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# Laser linewidth measurements using self-homodyne detection with short delay

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## Abstract

We compare delayed self-homodyne and self-heterodyne detection in the case of a time delay of the order of the coherence time of the laser. Both methods are found to yield similar results but the homodyne method is simpler to set up and gives a markedly better signal-to-noise ratio. With the short delay the low-frequency  $1/f$ -noise of the laser is effectively filtered out and the measurement provides a value for the Lorentzian contribution to the laser linewidth which is often of prime interest in coherent optical communication systems. © 1998 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Recent advances in laser technology have made it possible to fabricate diode lasers with emission linewidths less than 1 MHz. In external cavity configurations, laser linewidths of the order of kilohertz have been reported. The need to measure and characterize the narrow linewidth of these laser sources often arises in such applications as coherent optical communication, optical precision metrology, and high-resolution spectroscopy.

The delayed self-homodyne/heterodyne measurement technique is an established method for measuring the linewidth of diode lasers [1–3]. The basic idea of the technique is to convert the optical phase or frequency fluctuations of the laser into variations of light intensity in a Mach-Zehnder type interferometer. In the interferometer, the optical field is mixed with a delayed replica of itself and the interference signal is detected with a fast photodiode. The laser linewidth is then deduced from the recorded

power spectrum of the fluctuations of the photocurrent. In the heterodyne case, an acousto-optic modulator is used to shift the spectrum up in frequency in order to reject the detected DC signal in the photodiode and to allow the use of a standard RF spectrum analyzer to measure the spectrum of the photocurrent fluctuations.

Usually in the measurements, the optical path delay is chosen to be longer than the coherence length of the laser source. This makes the phase dynamics of the interfering fields become uncorrelated on the detector, which simplifies the interpretation of the measured power spectrum. For very narrow linewidths, however, the required length for the optical delay line may become unpractically long. Also, propagation losses of shorter wavelengths in optical fibers may set an upper limit for the usable length of the delay.

When the optical path delay in the interferometer is of the order of or shorter than the coherence length of the source, the phase dynamics of the interfering fields remain partially correlated. This complicates the analysis of the measurement results. Moreover, in the homodyne measurement, since the two interfering fields have the same mean

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optical frequency the measured beat signal will critically depend on the exact length difference between the two optical paths on the wavelength scale. Recently, a modified setup for self-homodyne laser linewidth measurements with a short delay fiber was developed to circumvent this problem [4]. It includes two modifications to the conventional homodyne technique. First, a phase modulator is inserted on the delay fiber to average out the critical dependence of the homodyne signal on the mean phase difference between the mixed beams. Secondly, the homodyne signal is electronically shifted up to a carrier frequency by an RF or microwave mixer to allow the use of a standard spectrum analyzer. With these modifications the self-homodyne measurement gives essentially the same signal as the heterodyne setup, except for a better signal-to-noise ratio, since the extra optical losses caused by the acousto-optic modulator are avoided.

In this paper, we compare the modified self-homodyne technique with the conventional self-heterodyne technique for laser linewidth measurements using interferometer delay lines of the order of or shorter than the coherence length of the laser. Moreover, the effects that the  $1/f$  noise of the laser has on the measured power spectra in the various configurations are investigated.

## 2. Theory of self-homodyne/heterodyne detection

The fundamental spectral width of the emission line of any laser originates from fluctuations of the phase of the optical field which are caused by spontaneous emission. Assuming the process driving the fluctuations to be a random Gaussian process with zero mean leads to a Lorentzian power spectrum for the laser output. According to the Shawlow-Townes formula for the laser linewidth, the width of the Lorentzian is inversely proportional to the output power of the laser. In semiconductor lasers, extra phase noise at low frequencies has been found to significantly contribute to the linewidth broadening of these lasers [5–7]. It has been observed that the spectral density of this noise falls off as  $1/f$  with frequency and that its contribution to the linewidth does not depend on the laser power.

