

# Physics 129L Final Project: Investigating the Outflow Dynamics of Active Galactic Nuclei through Computational Simulations

## AGN Galaxy Simulation

My goal of this project is to study the outflow dynamics of Active Galactic Nuclei (AGNs) since it is crucial for comprehending their roles in galaxy formation and evolution and aims to simulate the outflow properties of AGNs using computational techniques to understand their process and implications for galaxy evolution.

However, before starting analyzing the properties of AGN, we need some sample of AGN host galaxies to start with, and this is one of the most complex process since there are too many things to be considered. We start by considering some most important factors of AGN host galaxies in the simulation:

1. **Modeling the Black Holes:** Simulating the behavior of supermassive black holes at the centers of these galaxies, including their mass, spin, and accretion rates.
2. **Modeling the Gas Dynamics:** Modeling the accretion of gas onto the black hole and the resulting jets and winds.
3. **Consider the Galactic Interactions:** Considering the gravitational interactions between galaxies, especially in dense environments like galaxy clusters.
4. **Consider the Feedback Processes:** Including the effects of the AGN on its host galaxy, like outflows that can heat up or expel gas, affecting star formation.
5. **Consider the Fact that it is a Part of Our Universe (Cosmological Context):** Placing these galaxies within the larger structure of the universe, considering the expansion of the universe, dark matter distribution, and the cosmic web.
6. **Include the Radiation Processes:** Simulating the radiation emitted by the AGN, which includes a wide spectrum from radio to gamma rays.

I think the best way to simulate such galaxies is to create a galaxy class and define these above properties separately. So the first step is to initiate a class called galaxy with basic parameters.

```
In [93]: # First, import useful packages (subject to change, will be adding all the p
import numpy as np
import scipy as sp
import random
import matplotlib.pyplot as plt
import corner
import seaborn as sns
from scipy.stats import norm
from scipy.optimize import minimize
from scipy.optimize import curve_fit
from scipy.stats import gaussian_kde
from astropy import units as u
```

```
In [2]: # Second, define some constant
# (unit included by using the astropy package units)
'''Gravitational constant'''
G = 6.67430e-11 * u.m**3 / u.kg / u.s**2
'''speed of light'''
c = 299792458 * u.m / u.s
'''number of galaxies to be simulated'''
num_galaxies = 1000
'''time steps'''
time_steps = 100
```

```
In [3]: # Now we can start by defining a class of galaxy, and think about the structure
class Galaxy:
    # initialize the galaxy
    def __init__(self, mass, position, velocity, black_hole_mass):
        self.mass = mass
        self.position = position
        self.velocity = velocity
        self.black_hole_mass = black_hole_mass

    # black hole accretion model
    def accretion_model(black_hole_mass):
        """
        This will be an implement or accresion model for black hole accretion
        This should return accretion_rate, radiation, jets
        """
        pass

    # Gas Dynamics and Interstellar Medium
    def gas_dynamics(galaxy):
        """
        This models the implement model of the dynamics of gas in and around
        The processes like cooling, heating, and interaction with AGN radiat
        """
        pass

    # Galaxy Interaction and Dynamics
    def update_galaxy_dynamics(galaxies):
        """
        Implement gravitational interactions between galaxies.
        Update positions and velocities based on gravitational forces.
        """
        pass
```

## A) Accretion Model

The accretion model also contains many parts:

1. accretion rate
2. radiation
3. jet production

I will do these step by step by using and analyzing the equations that describe each model

## 1. Accretion Rate

The accretion rate of a black hole often depends on the **mass of the black hole**, the **density of the surrounding medium**, and the **velocity at which this medium is moving relative to the black hole**.

I here use the Bondi-Hoyle-Lyttleton accretion rate model:  
[\(https://en.wikipedia.org/wiki/Bondi\\_accretion\)](https://en.wikipedia.org/wiki/Bondi_accretion)

$$\dot{M} = \pi \left( \frac{2G^2 M^2 \rho_\infty}{(v_\infty^2 + c_s^2)^{3/2}} \right) \quad (1)$$

$\dot{M}$  : Accretion rate (1)

$G$  : Gravitational constant (2)

$M$  : Mass of the accreting object (3)

$\rho_\infty$  : Density of the ambient medium (far from the accreting object) (4)

$v_\infty$  : Relative velocity between the object and the ambient medium (5)

$c_s$  : Sound speed of the ambient medium (6)

Since I am working with AGN host galaxies, some of the parameters including  $\rho_\infty$ ,  $v_\infty$ , and  $c_s$  are constrained. However, they are also kind of randomized within the range.

I was thinking if I should use a constant or use a randomization function. However, since the blackhole mass are already an input, I will keep these three fixed first. Later on, I can also test how these values impact the output accretion rate solar masses per year. I made the default values  $\rho_\infty = 10^{-13} \text{ kg/m}^3$ ,  $v_\infty = 5 \times 10^7 \text{ m/s}$ , and  $c_s = 5 \times 10^5 \text{ m/s}$  just to start with.

```
In [4]: # based on this, we can define a accretion rate function called calculate_ac  
# returns the accretion rate solar masses per year  
def calculate_accretion_rate(black_hole_mass,  
                           rho = np.random.uniform(1e-14, 1e-13),  
                           v = np.random.uniform(4.5e7, 5.5e7),  
                           c_s = np.random.uniform(1e4, 2e6)):  
    ...  
    The input black hole mass should in the unit of solar mass  
    $\\rho_{\\infty} = 10^{-13} \\text{ kg/m}^3$, $v_{\\infty} = 5 \\times 10^7 \\text{ m/s}$, and $c_<  
    ...  
  
    G = 6.67430e-11 * u.m**3 / u.kg / u.s**2 # gravitational constant with u  
    M = black_hole_mass * 1.98847e30 * u.kg # solar masses to kg  
  
    rho = rho * u.kg / u.m**3 # density of the accreting material in kg/m^3  
    v = v * u.m / u.s # relative velocity in m/s  
    c_s = c_s * u.m / u.s # sound speed in the material in m/s  
  
    # Bondi-Hoyle-Lyttleton accretion rate  
    accretion_rate = (4 * np.pi * G**2 * M**2 * rho) / ((v**2 + c_s**2)**(3/2))  
  
    # Convert accretion rate to solar masses per year  
    accretion_rate_solar_masses_per_year = accretion_rate.to(u.kg / u.s) / (M * 365 * 24 * 60 * 60)  
  
    return accretion_rate_solar_masses_per_year
```

```
In [5]: # Test the function calculate_accretion_rate  
print(calculate_accretion_rate(100))
```

8.744656068435424e-15 yr / s

To test if this function works well, and if rho, c, c\_s contributes a lot to the function (if these parameters impacts it significantly, then we want to reconsider the definition of the function)

In [6]:

```
'''Plot Accretion Rate in term of Black Hole Masses with varies Parameters'''
# Randomize one parameter at a time, keeping the others fixed
num_points = 1000
fixed_rho = 1e-13 # Fixed density
fixed_v = 5e7      # Fixed relative velocity
fixed_c_s = 1e5    # Fixed sound speed

# Randomize black hole masses
black_hole_masses = np.random.uniform(1, 100, num_points)

# Randomizing rho while keeping v and c_s fixed
rho_values = np.random.uniform(5e-14, 1e-13, num_points)
accretion_rates_rho = [calculate_accretion_rate(mass, rho=rho, v=fixed_v, c_s=fixed_c_s) for mass in black_hole_masses]

# Randomizing v while keeping rho and c_s fixed
v_values = np.random.uniform(4.5e7, 5.5e7, num_points)
accretion_rates_v = [calculate_accretion_rate(mass, rho=fixed_rho, v=v, c_s=fixed_c_s) for mass in black_hole_masses]

# Randomizing c_s while keeping rho and v fixed
c_s_values = np.random.uniform(1e4, 2e6, num_points)
accretion_rates_c_s = [calculate_accretion_rate(mass, rho=fixed_rho, v=fixed_v, c_s=c_s) for mass in black_hole_masses]

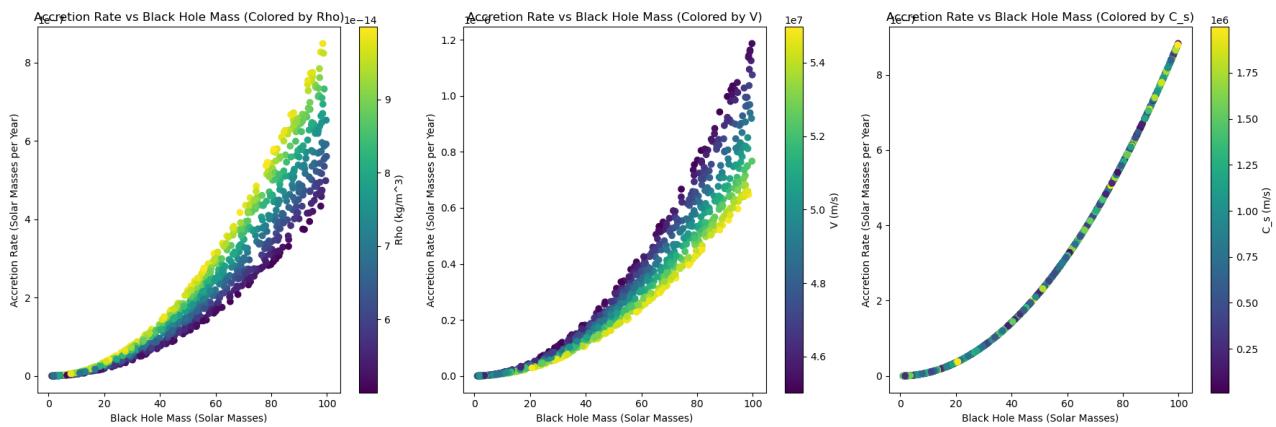
plt.figure(figsize=(18, 6))

# Scatter plot for rho
plt.subplot(131)
plt.scatter(black_hole_masses, accretion_rates_rho, c=rho_values, cmap='viridis')
plt.colorbar(label='Rho (kg/m^3)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Accretion Rate (Solar Masses per Year)')
plt.title('Accretion Rate vs Black Hole Mass (Colored by Rho)')

# Scatter plot for v
plt.subplot(132)
plt.scatter(black_hole_masses, accretion_rates_v, c=v_values, cmap='viridis')
plt.colorbar(label='V (m/s)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Accretion Rate (Solar Masses per Year)')
plt.title('Accretion Rate vs Black Hole Mass (Colored by V)')

# Scatter plot for c_s
plt.subplot(133)
plt.scatter(black_hole_masses, accretion_rates_c_s, c=c_s_values, cmap='viridis')
plt.colorbar(label='C_s (m/s)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Accretion Rate (Solar Masses per Year)')
plt.title('Accretion Rate vs Black Hole Mass (Colored by C_s)')

plt.tight_layout()
plt.show()
```



