

Examining the effectiveness of stock market indices in predicting the S&P 500

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# CROSS-EVALUATION OF STOCK MARKET INDICES AND THEIR PREDICTIVE EFFICIENCY ON THE S&P 500

Information plays a central role in modern finance. Investors are exposed to an ever-increasing amount of new facts, data and statistics every minute of the day. Stock market performance has historically worked as our strongest indicator of the economic conditions of a country. Indices such as the S&P 500 which tracks the stocks of 500 large-cap U.S. companies, are meant to represent the stock market's performance by reporting the risks and returns of the biggest companies. Investors use it as the benchmark of the overall market, to which all other investments are compared. Predicting stock performance through their returns, given their dynamic nature of the market can be incredibly worthwhile. However, the predictability of stock returns, requires forecasts on the basis of large sets of conditioning information and is still an .

This paper tries to establish an efficient model in predicting the S&P 500 returns, using information from 3 major stock exchanges- London, Tokyo and Frankfurt – through indices FTSE 100, Nikkei 225 and GDAXI respectively. I will try to do so by fitting the value to a multiple regression model as expressed by the equation:

$$y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_p x_{pi} + \varepsilon_i.$$

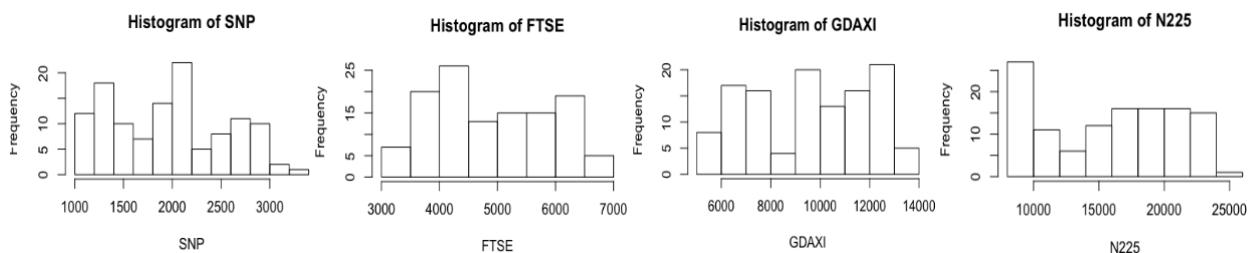
$$\text{return.SNP} = \beta_0 + \beta_1 * \text{return.FTSE} + \beta_2 * \text{return.GDAXI} + \beta_3 * \text{return.N225} + \varepsilon$$

## Data Collection

Historical data for stock exchange were fetched from the publicly available stock exchange database of Yahoo finance<sup>1</sup>. The collected data consists of monthly stock market prices of the S&P 500, the FTSE 100, the Nikkei 225 and the GDAXI index over a time of 10 years from 1st January 2010 to 1 January 2020.

## Data Analysis

Histograms of the prices of each Index can be found below:



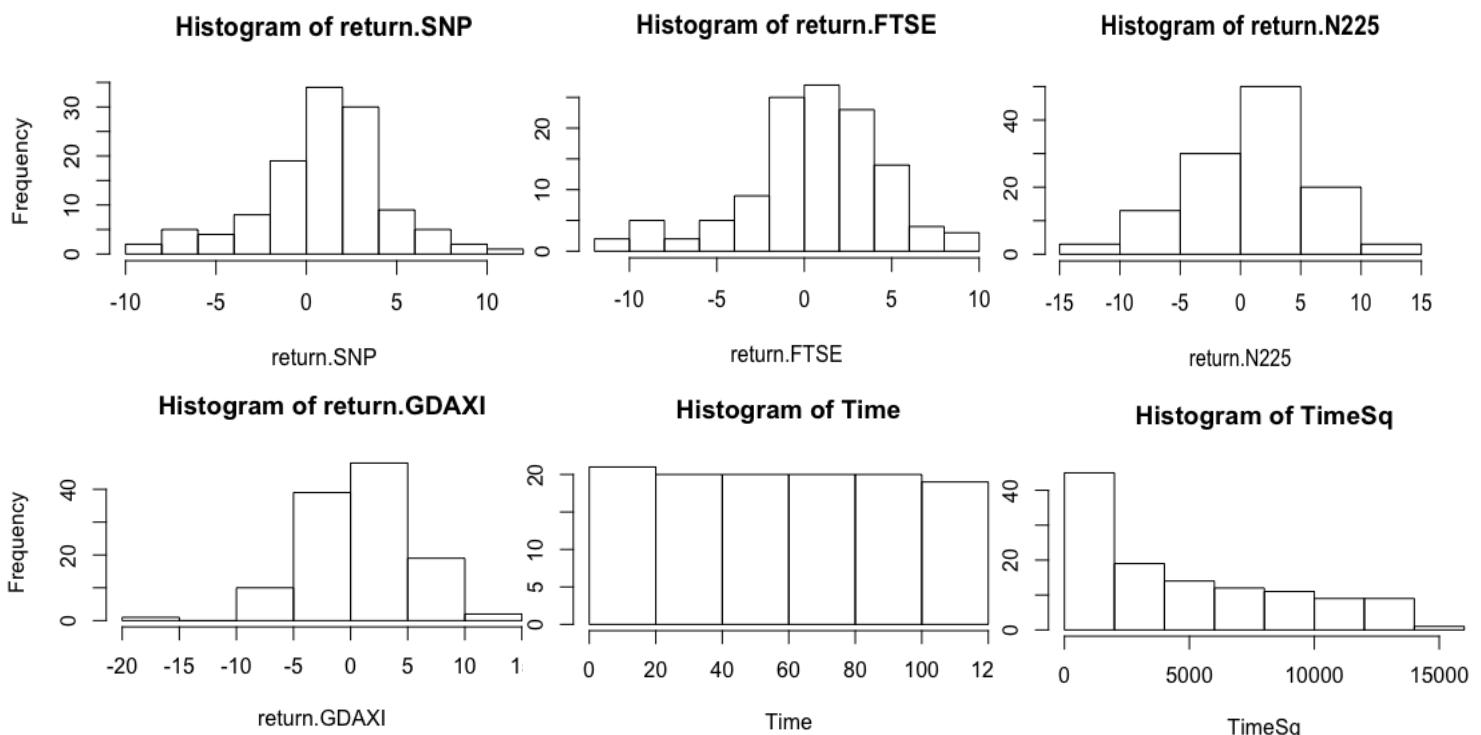
<sup>1</sup> <https://finance.yahoo.com/quote/%5EGSPC/history?period1=1262304000&period2=1577836800&interval=1mo&filter=history&frequency=1mo>

However, the prices themselves are not of interest to an investor. Instead I decided to explore the returns to each index

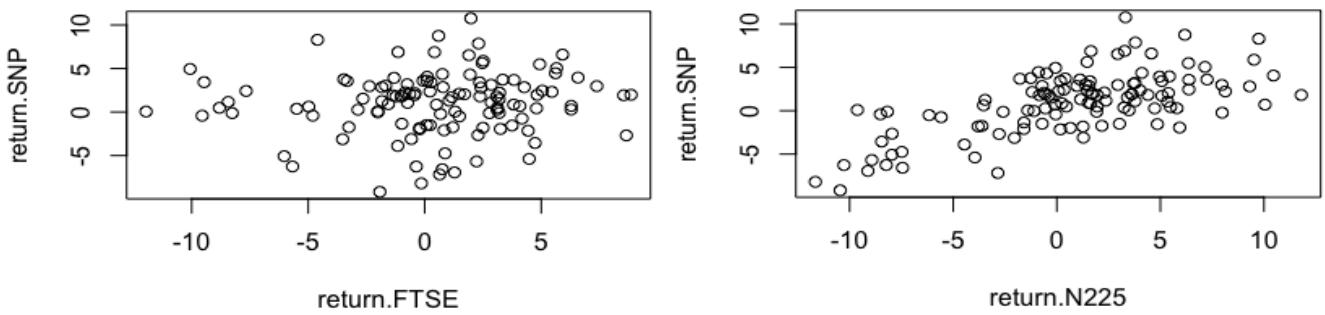
I calculated those using the formula

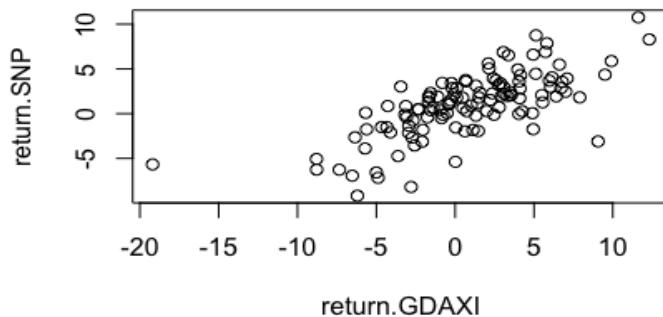
$$\text{return} = (\text{price} - \text{lag(price)})/\text{lag(price)} * 100$$

The histogram of the returns of each stock index can be found below



To explore the relationship of the predictors to the target variable on an preliminary level, I used scatter plot of the response variable versus each predictor. These describe the independent relationships however and not how the predictors will work together in our model.





Interestingly, it seems that the SNP 500 is more closely related to Japan's Nikkei and Germany's GDAXI, than the UK's FTSE 100 .