In the delayed self-homodyne/heterodyne linewidth measurement techniques, the fluctuations of the phase of the optical field are converted into intensity fluctuations in the Mach-Zehnder interferometer. For stationary fields, the photocurrent depends solely on the intensity correlation function of the laser field, which in the self-heterodyne case can be written in terms of the frequency noise spectrum  $S(\omega)$  of the laser as [8,9]:

$$G_E^{(2)}(\tau) = E_0^4 \left[ (1 + \alpha^2)^2 + 2\alpha^2 \cos \omega_m \tau e^{-2s(\tau, \tau_0)} \right], \quad (1)$$

where

$$s(\tau, \tau_0) = \frac{2}{\pi} \int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right) \sin^2\left(\frac{\omega\tau_0}{2}\right) S(\omega) \omega^{-2} d\omega. \quad (2)$$

In (1),  $E_0$  is the amplitude of the laser field entering the interferometer, assumed to be constant in time, and  $\tau_0$  is the delay difference of the two optical paths. The parameter  $\alpha$  is the amplitude ratio of the interfering fields and  $\omega_m$  is the RF-frequency of the acousto-optic modulator. The measured observable is the power spectrum of the photocurrent which is the Fourier transform of  $G_E^{(2)}(\tau)$ .

For the self-homodyne case, the intensity correlation function can be written as

$$G_E^{(2)}(\tau) = E_0^4 \left[ (1 + \alpha^2)^2 + 4\alpha(1 + \alpha^2) \cos \theta e^{-c(0, \tau_0)/2} + 2\alpha^2 [\cos 2\theta e^{-2c(\tau, \tau_0)} + e^{-2s(\tau, \tau_0)}] \right], \quad (3)$$

where

$$c(\tau, \tau_0) = \frac{2}{\pi} \int_{-\infty}^{\infty} \cos^2\left(\frac{\omega\tau}{2}\right) \sin^2\left(\frac{\omega\tau_0}{2}\right) S(\omega) \omega^{-2} d\omega, \quad (4)$$

and  $\theta$  is the mean phase difference of the interfering fields on the photodiode, which depends on the exact length difference of the optical paths of the interferometer. In the modified self-homodyne setup, the terms depending on  $\cos \theta$  and  $\cos 2\theta$  in Eq. (3) average out to zero. The correlation functions of Eqs. (1) and (3) then differ only in the extra  $\cos(\omega_m \tau)$ -factor entering the heterodyne formula as a result of the frequency shifting accomplished with the acousto-optic modulator.

For both cases, the power spectrum originating from pure white phase noise only consists of a narrow  $\delta$ -peak structure sitting on top of a broader quasi-Lorentzian pedestal. The degree of correlation of the optical phases of the interfering light fields depends on the length of the delay line. For a delay time  $\tau_0$  shorter than the coherence time  $\tau_c$  of the laser, the phases of the fields in the two branches are partially correlated. The pedestal then has an oscillating pattern superimposed on the Lorentzian line with the period of the oscillations given by  $1/\tau_0$ . An increase in the delay causes the signal strength to gradually shift from the  $\delta$ -peak to the pedestal. At the same time, the frequency of the oscillations on the wings of the power spectrum increases and their amplitude becomes smaller. For long delays,  $\tau_0 \gg \tau_c$ , the phases of the fields are completely uncorrelated and the power spectrum becomes strictly Lorentzian with a width twice the FWHM of the laser spectrum.

In this paper, we consider the frequency noise spectrum  $S(\omega)$  of the laser to consist of both a power-dependent white noise contribution  $S_0$  and a power-independent  $1/f$ -contribution of the form  $k/|\omega|$ , where  $k$  is a constant, i.e.:

$$S(\omega) = S_0 + \frac{k}{|\omega|}. \quad (5)$$

The contribution of the relaxation oscillations to the frequency noise at high frequencies will be neglected.

From the expressions for  $s(\tau, \tau_0)$  and  $c(\tau, \tau_0)$  of Eqs. (2) and (4) one can notice that the  $\sin^2(\omega\tau_0/2)$  term acts as a high-pass filter on the frequency noise spectrum. This term causes the amount of  $1/f$ -noise that enters the measured spectrum to depend on the length of the delay line. For short delays, the  $1/f$  component of the frequency noise spectrum is effectively filtered out whereas for longer delays the effect of  $1/f$ -noise becomes more prominent the longer the delay [9]. This fact can actually be used advantageously in measurements where the Lorentzian linewidth due to the white noise is the parameter of prime interest [10]. This type of measurement needs arise for instance when designing coherent communication systems where the optical power on the far wings of the laser's emission spectrum can severely limit the operability of the system.

### 3. Experimental setups

The experimental self-heterodyne and self-homodyne setups used to measure the laser linewidth are depicted in Fig. 1. The laser examined was a non-commercial multi-quantum-well DFB laser manufactured by NEC having a nominal wavelength of  $1.55 \mu\text{m}$ . The output power was 8 mW at an injection current of 150 mA and at a laser temperature of  $20.1^\circ\text{C}$ . A fibered optical isolator with 60 dB isolation was used in conjunction with the laser to reduce optical feedback. The output beam was fed into the single-mode-fiber Mach-Zehnder interferometer via a fiber coupler.

