
MINEX III Report Card

Matcher nbisminexiii+5001



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Participant Details

Company: NBIS

Provided **CBEFF PID**: 0000 5001

Date Submitted: 09/04/2015

Date Validated: 09/04/2015

Date Completed: 09/04/2015

Library	Size (bytes)	MD5 Checksum
libminexiii_nbisminexiii_5001.so	17514	76298e38dfa97940f6133f159ffaafded

Compliance Test Results

The following presents **PIV compliance** results per the criteria detailed in [NIST Special Publication 800-76-2: Biometric Specifications for Personal Identity Verification](#).

It also includes **MINEX III compliance** results per the criteria detailed in sections 4 through 8 of the [Minutiae Interoperability Exchange \(MINEX\) III Test Plan and Application Programming Interface](#).

PIV Level One: FAIL

- Must match templates from all certified template generators with an $\text{FNMR}_{\text{FMR}}(0.01) \leq 0.01$ using two fingers (4.5.2.1-4). **✗** (See Table 7)
- Average template comparison time must be no more than 10 milliseconds (6.4). **✓**

PIV Level Two: FAIL

- Must pass PIV level one compliance. **✗**
- Native template generator must pass level one compliance. **✗**
- Must match templates from native template generator with an $\text{FNMR}_{\text{FMR}}(0.0001) \leq 0.02$ using one finger (4.5.3-2) **✗**

MINEX III: FAIL

- Must pass MINEX III validation. **✓**
- Must pass PIV level two matcher compliance. **✗**
- Matcher must produce at least 256 distinct comparison scores over the entire dataset when comparing templates from different subjects. (30) **✗**

Notes

- This report will be updated as new matching algorithms and template generators pass the compliance test. These updates will not change the PASS/FAIL decision above.
- NIST reserves the right to decertify a matcher if it later discovers the matcher violates MINEX III or PIV specifications in some previously undetected way.

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1 Introduction

This report card presents measurements of performance and interoperability for a single fingerprint matching algorithm submitted to NIST as part of the ongoing MINEX III Evaluation. It reports whether the matcher passes the technical requirements for MINEX III as described in Section 8 of the [MINEX III Test Plan and Application Programming Interface](#). Full details on the ongoing MINEX III program can be found on the [MINEX III homepage](#). Questions should be directed to minex@nist.gov.

2 Methodology

Testing is performed at a NIST facility. Each participant's submission is validated by NIST (<http://www.nist.gov/itl/iad/ig/minexiii.participation.cfm>) before undergoing full testing to ensure it operates correctly. If the matcher passes the validation procedure, it is then used to compare standard fingerprint templates. Performance is assessed against templates created by a template generation algorithm submitted by the participant as well as templates created by other MINEX III compliant template generators.

2.1 Dataset

Testing is performed over a single dataset of sequestered fingerprint images. The images were collected by U.S. Visit at ports of entry into the United States. They consist of Live-scan plain impressions of left and right index fingers. WSQ [1] compression was applied to all images at a ratio of 15:1. The most recent capture of each subject was treated as the authentication sample, and the next most recent as the enrolled sample.

The dataset was divided into 533 767 mated and 1 067 530 non-mated subject pairings. Since both left and right index fingerprints are available for each subject, this provides 1 065 347 mated and 2 127 730 non-mated single-finger comparisons (after database consolidation). When left and right index fingers are fused at the score level [3, 7], the sets condense to 532 239 mated and 1 062 818 non-mated comparison scores.

2.2 Accuracy Metrics

Core matching accuracy is presented in the form of Detection Error Tradeoff (DET) plots [6], which show the trade-off between the False Match Rate (FMR) and the False Non-Match Rate (FNMR) as a decision threshold is adjusted. Formally, let m_i ($i = 1 \dots M$) be the i th mated comparison score, and n_j ($j = 1 \dots N$) the j th non-mated comparison score. Then the statistics are

$$\text{FNMR}(\tau) = \frac{1}{M} \sum_{i=1}^M \mathbb{1}\{m_i < \tau\}, \quad (1)$$

$$\text{FMR}(\tau) = \frac{1}{N} \sum_{j=1}^N \mathbb{1}\{n_j \geq \tau\}. \quad (2)$$

where $\mathbb{1}\{A\}$ is the indicator [4] of event A . Equations 1 and 2 define the curve parametrically with the decision threshold, τ , as the free parameter. In some figures and tables, FNMR is presented as a function of FMR. This relationship is determined by

$$\text{FNMR}_{\text{FMR}}(\alpha) = \min_{\tau} \{ \text{FNMR}(\tau) \mid \text{FMR}(\tau) \leq \alpha \}, \quad (3)$$

which reads as the smallest FNMR that can be achieved while maintaining an FMR less than or equal to α , the targeted FMR. This method of relating the two error statistics ensures FNMR is well-defined for all $0 \leq \alpha \leq 1$. When the matching algorithm produces only a few unique comparison scores, the maximum threshold, τ_0 , that elicits an $\text{FMR}(\tau_0) \leq \alpha$ may, in fact, be quite a bit lower than α . Thus, Equation 3 imposes a natural penalty on matching algorithms that produce overly discretized scores.

Some figures show *pooled* DET accuracy, which is a measure of the accuracy of the matcher against all compliant template generators. Accuracy is measured by concatenating all comparison scores involving the matcher together and computing FMR and FNMR using Equations 2 and 1. This roughly simulates performance for a biometric system that employs one matcher and templates created by several template generators.



Figure 1: MINEX III Interoperability Test Setup

2.3 Interoperability

Interoperability is tested in a manner similar to *Scenario 1* from the [MINEX Evaluation Report \[5\]](#) (see Figure 1). An enrolment template is prepared using submission X. Submission Y is used to prepare the authentication template and perform the match. The authentication template is always prepared by the same submission used to compare the templates. However, enrolment templates need not originate from the same submission. When they do, we refer to it as “native” mode.

2.4 Uncertainty Estimation

Some figures in this report include boxplots that convey the uncertainty associated with a statistic. The boxplots are intended to show the expected variation in the observed value if one assumes repeated iid sampling from the same population. They are not intended to reflect how the statistic might change over different test data or even different sampling strategies over the same data.

Estimates of uncertainty are computed using the Wilson Score method [8] which overcomes certain problems associated with applying the Central Limit Theorem to a discretized estimator. We make several simplifying assumptions when applying the method to biometric identification. Most notably, separate searches against the same enrollment database are treated as independent samples, yet we know positive correlations exist due to Doddingtons Zoo [2]. We also report estimates of the variability of FNIR at a fixed FPIR when in fact it is the decision threshold that is fixed. Uncertainty with respect to what decision threshold corresponds to the targeted FPIR results in increased uncertainty about the true value of FNIR. However, our estimates of FPIR are fairly tight due to the large number of non-mated searches performed, so they are not expected to have a large impact on the estimates.

3 Results

This section details the performance of matcher nbisminexiii+5001 when it compares verification templates created by its own template generator to enrolment templates created by all MINEX III compliant template generators. Sections 3.1 and 3.2 present accuracy results for single finger and two finger matching respectively. Sections 3.4 and 3.5 present potentially useful statistics not directly related to the performance of the matcher.

3.1 Single Finger

Single finger comparison results show the combined results for left and right index comparisons. For reference, *NIST Special Publication 800-76-2* requires that the matcher and template generator achieve a native accuracy of $\text{FNMR}_{\text{FMR}}(0.0001) \leq 0.02$.