Below are the results of a regression of S&P returns on the predictors:

```
lm(formula = return.SNP ~ return.FTSE + return.GDAXI + return.N225)
```

**Residuals:**

| Min    | 1Q     | Median | 3Q    | Max   |
|--------|--------|--------|-------|-------|
| -7.375 | -1.448 | -0.096 | 1.494 | 5.082 |

**Coefficients:**

|              | Estimate | Std. Error | t value | Pr(> t )     |
|--------------|----------|------------|---------|--------------|
| (Intercept)  | 0.48480  | 0.22415    | 2.163   | 0.032626 *   |
| return.FTSE  | -0.01634 | 0.05533    | -0.295  | 0.768323     |
| return.GDAXI | 0.38310  | 0.06381    | 6.004   | 2.3e-08 ***  |
| return.N225  | 0.23733  | 0.05995    | 3.959   | 0.000131 *** |

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.391 on 115 degrees of freedom

(1 observation deleted due to missingness)

Multiple R-squared: 0.5677, Adjusted R-squared: 0.5564

F-statistic: 50.33 on 3 and 115 DF, p-value: < 2.2e-16

### Regression Equation

$$\text{return.SNP} = .4848 - .01634 * \text{return.FTSE} + .38310 * \text{return.GDAXI} + .23733 * \text{return.N225}$$

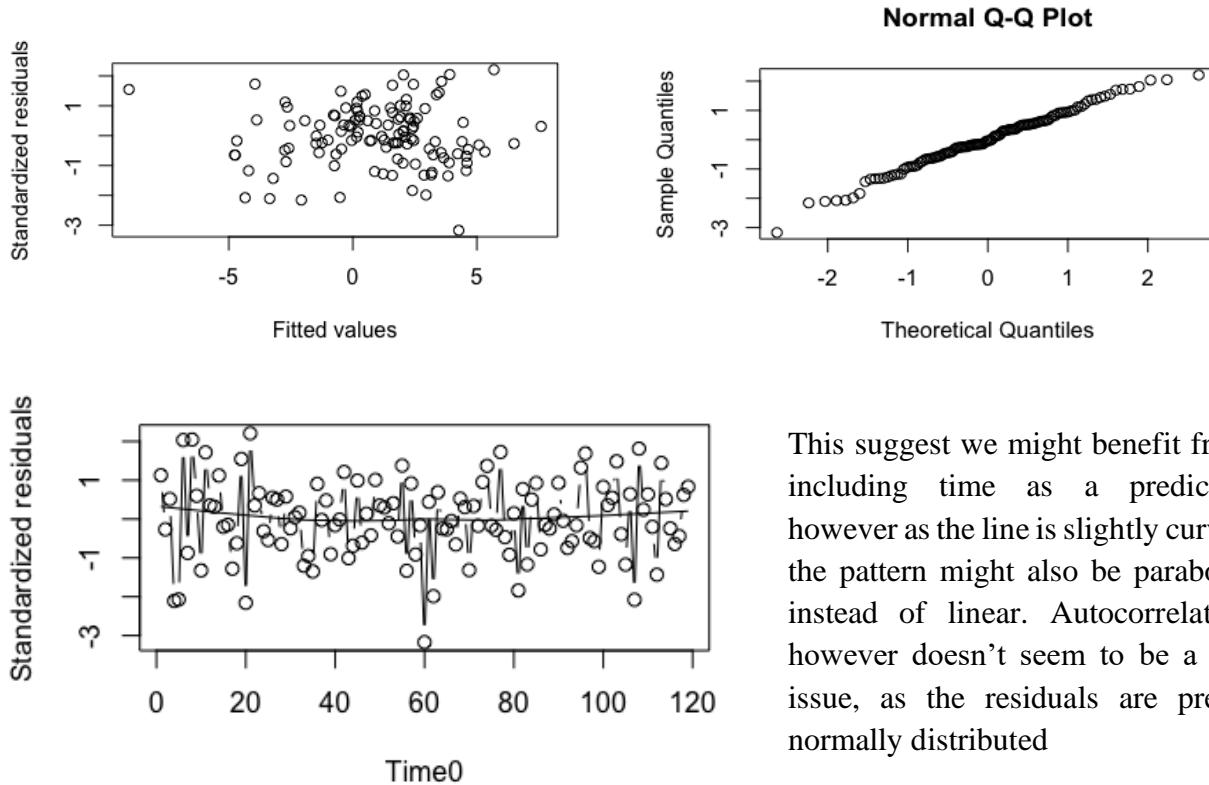
The regression is quite strong with an adjusted R-squared of .5564. Somewhat shockingly, it seems that given the other information, a FTSE's returns adds relatively little predictive power to the model, with its value actually being negative. This suggests when exploring the health of the US economy via the returns on the S&P 500, the performance of a trading partner such as the UK is not good point of comparison. Similarly, given the other values, the returns of the GDAXI and the Nikkei 225 seem to contribute insignificantly to the predictive power of the model. The coefficient for return.GDAXI shows that given the other 2 predictors are held constant, a one percentage increase in the index's return is associated with an estimated expected increase in the

S&P 500 index by .38310 percent. The coefficient for return.N225 says that given the other 2 predictors are held constant, a one-percentage increase is associated with an expected increase in the S&P score of .23733 percentage points. The value for the residual standard error implies that a rough 95% prediction interval for a the S&P 500 returns using this model is  $\pm 4.782$ .

To measure multicollinearity, I used the variance inflation factor (VIF), which assesses how much the variance of an estimated regression coefficient increases if the predictors are correlated. Each of these VIF values corresponds to an estimate of how much we think the variance of its predictor's  $\beta$  hat, the variance of its slope coefficient, has been inflated relative to what the variance would have been if our data had perfectly uncorrelated predictors. My VIF numbers are relatively close to 1, so collinearity doesn't seem to be a concern at this point.

| VIF VALUES  |              |             |
|-------------|--------------|-------------|
| return.FTSE | return.GDAXI | return.N225 |
| 1.027497    | 1.806864     | 1.840016    |

Looking now at the relevant residual plots, the standardized residual versus fitted values plot, the normal plot of the residuals and the standardized residuals versus index. The normal plot shows the some heavy-tailedness, with the right upper end of the normality plot going above the hypothetical straight and the left lower end going below it. The residuals versus fitted values plot also shows some non-constant variance, hinting at traces of a time trend.



This suggest we might benefit from including time as a predictor, however as the line is slightly curved the pattern might also be parabolic instead of linear. Autocorrelation however doesn't seem to be a big issue, as the residuals are pretty normally distributed

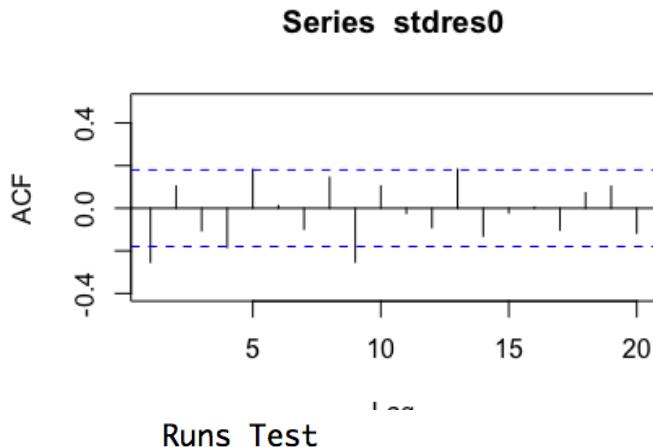
Looking at the standardized residuals graph, we can see the value at time zero seems to be an outlier. That value corresponds to January 2015. We will explore this later on.

The Durbin-Watson test verifies that preliminary evaluation with a value relatively close to 2, at 2.4887. As indicated by the high p value of 0.9963, we should not reject the no autocorrelation hypothesis.

### Durbin-Watson test

```
data: return.SNP ~ return.FTSE + return.GDAXI + return.N225
DW = 2.4887, p-value = 0.9963
alternative hypothesis: true autocorrelation is greater than 0
```

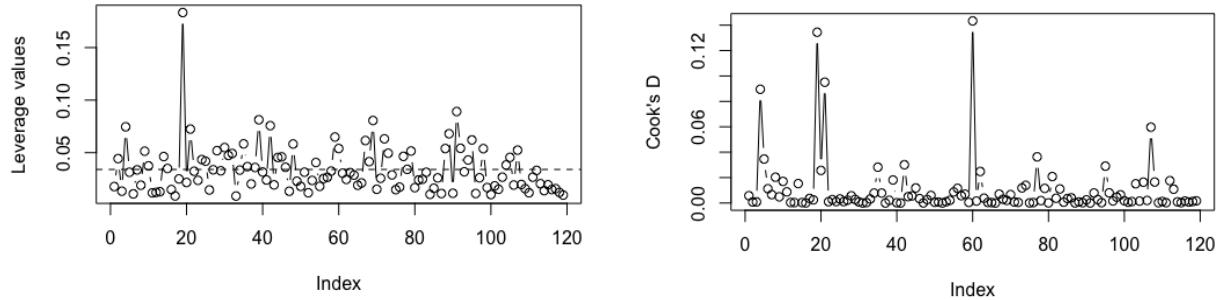
The test is contingent upon our assumptions of normality and constant variance, which do not seem to pose a big problem here. I decided however to explore the other test on the data. An ACF plot was created for the data



The ACF graph shows residuals that do not go down geometrically, therefore the DW test might not be effective. The values that are not all within the blue lines indicating to some autocorrelation, which seems to be negative. The Runs tests gives a p-value of .1125, according which the data is deemed to be random and we should not reject the no autocorrelation hypothesis.

```
data: stdres0
statistic = 1.5872, runs = 69, n1 = 57, n2 = 62, n = 119, p-value = 0.1125
alternative hypothesis: nonrandomness
```

To explore the potential existence of leverage points or influential points, I looked at Leverage values and Cook's distances; We can recognize here that a few points seem to have high leverage values and Cook's distance, especially the observation at index 20 and that at 60, therefore it is important to explore the implications of removing them on our model.