In the modified self-homodyne setup, a phase modulator was inserted on the delay fiber to average out the dependence on the mean phase difference  $\theta$ . The modulator was made of a low-voltage piezo-electric actuator (Tokin, AE0505D16) inserted in between two semi-circular disks of plastic with a diameter of 8 cm. Four turns of the delay fiber were wrapped around the circumference of the disks. The piezo-electric transducer was driven with a 225 Hz triangular wave generated by a signal generator and with an amplitude corresponding to a phase shift of  $2 \times 2\pi$ . A typical scan of 601 channels with the spectrum

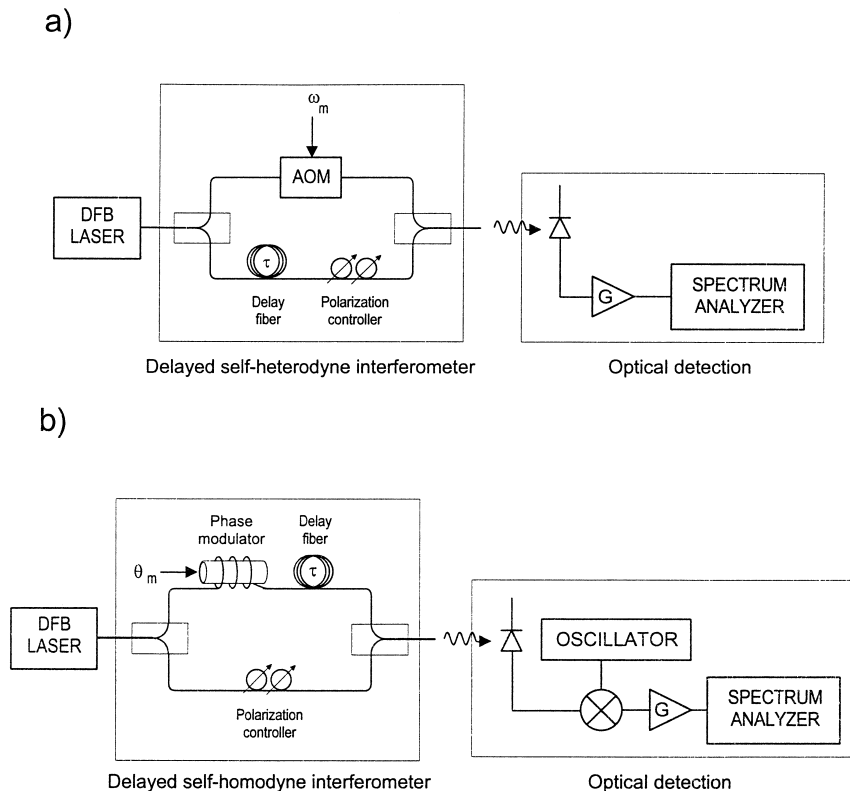


Fig. 1. Laser linewidth measurement setups. (a) Self-heterodyne setup with an acousto-optic modulator (AOM). (b) Modified self-homodyne setup with a phase modulator and electrical upconversion. ( $\omega_m$  – RF frequency of AOM and  $\theta_m$  – phase modulation frequency.)

analyzer was recorded in 16 s, i.e. the dwell-time per channel was 26.6 ms. With these settings  $\theta$  was effectively averaged over  $12 \times 2\pi$ .

In the self-heterodyne setup, an acousto-optic modulator (Crystal Technology, AOMO-3165-1) was inserted in one of the branches to shift the beat note in frequency. The modulator was driven at an RF-frequency of 200 MHz. The optical signal in one of the branches of the interferometer was coupled out of the fiber by a microscope objective with 40-fold magnification and a numerical aperture of 0.32 and, subsequently, directed through the acousto-optic modulator. The first-order diffractive beam generated by the acousto-optic modulator was then coupled back into the interferometer by another, identical  $40 \times$  objective. The losses incurred when coupling out of, and back into the optical fiber were of the order of 3 dB/coupling. When these losses together with the diffraction efficiency of the modulator are accounted for, a coupling ratio of the order of 10% is expected. In practice, it was found to be 7%.