According to the output plots, adjust the randomization to an acceptable range, and update the function calculate\_accretion\_rate with these randomization range.

```
In [7]: # observe the correlation between black hole mass, density of ambient object
# velocity between the object and the ambient medium, and Sound speed of the
def pearson_correlation(x, y):
    """
    Calculates the Pearson correlation coefficient between two arrays x and
    """
    mean_x = np.mean(x)
    mean_y = np.mean(y)
    numerator = np.sum((x - mean_x) * (y - mean_y))
    denominator = np.sqrt(np.sum((x - mean_x)**2) * np.sum((y - mean_y)**2))

    return numerator / denominator if denominator != 0 else 0

# Monte Carlo Simulation Parameters
num_simulations = 10000
black_hole_mass_range = (1, 100) # in solar masses
rho_range = (1e-14, 1e-13) # in kg/m^3
v_range = (4.5e7, 5.5e7) # in m/s
c_s_range = (1e6, 2e7) # in m/s

masses = np.random.uniform(*black_hole_mass_range, num_simulations)
rhos = np.random.uniform(*rho_range, num_simulations)
vs = np.random.uniform(*v_range, num_simulations)
c_ss = np.random.uniform(*c_s_range, num_simulations)
accretion_rates = np.array([calculate_accretion_rate(m, rho, v, c_s) for m,
                           rho, v, c_s in zip(masses, rhos, vs, c_ss)])

data = np.vstack([masses, rhos, vs, c_ss, accretion_rates]).T

# Labels for each parameter
labels = ['Mass (solar masses)', 'Density (kg/m³)', 'Velocity (m/s)', 'Sound Speed (m/s)']

figure = corner.corner(data, labels=labels, show_titles=True, title_kwargs={
    'color': 'skyblue',
    'hist_kwarg': {'density': True, 'histtype': 'step', 'linestyle': 'solid'},
    'plot_kwarg': {'color': 'red', 'marker': 'x', 'label': 'Data Points'},
    'plot_density': True,
    'plot_datapoints': False,
    'title': 'Accretion Rate vs Black Hole Mass (Colored by C_s)'})
```

```
        fill_contours=True,
        contour_kwarg={'colors': 'black', 'linewidths': 1})
plt.show()

# Monte Carlo Simulation Parameters
num_simulations = 10000
black_hole_mass_range = (1, 100) # in solar masses
rho_range = (1e-14, 1e-13) # in kg/m^3
v_range = (4.5e7, 5.5e7) # in m/s
c_s_range = (1e6, 2e7) # in m/s

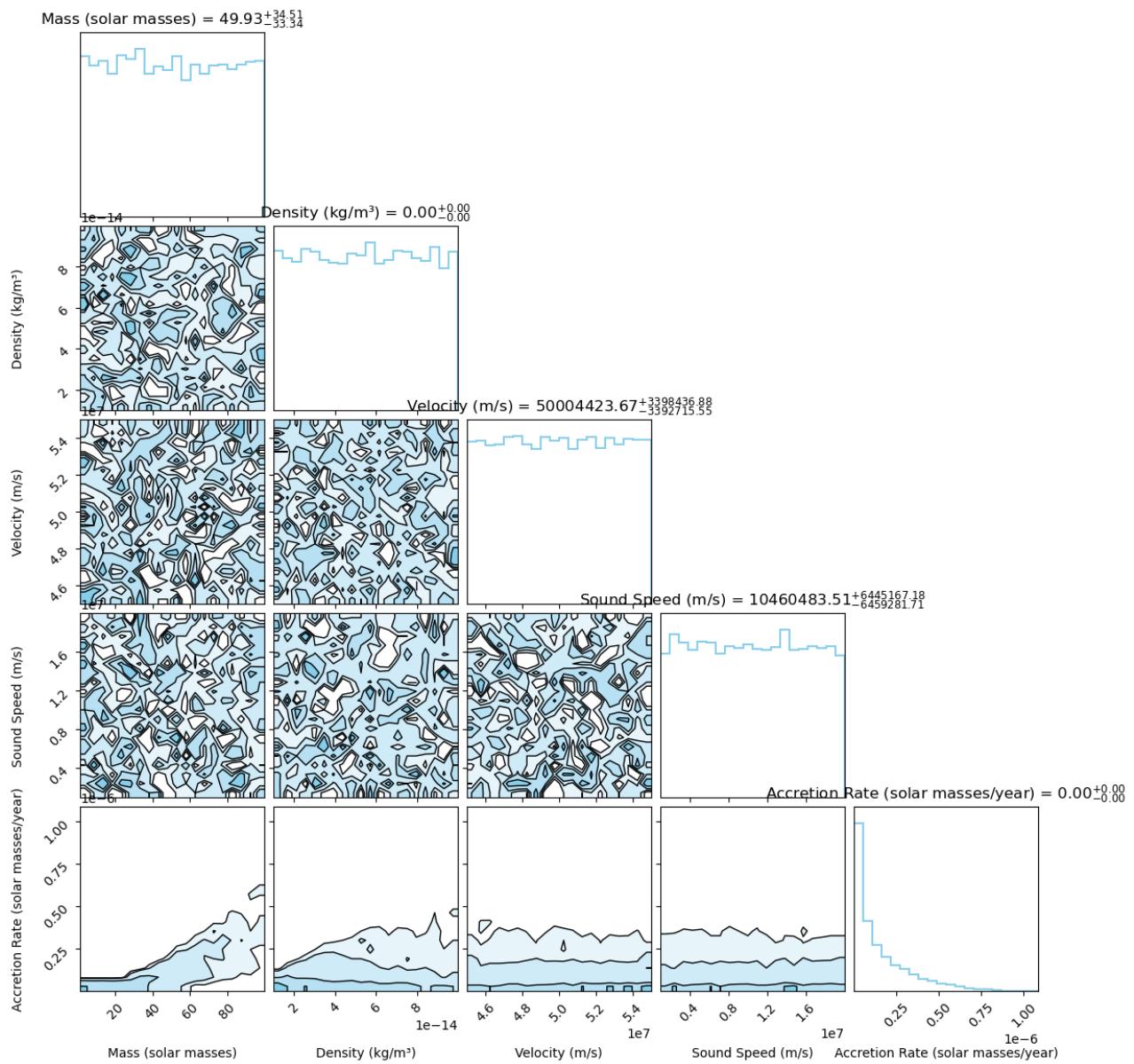
# Generating random data for each parameter
masses = np.random.uniform(*black_hole_mass_range, num_simulations)
rhos = np.random.uniform(*rho_range, num_simulations)
vs = np.random.uniform(*v_range, num_simulations)
c_ss = np.random.uniform(*c_s_range, num_simulations)

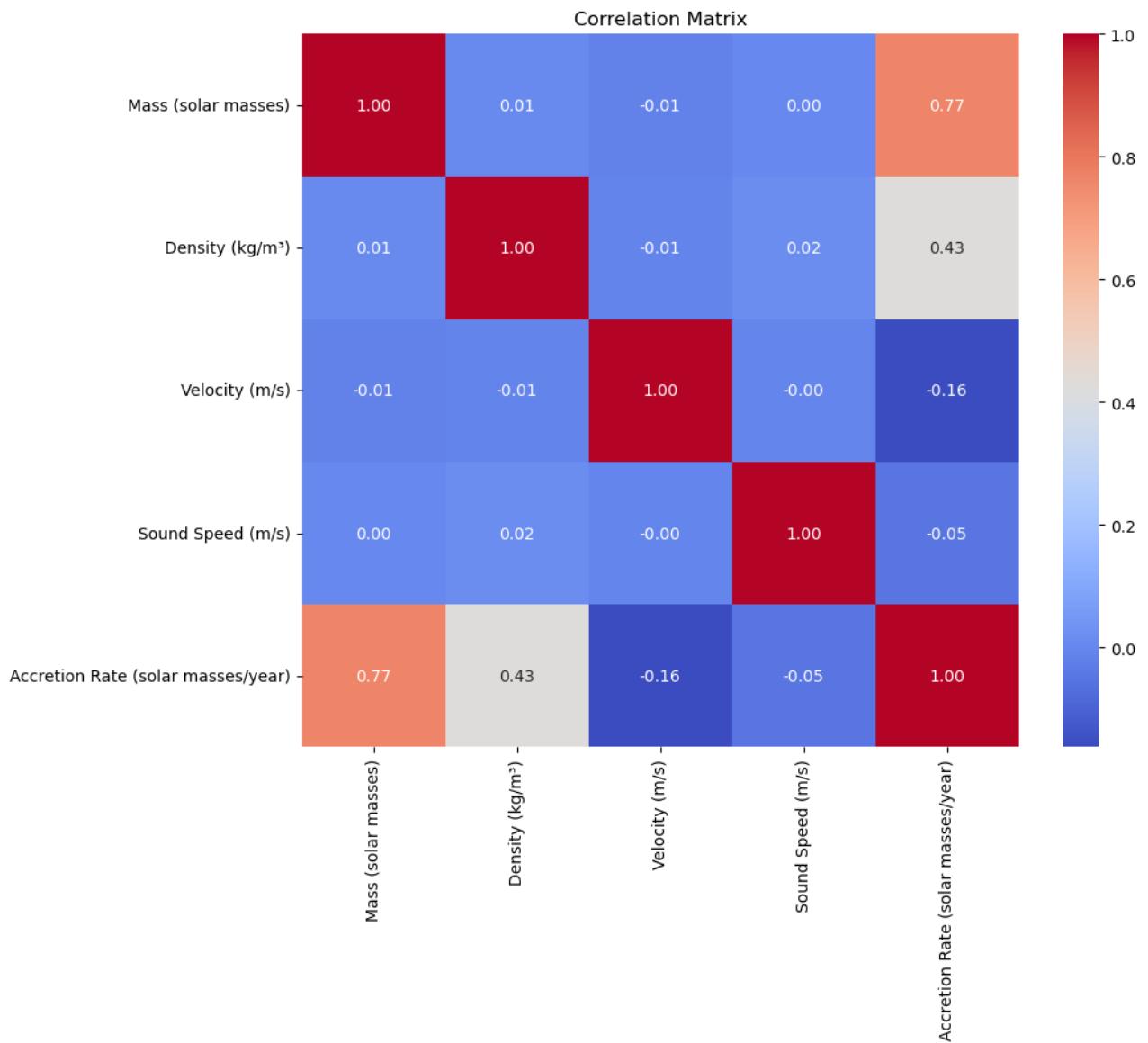
# Calculating accretion rates
accretion_rates = np.array([calculate_accretion_rate(m, rho, v, c_s) for m,

# Stacking the data and calculating the correlation matrix
data = np.vstack([masses, rhos, vs, c_ss, accretion_rates]).T
correlation_matrix = np.array([[pearson_correlation(data[:, i], data[:, j])]

# Labels for each parameter
labels = ['Mass (solar masses)', 'Density (kg/m³)', 'Velocity (m/s)', 'Sound

# Creating the correlation matrix heatmap
plt.figure(figsize=(10, 8))
sns.heatmap(correlation_matrix, annot=True, xticklabels=labels, yticklabels=labels,
            plt.title('Correlation Matrix')
            plt.show()
```





Here I run 10000 times monte carlo simulation to explore the correlation between our interested parameters. This is as what was expected: the mass of black hole contributes significantly to the accretion rate. However, note that the density also has a relatively high correlation with the accretion rate comparing to other parameters, so I should keep this in mind in the future work.

