A COMMON ERROR CONCERNING KURTOSIS

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In many texts[1] it is stated that a frequency curve with positive kurtosis is higher in the neighborhood of the mean than the corresponding normal curve, while one with negative kurtosis is lower. In the hope of clearing up this error, we offer in this note four examples showing that any combination of peakedness at the mean and kurtosis may occur. All four curves are symmetric about $x=0$, and have been arranged to have standard deviation unity; hence they are properly to be compared with the unit-normal distribution for which the fourth moment $\mu_4=3$, and the value at the mean $x=0$ is $1/\sqrt{2\pi}=.399$.

$$P(x) = \frac{1}{3\sqrt{\pi}}\left(\tfrac{9}{4}+x^4\right)e^{-x^2}. \tag{1}$$

$$Q(x) = \frac{3}{2\sqrt{2\pi}}e^{-x^2/2} - \frac{1}{6\sqrt{\pi}}\left(\tfrac{9}{4}+x^4\right)e^{-x^2}. \tag{2}$$

$$R(x) = \frac{1}{6\sqrt{\pi}}\left(e^{-x^2/4}+4e^{-x^2}\right). \tag{3}$$

$$S(x) = \frac{3\sqrt{3}}{16\sqrt{\pi}}(2+x^2)e^{-3x^2/4}. \tag{4}$$

In (1), $\mu_4=2.75$, $P_0 \doteq .423$.
In (2), $\mu_4=3.125$, $Q_0=.387$.
In (3), $\mu_4=4.5$, $R_0=.470$.
In (4), $\mu_4=8/3=2.667$, $S_0=.366$.

[1] At the request of the Editor the author has supplied the following examples from authoritative volumes. Many others could be cited.

Kenney, *Mathematics of Statistics*, part I, p. 106; H. L. Rietz, *Mathematical Statistics*, pp. 71–72; Yule and Kendall, *An Introduction to the Theory of Statistics*, p. 165.