In both setups, a polarization controller (Photonics, Lefevre's Loops) was placed in one of the branches of the interferometer to maximize the signal-to-noise ratio in the measurement. The beat signal was detected with a photodetector (Newport, D-100-FC) having a bandwidth of 4 GHz and a linearity of 0.03 dB per GHz. To reduce the effect of backreflection, particularly from the detector surface, an angled-polished fiber-optic connector was used. The beat signal was amplified with one low-noise amplifier in the homodyne case and with two amplifiers in the heterodyne setup. The bandwidth of the amplifiers was 1 GHz. The beat signal was detected with a spectrum ana-

lyzer with a resolution bandwidth of 30 kHz. In the modified self-homodyne setup, the photocurrent signal was mixed in a double-balanced mixer with a stable 200 MHz local oscillator.

#### 4. Results

Fig. 2 shows a self-heterodyne spectrum measured for a delay much longer than  $\tau_c$ , by using 6 km of delay fiber. The detector signal was amplified by 30 dB. A theoretical spectrum taking into account white noise only, i.e. the Fourier transform of Eq. (1) with  $S(\omega) = \text{const}$ , was fitted to the measured power spectrum by using a least-squares fitting procedure. The measured noise level of the detection system was subtracted from the data before the fitting. The fit which is shown by a dashed line in the figure clearly shows that the white noise model is not capable of explaining the measured spectrum. The fit which gives a FWHM value of 440 kHz for the Lorentzian linewidth of the laser is reasonably good on the far wings of the spectrum, but in the center of the line it deviates significantly from the measured data. By including a  $1/f$ -component  $k/|\omega|$  in the noise spectrum of the laser, the measured self-heterodyne spectrum for the long delay can be modeled in a more satisfactory manner for the central part of the line, too, as is indicated by the spectrum plotted with a solid line. The value of the parameter  $k$  in the fit was  $0.05 \times [(2\pi)^3/2] \times 10^{12} \text{ Hz}^2$ . Clearly, one should be cautious when characterizing the measured spectrum with its  $-3 \text{ dB}$  linewidth.

We also performed the linewidth measurement using the same setup but with a short delay line consisting of

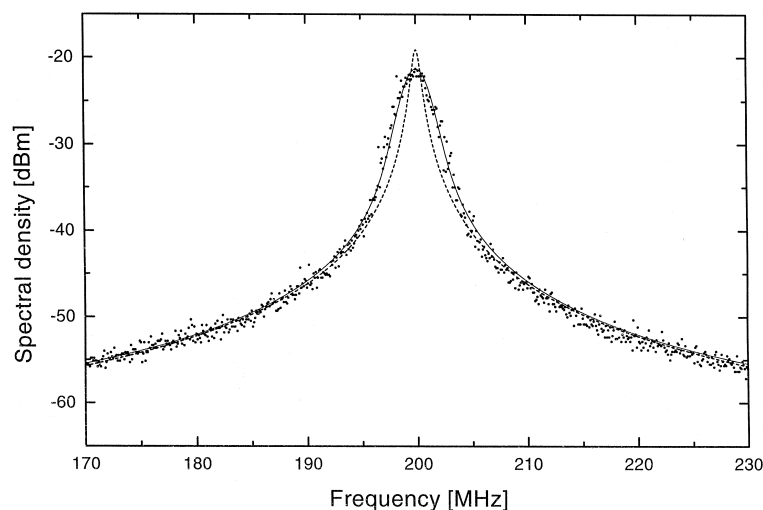


Fig. 2. Self-heterodyne linewidth measurement with a 6 km long delay fiber ( $\tau_0 = 29.34 \mu\text{s}$ ). The Fourier transform of Eq. (1) fitted to the measured power spectrum of the beat note gives a value of 440 kHz for the Lorentzian linewidth of the laser when only white noise is included in  $S(\omega)$  (dashed line). The solid line represents a fit to the experimental data with  $1/f$ -noise included in  $S(\omega)$  with parameter values  $S_0/2\pi = \Delta\nu_L = 410 \text{ kHz}$  and  $k = 0.05 \times [(2\pi)^3/2] \times 10^{12} \text{ Hz}^2$ . The amplitude ratio of the interfering fields was  $\alpha = 0.59$ .

