Applied Logistic Regression - Exercise Week 1

Yago Durán Cid

16/05/2015

WEEK 1

Exercise 1:
Use the Myopia Study (MYOPIA.dta) One variable that is clearly important is the initial value of spherical equivalent refraction (SPHEQ).

a. Write down the equation for the logistic regression model of SPHEQ on MYOPIA. Write down the equation for the logit transformation of this logistic regression model. What characteristic of the outcome variable, MYOPIA, leads us to consider the logistic regression model as opposed to the usual linear regression model to describe the relationship between MYOPIA and SPHEQ?

Given the data for myopia where y=0 if the subject has myopia and y=1 if the subject has not got myopia.

To evaluate the probability of a subject has not got myopia given a value of spherical equivalent refraction x we can write the following logistic regression model:

$\pi(x) = E(y|x) = \frac{e^{(\beta_0+\beta_1 x)}}{1+e^{(\beta_0+\beta_1 x)}}$

The odd ratio of the equation above would be:

$Odd\, Ratio = \frac{\pi(x)}{(1-\pi(x))}$

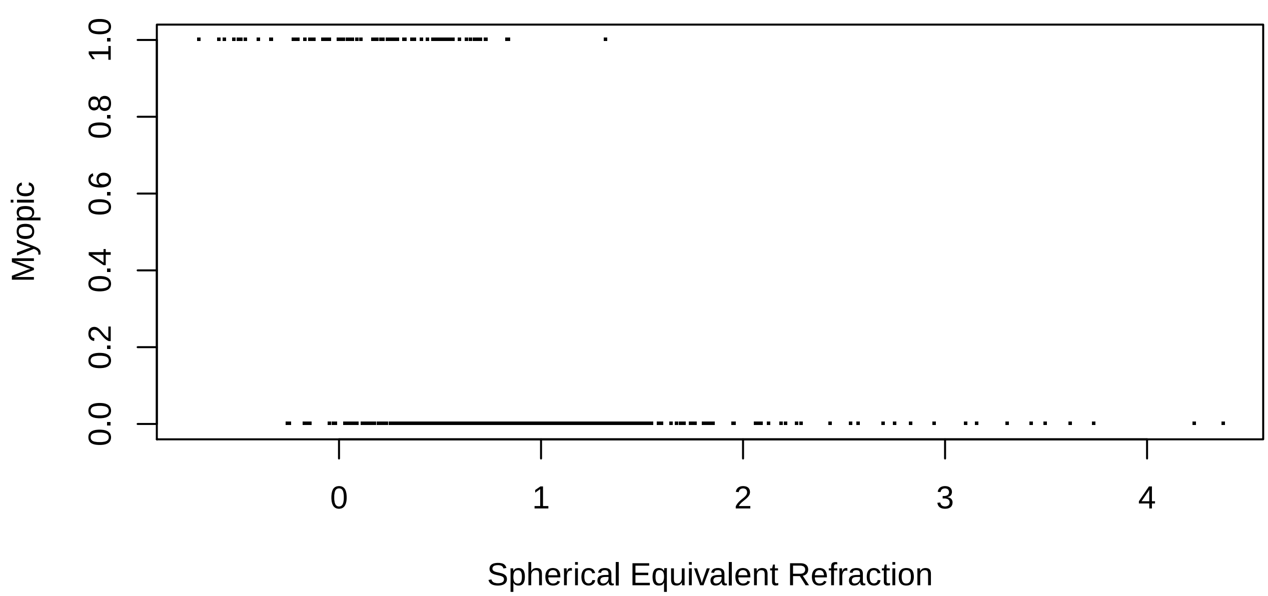
And the natural logarithm of the odd ratio would be:

$ln\left(\frac{\pi(x)}{(1-\pi(x))}\right) = \beta_0 + \beta_1 x$

The assumption in the case of a linear regression model is:

$y = E(y|x) + \varepsilon$ where $\varepsilon \to N(0, \sigma^2)$ and therefore $y|x\ N(E(y|x), \sigma^2)$ which in our case is not true given that we have only two possible outcomes (1 or 0)

b. Form a scatterplot of MYOPIA vs. SPHEQ.



c. Write down an expression for the likelihood and log likelihood for the logistic regression model in part (a) using the ungrouped, n = 618 , data. Obtain expressions for the two likelihood equations.