## 2. Radiation

Since radiation is emitted by the accretion process as luminosity, it is important to consider the radiation in doing the modeling. The luminosity is typically calculated by (<https://en.wikipedia.org/wiki/Luminosity#:~:text=Luminosity%20is%20an%20absolute%20r>

$$Radiation = \epsilon \times \dot{M} \times 1.98847 \times 10^{30} \times c^2$$

where

$\dot{M}$  : Efficiency factor - typically around 0.1 for non-rotating black holes (7)

$c$  : speed of light (8)

$R_{acc}$  : accretion rate (9)

```
In [8]: def calculate_radiation(accretion_rate, epsilon = 0.1):
    """
    The accretion rate is obtained from the previous function calculate_accretion_rate
    """
    c = 299792458 * u.m / u.s

    # calculate the luminosity (or radiation)
    radiation = epsilon * accretion_rate * 1.98847e30 * c**2 # Convert accretion rate to mass flow rate

    return radiation
```

```
In [9]: # Test the function calculate_radiation
print(calculate_radiation(calculate_accretion_rate(100)))

1.5627992069740066e+32 m2 yr / s3
```

```
In [10]: '''Plot Radiation in term of Black Hole Masses with varies Parameters'''
# Randomize the parameters and calculate accretion rates and radiation
num_points = 1000
fixed_rho = 1e-13 # Fixed density
fixed_v = 5e7      # Fixed relative velocity
fixed_c_s = 5e6    # Fixed sound speed

# Randomize black hole masses
black_hole_masses = np.random.uniform(1, 100, num_points)

# Randomizing rho while keeping v and c_s fixed
rho_values = np.random.uniform(5e-14, 1e-13, num_points)
accretion_rates_rho = [calculate_accretion_rate(mass, rho=rho, v=fixed_v, c_s=fixed_c_s) for mass in black_hole_masses]
radiation_rho = [calculate_radiation(rate) for rate in accretion_rates_rho]

# Randomizing v while keeping rho and c_s fixed
v_values = np.random.uniform(4.5e7, 5.5e7, num_points)
accretion_rates_v = [calculate_accretion_rate(mass, rho=fixed_rho, v=v, c_s=fixed_c_s) for mass in black_hole_masses]
radiation_v = [calculate_radiation(rate) for rate in accretion_rates_v]
```

```

radiation_v = [calculate_radiation(rate) for rate in accretion_rates_v]

# Randomizing c_s while keeping rho and v fixed
c_s_values = np.random.uniform(1e6, 2e7, num_points)
accretion_rates_c_s = [calculate_accretion_rate(mass, rho=fixed_rho, v=fixed_v) for mass in black_hole_masses]
radiation_c_s = [calculate_radiation(rate) for rate in accretion_rates_c_s]

radiation_rho = [x.value for x in radiation_rho]
radiation_v = [x.value for x in radiation_v]
radiation_c_s = [x.value for x in radiation_c_s]

# Plotting the results
plt.figure(figsize=(18, 6))

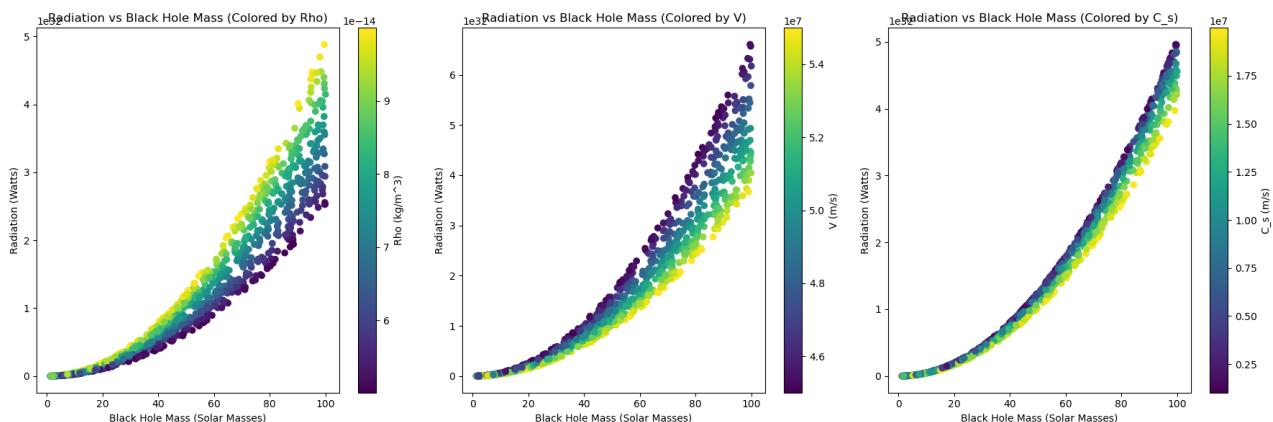
# Scatter plot for radiation vs black hole mass (Colored by Rho)
plt.subplot(131)
plt.scatter(black_hole_masses, radiation_rho, c=rho_values, cmap='viridis')
plt.colorbar(label='Rho (kg/m^3)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Radiation (Watts)')
plt.title('Radiation vs Black Hole Mass (Colored by Rho)')

# Scatter plot for radiation vs black hole mass (Colored by V)
plt.subplot(132)
plt.scatter(black_hole_masses, radiation_v, c=v_values, cmap='viridis')
plt.colorbar(label='V (m/s)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Radiation (Watts)')
plt.title('Radiation vs Black Hole Mass (Colored by V)')

# Scatter plot for radiation vs black hole mass (Colored by C_s)
plt.subplot(133)
plt.scatter(black_hole_masses, radiation_c_s, c=c_s_values, cmap='viridis')
plt.colorbar(label='C_s (m/s)')
plt.xlabel('Black Hole Mass (Solar Masses)')
plt.ylabel('Radiation (Watts)')
plt.title('Radiation vs Black Hole Mass (Colored by C_s)')

plt.tight_layout()
plt.show()

```



### 3. jet production

The jet which emits energy is determined by many factor and is usually very hard to model, so we here assume a simpler case where the jets are produced when the accretion rate and radiation exceed certain limit. We assume the limits to be:

1. accretion rate threshold =  $10^{-4}$  solar mass per year
2. radiation threshold =  $10^{36}$  watts

```
In [11]: def determine_jet_production(accretion_rate, radiation):
    # Limits when jet production takes place
    accretion_rate_threshold = 1e-4 # in solar masses per year
    radiation_threshold = 1e36

    # Determine jet production
    jets = accretion_rate > accretion_rate_threshold and radiation > radiation_threshold
    return jets
    # returns true or false
```

```
In [12]: # Examples for determine_jet_production
print(determine_jet_production(1e-7, 1e20)) # neither parameter exceeds the limit
print(determine_jet_production(1, 1e30)) # the accretion rate exceeds the limit
print(determine_jet_production(1e-5, 1e40)) # the radiation exceeds the limit
print(determine_jet_production(1, 1e40)) # both exceeds the limit
```

```
False
False
False
True
```

### combining the three function

```
In [13]: def accretion_model(black_hole_mass, accretion_radius, other_parameters):
    accretion_rate = calculate_accretion_rate(black_hole_mass, accretion_radius)
    radiation = calculate_radiation(accretion_rate, other_parameters)
    jets = determine_jet_production(accretion_rate, radiation, other_parameters)

    return accretion_rate, radiation, jets
```

Now I can add these functions to the original class as the first part of the code:

```
In [14]: class Galaxy:
    '''initialize the galaxy'''
    def __init__(self, mass, position, velocity, black_hole_mass):
        self.mass = mass
        self.position = position
        self.velocity = velocity
```

```

    self.black_hole_mass = black_hole_mass

'''black hole accretion model'''
def calculate_accretion_rate(black_hole_mass,
                               rho = np.random.uniform(1e-14, 1e-13),
                               v = np.random.uniform(4.5e7, 5.5e7),
                               c_s = np.random.uniform(1e6, 2e7)):
    # The input black hole mass should in the unit of solar mass
    G = 6.67430e-11 * u.m**3 / u.kg / u.s**2 # gravitational constant
    M = black_hole_mass * 1.98847e30 * u.kg # solar masses to kg
    rho = rho * u.kg / u.m**3 # density of the accreting material in kg
    v = v * u.m / u.s # relative velocity in m/s
    c_s = c_s * u.m / u.s # sound speed in the material in m/s
    # Bondi-Hoyle-Lyttleton accretion rate
    accretion_rate = (4 * np.pi * G**2 * M**2 * rho) / ((v**2 + c_s**2)**
    # Convert accretion rate to solar masses per year
    accretion_rate_solar_masses_per_year = accretion_rate.to(u.kg / u.s)
    return accretion_rate_solar_masses_per_year

def calculate_radiation(accretion_rate, epsilon = 0.1):
    # The accretion rate is obtained from the previous function calculated
    c = 299792458 * u.m / u.s
    # calculate the luminosity (or radiation)
    radiation = epsilon * accretion_rate * 1.98847e30 * c**2 # Convert
    return radiation

def accretion_model(black_hole_mass, accretion_radius, other_parameters):
    accretion_rate = calculate_accretion_rate(black_hole_mass, accretion_radius)
    radiation = calculate_radiation(accretion_rate, other_parameters)
    jets = determine_jet_production(accretion_rate, radiation, other_parameters)
    return accretion_rate, radiation, jets

# Gas Dynamics and Interstellar Medium
def gas_dynamics(galaxy):
    """
    This models the implement model of the dynamics of gas in and around
    The processes like cooling, heating, and interaction with AGN radiation
    """
    pass

# Galaxy Interaction and Dynamics
def update_galaxy_dynamics(galaxies):
    """
    Implement gravitational interactions between galaxies.
    Update positions and velocities based on gravitational forces.
    """
    pass

```

## B) Cooling and Heating

According to my research of other people's work, the fully modeled dynamic of mass cannot be easily computed from a local computer and requires much more computation work. Thus, for my work, I will be creating a easier model with more idealized condition than what is actually happening in AGN galaxies but keep all the key concepts in AGN gas dynamics simulation.

