1.2.3.

Show that

$$u(t,x,y) = e^{i(k_x x + k_y y - \omega t)} \qquad 1.6$$

satisfies equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \qquad 1.2$$

where:

ω: angular frequency

$\vec{k} = (k_x, k_y)$ wave vector in x and y direction recpectivly

Intermediate calculations:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{e^{i(k_x x + k_y y - \omega t)}}{\partial t}\right) = \quad e^{i(k_x x + k_y y - \omega t)} = e^{ik_x x} \cdot e^{ik_y y} \cdot e^{i\omega t}$$

$$= \frac{\partial}{\partial t}\left(-i\omega \cdot e^{ik_x x} \cdot e^{ik_y y} \cdot e^{-i\omega t}\right) =$$

$$= -\underbrace{i^2}_{=-1} \omega^2 \cdot e^{ik_x x} \cdot e^{ik_y y} \cdot e^{-i\omega t} = \omega^2 \cdot \underbrace{e^{i(k_x x + k_y y - \omega t)}}_{u(t,x,y)} = \omega^2 \cdot u \qquad (1)$$

$$\nabla^2 u = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial}{\partial x}\left(ik_x \cdot e^{ik_y y} \cdot e^{-i\omega t} \cdot e^{ik_x x}\right) = \qquad (2)$$

$$= \underbrace{i^2}_{=-1} k_x^2 \cdot e^{ik_y y} \cdot e^{-i\omega t} \cdot e^{ik_x x} = -k_x^2 \cdot \underbrace{e^{i(k_x x + k_y y - \omega t)}}_{u(t,x,y)}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(i \cdot k_y \cdot e^{ikx} \cdot e^{ik_y y} \cdot e^{-i\omega t}\right) =$$

$$= \underset{-1}{\underset{\shortparallel}{i^2}} k_y^2 \cdot e^{ikx} \cdot e^{ik_y y} \cdot e^{-i\omega t} = -k_y^2 \cdot \underbrace{e^{i(k_x x + k_y y - \omega t)}}_{u(x,y,t)} \qquad (3)$$

Combining 1) 2) and 3)

$$-\omega^2 u = c^2(-k_x^2 u - k_y^2 u)$$

$$\omega^2 = c^2(k_x^2 + k_y^2) \Rightarrow \omega = c\sqrt{k_x^2 + k_y^2} \qquad (4)$$

We end up with dispersion relation

$$\vec{k} = |k| = \sqrt{k_x^2 + k_y^2}$$

Thus 1.6 is solution to 1.2 as long as (4) is fulfilled