$\ell(\beta) = \prod_{i=1}^{n} [\pi(x_i)]^{y_i} [1 - \pi(x_i)]^{1-y_i}$

applying natural logarithm to expression above

$L(\beta) = ln(\ell(\beta)) = \sum_{i=1}^{n} y_i ln[\pi(x_i)] + (1 - y_i) ln[1 - \pi(x_i)]$

The likelihood equations are:

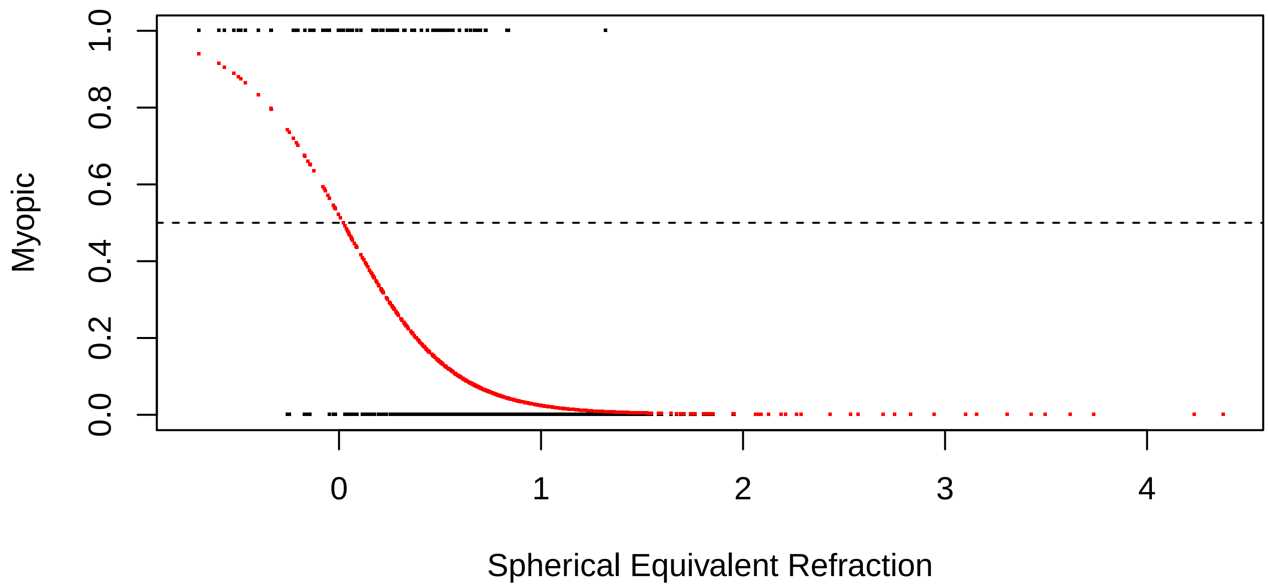
$\sum_{i=1}^{n} (y_i - \pi(x_i)) = 0$ $\sum_{i=1}^{n} x_i(y_i - \pi(x_i)) = 0$

d. Obtain the maximum likelihood estimates of the parameters of the logistic regression model in part (a). These estimates should be based on the ungrouped, n = 618 ,data. Using these estimates, write down the equation for the fitted values, that is, the estimated logistic probabilities. Plot the equation for the fitted values on the axes used in the scatterplots in parts (b) and (c).

```
##
## Call:
## glm(formula = MYOPIC ~ SPHEQ, family = "binomial", data = data)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6435  -0.4533  -0.2681  -0.1029   3.1602
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.05397    0.20675   0.261    0.794
## SPHEQ       -3.83310    0.41837  -9.162   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 480.08  on 617  degrees of freedom
## Residual deviance: 337.34  on 616  degrees of freedom
## AIC: 341.34
##
## Number of Fisher Scoring iterations: 6
```

Substituting with the estimated values we get:

$\pi_e(x) = \frac{e^{(0.05397 - 3.8331x)}}{(1 + e^{(0.05397 - 3.8331x)})}$



Exercise 2:
Use the ICU study (icu.dta) The ICU data set consists of a sample of 200 subjects who were part of a much larger study on survival of patients following admission to an adult intensive care unit (ICU). The major goal of this study was to develop a logistic regression model to predict the probability of survival to hospital discharge of these patients. A number of publications have appeared which have focused on various facets of this problem.

a. Write down the equation for the logistic regression model of STA on AGE. Write down the equation for the logit transformation of this logistic regression model. What characteristic of the outcome variable, STA, leads us to consider the logistic regression model as opposed to the usual linear regression model to describe the relationship between STA and AGE?