The temperature change within the galaxy is another important thing to consider. From previous study (Li et al 2016), they analyzed a battle between radiative cooling and AGN heating. Thus the cooling and heating are happening at the same time within the AGN host galaxy. That being said, we want to set different rate of cooling and heating in terms of other parameters.

This is complex as a lot of parameters could contribute to the cooling/ heating rates, such as mass, cooling coefficient, environment, AGN activity, and many more. We here consider some of the most significant factor in cooling and heating, which are:

1. **density**: Cooling processes in the ISM are significantly influenced by the gas's density, and it is usually proportional to density square
2. **cooling coefficient**
3. **temperature**: Cooling processes in the ISM are inversely proportional to temperature
4. **time**: The rate might decrease over time as the galaxy loses energy
5. **composition**: The composition of AGN galaxies are really hard to determine, I will keep this away from the simulation for now, but may come back to it as this project becomes more developed
6. **metallicity**
7. **star formation rate**
8. **AGN luminosity**: this was calculated above with the radiation

```
In [15]: # combining what has been discussed above, cooling rate is proportional to density
# and inversely proportional to time and temperature
def cooling_function(temperature):
    if temperature < 1e4:
        # Low-temperature regime
        new_temp = 1e-22 * temperature
        return new_temp
    else:
        # High-temperature regime
        new_temp = 1e-24 * temperature**1.5
        return new_temp

def heating_function(star_formation_initial, agn_luminosity_initial, density,
                    decay_factor = 0.05 # Represents a decay over time
                    # Star formation rate and AGN luminosity decrease over time
                    star_formation_rate = star_formation_initial / (1 + decay_factor * time)
                    agn_luminosity = agn_luminosity_initial / (1 + decay_factor * time))

    return 1e-35 * (star_formation_rate + agn_luminosity) * density

def cooling_rate(new_temp, density):
    cooling_rate = new_temp * density**2
    return cooling_rate

def ism_temperature_evolution(initial_temperature, density, metallicity, star_formation_rate,
                             agn_luminosity, total_time, timestep):
    temperature = initial_temperature

    for time in range(total_time):
        # cooling and heating rates
        cooling_rate = cooling_function(temperature) * density**2
        heating_rate = heating_function(star_formation_rate, agn_luminosity, density)

        # Update temperature
        temperature_change = (heating_rate - cooling_rate) * timestep
        temperature += temperature_change

        if temperature < 10: # 10 K is a reasonable lower limit for ISM temperature
            temperature = 10

    return temperature
```

```
In [16]: # Test the function
initial_temperature = 1000000 # in Kelvin
density = 1.0e5 # in particles per cubic centimeter
metallicity = 1.0 # Solar metallicity
star_formation_rate = 2.0 # Solar masses per year
agn_luminosity = 1e44 # in ergs per second
timestep = 1e3 # in years
total_time = 1000 # number of timesteps
final_temperature = ism_temperature_evolution(initial_temperature, density, metallicity, star_formation_rate, agn_luminosity, timestep, total_time)
print(f"Final ISM Temperature: {final_temperature} K")
```

Final ISM Temperature: 3.669852783696508e+17 K

## Week 2 (Milestone2 start)

In [17]:

```
def ism_temperature_evolution_over_time(initial_temperature,
                                         density,
                                         metallicity,
                                         star_formation_initial,
                                         agn_luminosity_initial,
                                         timestep,
                                         total_time):
    temperature = initial_temperature
    temperatures, cooling_rates, heating_rates = [], [], []

    for time in range(total_time):
        cooling_rate = cooling_function(temperature) * density**2
        heating_rate = heating_function(star_formation_initial, agn_luminosity)

        temperature_change = (heating_rate - cooling_rate) * timestep
        # print(heating_rate, cooling_rate)
        temperature += temperature_change
        if temperature < 10:
            temperature = 10

        temperatures.append(temperature)
        cooling_rates.append(cooling_rate)
        heating_rates.append(heating_rate)

    return temperatures, cooling_rates, heating_rates
```

In [18]:

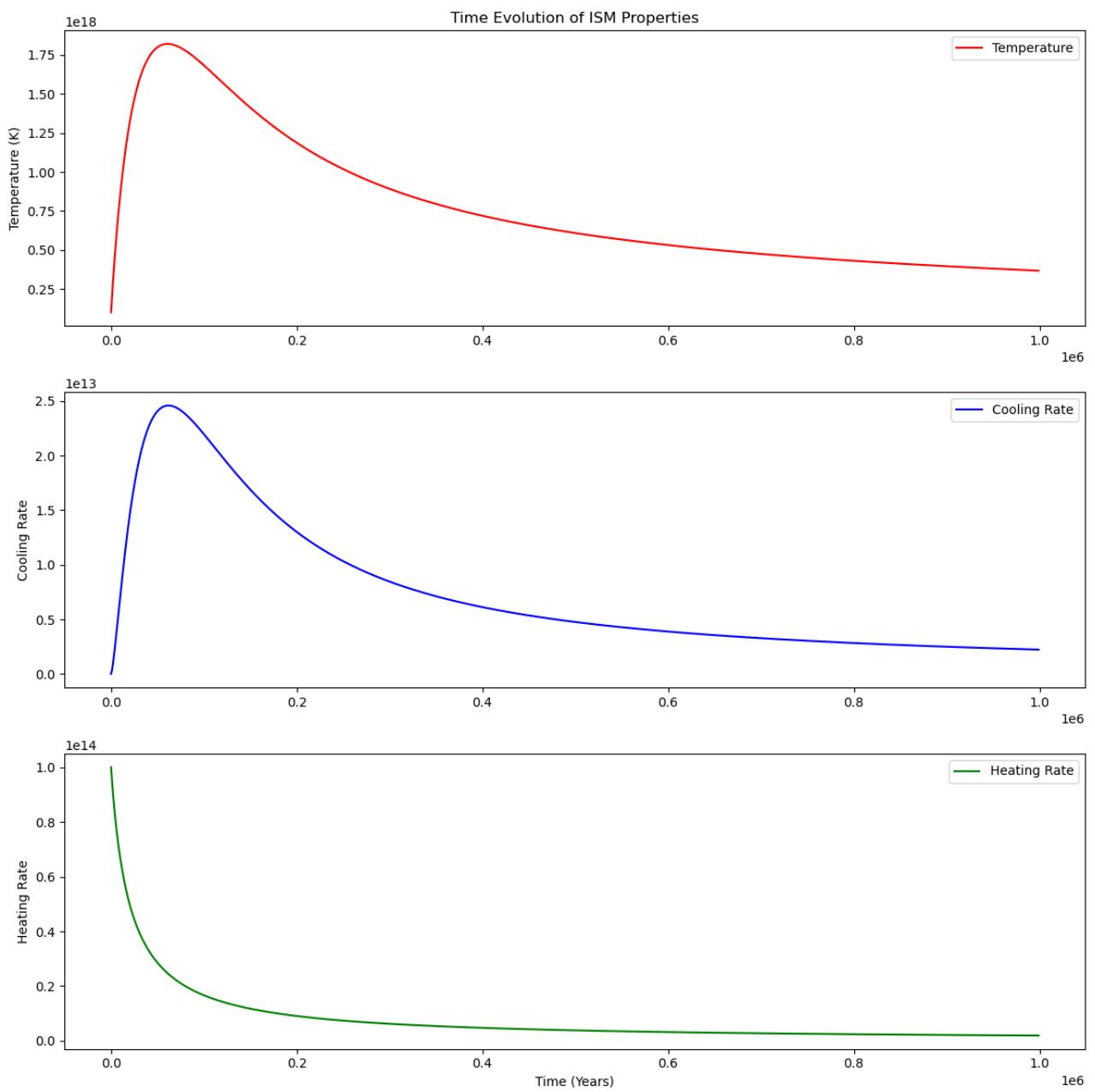
```
# simulation
temperatures, cooling_rates, heating_rates = ism_temperature_evolution_over_
time_axis = [timestep * i for i in range(total_time)]
plt.figure(figsize=(12, 12))

# Temperature
plt.subplot(3, 1, 1)
plt.plot(time_axis, temperatures, label='Temperature', color='red')
plt.ylabel('Temperature (K)')
plt.title('Time Evolution of ISM Properties')
plt.legend()

# Cooling
plt.subplot(3, 1, 2)
plt.plot(time_axis, cooling_rates, label='Cooling Rate', color='blue')
plt.ylabel('Cooling Rate')
plt.legend()

# Heating
plt.subplot(3, 1, 3)
plt.plot(time_axis, heating_rates, label='Heating Rate', color='green')
plt.ylabel('Heating Rate')
plt.xlabel('Time (Years)')
plt.legend()

plt.tight_layout()
plt.show()
```



From this plot, it seems like the temperature follows similar evolution as the cooling rate, however, when I did the attempts of changing the value of density, this trend varies a lot. This is impressive and should be further explored, so I want to plot out the evolution of ISM temperature and cooling/heating rates corresponding to different density environment. This is crucial as it reveals potential environmental impact on AGN galaxy evolution. Some methods I could use is to