Below are the standard residuals, hat values and cook's distance for each of the data points

```
> cbind(stdres0, hatvalues(IndeicesR0), cooks.distance(IndeicesR0))
   stdres0
2  1.12869938 0.017740753 5.752313e-03
3 -0.26603235 0.044340585 8.209320e-04
4  0.51540669 0.013300133 8.951814e-04
5 -2.10816626 0.074575506 8.953750e-02
6 -2.06936718 0.031324167 3.461913e-02
7  2.03404366 0.010742001 1.123146e-02
8 -0.87328442 0.033578949 6.624486e-03
9  2.04777212 0.019008798 2.031388e-02
10 0.59953121 0.051387946 4.867839e-03
11 -1.32273427 0.037409632 1.699915e-02
12 1.71925855 0.011730452 8.771255e-03
13 0.35363577 0.012162324 3.849314e-04
14 0.31519573 0.012757850 3.209626e-04
15 1.12246062 0.046269377 1.528094e-02
16 -0.20003104 0.034933926 3.620972e-04
17 -0.14727770 0.014841453 2.169289e-05
18 -1.28012049 0.008767917 3.623788e-03
19 -0.62318007 0.025010344 2.490501e-03
20 1.54762042 0.183364106 1.344481e-01
21 -2.15732232 0.021585019 2.566844e-02
22 2.20721755 0.072430350 9.510522e-02
23 0.35008650 0.032298837 1.022672e-03
24 0.66501113 0.023623702 2.675029e-03
25 -0.31192059 0.043408421 1.103762e-03
26 -0.53938205 0.041860585 3.177676e-03
27 0.55128964 0.014473209 1.115825e-03
28 0.50283377 0.033700216 2.204498e-03
29 -0.64538993 0.051894270 5.699635e-03
30 0.57837980 0.032724963 2.829407e-03
31 -0.24054052 0.054851513 8.394698e-04
32 0.05047848 0.047359183 3.166851e-05
33 0.16468851 0.049180778 3.507228e-04
34 -1.20100212 0.008670063 3.153781e-03
35 -0.95210816 0.033393673 7.829376e-03
36 -1.34924129 0.058373364 2.821339e-02
37 0.90743587 0.036696959 7.842220e-03
38 -0.02500207 0.020177527 3.218195e-06
39 0.47962917 0.035858985 2.138989e-03
40 -0.90524477 0.081294356 1.812826e-02
41 -0.16056839 0.031565697 2.100900e-04
42 -0.01190305 0.023956946 8.693993e-07
43 1.21287730 0.075690474 3.011602e-02
44 -1.00814298 0.019319547 5.005572e-03
45 -0.68623593 0.045430669 5.603102e-03
   46  0.99229469 0.045977743 1.186344e-02
47 -0.60701450 0.036099888 3.449943e-03
48  0.13608204 0.013255108 6.218992e-05
49 -0.42011580 0.058234619 2.728454e-03
50  1.01050383 0.022807975 5.958305e-03
51  0.35220357 0.017895359 5.650803e-04
52  0.29856652 0.031406245 7.225977e-04
53 -0.10886249 0.011827469 3.546138e-05
54  0.40805528 0.023541413 1.003591e-03
55 -0.44785862 0.040615438 2.122855e-03
56  1.37880899 0.017936199 8.680384e-03
57 -1.33203734 0.025396431 1.155893e-02
58  0.91912216 0.026530563 5.755865e-03
59 -0.91323553 0.032387398 6.978790e-03
60 -0.16194913 0.064854624 4.547357e-04
61 -3.17128100 0.053936485 1.433415e-01
62  0.44281808 0.030429101 1.538510e-03
63 -1.98649397 0.024542218 2.482103e-02
64  0.69143441 0.031013571 3.825393e-03
65 -0.24626152 0.028664544 4.474133e-04
66 -0.26214306 0.018977512 3.323357e-04
67 -0.04894825 0.021151305 1.294303e-05
68 -0.65475383 0.061526716 7.026482e-03
69  0.52503478 0.041438932 2.979236e-03
70  0.30837521 0.080580421 2.083602e-03
71 -1.31409336 0.015138544 6.635924e-03
72  0.33523416 0.025599565 7.381280e-04
73 -0.17184803 0.063148577 4.976477e-04
74  0.95586837 0.049424231 1.187652e-02
75  1.37103128 0.028792300 1.393154e-02
76 -0.17397267 0.014774143 1.134667e-04
77 -0.27836062 0.017194239 3.388949e-04
78  1.72745664 0.046462612 3.635132e-02
79 -0.46540354 0.034119838 1.912860e-03
80 -0.92039907 0.051536201 1.150758e-02
81  0.14656187 0.016683767 9.111354e-05
82 -1.83851897 0.024066821 2.083890e-02
83  0.77185171 0.024686477 3.769837e-03
84 -1.16992053 0.031297366 1.105529e-02
85  0.50161920 0.010305829 6.550437e-04
86  0.92749885 0.016117579 3.523087e-03
87 -0.78196867 0.025957501 4.073837e-03
88 -0.14775904 0.011057051 6.102618e-05
89 -0.24566317 0.053888089 8.593506e-04
90  0.11731119 0.067978416 2.509366e-04
91  0.93134881 0.011338714 2.487030e-03
```

I decide to add a variable for time, to account for the potential growth in the economy we would expect to see. Below are the results of a regression of S&P returns on the predictors, having added a variable for time:

```
lm(formula = return.SNP ~ return.FTSE + return.GDAXI + return.N225 +
    Time + TimeSq)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.0807 -1.3633  0.0853  1.3823  5.0841 

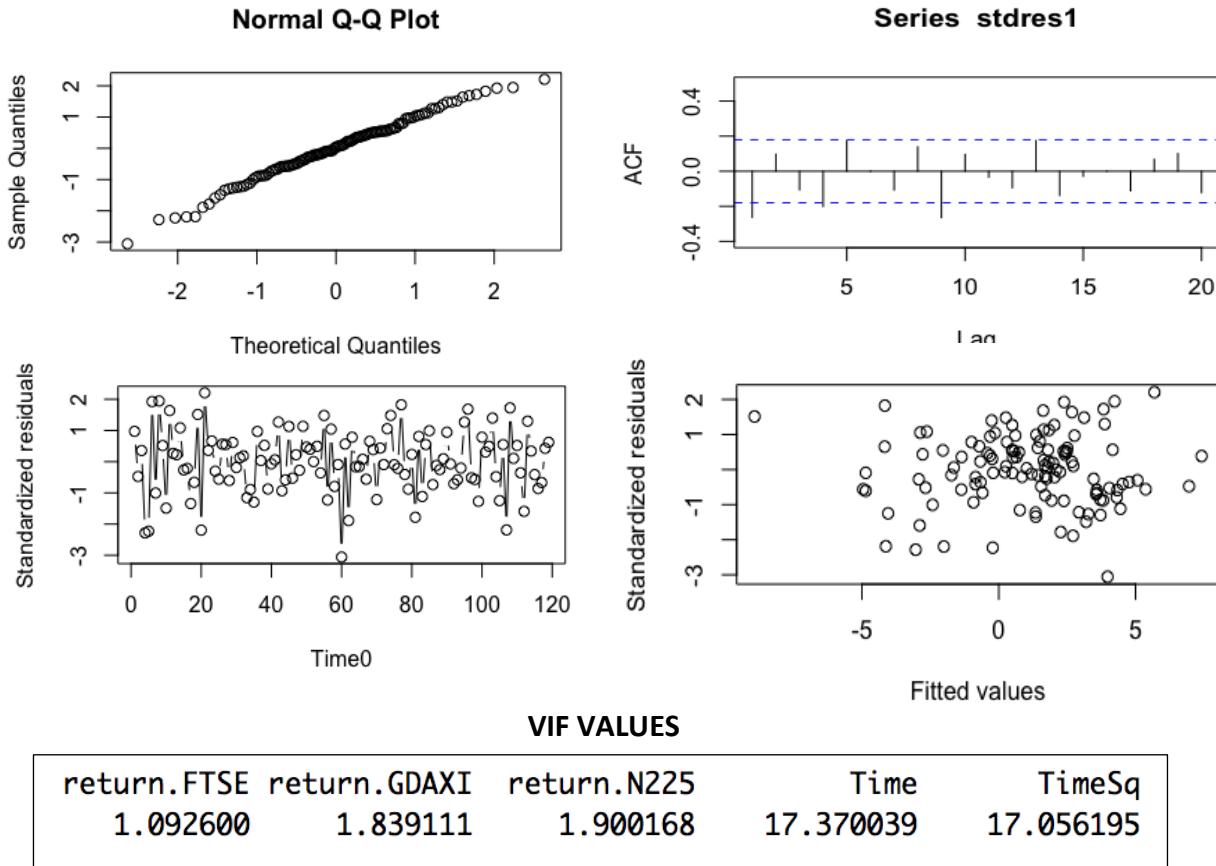
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.9851818  0.7026867   1.402 0.163650  
return.FTSE -0.0255221  0.0572844  -0.446 0.656787  
return.GDAXI  0.3772745  0.0646347   5.837 5.18e-08 *** 
return.N225   0.2463002  0.0611684   4.027 0.000103 *** 
Time         -0.0258506  0.0266985  -0.968 0.334990  
TimeSq        0.0002202  0.0002136   1.031 0.304780  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.4 on 113 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared:  0.5718,    Adjusted R-squared:  0.5528 
F-statistic: 30.17 on 5 and 113 DF,  p-value: < 2.2e-16
```

### Regression Equation

$$\begin{aligned} \text{return.SNP} = & .9852 - .0255*\text{return.FTSE} + .3773*\text{return.GDAXI} \\ & + .2463*\text{return.N225} - .02585 * \text{Time} + .0002 * \text{TimeSq} \end{aligned}$$

The regression is actually less strong, with an adjusted R-squared of .5528. Somewhat shockingly, it seems that given the other information, Time and Time squared add virtually no predictive power to the model. The multicollinearity of Time and TimeSq is expected and not worrying



### Runs Test

```
data: stdres1
statistic = 1.2045, runs = 67, n1 = 61, n2 = 58, n = 119, p-value = 0.2284
alternative hypothesis: nonrandomness
```

### Durbin-Watson test

```
data: return.SNP ~ return.FTSE + return.GDAXI + return.N225 + Time + TimeSq
DW = 2.5137, p-value = 0.9958
alternative hypothesis: true autocorrelation is greater than 0
```

The Durbin-Watson test verifies that preliminary evaluation with a value relatively close to 2, at 2.5137. As indicated by the high p value of 0.9958, we should not reject the no autocorrelation hypothesis. The Runs tests gives a p-value of .2284, the data is deemed to be random and we should not reject the no autocorrelation hypothesis. The values at the ACF plot though are still not within the blue lines, hinting at the existence of autocorrelation.