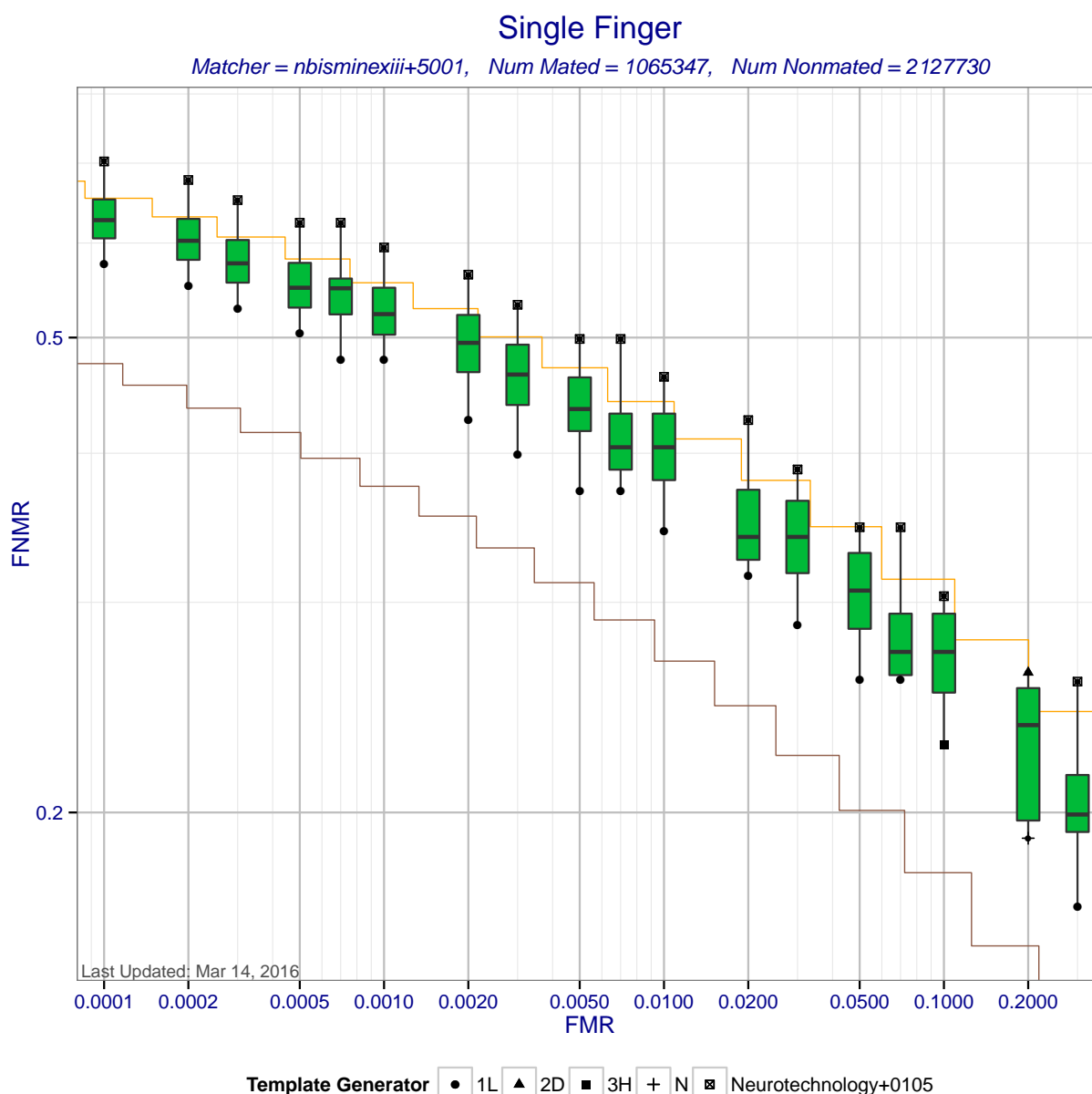


Figure 2: Single finger DET statistics for matcher nbisminexiii+5001. Each box shows the distribution of FNMRs at a fixed FMR across all MINEX III compliant template generators. The ends of the whiskers show the minimum and maximum FNMRs. The orange DET curve shows pooled performance against all template generators.

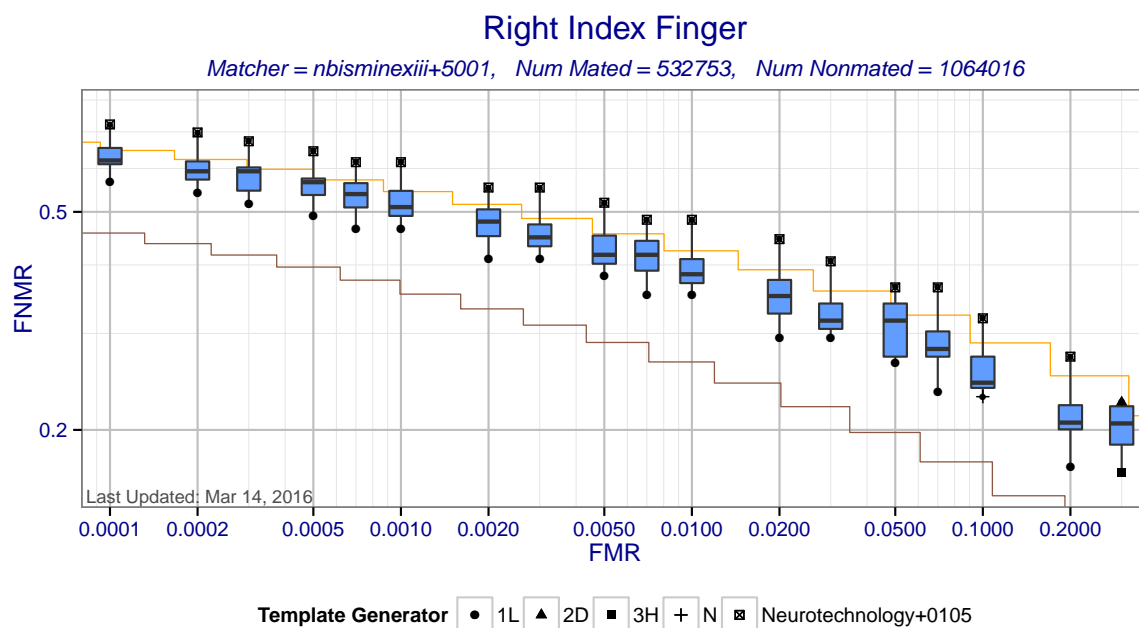


Figure 3: Right index finger DET statistics for matcher nbisminexiii+5001. Each box shows the distribution of FNMR at a fixed FMR across all MINEX III compliant template generators. The ends of the whiskers show the minimum and maximum FNMRs. The orange DET curve shows pooled performance against all template generators.