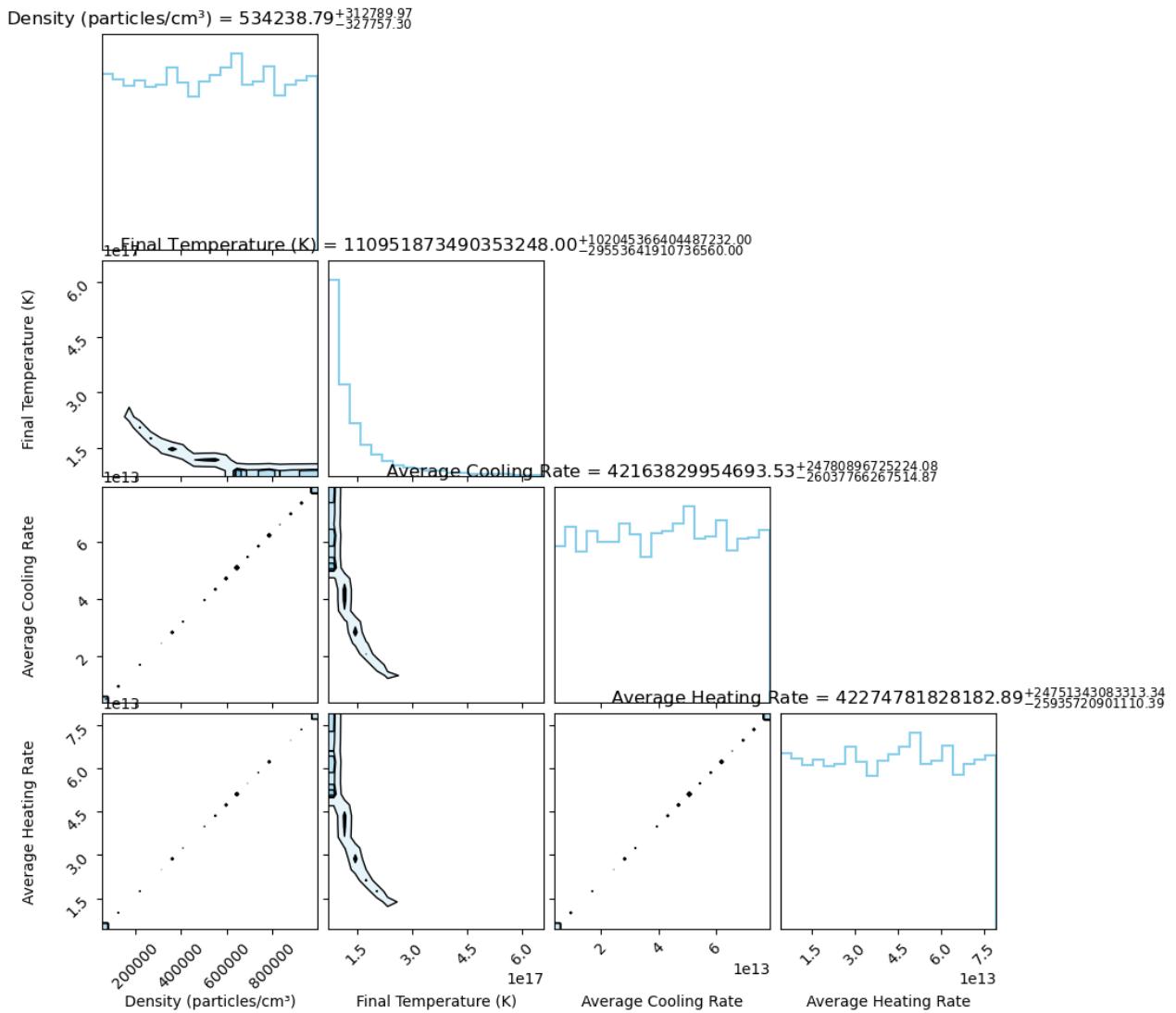
1. again, use the monte carlo simulation to find the correlation, and
2. plot out the rates and temperature at different density to observe the trends.

```
In [19]: # monte carlo simulation to find the correlation between density and rates/t
num_simulations = 10000
density_range = (5.5e4, 1e6)

initial_temperature = 1000000 # in Kelvin
metallicity = 1.0 # Solar metallicity
star_formation_rate = 2.0 # Solar masses per year
agn_luminosity = 1e44 # in ergs per second
timestep = 1e3 # in years
total_time = 1000 # number of timesteps

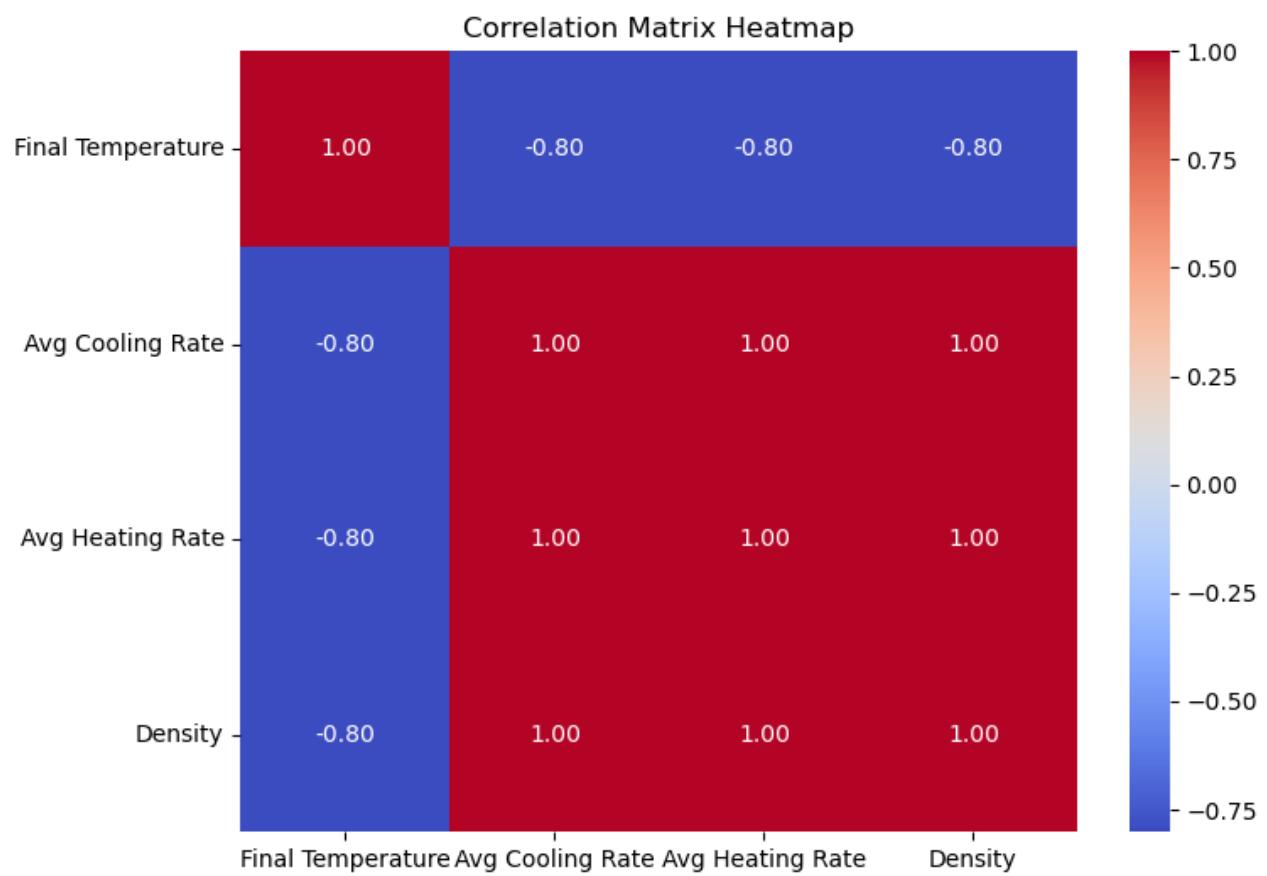
densities = np.random.uniform(*density_range, num_simulations)
final_temperatures = []
average_cooling_rates = []
average_heating_rates = []

for density in densities:
    temperatures, cooling_rates, heating_rates = ism_temperature_evolution_c
        initial_temperature, density, metallicity, star_formation_rate,
        agn_luminosity, timestep, total_time
    )
    final_temperatures.append(temperatures[-1])
    average_cooling_rates.append(np.mean(cooling_rates))
    average_heating_rates.append(np.mean(heating_rates))
simulation_data = np.vstack([densities, final_temperatures, average_cooling_
corner_labels = ['Density (particles/cm³)', 'Final Temperature (K)', 'Averag
corner_figure = corner.corner(simulation_data, labels=corner_labels, show_t
                                color='skyblue',
                                hist_kwarg
                                plot_densit
                                plot_datapoint
                                fill_contour
                                contour_kwarg
plt.show()
```



```
In [20]: # the heatmap for correlations between
# temperature, cooling rate, heating rate, and density
simulation_data_array = np.array([final_temperatures, average_cooling_rates,
correlation_matrix = np.corrcoef(simulation_data_array)
heatmap_labels = ['Final Temperature', 'Avg Cooling Rate', 'Avg Heating Rate']

plt.figure(figsize=(8, 6))
sns.heatmap(correlation_matrix, annot=True, xticklabels=heatmap_labels, ytic
plt.title('Correlation Matrix Heatmap')
plt.show()
```



```
In [21]: selected_densities = np.linspace(density_range[0], density_range[1], 10)

plt.figure(figsize=(12, 18))

for density in selected_densities:
    temperatures, cooling_rates, heating_rates = ism_temperature_evolution_c
        initial_temperature, density, metallicity, star_formation_rate,
        agn_luminosity, timestep, total_time
    )
    time_axis = [timestep * i for i in range(total_time)]

    # Temperature
    plt.subplot(3, 1, 1)
    plt.plot(time_axis, temperatures, label=f'Density {density:.0e} particles/cm³')

    # cooling
    plt.subplot(3, 1, 2)
    plt.plot(time_axis, cooling_rates, label=f'Density {density:.0e} particles/cm³')

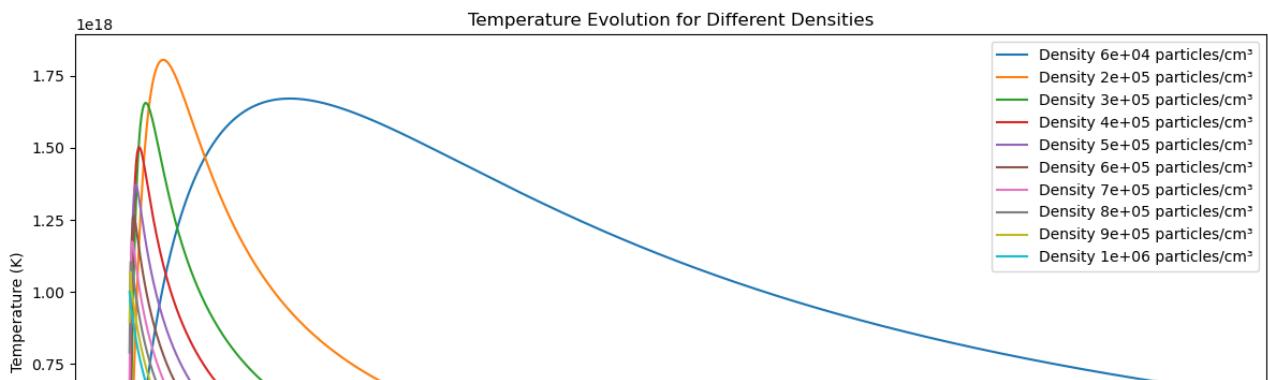
    # heating
    plt.subplot(3, 1, 3)
    plt.plot(time_axis, heating_rates, label=f'Density {density:.0e} particles/cm³')

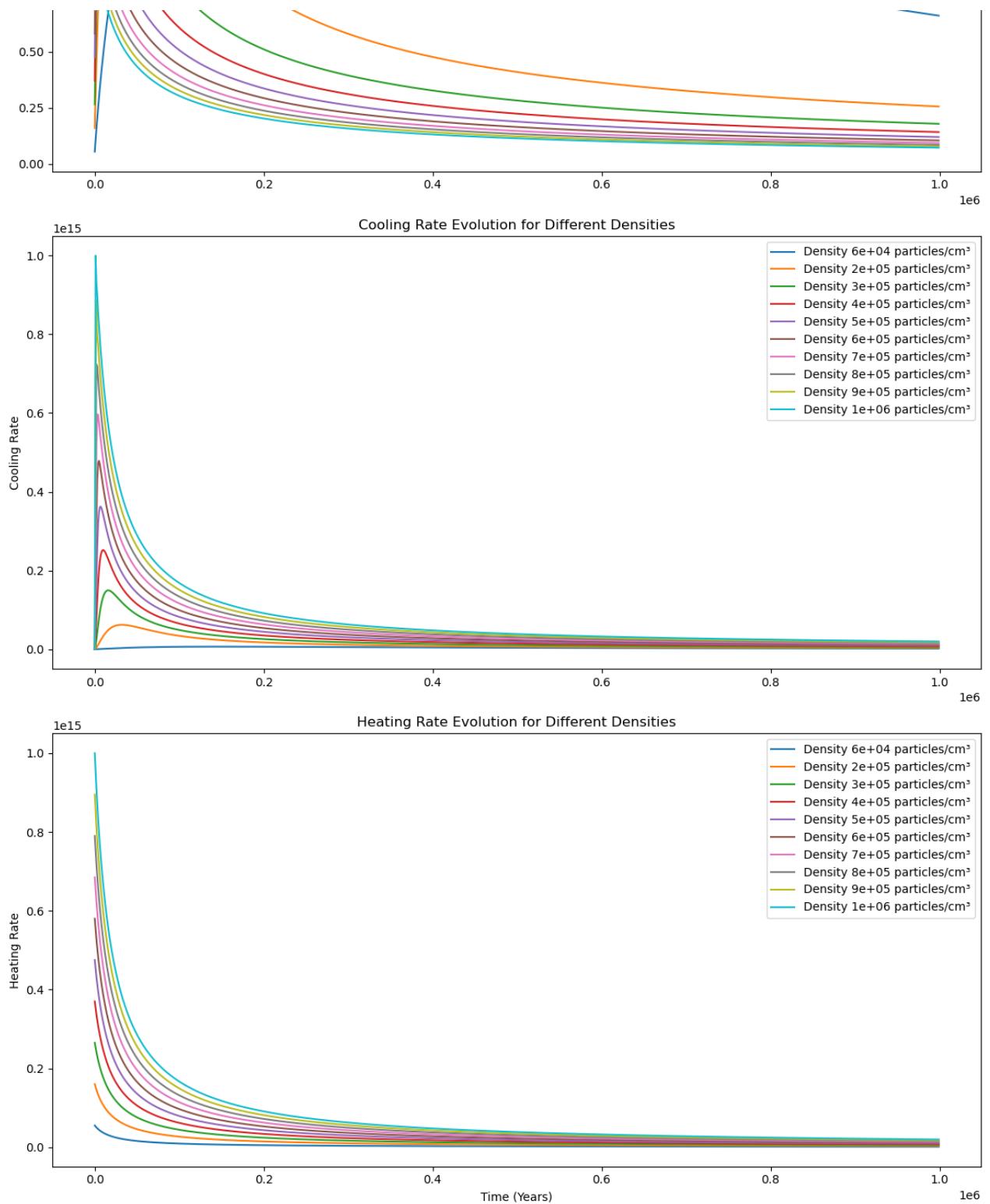
plt.subplot(3, 1, 1)
plt.ylabel('Temperature (K)')
plt.title('Temperature Evolution for Different Densities')
plt.legend()

plt.subplot(3, 1, 2)
plt.ylabel('Cooling Rate')
plt.title('Cooling Rate Evolution for Different Densities')
plt.legend()

plt.subplot(3, 1, 3)
plt.ylabel('Heating Rate')
plt.xlabel('Time (Years)')
plt.title('Heating Rate Evolution for Different Densities')
plt.legend()

plt.tight_layout()
plt.show()
```