Here is a best subset regression including the Time and Time squared as potential predictors.

```
> leaps(cbind(return.FTSE,return.GDAXI,Time, TimeSq,return.N225)[2:n,],return.SNP[2:n],nbest=2)
$which
  1   2   3   4   5
1 FALSE TRUE FALSE FALSE FALSE
1 FALSE FALSE FALSE FALSE TRUE
2 FALSE TRUE FALSE FALSE TRUE
2 FALSE TRUE FALSE TRUE FALSE
3 FALSE TRUE FALSE TRUE TRUE
3 TRUE  TRUE FALSE FALSE TRUE
4 FALSE TRUE  TRUE TRUE TRUE
4 TRUE  TRUE FALSE TRUE TRUE
5 TRUE  TRUE TRUE TRUE TRUE

$label
[1] "(Intercept)" "1"          "2"          "3"          "4"          "5"

$size
[1] 2 2 3 3 4 4 5 5 6

$Cp
[1] 14.685202 35.022734 1.165791 16.392359 3.000614 3.079307 4.198499 4.940293 6.000000

$adjr2
[1] 0.5043202 0.4265865 0.5598764 0.5011760 0.5566915 0.5563855 0.5559494 0.5530395 0.5528053

$r2
[1] 0.5085209 0.4314460 0.5673361 0.5096307 0.5679621 0.5676639 0.5710019 0.5681907 0.5717542
```

The simplest model that seems viable (achieving the maximal adjusted and predicted Rsquared values respectively) is the two predictor model, using the returns of the GDAXI and the N225. The regression follows

```
lm(formula = return.SNP ~ return.GDAXI + return.N225)
```

Residuals:

| Min     | 1Q      | Median  | 3Q     | Max    |
|---------|---------|---------|--------|--------|
| -7.3600 | -1.4525 | -0.0903 | 1.4624 | 5.0593 |

Coefficients:

|              | Estimate | Std. Error | t value | Pr(> t )     |
|--------------|----------|------------|---------|--------------|
| (Intercept)  | 0.47854  | 0.22227    | 2.153   | 0.033390 *   |
| return.GDAXI | 0.38353  | 0.06354    | 6.036   | 1.94e-08 *** |
| return.N225  | 0.23492  | 0.05916    | 3.971   | 0.000125 *** |

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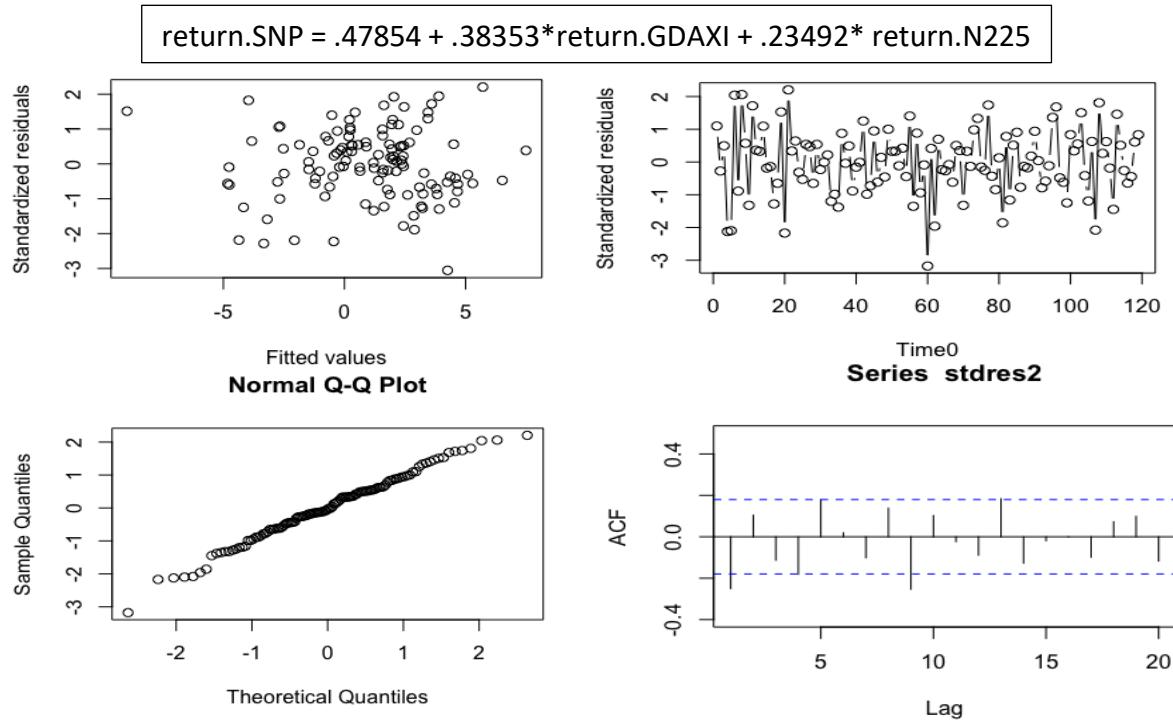
Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.381 on 116 degrees of freedom  
(1 observation deleted due to missingness)

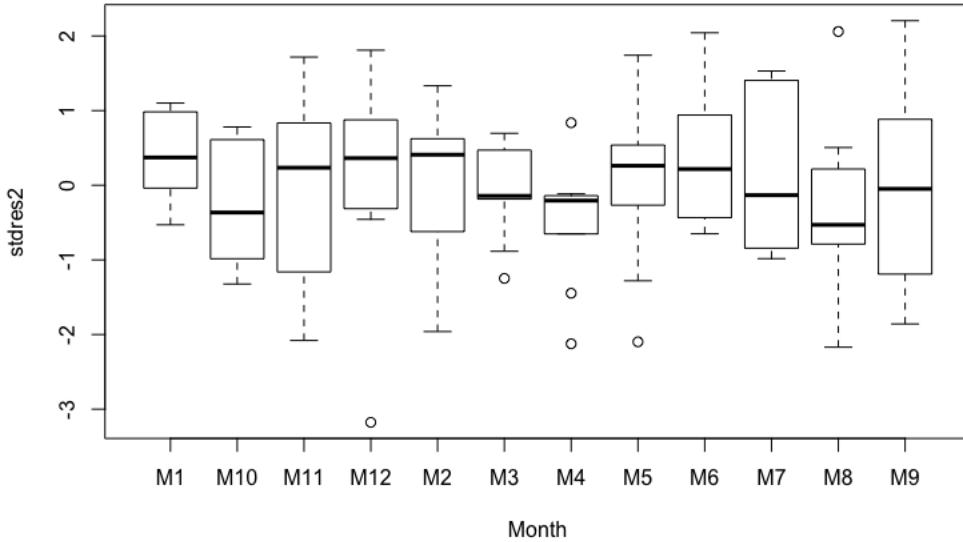
Multiple R-squared: 0.5673, Adjusted R-squared: 0.5599