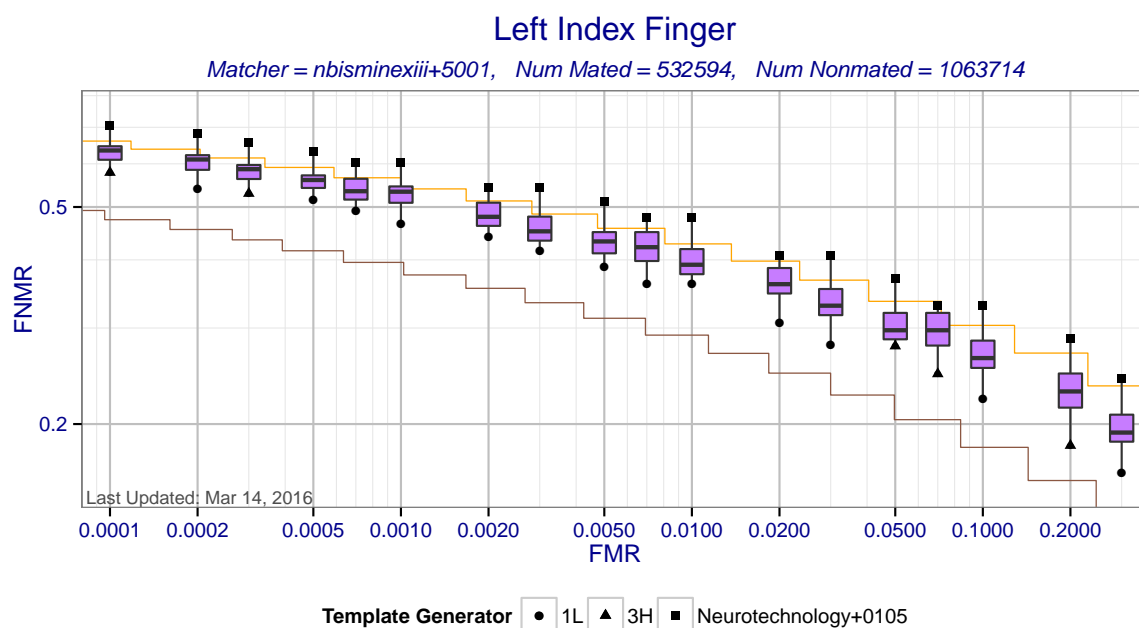


Figure 4: Left index finger DET statistics for matcher nbisminexiii+5001. Each box shows the distribution of FNMRs at a fixed FMR across all MINEX III compliant template generators. The ends of whiskers show the minimum and maximum FNMRs. The orange DET curve shows pooled performance against all template generators.

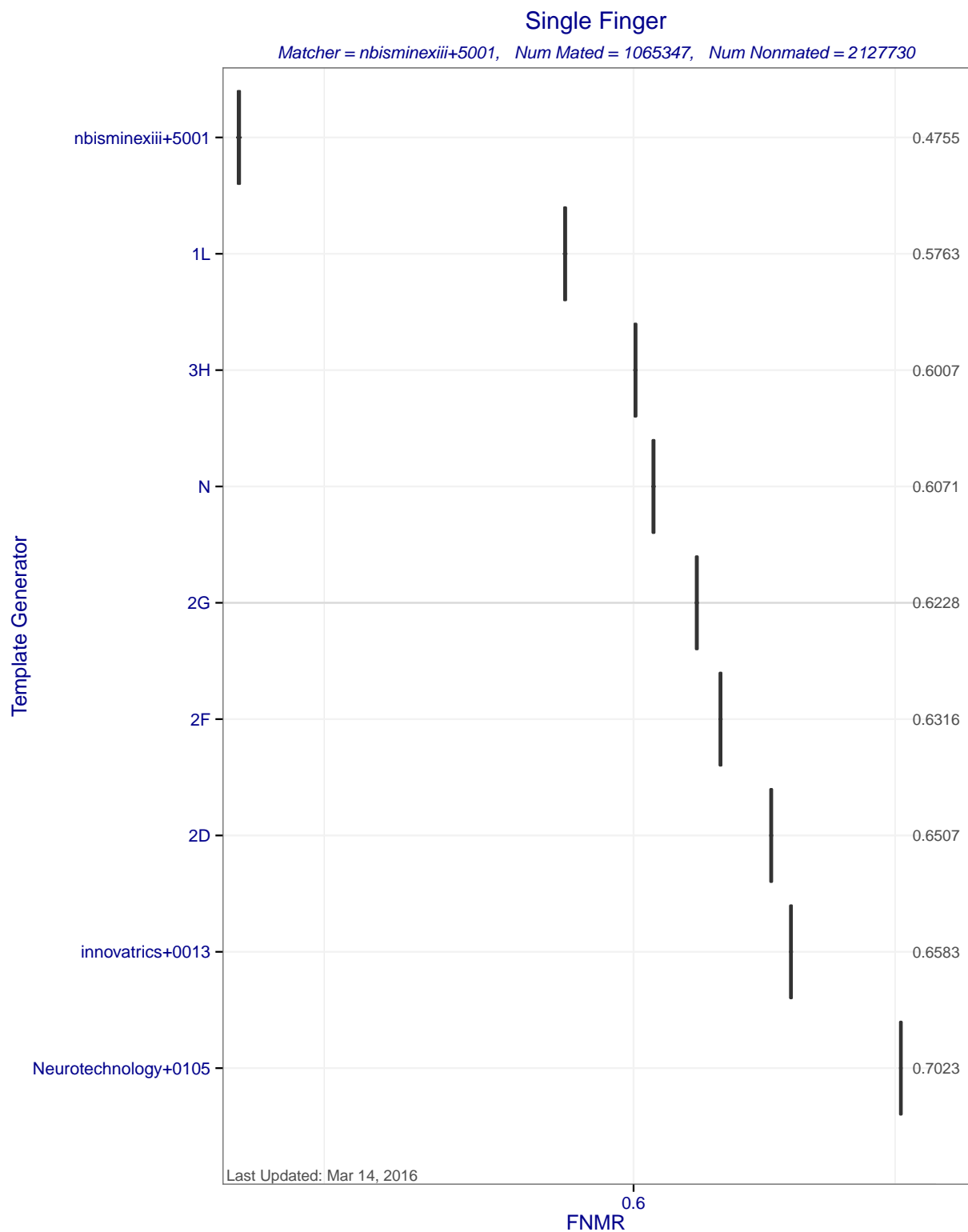


Figure 5: Single finger FNMRs at FMR = 0.0001 when matcher nbisminexiii+5001 compares templates created by different template generators. The ends of the whiskers show the minimum and maximum FNMRs. Each box represents uncertainty about the true FNMR. The box edges mark the 50% confidence intervals while the whiskers mark the 90% confidence intervals. The numbers on the right show the actual computed FNMRs.