As density increases, the heating rate increases, the cooling rate increases as well, and the overall temperature peaks lower and decays faster.

Also, from the correlation matrix heat map, it is clear how things are closely related in this cooling and heating system. It is as expected because environment itself is a very deterministic factor.

The density =  $1e5$  to  $1e6$  is a fair estimation for the average density inside of an AGN galaxy, but note that the density varies a lot in different parts of a galaxy. For my targets which are AGN galaxies, they are extremely powerful towards the center mainly due to the super massive blackhole. Thus the density at broad line region (the region close to black hole) of AGN galaxies can exceed  $1e9$  particles per cubic centimeter. As the radius moves away from the center of the galaxy, the narrow line region typically has densities in the range of  $1e2$  to  $1e6$ .

To make the system more realistic, I can write a function of density in terms of the radius. The typical relationship between density and radius can be expressed as

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}$$

where

$\rho(r)$  : The density as a function of radius from the center

$\rho_0$  : the reference density, I here take it to be  $1.0 \times 10^5$  particles per cubic cm

$r_0$  : reference radius, I here take it to be 1 parsec

$r$  : the radius from the center, in unit of parsec

$\alpha$  : the index of this power law, I here take it to be 1.5, this varies for different galaxies

```
In [22]: # we can define the function density
def density_radius(r, rho_0 = 1e5, r_0 = 1, alpha = 1.5):
    """
    Density as a function of radius for an AGN environment.
    r: Radius in parsec
    rho_0: Reference density at reference radius r_0
    r_0: Reference radius
    alpha: Power-law index
    """
    rho_r = rho_0 * (r / r_0) ** (-alpha)
    return rho_r
```

Then, I can update the previous functions:

```
In [23]: def heating_function(star_formation_initial,
```

```
r,
time,
agn_luminosity_initial = 1e44,
rho_0 = 1e5,
r_0 = 1,
alpha = 1.5):
decay_factor = 0.05
star_formation_rate = star_formation_initial / (1 + decay_factor * time)
agn_luminosity = agn_luminosity_initial / (1 + decay_factor * time)
density = density_radius(r, rho_0, r_0, alpha)
new_temp = 1e-35 * (star_formation_rate + agn_luminosity) * density
return new_temp

def cooling_function(temperature, r, rho_0 = 1e5, r_0 = 1, alpha = 1.5):
    density = density_radius(r, rho_0, r_0, alpha)
    if temperature < 1e4:
        new_temp = 1e-22 * temperature * density**2
    else:
        new_temp = 1e-24 * temperature**1.5 * density**2
    return new_temp

def cooling_rate(new_temp, density):
    cooling_rate = new_temp * density**2
    return cooling_rate

def ism_temperature_evolution(initial_temperature,
                               r,
                               metallicity,
                               star_formation_rate,
                               agn_luminosity, timestep, total_time):
    temperature = initial_temperature
    density = density_radius(r, rho_0, r_0, alpha)
    for time in range(total_time):
        # cooling and heating rates
        cooling_rate = cooling_function(temperature, r, rho_0 = 1e5, r_0 = 1)
        heating_rate = heating_function(star_formation_rate, agn_luminosity,
                                         temperature_change = (heating_rate - cooling_rate) * timestep
                                         temperature += temperature_change
        if temperature < 10: # 10 K is a reasonable lower limit for ISM temperature
            temperature = 10
    return temperature

def ism_temperature_evolution_over_time(initial_temperature,
                                        r,
                                        rho_0=1e5,
                                        r_0=1,
                                        alpha=1.5,
                                        metallicity=1.0,
                                        star_formation_initial=2.0,
                                        agn_luminosity_initial=1e44,
                                        timestep=1e3,
                                        total_time=10000):
    temperature = initial_temperature
```

```

temperatures, cooling_rates, heating_rates = [], [], []
for time in range(total_time):
    density = density_radius(r, rho_0, r_0, alpha)
    cooling_rate = cooling_function(temperature, r, rho_0, r_0, alpha) *
    heating_rate = heating_function(star_formation_initial, agn_luminosity)
    temperature_change = (heating_rate - cooling_rate) * timestep
    temperature += temperature_change
    if temperature < 10:
        temperature = 10
    temperatures.append(temperature)
    cooling_rates.append(cooling_rate)
    heating_rates.append(heating_rate)
return temperatures, cooling_rates, heating_rates

```

In [24]:

```

# Example
r = 2.0 # radius in parsecs
initial_temperature = 100000 # in Kelvin
temperatures, cooling_rates, heating_rates = ism_temperature_evolution_over_
temperatures[:10]

```

Out [24]:

```
[50589.411559869055,
 32810.32404018419,
 23524.171671272197,
 17886.6070103183,
 14148.842176469958,
 11519.171404028086,
 9587.417107373523,
 8089.3831843464095,
 6825.417061792283,
 5758.945645887238]
```

Add these to the previously defined class:

In [25]:

```

class Galaxy:
    '''initialize the galaxy'''
    def __init__(self, mass, position, velocity, black_hole_mass):
        self.mass = mass
        self.position = position
        self.velocity = velocity
        self.black_hole_mass = black_hole_mass

    '''black hole accretion model'''
    def calculate_accretion_rate(black_hole_mass,
                                  rho = np.random.uniform(1e-14, 1e-13),
                                  v = np.random.uniform(4.5e7, 5.5e7),
                                  c_s = np.random.uniform(1e6, 2e7)):
        # The input black hole mass should be in the unit of solar mass
        G = 6.67430e-11 * u.m**3 / u.kg / u.s**2 # gravitational constant
        M = black_hole_mass * 1.98847e30 * u.kg # solar masses to kg
        rho = rho * u.kg / u.m**3 # density of the accreting material in kg
        v = v * u.m / u.s # relative velocity in m/s
        c_s = c_s * u.m / u.s # sound speed in the material in m/s
        # Bondi-Hoyle-Lyttleton accretion rate

