

$(1)^2+4 = 5$

$\left.\begin{array}{l} x^2+4 = \boxed{8} \\ 2^3 = \boxed{8} \end{array}\right\}$ same

discontinuous = $x=1$

continuous, $x=2$

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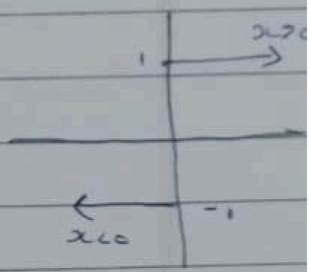
Jump discontinuity

→ left & right are not connect
→ step function



Eg:- $f(x) = \frac{|x|}{x}$ $x \neq 0$

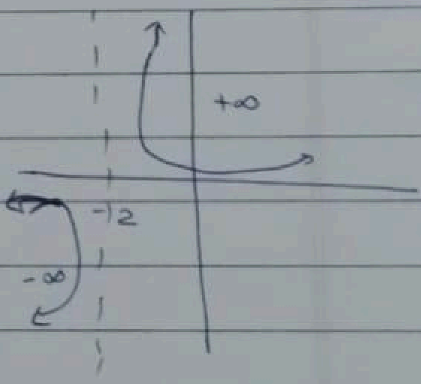
$\lim_{x \to 1} \frac{|1|}{1} = 1$ $\lim_{x \to -1} \frac{|-1|}{-1} = -1$

$x > 0$ $x < 0$

Infinite discontinuity

Eg:- $f(x) = \frac{5}{x+2}$

$x + 2 \neq 0$
$x = -2$

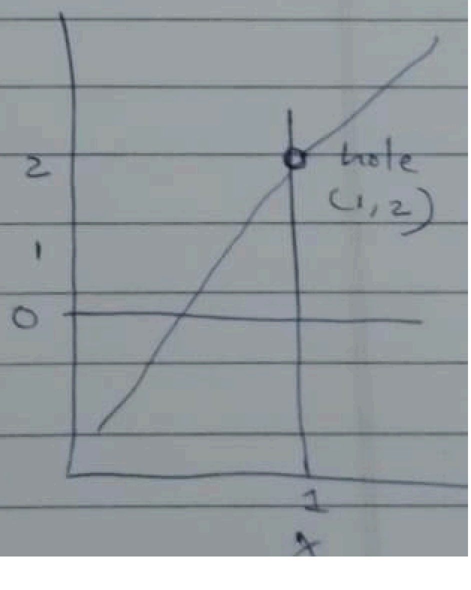


Removable discontinuity

$f(x) = \frac{x^2-1}{x-1}$, $x - 1 \neq 0$

$f(x) = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1$

$\lim_{x \to 1} (x+1) = 1+1 = \underline{\underline{2}}$



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CHAIN RULE

$$y = f(g(x))$$

nx^{n-1}

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

$$y = (3x^2+1)^5$$

outer :- $f(u) = u^5$ $\quad f'(u) = 5u^4$ $\quad nu^{n-1}$

$5u^4$

Inner :- $g(x) = 3x^2+1$ $\quad g'(x) = 6x$ $\quad nx^{n-1}$

$3x \quad 2u+1$

$$\frac{dy}{dx} = \cancel{5(3x^2+1)6x}$$

$$= 5(3x^2+1)^4 6x$$

$$= 30x(3x^2+1)^4$$

nx^{n-1}

Loss & Cost Functions in ML

Loss vs Cost

- Loss function: Error for one data sample.
- Cost function: Average of loss across dataset.

Common Functions

- MSE (Mean Squared Error):
  $$MSE = 1 / n \sum (yi - yi\hat{})$$
- MAE (Mean Absolute Error):
  $MAE = 1 / n \sum |yi - yi\hat{}|$

Graphical Representation:

- MSE curve is quadratic (smooth).
- MAE has sharp edges (less sensitive to outliers).

Gradient Descent: Intuition

Idea:

- Gradient descent minimizes cost by moving opposite to slope of gradient.

Update Rule:

$\theta = \theta - \alpha \cdot \partial J / \partial \theta$, where α = learning rate.

Visualization:

- Think of a ball rolling down a hill until it reaches the lowest point (minimum).

Multivariable Calculus & ML Integration

Partial Derivatives:

- For f (x,y), derivative w.r.t x treats y constant:

    - Example: $f(x,y) = x^2y \rightarrow \partial f / \partial x = 2xy$

Gradient Vector:

- Collection of all partial derivatives:
  $\nabla f(x,y) = [\partial f / \partial x , \partial f / \partial y]$

Application in ML:

- Used in optimizing weights of models.

- Gradient descent in multivariable case: update all parameters simultaneously.

ML Connection:

- Functions → models
- Derivatives → gradients
- Loss functions → error measurement
- Gradient descent → optimization
- Chain rule → backpropagation

NumPy (Numerical Python)

- Definition: NumPy is a Python library used for numerical computing. It provides support for large multidimensional arrays and matrices along with a collection of high-level mathematical functions.

- Why it's used:

    - Fast mathematical operations (addition, multiplication, trigonometry, statistics, etc.).
    - Memory-efficient handling of large datasets.
    - Foundation for many other libraries (like Pandas, SciPy, scikit-learn, TensorFlow, etc.).

- Key Features:
    - ndarray: a powerful n-dimensional array object.
    - Broadcasting (apply operations across arrays of different shapes).
    - Linear algebra, Fourier transforms, random number generation.

Pandas (Python Data Analysis Library)

- Definition: Pandas is a Python library built on top of NumPy. It is mainly used for data manipulation and analysis, especially with tabular or labeled data (like an Excel sheet or SQL table).

- Why it's used:
    - Handles structured data (rows & columns).
    - Easier data cleaning, transformation, and visualization prep.
    - Works well with CSV, Excel, SQL, JSON, and more.

- Key Features:
    - Series: 1D labeled array (like a single column).
    - DataFrame: 2D labeled structure (like an Excel sheet).
    - Handling of missing data.
    - Grouping, merging, reshaping, filtering, time-series operations.