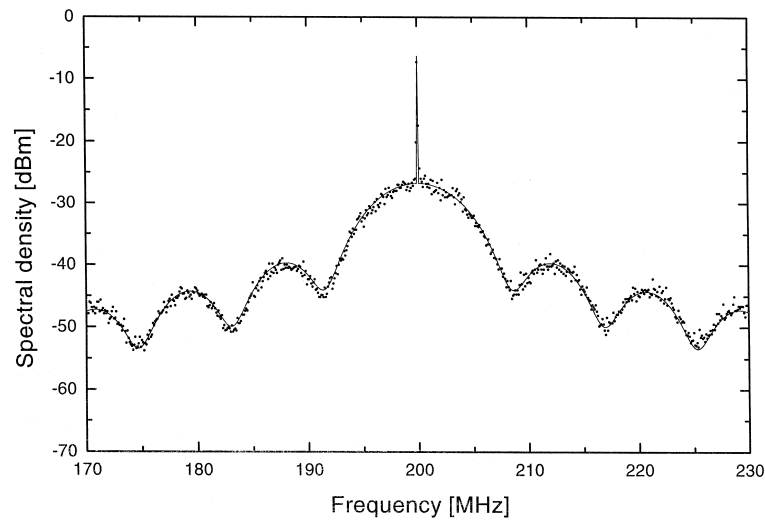


Fig. 3. Self-heterodyne linewidth measurement with a short delay line of 24.3 m ( $\tau_0 = 119$  ns). A value of  $\Delta\nu_L = 450$  kHz for the Lorentzian linewidth was obtained by fitting the Fourier transform of Eq. (1) to the measured data and including white noise only in  $S(\omega)$ . The amplitude ratio of the interfering fields was  $\alpha = 0.65$ .

24.3 m of fiber. The measured power spectrum is shown in Fig. 3. In contrast to the previous result with a long delay the spectrum in Fig. 3 can be satisfactorily interpreted by assuming the phase noise to consist of white noise only. Due to the short delay the contribution of the  $1/f$  noise of the laser at low frequencies is filtered out from the measured spectrum. The white noise fit, which is indicated by the solid line in the figure, gives a value of 450 kHz for the FWHM Lorentzian linewidth of the laser.

The low signal level in the heterodyne measurements made it necessary to amplify the detector signal by 44 dB.

By applying the modified self-homodyne technique of Fig. 1b the same spectral information could be obtained with a simpler setup and a better signal-to-noise ratio. The length of the delay line was 20.4 m of fiber. The measured power spectrum is shown in Fig. 4. A fit using the white noise model gives a value of 410 kHz for the FWHM Lorentzian linewidth of the laser. With this setup only one amplifier with an amplification of 22 dB was needed.

Another measurement with the homodyne setup was performed using a delay line of 71.7 m, slightly less than the coherence length of  $\sim 100$  m of the laser. The result is

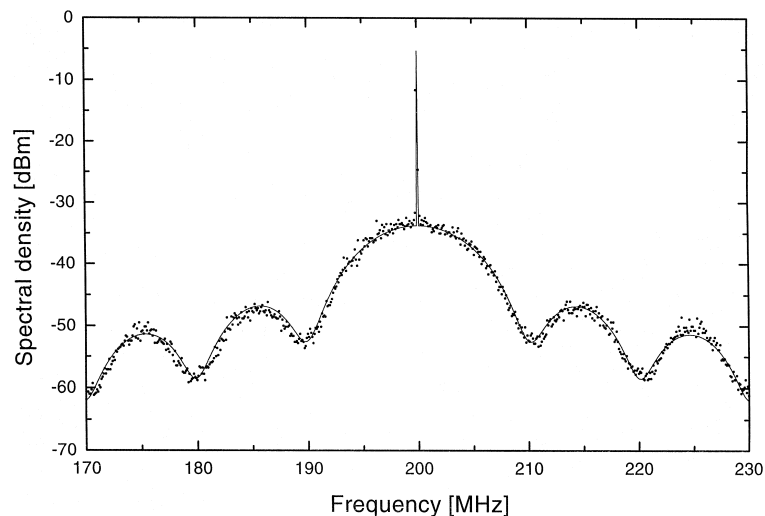


Fig. 4. Result of a measurement performed with the modified self-homodyne technique with a short delay line of 20.4 m ( $\tau_0 = 99.8$  ns). The Lorentzian linewidth was found to be  $\Delta\nu_L = 410$  kHz. The amplitude ratio was  $\alpha = 0.85$ .