The logistic regression model is:

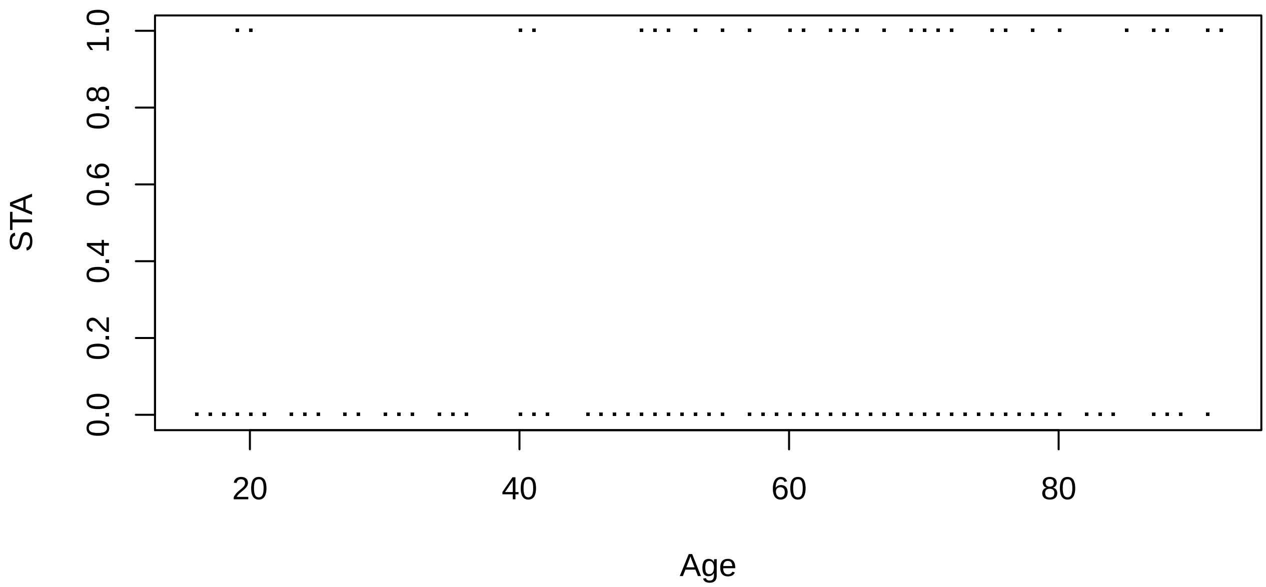
$\pi(x) = E(y|x) = \frac{e^{(\beta_0+\beta_1 x)}}{1+e^{(\beta_0+\beta_1 x)}}$ $where\, x = AGE\, and\, \pi(x) = Prob(STA = 1|AGE)$

Being the logit transformation:

$ln\left(\frac{\pi(x)}{(1-\pi(x))}\right) = \beta_0 + \beta_1 x$

Usual linear regression analysis is not recommended given the binary nature of STA

b. Form a scatterplot of STA versus AGE.c. Write down an expression for the likelihood and log likelihood for the logistic regression model in part (a) using the ungrouped, n = 200 , data. Obtain expressions for the two likelihood equations.