Single Finger

Matcher = nbisminexiii+5001, Num Mated = 1065347, Num Nonmated = 2127730

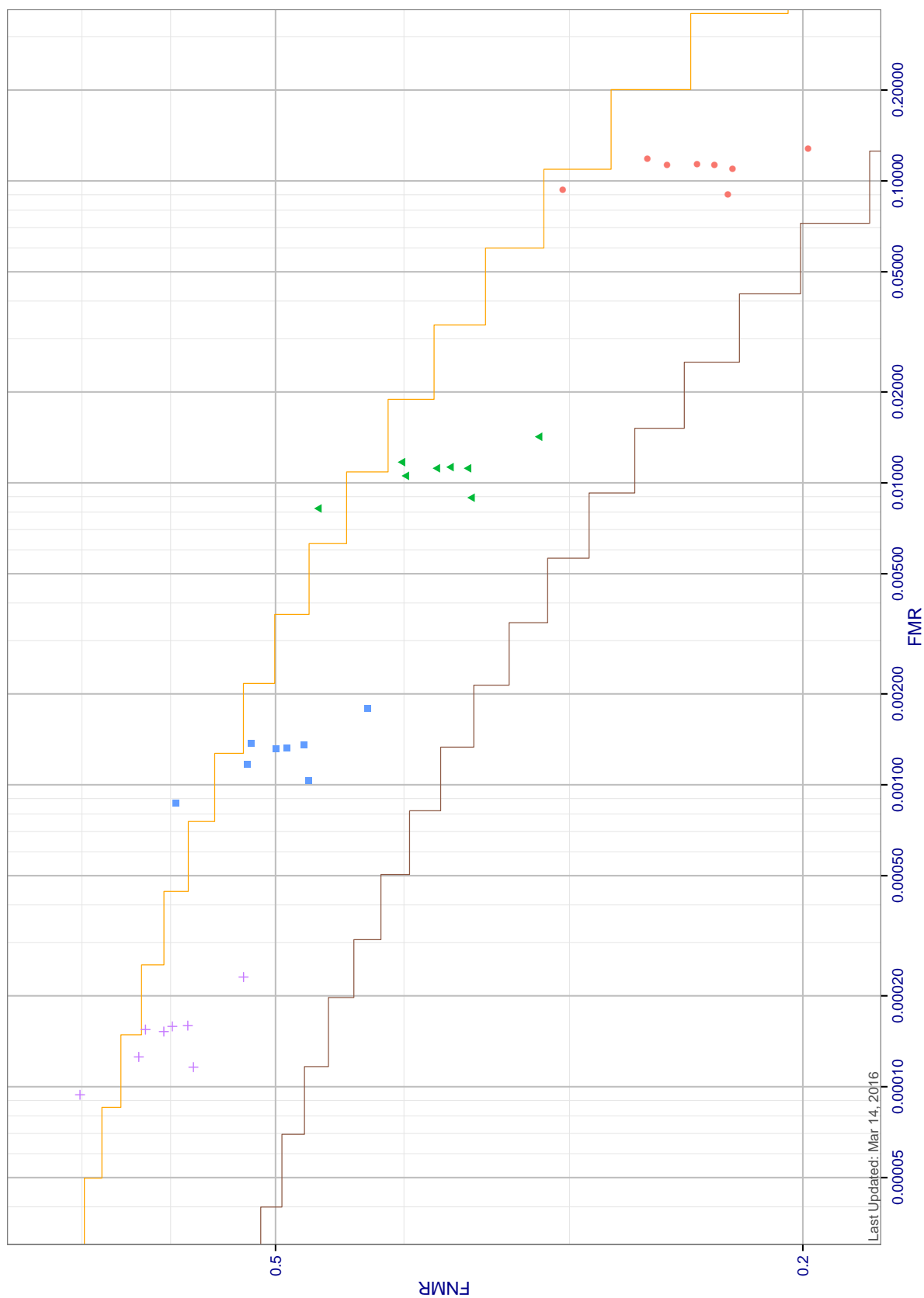


Figure 6: Single finger DET accuracy for matcher nbisminexiii+5001. Each cluster of points represents the variation in FMR and FNMR across MINEX III compliant template generators at a fixed decision threshold. Each point corresponds to an (FMR, FNMR) pair for a specific template generator at a particular decision threshold. Four clusters are produced corresponding to four decision thresholds which produce pooled FMRs of 10^{-1} , 10^{-2} , 10^{-3} , and 10^{-4} . The orange DET curve shows pooled performance against all template generators.

3.2 Two Finger

This section presents accuracy when matcher nbisminexiii+5001 compares templates created by all MINEX III compliant template generators. Two-finger fusion is achieved by averaging the scores for left and right index fingers for each person. *NIST Special Publication 800-76-2* requires the matcher to achieve an accuracy of $\text{FNMR}_{\text{FMR}(0.01)} \leq 0.01$ for all MINEX III compliant template generators.

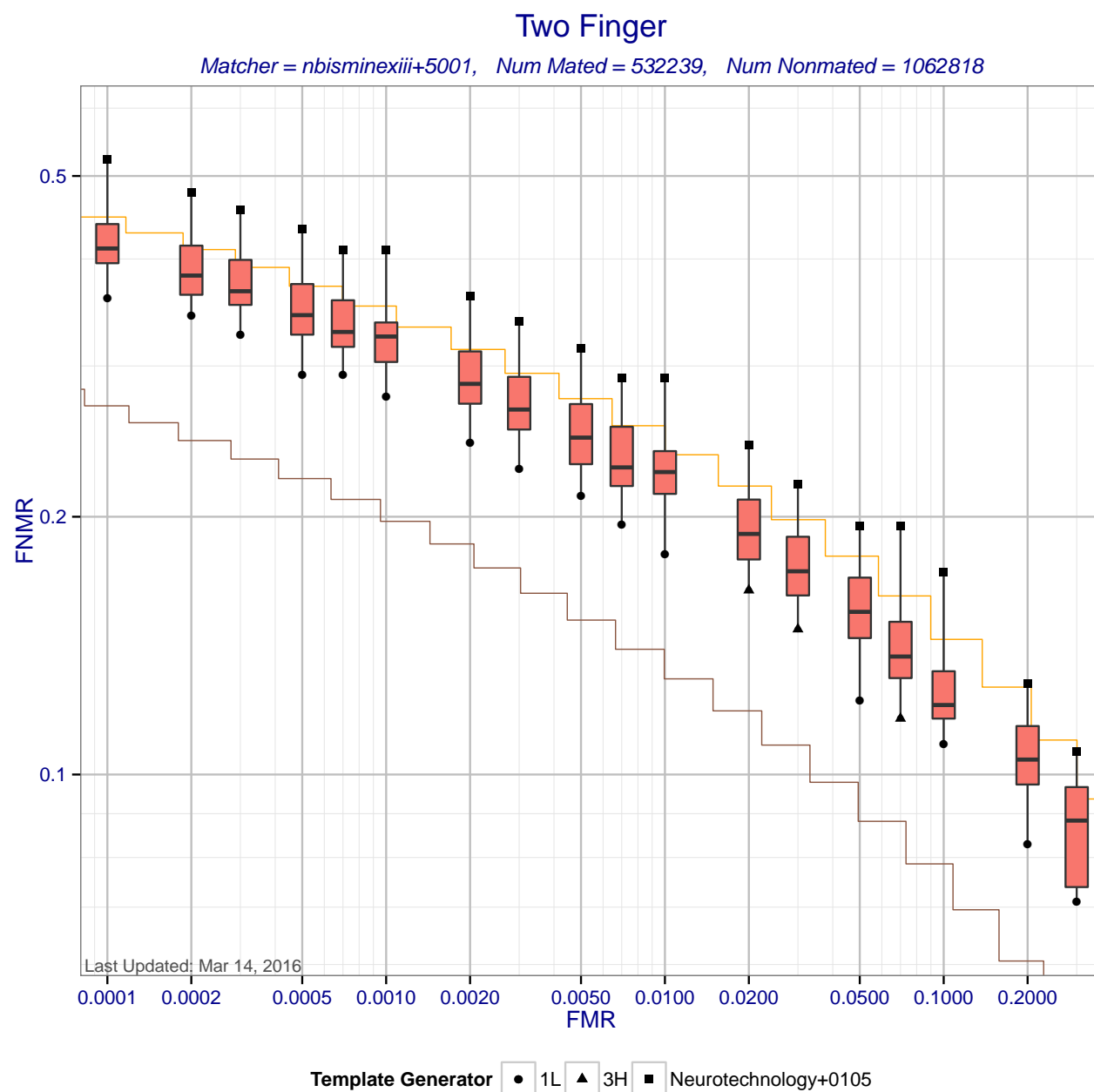


Figure 7: Two finger DET statistics for matcher nbisminexiii+5001. Each box shows the distribution of FNMRs at a fixed FMR across all MINEX III compliant template generators. The whisker ends show the minimum and maximum FNMRs. The orange DET curve shows pooled performance against all template generators. Score-level fusion is achieved by averaging the scores for left and right index fingers.