```

```
accretion_rate = (4 * np.pi * G**2 * M**2 * rho) / ((v**2 + c_s**2)*  
# Convert accretion rate to solar masses per year  
accretion_rate_solar_masses_per_year = accretion_rate.to(u.kg / u.s)  
return accretion_rate_solar_masses_per_year  
  
def calculate_radiation(accretion_rate, epsilon = 0.1):  
    # The accretion rate is obtained from the previous function calculate  
    c = 299792458 * u.m / u.s  
    # calculate the luminosity (or radiation)  
    radiation = epsilon * accretion_rate * 1.98847e30 * c**2 # Convert  
    return radiation  
  
def accretion_model(black_hole_mass, accretion_radius, other_parameters):  
    accretion_rate = calculate_accretion_rate(black_hole_mass, accretion  
    radiation = calculate_radiation(accretion_rate, other_parameters)  
    jets = determine_jet_production(accretion_rate, radiation, other_par  
    return accretion_rate, radiation, jets  
  
'''cooling and heating process in different radius of the galaxy'''  
def heating_function(star_formation_initial,  
                     r,  
                     time,  
                     agn_luminosity_initial = 1e44,  
                     rho_0 = 1e5,  
                     r_0 = 1,  
                     alpha = 1.5):  
    decay_factor = 0.05  
    star_formation_rate = star_formation_initial / (1 + decay_factor * t  
    agn_luminosity = agn_luminosity_initial / (1 + decay_factor * time)  
    density = density_radius(r, rho_0, r_0, alpha)  
    new_temp = 1e-35 * (star_formation_rate + agn_luminosity) * density  
    return new_temp  
  
def cooling_function(temperature, r, rho_0 = 1e5, r_0 = 1, alpha = 1.5):  
    density = density_radius(r, rho_0, r_0, alpha)  
    if temperature < 1e4:  
        new_temp = 1e-22 * temperature * density**2  
    else:  
        new_temp = 1e-24 * temperature**1.5 * density**2  
    return new_temp  
  
def cooling_rate(new_temp, density):  
    cooling_rate = new_temp * density**2  
    return cooling_rate  
  
def ism_temperature_evolution(initial_temperature,  
                               r,  
                               metallicity,  
                               star_formation_rate,  
                               agn_luminosity, timestep, total_time):  
    temperature = initial_temperature  
    density = density_radius(r, rho_0, r_0, alpha)  
    for time in range(total_time):
```

```
# cooling and heating rates
cooling_rate = cooling_function(temperature, r, rho_0 = 1e5, r_0
heating_rate = heating_function(star_formation_rate, agn_luminos
temperature_change = (heating_rate - cooling_rate) * timestep
temperature += temperature_change
if temperature < 10: # 10 K is a reasonable lower limit for ISM
    temperature = 10
return temperature

def ism_temperature_evolution_over_time(initial_temperature,
                                         r,
                                         rho_0=1e5,
                                         r_0=1,
                                         alpha=1.5,
                                         metallicity=1.0,
                                         star_formation_initial=2.0,
                                         agn_luminosity_initial=1e44,
                                         timestep=1e3,
                                         total_time=10000):
    temperature = initial_temperature
    temperatures, cooling_rates, heating_rates = [], [], []
    for time in range(total_time):
        density = density_radius(r, rho_0, r_0, alpha)
        cooling_rate = cooling_function(temperature, r, rho_0, r_0, alpha)
        heating_rate = heating_function(star_formation_initial, agn_lumi
temperature_change = (heating_rate - cooling_rate) * timestep
temperature += temperature_change
if temperature < 10:
    temperature = 10
temperatures.append(temperature)
cooling_rates.append(cooling_rate)
heating_rates.append(heating_rate)
return temperatures, cooling_rates, heating_rates

# Gas Dynamics and Interstellar Medium
def gas_dynamics(galaxy):
    """
    This models the implement model of the dynamics of gas in and around
    The processes like cooling, heating, and interaction with AGN radiat
    """
    pass

# Galaxy Interaction and Dynamics
def update_galaxy_dynamics(galaxies):
    """
    Implement gravitational interactions between galaxies.
    Update positions and velocities based on gravitational forces.
    """
    pass
```

## C) Spectra Modeling for AGN galaxies

Spectra modeling is significant for studying galaxies because it gives us information about various things. Some typical ones include:

1. **composition and chemical elemental abundances:** from analyzing a spectra of a galaxy, we can tell from the emission and absorption lines which components exist in the galaxies, also, the ratio between certain emission lines give us the chemical elemental abundances, which reveals the history of the galaxy because some elements are more produced in certain age or phase of galaxy evolution.
2. **redshift:** by looking at the obvious emission and absorption lines from a galaxy's spectrum, we can compare it with the wavelength of that element in the rest frame spectrum, and by using  $1 + z = \frac{\lambda_{observed}}{\lambda_e}$  we can find the redshift of the galaxy, and thus how far the galaxy is from us.
3. **stellar population:** since different stars have very different physical properties, we can tell from the spectra which kind of stars do the galaxy have and thus better understand the stellar populations of certain types of galaxies.
4. Spectra modeling also plays critical roles in **understanding black holes of AGN, studying dark matter and dark matter halo, and providing insights into the temperature and density.** Thus we want to understand how AGN galaxies look like and potentially build AGN galaxy spectrum.

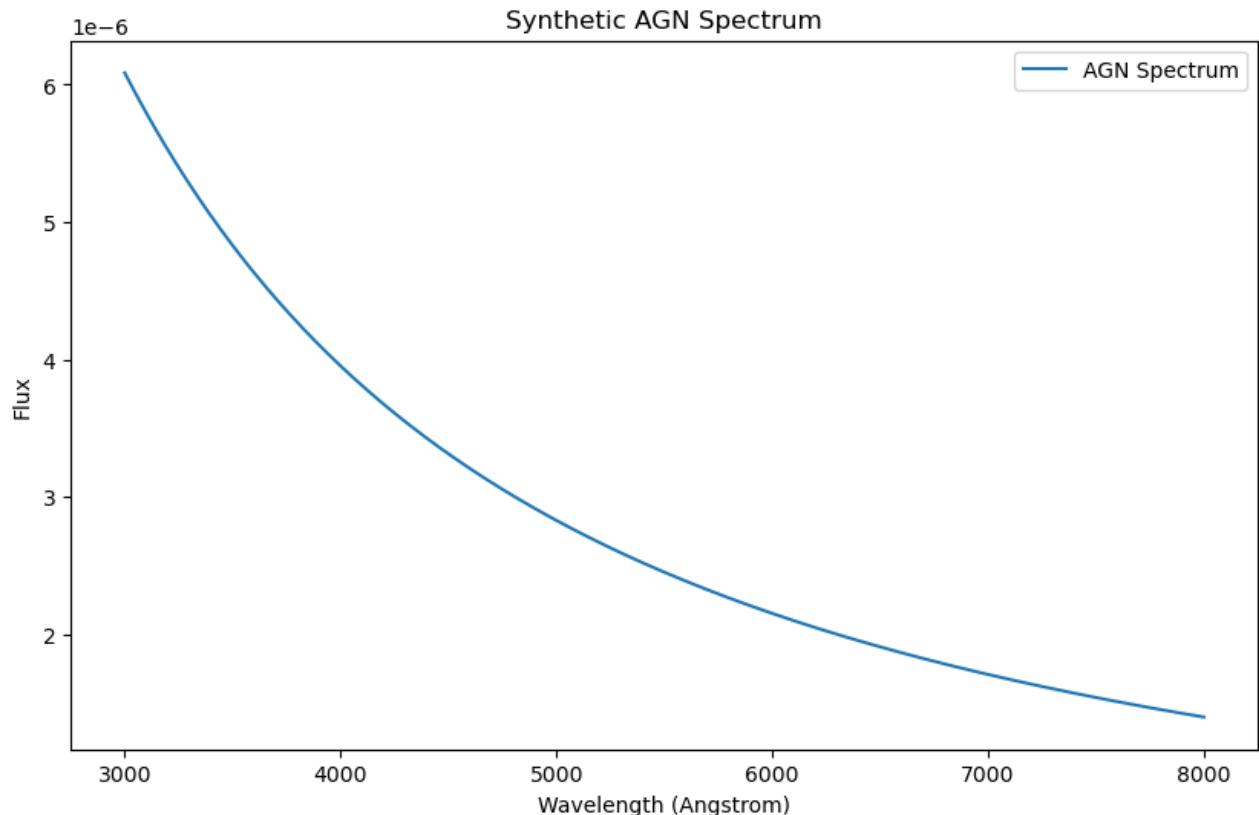
There are several things and steps I need to consider towards doing spectra modeling:

1. I need to **generate a power-law continuum**
2. consider **emission lines**
3. add in **noise**
4. consider **absorption feature** I here use units that is typically used in experimental astrophysics, which are arbitrary unit for flux (we do not actually care about the unit of flux, what we care is the intensity and the ratios)

In [26]:

```
# Step 1
# we first assumed redshift 0 and generalize the power-law continuum
# most of the emission lines and absorption features are within wavelength range
# so I choose the wavelength range from 3000 to 10000
# alpha is the power-law slope
wavelengths = np.linspace(3000, 8000, 10000)
alpha = -1.5
flux_continuum = wavelengths ** alpha

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(wavelengths, flux_continuum, label='AGN Spectrum')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('Flux')
plt.title('Synthetic AGN Spectrum')
plt.legend()
plt.show()
```



The **width** and **height** of each emission line is usually based on observation, these are determined by a various factor such as pressure, abundance, temperature, density, and ionization state. I here randomize them, and for next week, I want to use the randomized spectra to observe the correlation between these parameters. The emission lines that are usually visible are:

1. H\_alpha = 6563 # H-alpha
2. H\_beta = 4861 # H-beta
3. OIII\_5007 = 5007 # O III line
4. OIII\_4959 = 4959 # O III line
5. NII\_6584 = 6584 # N II line
6. SII\_6716 = 6716 # S II line
7. SII\_6731 = 6731 # S II line

among them, for AGN host galaxies, elements like Hydrogen, Oxygen, and Nitrogen are commonly observed with relatively intense lines, so I time their flux by a factor of 1.5