F-statistic: 76.05 on 2 and 116 DF, p-value: < 2.2e-16

## Regression Equation



The autocorrelation evident in the ACF plot demonstrates negative autocorrelation at multiples of lags of four, which indicates potential seasonality in the data. This is something that would not show up on the Durbin-Watson test, as this is non-AR(1) autocorrelation. I decided to try and take the seasonality into account by creating a variable for each month



```

lm(formula = return.SNP ~ return.FTSE + return.GDAXI + return.N225 +
  Time + TimeSq + Indices$M1 + Indices$M2 + Indices$M3 + Indices$M4 +
  Indices$M5 + Indices$M6 + Indices$M7 + Indices$M8 + Indices$M9 +
  Indices$M10 + Indices$M11)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.3982 -1.4235  0.1727  1.4108  5.2703 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.0346605  1.0444896   0.991  0.324230  
return.FTSE -0.0216735  0.0623860  -0.347  0.728999  
return.GDAXI  0.3657709  0.0703670   5.198  1.04e-06 *** 
return.N225   0.2543097  0.0651199   3.905  0.000169 *** 
Time        -0.0267019  0.0278563  -0.959  0.340048  
TimeSq       0.0002276  0.0002192   1.039  0.301485  
Indices$M1   0.3877216  1.1537007   0.336  0.737510  
Indices$M2   0.9087749  1.1061742   0.822  0.413250  
Indices$M3   0.1979577  1.1212645   0.177  0.860213  
Indices$M4   -0.2359442  1.1003751  -0.214  0.830646  
Indices$M5   -1.0752333  1.1085126  -0.970  0.334352  
Indices$M6   0.1769577  1.0983019   0.161  0.872318  
Indices$M7   0.7749680  1.1019649   0.703  0.483496  
Indices$M8   0.0864445  1.1146853   0.078  0.938338  
Indices$M9   -0.7619477  1.1154013  -0.683  0.496083  
Indices$M10  -0.0745523  1.1334141  -0.066  0.947684  
Indices$M11  -0.7477434  1.1137526  -0.671  0.503501  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

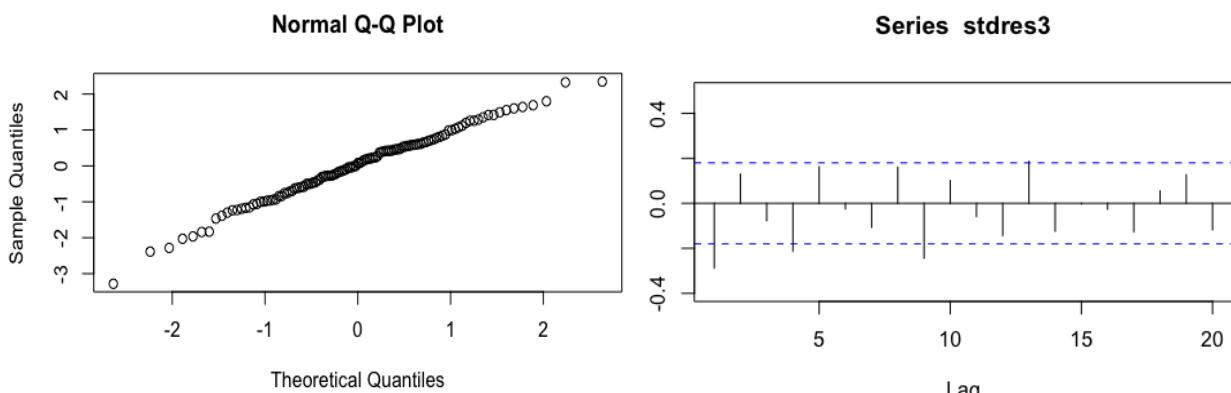
Residual standard error: 2.449 on 102 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.5978,    Adjusted R-squared:  0.5347 
F-statistic: 9.474 on 16 and 102 DF,  p-value: 6.057e-14

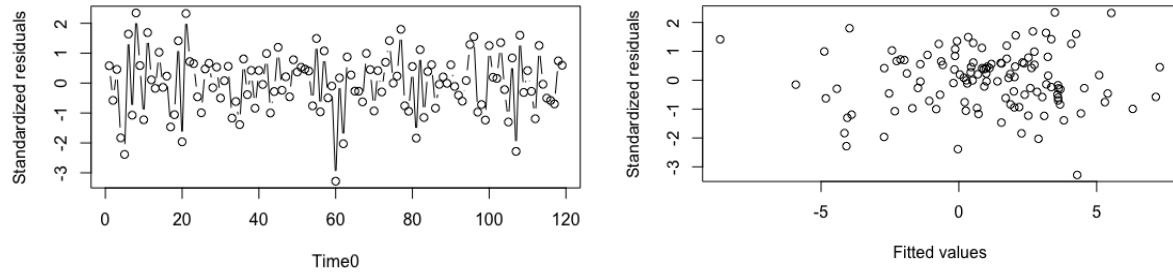
```

## Regression Equation

```
[1] "Y = 1.03 + -0.02 * return.FTSE + 0.37 * return.GDAXI + 0.25 * return.N225 + -0.03 * Time + 0 * TimeSq + 0.39 *
  Indices$M1 + 0.91 * Indices$M2 + 0.2 * Indices$M3 + -0.24 * Indices$M4 + -1.08 * Indices$M5 + 0.18 * Indices$M6 +
  0.77 * Indices$M7 + 0.09 * Indices$M8 + -0.76 * Indices$M9 + -0.07 * Indices$M10 + -0.75 * Indices$M11 + e"
```

The R squared remains strong with an adjusted R squared of 53.47



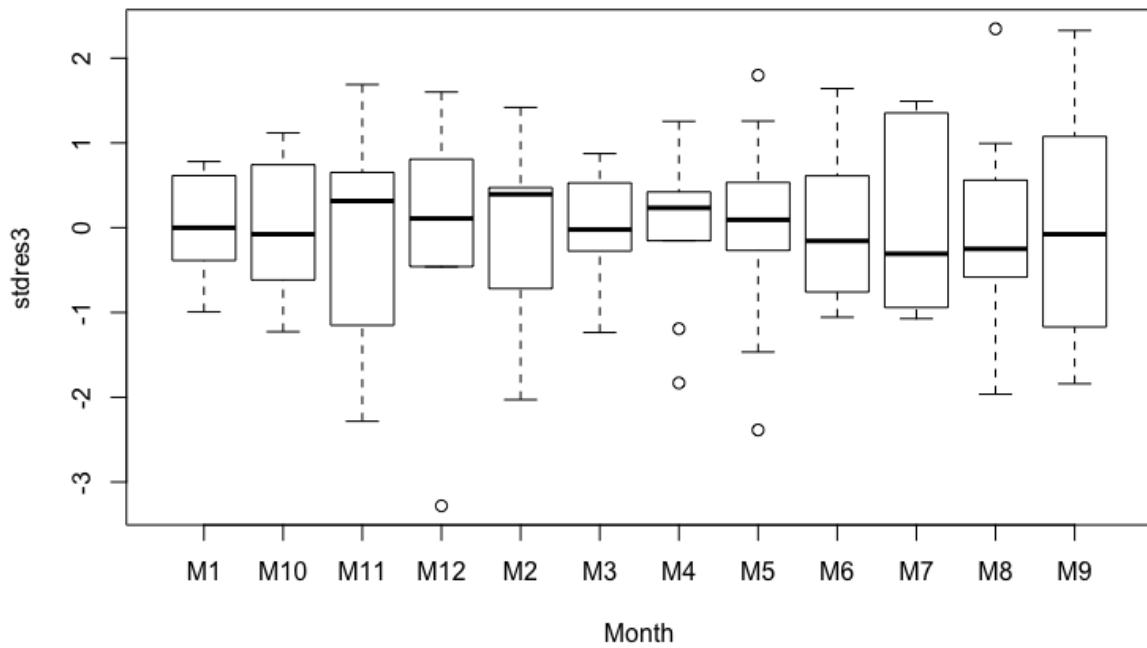


#### Durbin-Watson test

```
data: return.SNP ~ return.FTSE + return.GDAXI + return.N225 + Time +      TimeSq + Indices$M1 + Indices$M2 +
  Indices$M3 + Indices$M4 +      Indices$M5 + Indices$M6 + Indices$M7 + Indices$M8 + Indices$M9 +      Indices
$M10 + Indices$M11
DW = 2.5691, p-value = 0.9979
alternative hypothesis: true autocorrelation is greater than 0
```

#### Runs Test

```
data: stdres3
statistic = 3.039, runs = 77, n1 = 60, n2 = 59, n = 119, p-value = 0.002374
alternative hypothesis: nonrandomness
```



With a **A p-value**  $\leq 0.05$ , at .002374, there is strong evidence against the null hypothesis of randomness in the data, so the Runs tests is signaling to some autocorrelation too. However, when comparing the boxplots of the residuals before and after there seems to be a marginal improvement.

I run a best subsets test on all the potential predictors:

```
> leaps(cbind( return.FTSE,return.GDAXI,return.N225,Time ,TimeSq ,Indices$M1,Indices$M2,Indices$M3,Indices$M4,Indices$M5,
  Indices$M6,Indices$M7,Indices$M8,Indices$M9,Indices$M10,Indices$M11)[2:n],return.SNP[2:n],nbest=2,method="r2")
   1   2   3   4   5   6   7   8   9   A   B   C   D   E   F   G
1 FALSE TRUE FALSE FALSE
1 FALSE FALSE TRUE FALSE FALSE
2 FALSE TRUE TRUE FALSE FALSE
2 FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
3 FALSE TRUE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE
3 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE
4 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
4 FALSE TRUE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE
5 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE
5 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE TRUE FALSE FALSE TRUE FALSE FALSE
6 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE TRUE FALSE FALSE TRUE FALSE TRUE
6 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE TRUE FALSE FALSE TRUE FALSE TRUE
7 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
7 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE
8 FALSE TRUE TRUE TRUE FALSE TRUE FALSE TRUE FALSE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE
8 FALSE TRUE TRUE FALSE TRUE FALSE TRUE FALSE FALSE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
9 FALSE TRUE TRUE TRUE FALSE TRUE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
9 FALSE TRUE TRUE TRUE FALSE TRUE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
10 FALSE TRUE TRUE TRUE FALSE TRUE FALSE
10 FALSE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
11 TRUE TRUE TRUE TRUE FALSE TRUE FALSE
11 FALSE TRUE TRUE TRUE TRUE FALSE TRUE
12 TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE
12 TRUE TRUE TRUE TRUE TRUE FALSE TRUE
13 TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE
13 TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
14 TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE TRUE
14 TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE
15 TRUE FALSE TRUE FALSE TRUE FALSE TRUE FALSE
15 TRUE FALSE TRUE FALSE TRUE FALSE TRUE
16 TRUE TRUE

$label
[1] "(Intercept)" "1"          "2"          "3"          "4"          "5"          "6"          "D"
[8] "7"            "8"          "9"          "A"          "B"          "C"          "E"          "G"
[15] "E"            "F"          "G"

$size
[1] 2 2 3 3 4 4 5 5 6 6 7 7 8 8 9 9 10 10 11 11 12 12 13 13 13 14 14 15 15 16 16 17

$Cp
[1] 9.6345281 29.1800017 -3.2804690 7.7998671 -3.2810377 -2.8716389 -2.5813052 -2.5617903
[9] -1.9456941 -1.8139031 -0.7577090 -0.6299872 0.4769914 0.9703373 2.2488004 2.2632737
[17] 3.4915675 3.9697886 5.3285372 5.3369093 7.2092845 7.2131347 9.0877413 9.1220524
[25] 11.0396892 11.0547472 13.0185059 13.0259666 15.0043266 15.0060141 17.0000000

$r2
[1] 0.5085209 0.4314460 0.5673361 0.5236423 0.5752251 0.5736107 0.5803525 0.5802755 0.5857328
[10] 0.5852131 0.5889348 0.5884312 0.5919527 0.5900073 0.5928525 0.5927955 0.5958386 0.5939528
[19] 0.5964815 0.5964485 0.5969517 0.5969365 0.5974310 0.5972957 0.5976205 0.5975611 0.5977040
[28] 0.5976746 0.5977600 0.5977533 0.5977770

$adjr2
[1] 0.5043202 0.4265865 0.5598764 0.5154293 0.5641440 0.5624875 0.5656280 0.5655484 0.5674024
[10] 0.5668597 0.5669135 0.5663829 0.5662200 0.5641519 0.5632418 0.5631806 0.5624675 0.5604260
[19] 0.5591186 0.5590826 0.5555169 0.5555001 0.5518572 0.5517066 0.5478021 0.5477354 0.5435488
[28] 0.5435154 0.5391813 0.5391737 0.5346832
```

Best subsets seem to suggest the best model to be one with 3 predictors, the return of the GDAXI, the return of the Nikkei 225 and the Variable for month 5.