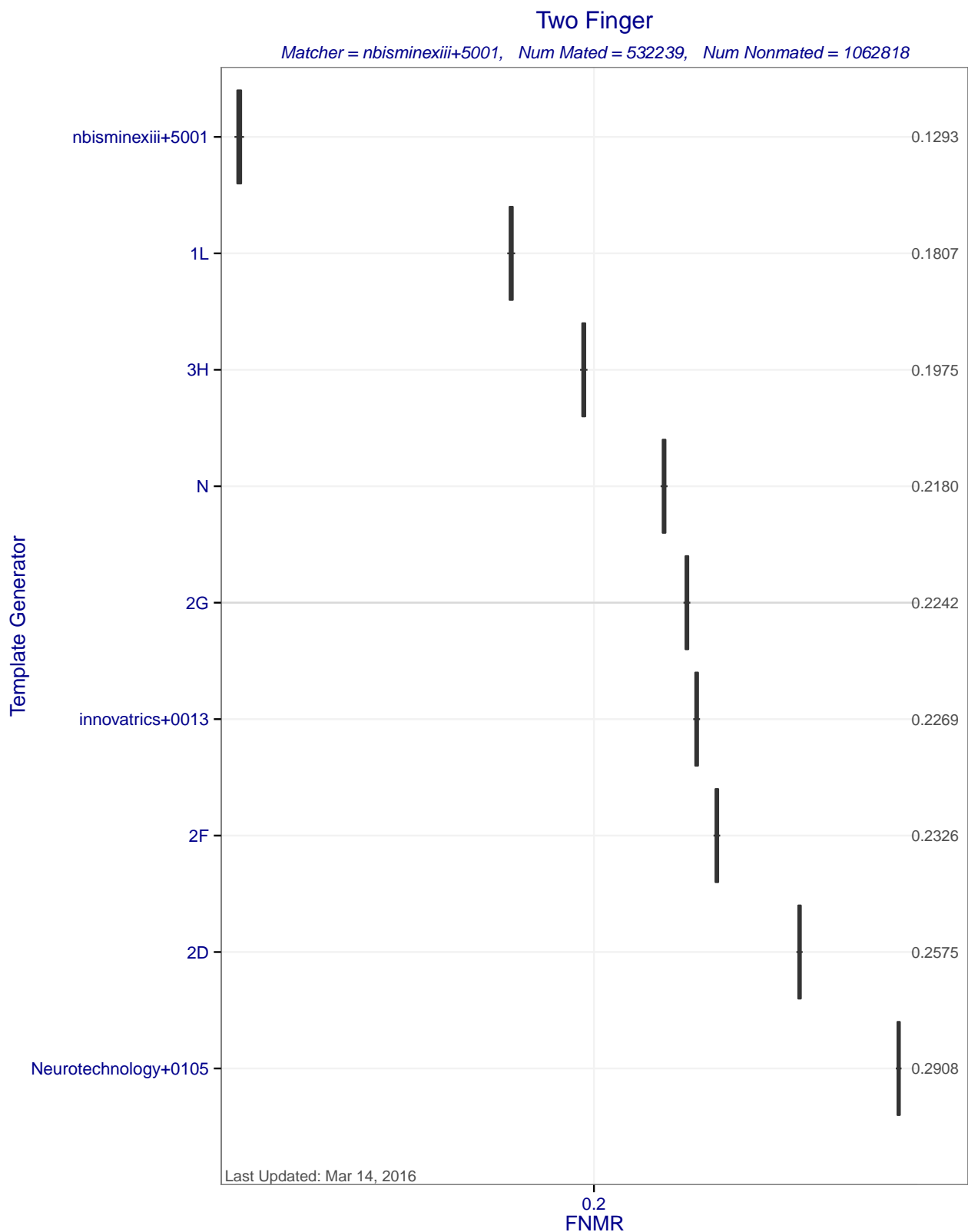


Figure 8: Two finger FNMR at FMR=0.01 when matcher nbisminexiii+5001 compares templates created by different template generators. Each box represents uncertainty about the true FNMR. The box edges mark the 50% confidence intervals while the whiskers mark the 90% confidence intervals. The numbers on the right show the actual computed FNMRs. Score-level fusion is achieved by averaging the scores for left and right index fingers.

Two Finger

Matcher = nbisminexiii+5001, Num Mated = 532239, Num Nonmated = 1062818

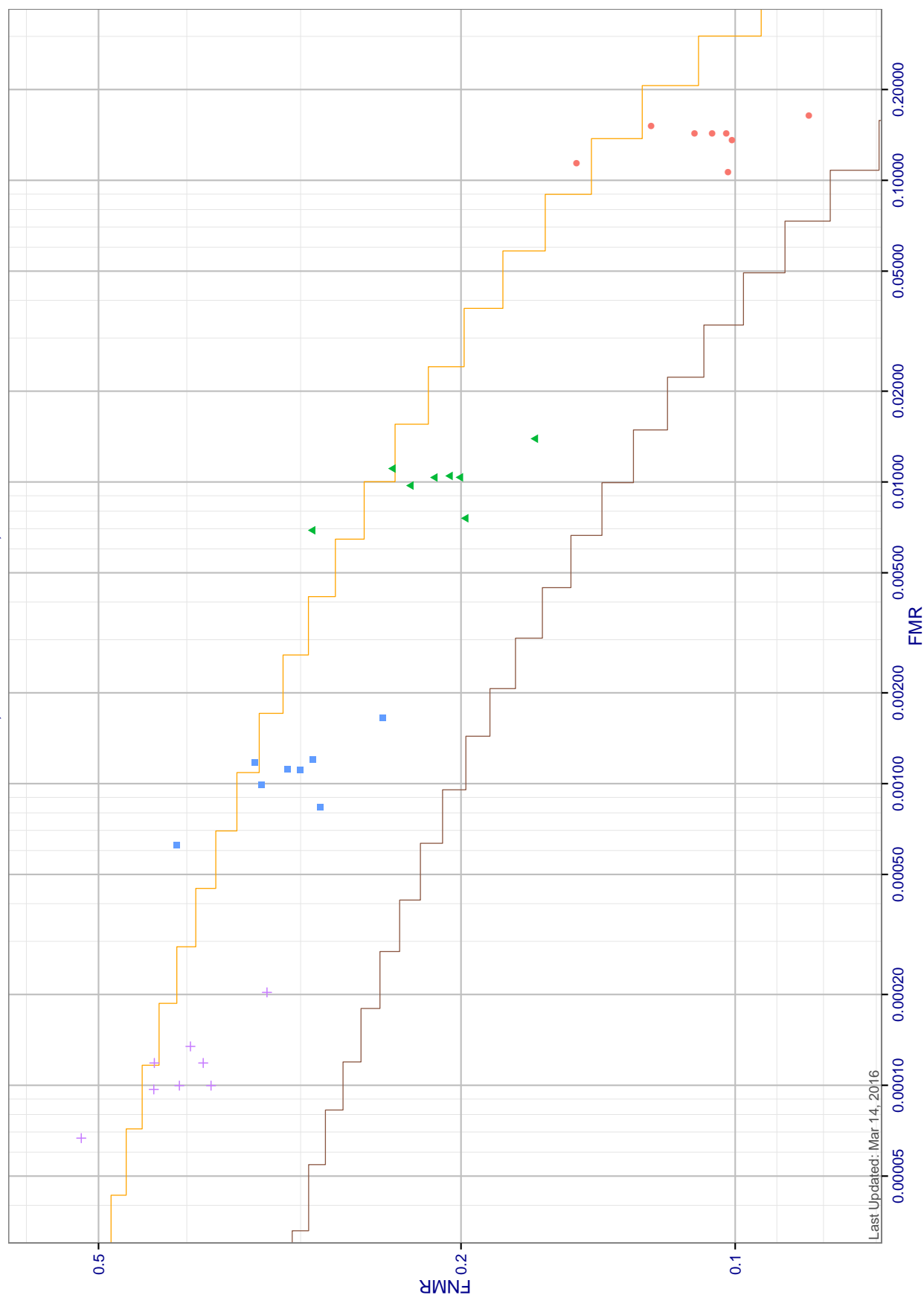


Figure 9: Two finger DET accuracy for matcher nbisminexiii+5001. Each cluster of points represents the variation in FMR and FNMR across MINEX III compliant template generators at a fixed decision threshold. Each point corresponds to an (FMR, FNMR) pair for a specific template generator at a particular decision threshold. Four clusters are produced corresponding to four decision thresholds which produce pooled FMRs of 10^{-1} , 10^{-2} , 10^{-3} , and 10^{-4} . The orange DET curve shows pooled performance against all template generators. Score-level fusion is achieved by averaging the scores for left and right index fingers.