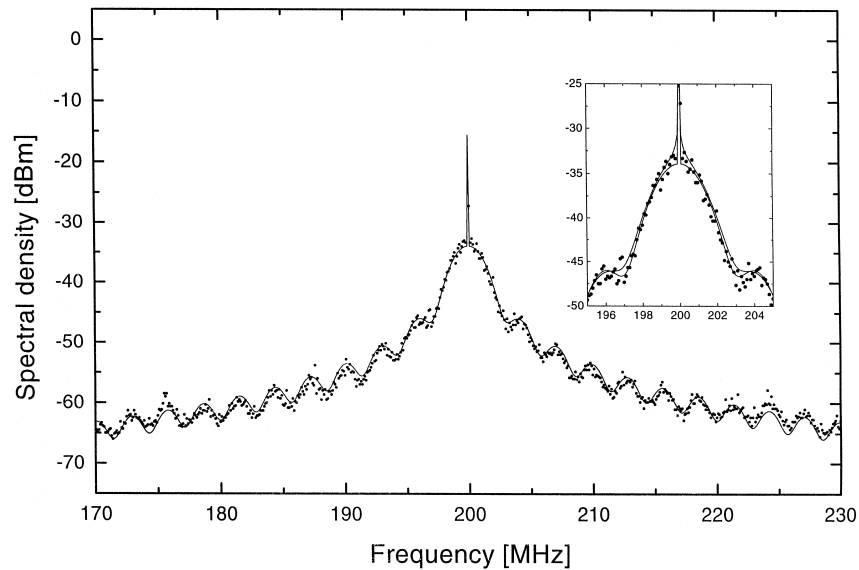


Fig. 5. Result of a measurement performed with the modified self-homodyne technique with a short delay line of 71.7 m ( $\tau_0 = 351$  ns). The Lorentzian linewidth was found to be  $\Delta\nu_L = 460$  kHz. The amplitude ratio was  $\alpha = 0.97$ . The high power in the  $\delta$ -peak is due to imperfect balancing of the RF-mixer.

shown in Fig. 5. In this measurement, a value of 460 kHz was found for the FWHM Lorentzian linewidth of the laser when assuming white phase noise only. The fit is not, however, as good as with the shorter delays. Adding a  $1/f$ -noise contribution to  $S(\omega)$  with  $k = 0.05 \times [(2\pi)^3/2] \times 10^{12} \text{ Hz}^2$  improves the fit slightly, but only at the very center of the spectrum as is shown in the inset in Fig. 5.

## 5. Discussion

The delayed self-homodyne/heterodyne technique gives a convenient method to study the linewidth of laser light sources. When only phase noise due to spontaneous emission is present, the use of a delay line longer than the coherence length of the source results in a Lorentzian line profile for the laser and a homodyne/heterodyne spectrum where the linewidth can be directly observed from the measured spectrum. It is, however, well known that the phase noise of semiconductor lasers includes other contributions besides the fundamental quantum noise contribution. The results presented in this work also clearly show that the self-heterodyne spectrum of the DFB-laser measured by applying a long delay line is not satisfactorily explained by the model based on white phase noise only. The longer the delay the more prominent the effects of the  $1/f$ -noise of the laser. The interpretation of the measured spectra then requires a careful analysis and fitting of a theoretical model spectrum to the measurement data. Moreover, when the linewidth of the source starts to be very narrow, the required length of the delay line may

become unpractically long or the overall propagation losses too high.

Often for example in coherent communication systems, the Lorentzian contribution to the linewidth broadening due to the white phase noise is the parameter of importance. Therefore, measurements with short delays, where the low-frequency  $1/f$ -noise is effectively filtered out, may be more preferable than the use of long delays combined with a complicated analysis of the measured spectra. As a matter of fact, we were able to describe the spectra measured with delays a few times shorter than the coherence length of the laser satisfactorily by applying the simple white noise model in fits where the Lorentzian linewidth of the laser was a free parameter.

Of the two techniques applied with the short delays, the modified self-homodyne setup offers some clear advantages. It avoids the use of the acousto-optic modulator, which necessarily introduces extra losses and makes the system more cumbersome to operate. In principle, the spectrum measured with the modified homodyne technique is susceptible to the  $1/f$ -noise in the photodiode. However, in our measurements this was not a noticeable effect. Within the overall accuracy of the fitting procedure, which was estimated to be  $\pm 50$  kHz for our spectra, the values for the Lorentzian linewidth were the same for the two techniques.

## 6. Conclusion

We have studied in detail the use of short delays in both self-homodyne and self-heterodyne detection of the

phase noise dynamics of a DFB-laser. When only the high-frequency behavior of the phase noise spectrum is of interest, the use of a short delay line gives a viable method to determine the Lorentzian linewidth of the laser. In particular, we found that the modified self-homodyne technique offers some clear technical advantages over the conventional heterodyne method and should be considered as an alternative way to perform this type of linewidth measurements.

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