NRSF2 Design Document Appendix: Strain and Stress Calculation

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Strain, aka, unconstrained strain, is measured as the fraction change from a reference state (d_0).

$$\epsilon_{ij} = \frac{d_{ij} - d_0}{d_0} \tag{1}$$

Residual stress is determined by measuring stress along *3* orthogonal directions

$$\sigma_{ij} = \frac{E}{(1+\nu)}[\epsilon_{ij} + \frac{\nu}{1-2\nu}(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})] \tag{2}$$

where

- ν is *Poisson's Ratio*.
- E is *Young's Modulus*.
- ϵ_{ij} are strains. Be noted that
  - ϵ_{ij} with $i = j$ are principle strains. But not all all three orthogonal strains are equivalent to principle strains.
  - The off-diagonal strain component, i.e., ϵ_{ij} with $i \neq j$ are all set to **zero**. It is very hard to measure these values in HB2B's setup.

In **plane strain**, ϵ_{33} is **zero**.

In **plane stress**, σ_{33} is **zero**. Therefore, ϵ_{33} can be calculated from ϵ_{11} and ϵ_{22} from $\sigma_{33} = 0$.

$$\sigma_{33} = 0 \tag{3}$$

$$= \frac{E}{(1+\nu)}[\epsilon_{33} + \frac{\nu}{1-2\nu}(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})] \tag{4}$$

$$\epsilon_{33} = -\frac{\nu}{(1-2\nu)}(\epsilon_{11} + \epsilon_{22}) + -\frac{\nu}{(1-2\nu)}\epsilon_{33} \tag{5}$$

$$\epsilon_{33} = \frac{\nu}{\nu - 1}(\epsilon_{11} + \epsilon_{22}) \tag{6}$$

Therefore, for both plain stress and plain strain, d from 2 principle directions are enough, but not 3.