$\ell(\beta) = \prod_{i=1}^{n} [\pi(x_i)]^{y_i} [1 - \pi(x_i)]^{1-y_i}$
$where\, x = AGE\, and\, y_i = 0\, if\, the\, patient\, lived\, or\, y_i = 1\, if\, the\, patient\, died$

applying natural logarithm to expression above

$L(\beta) = ln(\ell(\beta)) = \sum_{i=1}^{n} y_i ln[\pi(x_i)] + (1 - y_i) ln[1 - \pi(x_i)]$

The likelihood equations are:

$\sum_{i=1}^{n} (y_i - \pi(x_i)) = 0$ $\sum_{i=1}^{n} x_i (y_i - \pi(x_i)) = 0$

d. Obtain the maximum likelihood estimates of the parameters of the logistic regression model in part (a). These estimates should be based on the ungrouped, n = 200 , data. Using these estimates, write down the equation for the fitted values, that is, the estimated logistic probabilities. Plot the equation for the fitted values on the axes used in the scatterplots in part (b).

```
##
## Call:
## glm(formula = STA ~ AGE, family = "binomial", data = icu)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.9536  -0.7391  -0.6145  -0.3905   2.2854
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.05851    0.69608  -4.394 1.11e-05 ***
## AGE          0.02754    0.01056   2.607  0.00913 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 200.16  on 199  degrees of freedom
## Residual deviance: 192.31  on 198  degrees of freedom
## AIC: 196.31
```

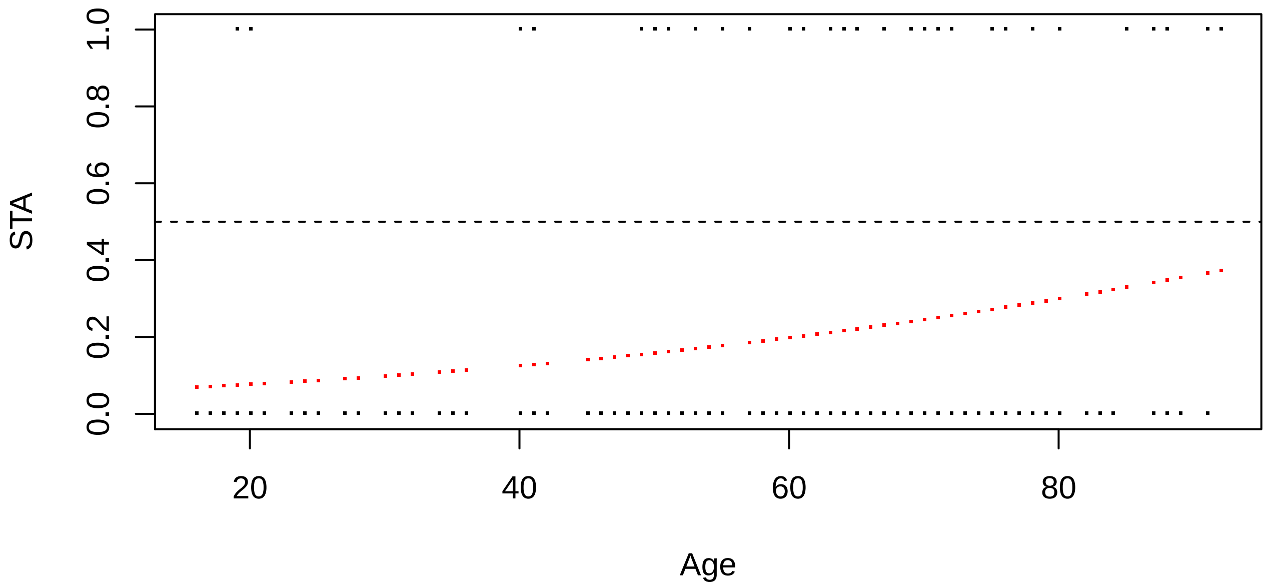
```
##
## Number of Fisher Scoring iterations: 4
```

Substituting with the estimated values we get:

$\pi_e(x) = \frac{e^{(-3.05851+0.02754AGE)}}{(1+e^{(-3.05851-0.02754AGE)})}$

And the logit transformation:

$g(x) = -3.05851 - 0.02754AGE$



e. Summarize (describe in words) the results presented in the plot obtained from parts (b) and (d).

Given the dichotomous nature of the STA variable (patients or live or die) Logistic Regression is appropriate to understand the impact of age on the probability of surviving hospital discharge.
From estimated parameters it seems that as the age increases the probability of survive to hospital discharge reduces.