3.3 Match Times

To achieve PIV compliance, the matcher must average no more than 10 milliseconds (0.01 seconds) per comparison. Speeds are timed on a machine with an Intel Xeon E5-2680 CPU.

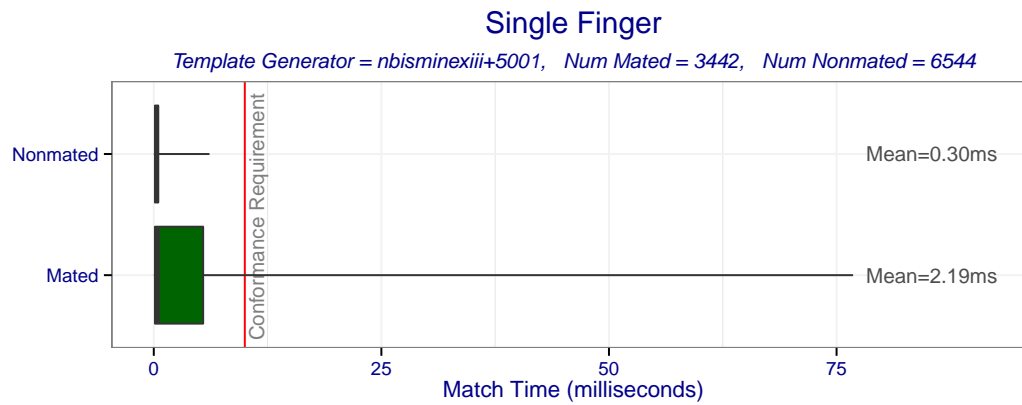


Figure 10: Boxplot of match times for single finger comparisons. The box edges mark the 10th and 90th percentiles while the whiskers mark the maximum and minimum comparison times.

3.4 Threshold Statistics

Results in this section are computed by concatenating comparison scores for matcher nbisminexiii+5001 across all MINEX III compliant template generators.

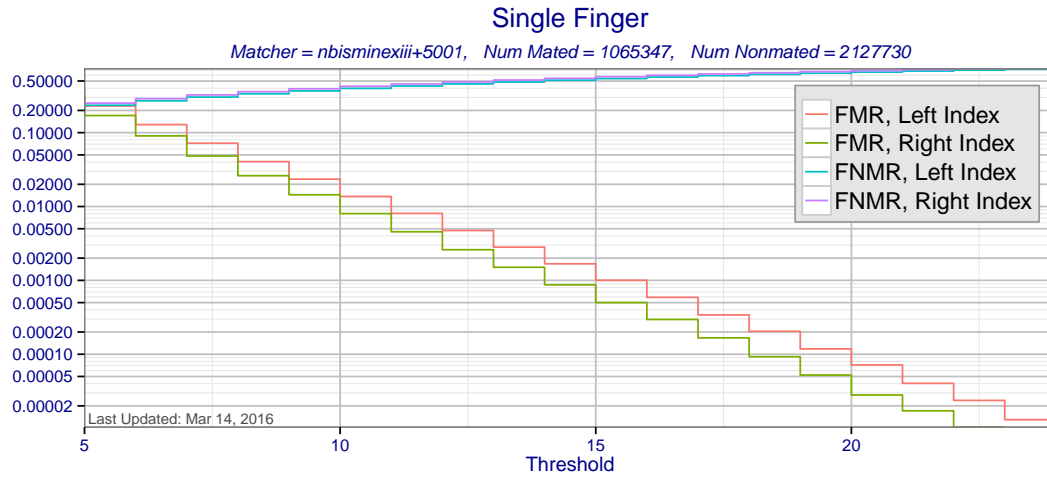


Figure 11: Single finger FMR and FNMR as a function of score threshold for matcher nbisminexiii+5001 using templates created by all MINEX III compliant template generators. Separate curves are presented for left and right index fingers.

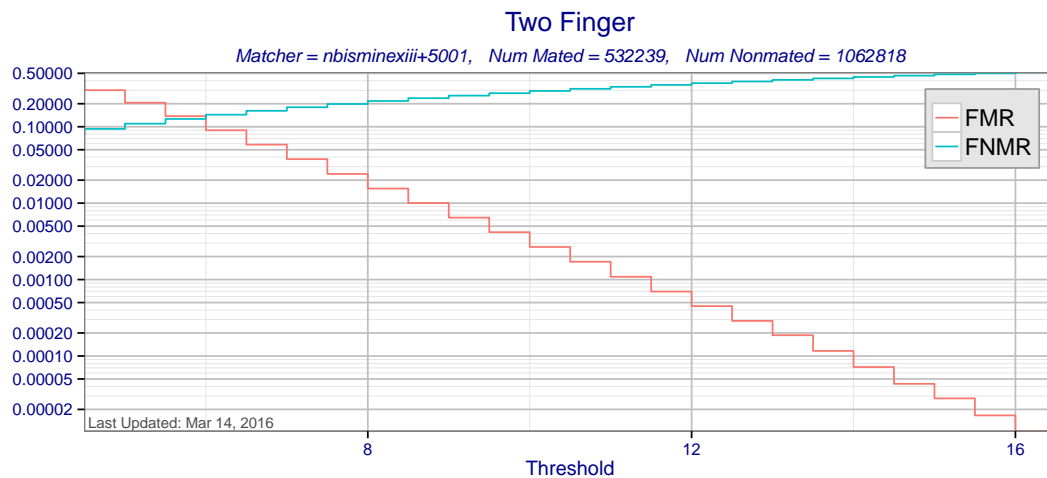


Figure 12: Two finger FMR and FNMR as a function of score threshold for matcher nbisminexiii+5001 using templates created by all MINEX III compliant template generators. Score-level fusion is achieved by averaging scores for the left and right index fingers.

	FMR=0.1	FMR=0.01	FMR=0.001	FMR=0.0001
Right index finger	6	10	14.0	18
Left index finger	7	11	16.0	20
Single finger	7	11	15.0	19
Two finger	6	9	11.5	14

Table 1: Threshold calibration table. The cells show the thresholds corresponding to the FMR indicated by the column header.

3.5 Q-Q Plot

The Q-Q plot compares two probability distributions. It plots the quantile of one distribution as a function of the other. If the curve follows the $y = x$ line, then the distributions are identical. If the FMR curve is above the $y = x$ line, then the left index finger tends to produce lower non-mated scores than the right index finger. If the FNMR curve is above the $y = x$ line, then the left index finger tends to produce lower mated scores than the right index finger. A jagged and/or truncated curve is indicative of discretized scores.

