Strain, aka, unconstrained strain, is measured as the fraction change from a reference state (d_0).

$$\epsilon_{ij} = \frac{d_{ij} - d_0}{d_0} \tag{1}$$

Residual stress is determined by measuring stress along *3* orthogonal directions

$$\sigma_{ij} = \frac{E}{(1+\nu)} \left[\epsilon_{ij} + \frac{\nu}{1-2\nu} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) \right] \tag{2}$$

where

- ν is *Poisson's Ratio.*
- E is *Young's Modulus.*
- ϵ_{ij} are strains. Be noted that
  - ϵ_{ij} with $i = j$ are principle strains. But not all all three orthogonal strains are equivalent to principle strains.
  - The off-diagonal strain component, i.e., ϵ_{ij} with $i \neq j$ are all set to **zero**. It is very hard to measure these values in HB2B's setup.

Therefore the stress that is calculated is

$$\sigma_{ii} = \frac{E}{(1+\nu)} \left[\epsilon_{ii} + \frac{\nu}{1-2\nu} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) \right] \tag{3}$$

where the second term in the sum is the same between all 3 principle strain directions, (σ_{11}, σ_{22}, and σ_{33}).

There are also two simplified cases when only two strain components are measured. The first is **in-plane strain**, where $\epsilon_{33} = 0$. Then the strain equations become

$$\sigma_{11} = \frac{E}{(1+\nu)} \left[\epsilon_{11} + \frac{\nu}{1-2\nu} (\epsilon_{11} + \epsilon_{22}) \right] \tag{4}$$

$$\sigma_{22} = \frac{E}{(1+\nu)} \left[\epsilon_{22} + \frac{\nu}{1-2\nu} (\epsilon_{11} + \epsilon_{22}) \right] \tag{5}$$

$$\sigma_{33} = \frac{E\nu(\epsilon_{11} + \epsilon_{22})}{(1+\nu)(1-2\nu)} \tag{6}$$

The **in-plane stress**, assumes $\sigma_{33} = 0$. Therefore, ϵ_{33} can be calculated from ϵ_{11} and ϵ_{22} from $\sigma_{33} = 0$. Then the missing strain can be determined to be

$$\epsilon_{33} = \frac{\nu}{\nu - 1} (\epsilon_{11} + \epsilon_{22}) \tag{7}$$

With that relation, the stresses (with the in-plane stress assumption) are

$$\sigma_{11} = \frac{E}{(1+\nu)}\left[\epsilon_{11} + \frac{\nu(\epsilon_{11}+\epsilon_{22})}{1-\nu}\right] \tag{8}$$

$$\sigma_{22} = \frac{E}{(1+\nu)}\left[\epsilon_{22} + \frac{\nu(\epsilon_{11}+\epsilon_{22})}{1-\nu}\right] \tag{9}$$

$$\sigma_{33} = 0 \tag{10}$$