The regression follows

```
lm(formula = return.SNP ~ +return.GDAXI + return.N225 + Indices$M5)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.473 -1.506  0.351  1.357  4.965 

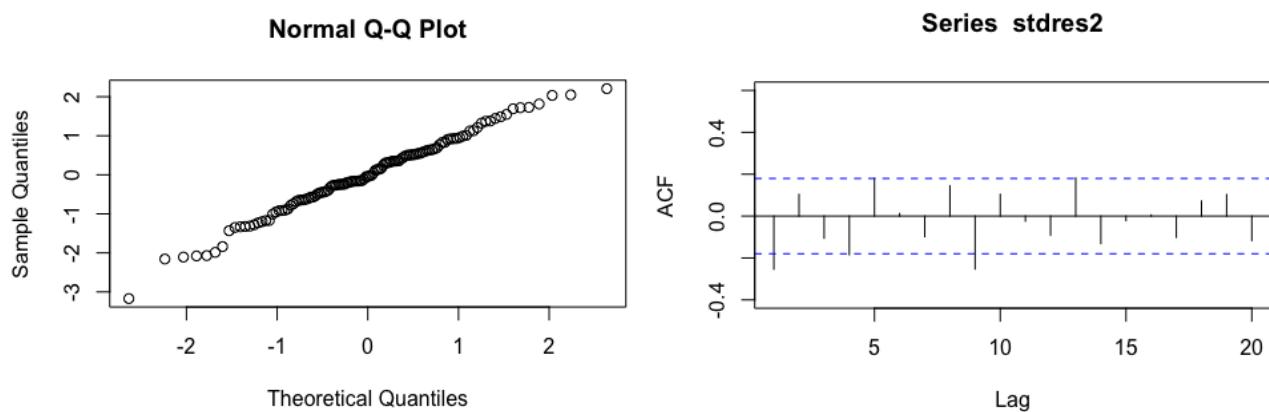
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.58451   0.23277   2.511  0.013424 *  
return.GDAXI  0.38609   0.06326   6.104  1.44e-08 *** 
return.N225   0.22234   0.05950   3.737  0.000292 *** 
Indices$M5    -1.16153   0.79479  -1.461  0.146623    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

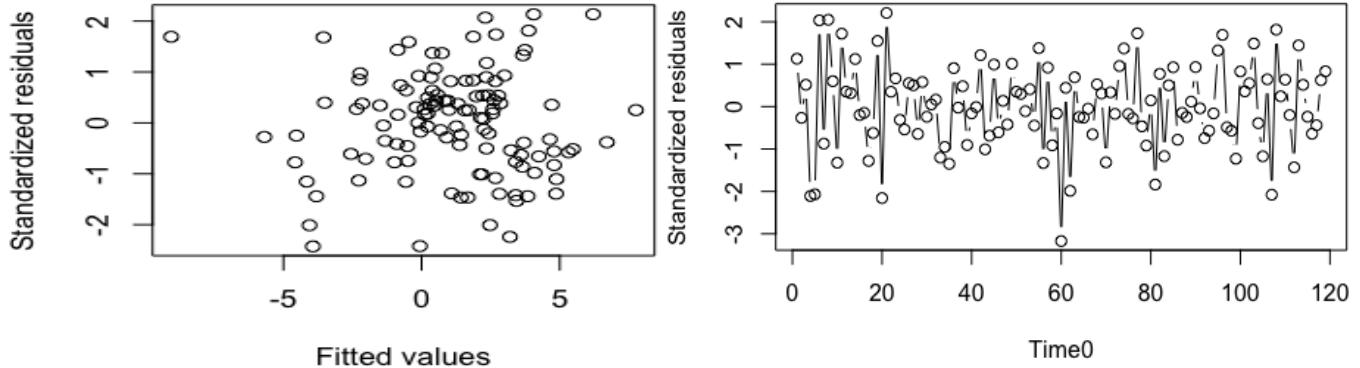
Residual standard error: 2.37 on 115 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared:  0.5752,    Adjusted R-squared:  0.5641 
F-statistic: 51.91 on 3 and 115 DF,  p-value: < 2.2e-16
```

Adjusted R squared is marginally better at 56.41%. The run test gives us a p value close enough to the .05 limit that signals there is still autocorrelation remaining in the data. Similarly, autocorrelation is evident in the ACF plot.

#### Runs Test

```
data: stdres4
statistic = 1.9561, runs = 71, n1 = 62, n2 = 57, n = 119, p-value = 0.05045
alternative hypothesis: nonrandomness
```





In order to account for the outlier and influential points as identified by the cook's distance and leverage values, also evident in the standardized residuals plot above, I created two new indicator variables to identify each of those points. The regression follows:

```
lm(formula = return.SNP ~ return.FTSE + return.GDAXI + return.N225 +
  Time + TimeSq + Indices$M1 + Indices$M2 + Indices$M3 + Indices$M4 +
  Indices$M5 + Indices$M6 + Indices$M7 + Indices$M8 + Indices$M9 +
  Indices$M10 + Indices$M11 + Indices$J15 + Indices$S11)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.4946 -1.4644  0.0289  1.2957  4.9364 

Coefficients: (1 not defined because of singularities)
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.4485839  0.8357342  1.733  0.086093 .
return.FTSE -0.0268986  0.0583030 -0.461  0.645533
return.GDAXI  0.4006405  0.0672347  5.959 3.74e-08 ***
return.N225   0.2278868  0.0612152  3.723  0.000325 ***
Time        -0.0212712  0.0262960 -0.809  0.420468
TimeSq       0.0001698  0.0002064  0.823  0.412507
Indices$M1    0.8010402  0.9676168  0.828  0.409708
Indices$M2      NA        NA        NA        NA      
Indices$M3   -0.2839335  0.9007912 -0.315  0.753257
Indices$M4   -0.7127455  0.8880696 -0.803  0.424103
Indices$M5   -1.5770551  0.9006557 -1.751  0.082981 .
Indices$M6   -0.2469442  0.8916401 -0.277  0.782381
Indices$M7    0.2842691  0.8893378  0.320  0.749900
Indices$M8   -0.3411114  0.9193255 -0.371  0.711381
Indices$M9   -0.7046853  0.9365117 -0.752  0.453526
Indices$M10   -0.5835146  0.9117372 -0.640  0.523620
Indices$M11   -1.1531007  0.9010104 -1.280  0.203552
Indices$J15   -8.6269503  2.4872173 -3.469  0.000771 ***
Indices$S11   -4.9235488  2.4684427 -1.995  0.048782 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.29 on 101 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared:  0.6517,    Adjusted R-squared:  0.593 
F-statistic: 11.11 on 17 and 101 DF,  p-value: 2.507e-16
```

## Regression Equation

```
[1] "Y = 1.45 + -0.03 * return.FTSE + 0.4 * return.GDAXI + 0.23 * return.N225 + -0.02 * Time +
Indices$M1 + NA * Indices$M2 + -0.28 * Indices$M3 + -0.71 * Indices$M4 + -1.58 * Indices$M5 +
+ 0.28 * Indices$M7 + -0.34 * Indices$M8 + -0.7 * Indices$M9 + -0.58 * Indices$M10 + -1.15 * ]
* Indices$J15 + -4.92 * Indices$S11 + e"
```