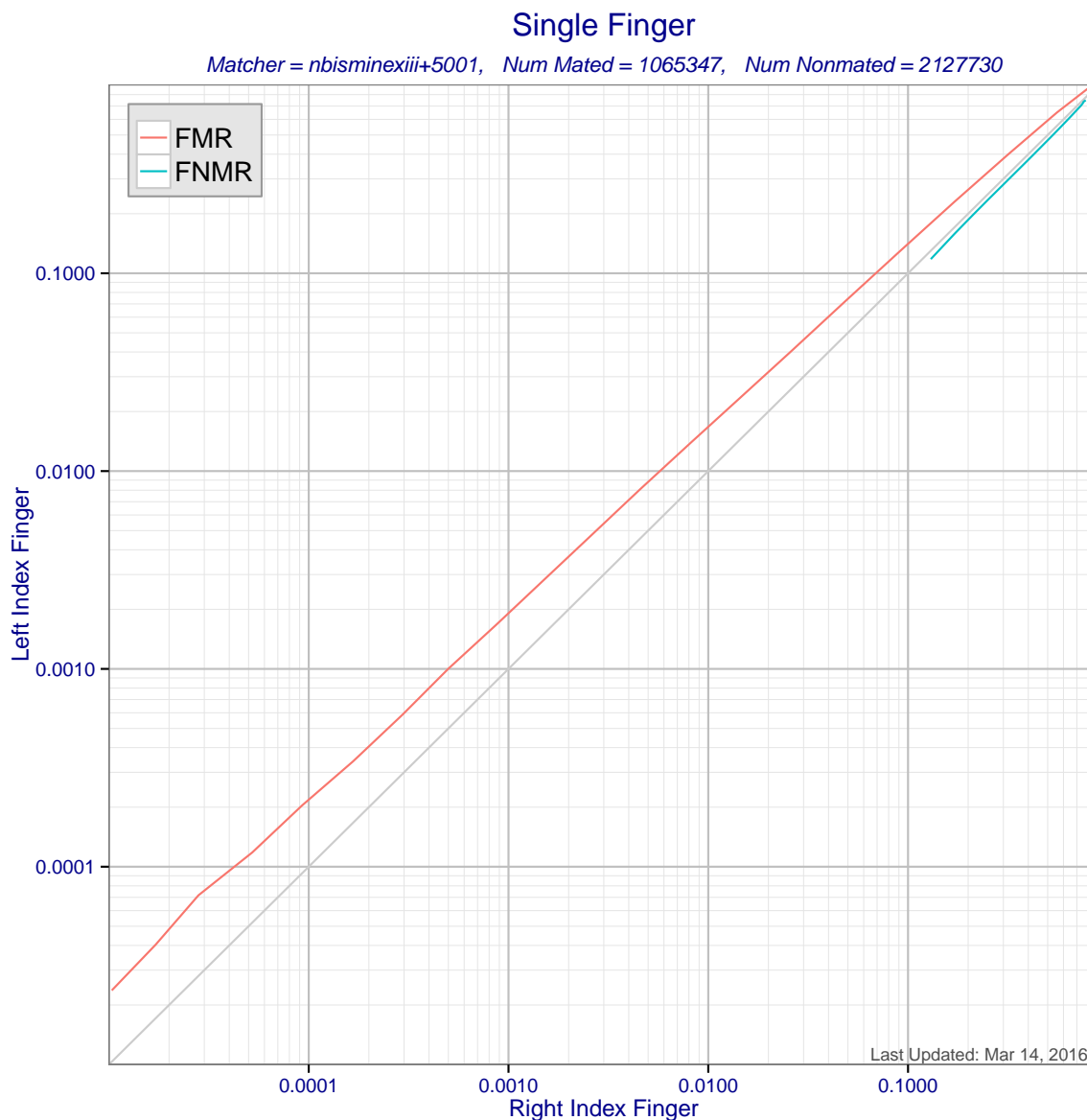


Figure 13: Q-Q plot comparing score distributions for left and right index fingers.

3.6 Effect of Minutia Count on Accuracy

This section shows how the number of minutia found in the samples affects recognition accuracy. To be robust to spoofing and other active attacks, the algorithm should not allow FMR to rise sharply as the number of available minutia decreases. Nor should it allow FMR to rise sharply as the number of detected minutia increases.

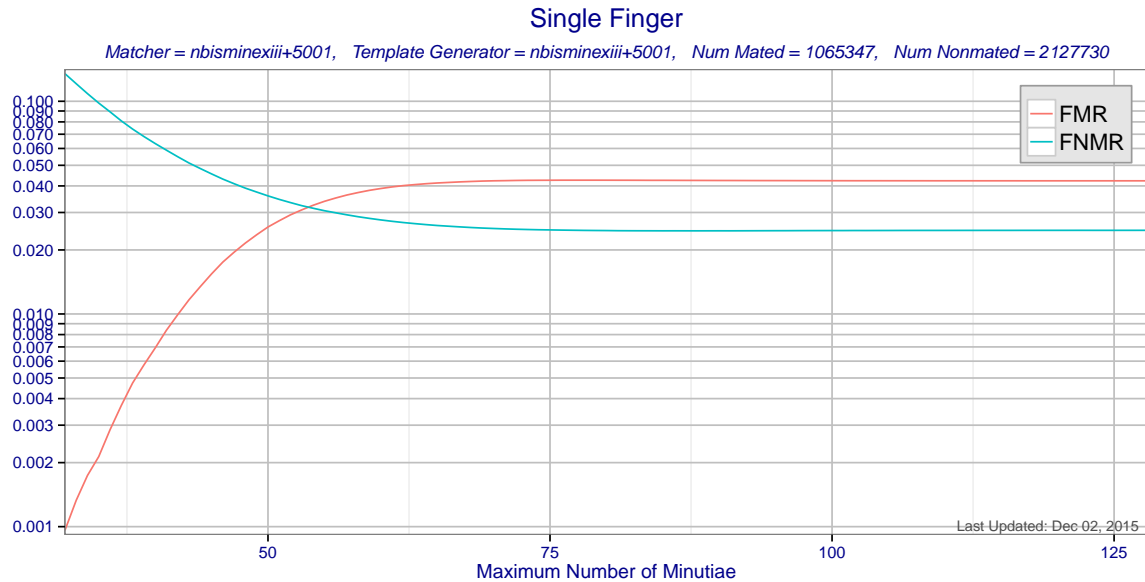


Figure 14: FNMR and FMR as a function of the number of minutia found by the template generator. The vertical axis defines a filter criterion such that FNMR and FMR are computed over only those comparisons where at least one of the compared templates has no more than the specified number of minutia. The threshold is fixed separately for FNMR and FMR to elicit an error rate of approximately 0.01 over unfiltered comparisons.

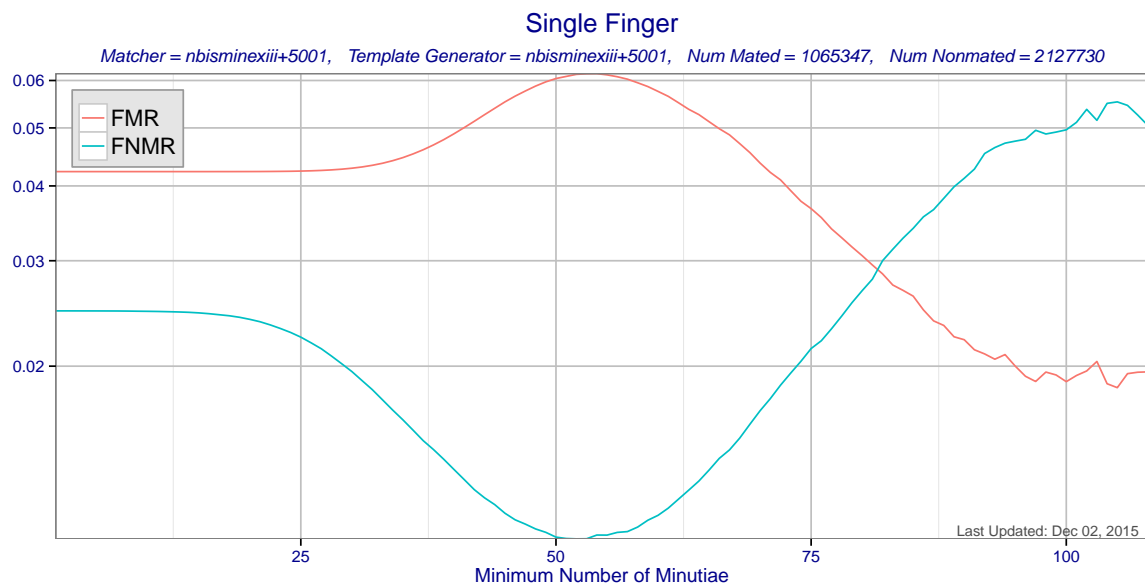


Figure 15: FNMR and FMR as a function of the number of minutia found by the template generator. The vertical axis defines a filter criterion such that FNMR and FMR are computed over only those comparisons where at least one of the compared templates has at least the indicated number of minutia. The threshold is fixed separately for FNMR and FMR to elicit an error rate of approximately 0.01 over unfiltered comparisons.