I run a best subset test on the potential predictors:

```
> leaps(cbind(return.FTSE,return.GDAXI,return.N225,Time,TimeSq,Indices$M1,Indices$M5,Indices$M6,Indices$M7,Indices$M10,Indices$M11,Indices$J15
+Indices$S11)[2:n],return.SNP[2:n],nbest=2)
$which
 1   2   3   4   5   6   7   8   9   A   B   C
1 FALSE TRUE FALSE FALSE
1 FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
2 FALSE TRUE FALSE TRUE
2 FALSE TRUE FALSE FALSE
3 FALSE TRUE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE
3 FALSE TRUE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE TRUE
4 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE TRUE
4 FALSE TRUE TRUE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE TRUE
5 FALSE TRUE TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE TRUE
5 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE FALSE FALSE TRUE
6 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE FALSE FALSE TRUE
6 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE FALSE FALSE TRUE
7 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
7 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE FALSE TRUE TRUE
8 TRUE TRUE TRUE FALSE FALSE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
8 FALSE TRUE TRUE FALSE FALSE TRUE TRUE FALSE TRUE TRUE TRUE
9 FALSE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
9 TRUE TRUE TRUE TRUE TRUE TRUE TRUE FALSE FALSE TRUE TRUE
10 TRUE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
10 FALSE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE TRUE TRUE
11 TRUE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
11 TRUE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE FALSE TRUE TRUE
12 TRUE TRUE

$label
[1] "(Intercept)" "1"      "2"      "3"      "4"      "5"      "6"      "7"      "8"      "9"
[11] "A"          "B"      "C"

$size
[1] 2 2 3 3 4 4 5 5 6 6 7 7 8 8 9 9 10 10 11 11 12 12 13

$Cp
[1] 31.882308 54.916730 15.226185 16.304922 2.614992 12.005147 1.600672 3.009580 2.058125 2.365992 3.006778 3.387711 4.253111 4.7530
99
[15] 6.083314 6.105428 7.455504 7.774944 9.088790 9.320162 11.002732 11.081300 13.000000

$adjr2
[1] 0.5043202 0.4265865 0.5635481 0.5598764 0.6099184 0.5776786 0.6169367 0.6120570 0.6189366 0.6178609 0.6192406 0.6178977 0.6184912
[14] 0.6167127 0.6156324 0.6155531 0.6143803 0.6132232 0.6121504 0.6113045 0.6088432 0.6085533 0.6051632

$r2
[1] 0.5085209 0.4314460 0.5709456 0.5673361 0.6198358 0.5884156 0.6299219 0.6252076 0.6350834 0.6340532 0.6386013 0.6373266 0.6411231
[14] 0.6394501 0.6416913 0.6416173 0.6437920 0.6427231 0.6450190 0.6442448 0.6453070 0.6450441 0.6453161
```

Best subsets seem to suggest the best model to be one with 5 predictors, the return of the GDAXI, the return of the Nikkei 225, the Variable for month 5 and the variables accounting for the outliers.

The regression follows:

```
lm(formula = return.SNP ~ return.GDAXI + return.N225 + Indices$M5 +
  Indices$J15 + Indices$S11)
```

Residuals:

| Min     | 1Q      | Median | 3Q     | Max    |
|---------|---------|--------|--------|--------|
| -5.3266 | -1.4095 | 0.2141 | 1.2017 | 4.6999 |

Coefficients:

|              | Estimate | Std. Error | t value | Pr(> t )     |
|--------------|----------|------------|---------|--------------|
| (Intercept)  | 0.70329  | 0.22071    | 3.187   | 0.001862 **  |
| return.GDAXI | 0.41794  | 0.06106    | 6.845   | 4.18e-10 *** |
| return.N225  | 0.19565  | 0.05644    | 3.467   | 0.000746 *** |
| Indices\$M5  | -1.31348 | 0.74724    | -1.758  | 0.081495 .   |
| Indices\$J15 | -7.84628 | 2.28874    | -3.428  | 0.000849 *** |
| Indices\$S11 | -5.27919 | 2.25126    | -2.345  | 0.020773 *   |

---

Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.225 on 113 degrees of freedom

(1 observation deleted due to missingness)

Multiple R-squared: 0.632, Adjusted R-squared: 0.6157

F-statistic: 38.81 on 5 and 113 DF, p-value: < 2.2e-16

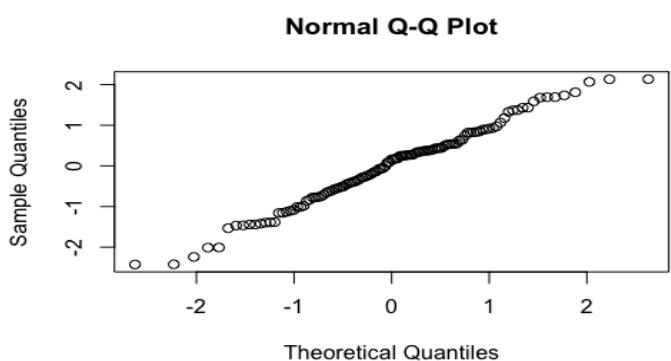
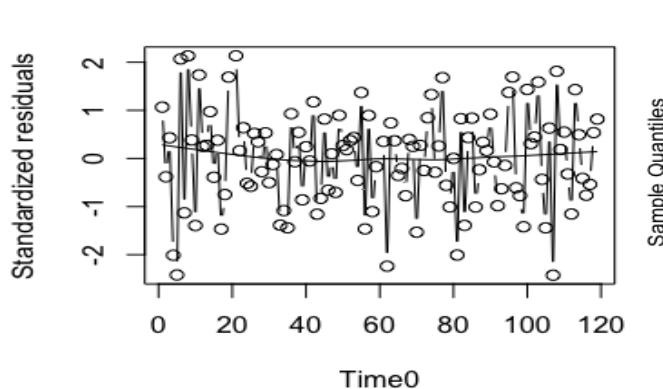
### Regression Equation

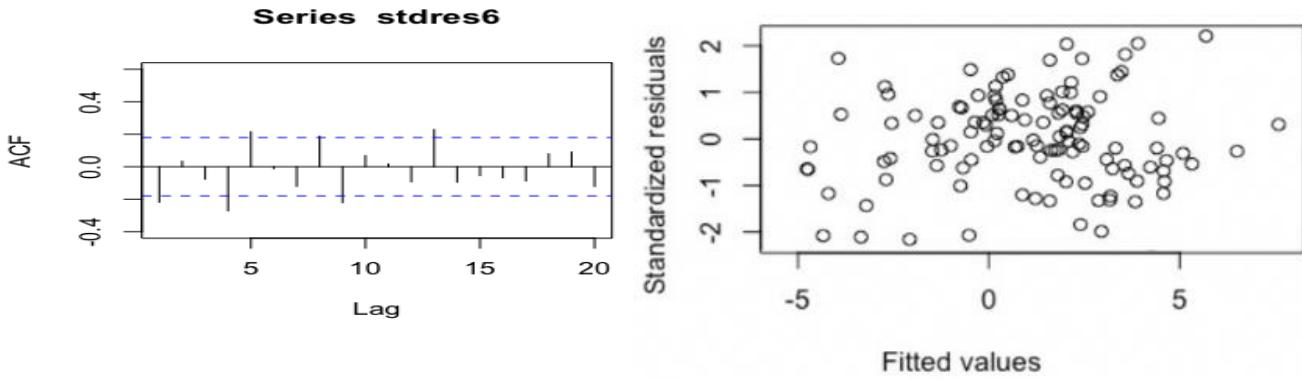
$$\text{return.SNP} = .70329 + .41794 * \text{return.GDAXI} + .19565 * \text{return.N225} - 1.31348 * \text{Month5} - 7.84628 * \text{January2015} - 5.27919 * \text{September2011}$$

### VIF VALUES

| return.GDAXI | return.N225 | Indices\$M5 | Indices\$J15 | Indices\$S11 |
|--------------|-------------|-------------|--------------|--------------|
| 1.910009     | 1.882253    | 1.032826    | 1.048942     | 1.014867     |

As shown by the VIF values, multicollinearity does not seem to be an issue here.



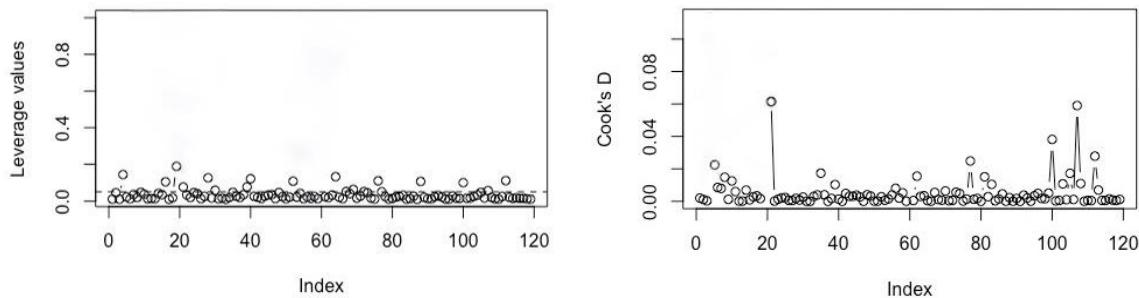


### Runs Test

```
data: stdres6
statistic = 1.8096, runs = 69, n1 = 62, n2 = 55, n = 117, p-value = 0.07036
alternative hypothesis: nonrandomness
```

### Durbin-Watson test

```
data: return.SNP ~ return.FTSE + return.GDAXI + return.N225
DW = 2.4887, p-value = 0.9963
alternative hypothesis: true autocorrelation is greater than 0
```



The adjusted R squared with this model is the highest at 61.57%. Autocorrelation unfortunately remain an issue, as evident by the ACF plot, however the increased p-value from the Runs test shows a decreased level of autocorrelation. The Durbin-Watson is insisting on no autocorrelation, but this can be attributed to non AR(1) autocorrelation being here. It seems like the methods that we have explored so far for correcting autocorrelation have not been sufficient for this data.

The equation tells us that given the return for the GDAXI and the index variables for the 5<sup>th</sup> Month and the outliers are held fixed, a one percentage increase in the return of the N225 index is associated with a .19565% increase in the return of the S&P. Similarly, a one percentage point increase is associated with a .41794% increase in the return of the S&P, holding all else constant. The regression is somewhat significant with 61.57% of the variability in the return of the S&P is being accounted for by the predictors. A prediction

interval for the return of the S&P is  $\pm 2\sigma \approx 4.45$ ; that is 95% of the time the logged inbound tourism is known within  $\pm 4.45$ . The normal plot shows the some heavy-tailedness, with the right upper end of the normality plot going above the hypothetical straight and the left lower end going below it. The residuals versus fitted values plot also shows some non-constant variance.