3.7 Comparison to Ongoing MINEX

MINEX III uses a larger set of comparisons than the older ongoing MINEX evaluation. Although this is generally good because it provides more accurate estimates of performance in MINEX III, it makes it more difficult to directly compare the results in this report to the archived ones from ongoing MINEX. The tables below report DET accuracy at fixed FMRs computed over the same set of comparisons that were used in ongoing MINEX. Ongoing MINEX reported FNMR at FMR = 0.01 for two-finger, and FNMR at FMR = 0.0001 for native single-finger.

Table 2: *Single finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.*

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.361 \pm 0.002	0.471 \pm 0.002	0.595 \pm 0.002
2D	0.426 \pm 0.002	0.547 \pm 0.002	0.652 \pm 0.002
2F	0.395 \pm 0.002	0.547 \pm 0.002	0.626 \pm 0.002
2G	0.383 \pm 0.002	0.536 \pm 0.002	0.639 \pm 0.002
3H	0.365 \pm 0.002	0.483 \pm 0.002	0.612 \pm 0.002
innovatrics+0013	0.412 \pm 0.002	0.541 \pm 0.002	0.651 \pm 0.002
N	0.375 \pm 0.002	0.524 \pm 0.002	0.627 \pm 0.002
Neurotechnology+0105	0.478 \pm 0.002	0.610 \pm 0.002	0.716 \pm 0.001
nbisminexiii+5001	0.284 \pm 0.001	0.395 \pm 0.002	0.496 \pm 0.002

Table 3: *Two finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.*

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.194 \pm 0.002	0.278 \pm 0.002	0.364 \pm 0.002
2D	0.263 \pm 0.002	0.367 \pm 0.002	0.447 \pm 0.002
2F	0.231 \pm 0.002	0.331 \pm 0.002	0.411 \pm 0.002
2G	0.219 \pm 0.002	0.318 \pm 0.002	0.436 \pm 0.002
3H	0.207 \pm 0.002	0.297 \pm 0.002	0.391 \pm 0.002
innovatrics+0013	0.240 \pm 0.002	0.327 \pm 0.002	0.435 \pm 0.002
N	0.217 \pm 0.002	0.313 \pm 0.002	0.408 \pm 0.002
Neurotechnology+0105	0.281 \pm 0.002	0.405 \pm 0.002	0.519 \pm 0.002
nbisminexiii+5001	0.143 \pm 0.002	0.216 \pm 0.002	0.279 \pm 0.002

4 Performance Tables

The following tables present accuracy numbers, including estimates of uncertainty in the form of 90% confidence bounds. These tables are provided because most of the figures in the main body of this report do not present numerical results.

Table 4: *Single finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.*

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.3442 ± 0.0008	0.4787 ± 0.0008	0.5763 ± 0.0008
2D	0.4322 ± 0.0008	0.5493 ± 0.0008	0.6507 ± 0.0008
2F	0.4092 ± 0.0008	0.5278 ± 0.0008	0.6316 ± 0.0008
2G	0.4001 ± 0.0008	0.5186 ± 0.0008	0.6228 ± 0.0008
3H	0.3552 ± 0.0008	0.4993 ± 0.0008	0.6007 ± 0.0008
innovatrics+0013	0.4316 ± 0.0008	0.5540 ± 0.0008	0.6583 ± 0.0008
N	0.3883 ± 0.0008	0.5040 ± 0.0008	0.6071 ± 0.0008
Neurotechnology+0105	0.4635 ± 0.0008	0.5950 ± 0.0008	0.7023 ± 0.0007
nbisminexiii+5001	0.2678 ± 0.0007	0.3753 ± 0.0008	0.4755 ± 0.0008

Table 5: *Right index finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.*

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.353 ± 0.001	0.465 ± 0.001	0.566 ± 0.001
2D	0.409 ± 0.001	0.562 ± 0.001	0.665 ± 0.001
2F	0.389 ± 0.001	0.514 ± 0.001	0.622 ± 0.001
2G	0.381 ± 0.001	0.506 ± 0.001	0.615 ± 0.001
3H	0.373 ± 0.001	0.493 ± 0.001	0.599 ± 0.001
innovatrics+0013	0.412 ± 0.001	0.541 ± 0.001	0.651 ± 0.001
N	0.365 ± 0.001	0.487 ± 0.001	0.619 ± 0.001
Neurotechnology+0105	0.484 ± 0.001	0.616 ± 0.001	0.722 ± 0.001
nbisminexiii+5001	0.2661 ± 0.0010	0.354 ± 0.001	0.457 ± 0.001

Table 6: Left index finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.362 ± 0.001	0.466 ± 0.001	0.585 ± 0.001
2D	0.423 ± 0.001	0.563 ± 0.001	0.659 ± 0.001
2F	0.397 ± 0.001	0.540 ± 0.001	0.640 ± 0.001
2G	0.387 ± 0.001	0.529 ± 0.001	0.630 ± 0.001
3H	0.366 ± 0.001	0.478 ± 0.001	0.578 ± 0.001
innovatrics+0013	0.417 ± 0.001	0.537 ± 0.001	0.642 ± 0.001
N	0.380 ± 0.001	0.520 ± 0.001	0.619 ± 0.001
Neurotechnology+0105	0.478 ± 0.001	0.603 ± 0.001	0.706 ± 0.001
nbisminexiii+5001	0.291 ± 0.001	0.396 ± 0.001	0.474 ± 0.001

Table 7: *Two finger FNMRs at various FMRs when matcher nbisminexiii+5001 compares templates created by its template generator and PIV-compliant template generators.*

Enroller	FNMR @ FMR=0.01	FNMR @ FMR=0.001	FNMR @ FMR=0.0001
1L	0.1807 ± 0.0009	0.276 ± 0.001	0.360 ± 0.001
2D	0.2575 ± 0.0010	0.356 ± 0.001	0.453 ± 0.001
2F	0.2326 ± 0.0010	0.330 ± 0.001	0.408 ± 0.001
2G	0.2242 ± 0.0009	0.320 ± 0.001	0.415 ± 0.001
3H	0.1975 ± 0.0009	0.286 ± 0.001	0.376 ± 0.001
innovatrics+0013	0.2269 ± 0.0009	0.331 ± 0.001	0.435 ± 0.001
N	0.2180 ± 0.0009	0.309 ± 0.001	0.402 ± 0.001
Neurotechnology+0105	0.291 ± 0.001	0.410 ± 0.001	0.522 ± 0.001
nbisminexiii+5001	0.1293 ± 0.0008	0.1976 ± 0.0009	0.269 ± 0.001

5 References

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