## Appendices

## Appendix A.

|    | A          | B     | C       | D      | E        | F        | G    | H      | I  | J  | K  | L  | M  | N  | O  | P  | Q  | R   | S   |
|----|------------|-------|---------|--------|----------|----------|------|--------|----|----|----|----|----|----|----|----|----|-----|-----|
| 1  | Date       | Month | SNP     | FTSE   | GDAXI    | N25      | Time | TimeSq | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | M9 | M10 | M11 |
| 2  | 01/01/2010 | M1    | 1073.87 | 3009.3 | 5608.79  | 10198.04 | 0    | 0      | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 3  | 01/02/2010 | M2    | 1104.49 | 3137.9 | 5598.46  | 10126.03 | 1    | 1      | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 4  | 01/03/2010 | M3    | 1169.43 | 3216.7 | 6153.55  | 11089.94 | 2    | 4      | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 5  | 01/04/2010 | M4    | 1186.69 | 3119.4 | 6135.7   | 11057.4  | 3    | 9      | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 6  | 01/05/2010 | M5    | 1089.41 | 3114.6 | 5964.33  | 9768.7   | 4    | 16     | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 7  | 01/06/2010 | M6    | 1030.71 | 3463.3 | 5965.52  | 9382.64  | 5    | 25     | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 8  | 01/07/2010 | M7    | 1101.6  | 3477.8 | 6147.97  | 9537.3   | 6    | 36     | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0   | 0   |
| 9  | 01/08/2010 | M8    | 1049.33 | 3508.2 | 5925.22  | 8824.06  | 7    | 49     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 10 | 01/09/2010 | M9    | 1141.2  | 3529.1 | 6229.02  | 9369.35  | 8    | 64     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0   | 0   |
| 11 | 01/10/2010 | M10   | 1183.26 | 3664.3 | 6601.37  | 9202.45  | 9    | 81     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 0   |
| 12 | 01/11/2010 | M11   | 1180.55 | 3689.3 | 6688.49  | 9937.04  | 10   | 100    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 1   |
| 13 | 01/12/2010 | M12   | 1257.64 | 3759.3 | 6914.19  | 10228.92 | 11   | 121    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 14 | 01/01/2011 | M1    | 1286.12 | 3727.6 | 7077.48  | 10237.92 | 12   | 144    | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 15 | 01/02/2011 | M2    | 1327.22 | 3699.7 | 7272.32  | 10624.09 | 13   | 169    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 16 | 01/03/2011 | M3    | 1325.83 | 3817.9 | 7041.31  | 9755.1   | 14   | 196    | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 17 | 01/04/2011 | M4    | 1363.61 | 3747.8 | 7514.46  | 9849.74  | 15   | 225    | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 18 | 01/05/2011 | M5    | 1345.2  | 3711   | 7293.69  | 9693.73  | 16   | 256    | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 19 | 01/06/2011 | M6    | 1320.64 | 3703.2 | 7376.24  | 9816.09  | 17   | 289    | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 20 | 01/07/2011 | M7    | 1292.28 | 3867.6 | 7158.77  | 9833.03  | 18   | 324    | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0   | 0   |
| 21 | 01/08/2011 | M8    | 1218.89 | 3953.7 | 5784.85  | 8955.2   | 19   | 361    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 22 | 01/09/2011 | M9    | 1311.42 | 3979.1 | 5502.02  | 8700.29  | 20   | 400    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 23 | 01/10/2011 | M10   | 1253.3  | 4058   | 6141.34  | 8988.39  | 21   | 441    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0   | 0   |
| 24 | 01/11/2011 | M11   | 1246.96 | 4118.5 | 6088.84  | 8434.61  | 22   | 484    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 0   |
| 25 | 01/12/2011 | M12   | 1257.6  | 4275.8 | 5898.35  | 8455.35  | 23   | 529    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 26 | 01/01/2012 | M1    | 1312.41 | 4308.3 | 6458.91  | 8802.51  | 24   | 576    | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 27 | 01/02/2012 | M2    | 1365.68 | 4312.9 | 6856.08  | 9723.24  | 25   | 625    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 28 | 01/03/2012 | M3    | 1408.47 | 4436   | 6946.83  | 10083.56 | 26   | 676    | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 29 | 01/04/2012 | M4    | 1397.91 | 4621.3 | 6761.19  | 9520.89  | 27   | 729    | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 30 | 01/05/2012 | M5    | 1310.33 | 4604.6 | 6264.38  | 8542.73  | 28   | 784    | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 31 | 01/06/2012 | M6    | 1362.16 | 4907.8 | 6416.28  | 9006.78  | 29   | 841    | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 32 | 01/07/2012 | M7    | 1379.32 | 4817.5 | 6772.26  | 8695.06  | 30   | 900    | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0   | 0   |
| 33 | 01/08/2012 | M8    | 1406.58 | 5244.8 | 6970.79  | 8839.91  | 31   | 961    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 34 | 01/09/2012 | M9    | 1440.67 | 4842.3 | 7216.15  | 8870.16  | 32   | 1024   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0   | 0   |
| 35 | 01/10/2012 | M10   | 1412.16 | 4831.8 | 7260.63  | 8928.29  | 33   | 1089   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 0   |
| 36 | 01/11/2012 | M11   | 1416.18 | 5135.5 | 7405.5   | 9446.01  | 34   | 1156   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 1   |
| 37 | 01/12/2012 | M12   | 1426.19 | 5458.5 | 7612.39  | 10395.18 | 35   | 1225   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 38 | 01/01/2013 | M1    | 1498.11 | 5767.3 | 7776.05  | 11138.66 | 36   | 1296   | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 39 | 01/02/2013 | M2    | 1514.68 | 5932.2 | 7747.17  | 11559.36 | 37   | 1369   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 40 | 01/03/2013 | M3    | 1569.19 | 5928.4 | 7795.31  | 12397.91 | 38   | 1444   | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 41 | 01/04/2013 | M4    | 1597.57 | 5870.7 | 7913.71  | 13860.86 | 39   | 1521   | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 42 | 01/05/2013 | M5    | 1630.74 | 5832.6 | 8348.84  | 13774.54 | 40   | 1600   | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 43 | 01/06/2013 | M6    | 1606.28 | 5837.1 | 7959.22  | 13677.32 | 41   | 1681   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 44 | 01/07/2013 | M7    | 1685.73 | 5249.4 | 8275.97  | 13668.32 | 42   | 1764   | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0   | 0   |
| 45 | 01/08/2013 | M8    | 1632.97 | 5064.4 | 8103.15  | 13388.86 | 43   | 1849   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 46 | 01/09/2013 | M9    | 1681.55 | 5438.4 | 8594.4   | 14455.8  | 44   | 1936   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0   | 0   |
| 47 | 01/10/2013 | M10   | 1756.54 | 5743.9 | 9039.92  | 14327.94 | 45   | 2025   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 0   |
| 48 | 01/11/2013 | M11   | 1805.81 | 5882.6 | 9405.3   | 15661.87 | 46   | 2116   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 1   |
| 49 | 01/12/2013 | M12   | 1848.36 | 5896   | 9552.16  | 16291.31 | 47   | 2209   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 50 | 01/01/2014 | M1    | 1782.59 | 5175.1 | 9306.48  | 14914.53 | 48   | 2304   | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 51 | 01/02/2014 | M2    | 1859.45 | 6295.3 | 9692.08  | 14841.07 | 49   | 2401   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 52 | 01/03/2014 | M3    | 1872.34 | 6552.2 | 9555.91  | 14827.83 | 50   | 2500   | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 53 | 01/04/2014 | M4    | 1883.95 | 6226.2 | 9603.23  | 14304.11 | 51   | 2601   | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 54 | 01/05/2014 | M5    | 1923.57 | 6318.5 | 9943.38  | 14632.38 | 52   | 2704   | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 55 | 01/06/2014 | M6    | 1960.23 | 6231.9 | 9833.07  | 15162.1  | 53   | 2809   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 56 | 01/07/2014 | M7    | 1930.67 | 6246.4 | 9407.48  | 15620.77 | 54   | 2916   | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0   | 0   |
| 57 | 01/08/2014 | M8    | 2003.37 | 6029.8 | 9470.17  | 15424.59 | 55   | 3025   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0   | 0   |
| 58 | 01/09/2014 | M9    | 1972.29 | 6225.7 | 9474.3   | 16173.52 | 56   | 3136   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0   | 0   |
| 59 | 01/10/2014 | M10   | 2018.05 | 6597.2 | 9326.87  | 16413.76 | 57   | 3249   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 0   |
| 60 | 01/11/2014 | M11   | 2067.56 | 6930.2 | 9980.85  | 17459.85 | 58   | 3364   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 1   |
| 61 | 01/12/2014 | M12   | 2058.9  | 6268.5 | 9805.55  | 17450.77 | 59   | 3481   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 62 | 01/01/2015 | M1    | 1994.99 | 6232.6 | 10694.32 | 17674.39 | 60   | 3500   | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 63 | 01/02/2015 | M2    | 2104.45 | 6540.2 | 11401.66 | 18797.94 | 61   | 3721   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 64 | 01/03/2015 | M3    | 2067.89 | 6322.5 | 11966.17 | 19206.99 | 62   | 3844   | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 65 | 01/04/2015 | M4    | 2085.51 | 6359.4 | 11454.3  | 19520.01 | 63   | 3969   | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0   | 0   |
| 66 | 01/05/2015 | M5    | 2107.39 | 6312.7 | 11413.82 | 20563.15 | 64   | 4096   | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0   | 0   |
| 67 | 01/06/2015 | M6    | 2063.11 | 6365.3 | 10944.97 | 2035.73  | 65   | 4225   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0   | 0   |
| 68 | 01/07/2015 |       |         |        |          |          |      |        |    |    |    |    |    |    |    |    |    |     |     |