In [27]:

```
# Step 2
# now I want to add in some typical emission lines for an AGN host galaxies
wavelengths = np.linspace(3700, 8000, 1000)
alpha = -15
flux_continuum = wavelengths ** alpha
# the emission lines include:
# Randomization ranges
width_range = (2, 10) # Width range in Angstroms
base_height_range = (10, 20) # Base height range for less intense lines

emission_lines = {
    "H_alpha": {"center": 6563, "element": "H"},
    "H_beta": {"center": 4861, "element": "H"},
    "OIII_5007": {"center": 5007, "element": "O"}, 
    "OIII_4959": {"center": 4959, "element": "O"}, 
    "NII_6584": {"center": 6584, "element": "N"}, 
    "SII_6716": {"center": 6716, "element": "S"}, 
    "SII_6731": {"center": 6731, "element": "S"}
}

def add_emission_line(wavelength_center, width, height):
    return height * norm.pdf(wavelengths, wavelength_center, width)

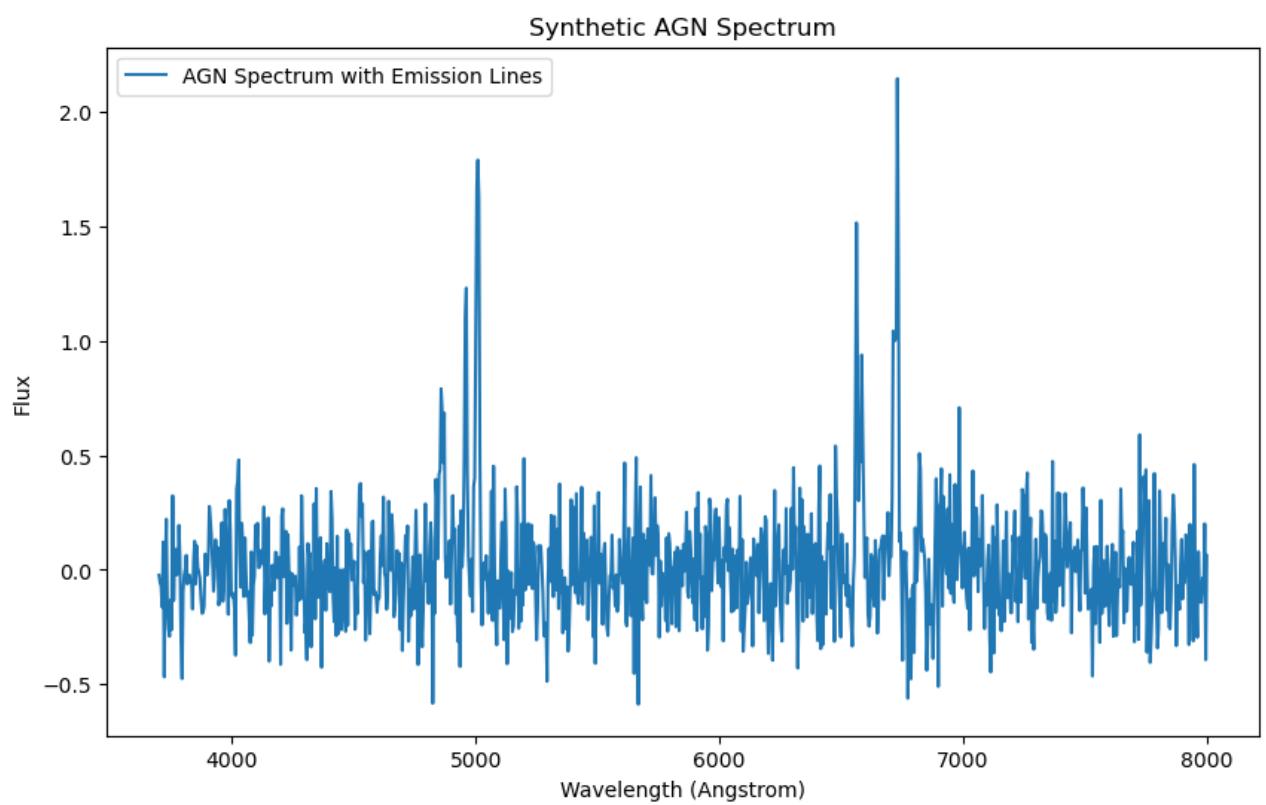
for line in emission_lines.values():
    width = random.uniform(*width_range)
    height = random.uniform(*base_height_range)

    if line["element"] in ["H", "O", "N"]:
        height *= 1.5

    flux_continuum += add_emission_line(line["center"], width, height)

# Step 3: The spectrum usually also have noise, I also want to include that:
noise_level = 0.2
noise = np.random.normal(0, noise_level, wavelengths.shape)
flux_continuum += noise

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(wavelengths, flux_continuum, label='AGN Spectrum with Emission Line')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('Flux')
plt.title('Synthetic AGN Spectrum')
plt.legend()
plt.show()
```



```
In [28]: wavelengths = np.linspace(3700, 8000, 1000)
alpha = -15
flux_continuum = wavelengths ** alpha

emission_lines = {
    "H_alpha": {"center": 6563, "element": "H"},
    "H_beta": {"center": 4861, "element": "H"},
    "OIII_5007": {"center": 5007, "element": "O"}, 
    "OIII_4959": {"center": 4959, "element": "O"}, 
    "NII_6584": {"center": 6584, "element": "N"}, 
    "SII_6716": {"center": 6716, "element": "S"}, 
    "SII_6731": {"center": 6731, "element": "S"}
}

def add_emission_line(wavelength_center, width, height):
    return height * norm.pdf(wavelengths, wavelength_center, width)

width_range = (2, 10)
base_height_range = (10, 20)

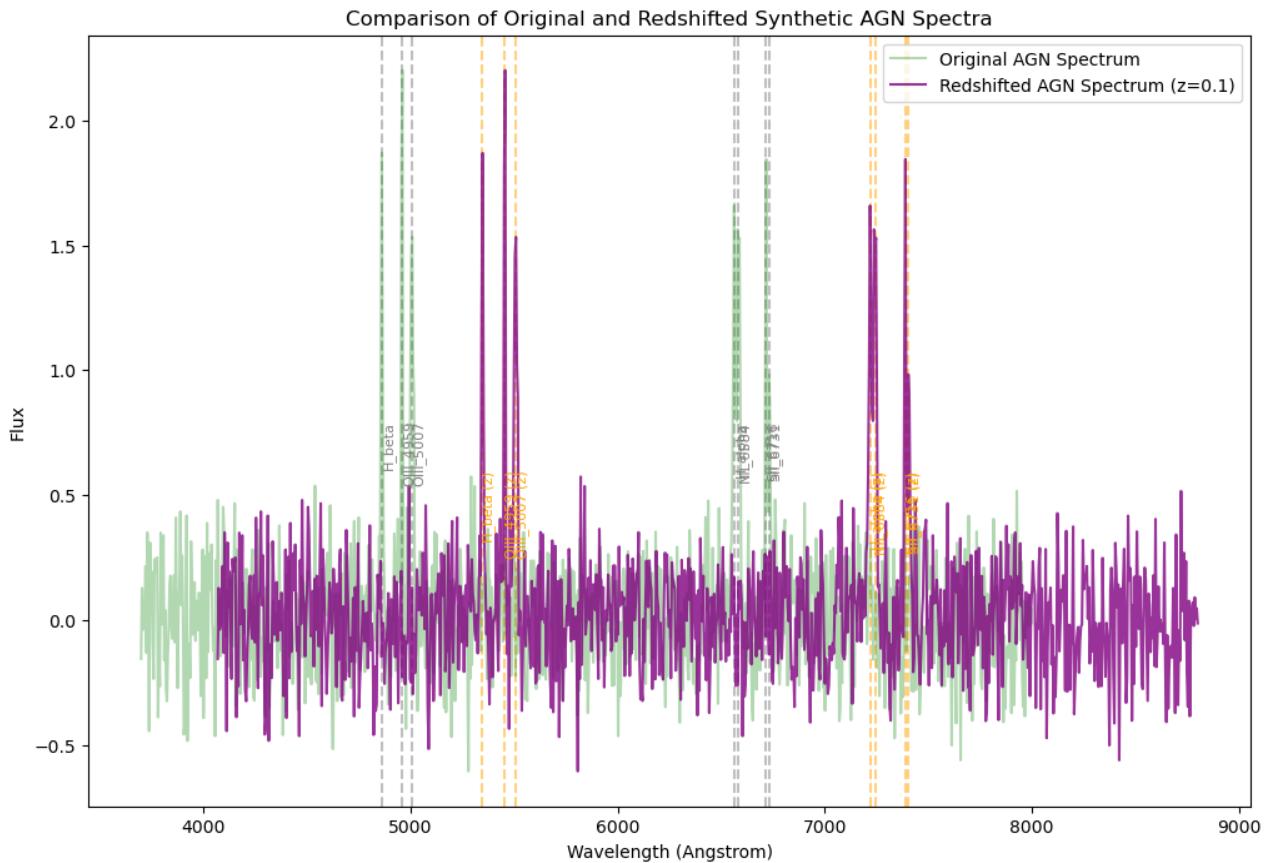
for line in emission_lines.values():
    width = random.uniform(*width_range)
    height = random.uniform(*base_height_range)
    if line["element"] in ["H", "O", "N"]:
        height *= 1.5
    flux_continuum += add_emission_line(line["center"], width, height)

noise_level = 0.2
noise = np.random.normal(0, noise_level, wavelengths.shape)
flux_continuum += noise

z = 0.1
original_flux_continuum = flux_continuum.copy()
redshifted_wavelengths = wavelengths * (1 + z)
redshifted_flux_continuum = original_flux_continuum

plt.figure(figsize=(12, 8))
for line_name, line in emission_lines.items():
    original_center = line["center"]
    redshifted_center = original_center * (1 + z)
    plt.axvline(x=original_center, color='gray', linestyle='--', alpha=0.5)
    plt.axvline(x=redshifted_center, color='orange', linestyle='--', alpha=0.5)
    plt.text(original_center, plt.ylim()[1]*0.8, line_name, rotation=90, vertical-align='bottom')
    plt.text(redshifted_center, plt.ylim()[1]*0.6, f'{line_name} (z)', rotation=90, vertical-align='bottom')

plt.plot(wavelengths, original_flux_continuum, label='Original AGN Spectrum')
plt.plot(redshifted_wavelengths, redshifted_flux_continuum, label=f'Redshifted {line_name} Spectrum')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('Flux')
plt.title('Comparison of Original and Redshifted Synthetic AGN Spectra')
plt.legend()
plt.show()
```



```
In [29]: # define a function that takes a single input of redshift and produce the spectrum
def generate_redshifted_spectrum(redshift):
    wavelengths = np.linspace(3700, 8000, 1000)
    alpha = -15
    flux_continuum = wavelengths ** alpha

    emission_lines = {
        "H_alpha": {"center": 6563, "element": "H"},
        "H_beta": {"center": 4861, "element": "H"},
        "OIII_5007": {"center": 5007, "element": "O"}, 
        "OIII_4959": {"center": 4959, "element": "O"}, 
        "NII_6584": {"center": 6584, "element": "N"}, 
        "SII_6716": {"center": 6716, "element": "S"}, 
        "SII_6731": {"center": 6731, "element": "S"}
    }

    def add_emission_line(wavelength_center, width, height):
        return height * norm.pdf(wavelengths, wavelength_center, width)

    width_range = (2, 10)
    base_height_range = (10, 20)

    for line in emission_lines.values():
        width = random.uniform(*width_range)
        height = random.uniform(*base_height_range)
        if line["element"] in ["H", "O", "N"]:
            line["width"] = width
            line["height"] = height
        else:
            line["width"] = 0
            line["height"] = 0
    return flux_continuum, wavelengths, emission_lines
```

```
height *= 1.5
flux_continuum += add_emission_line(line["center"], width, height)

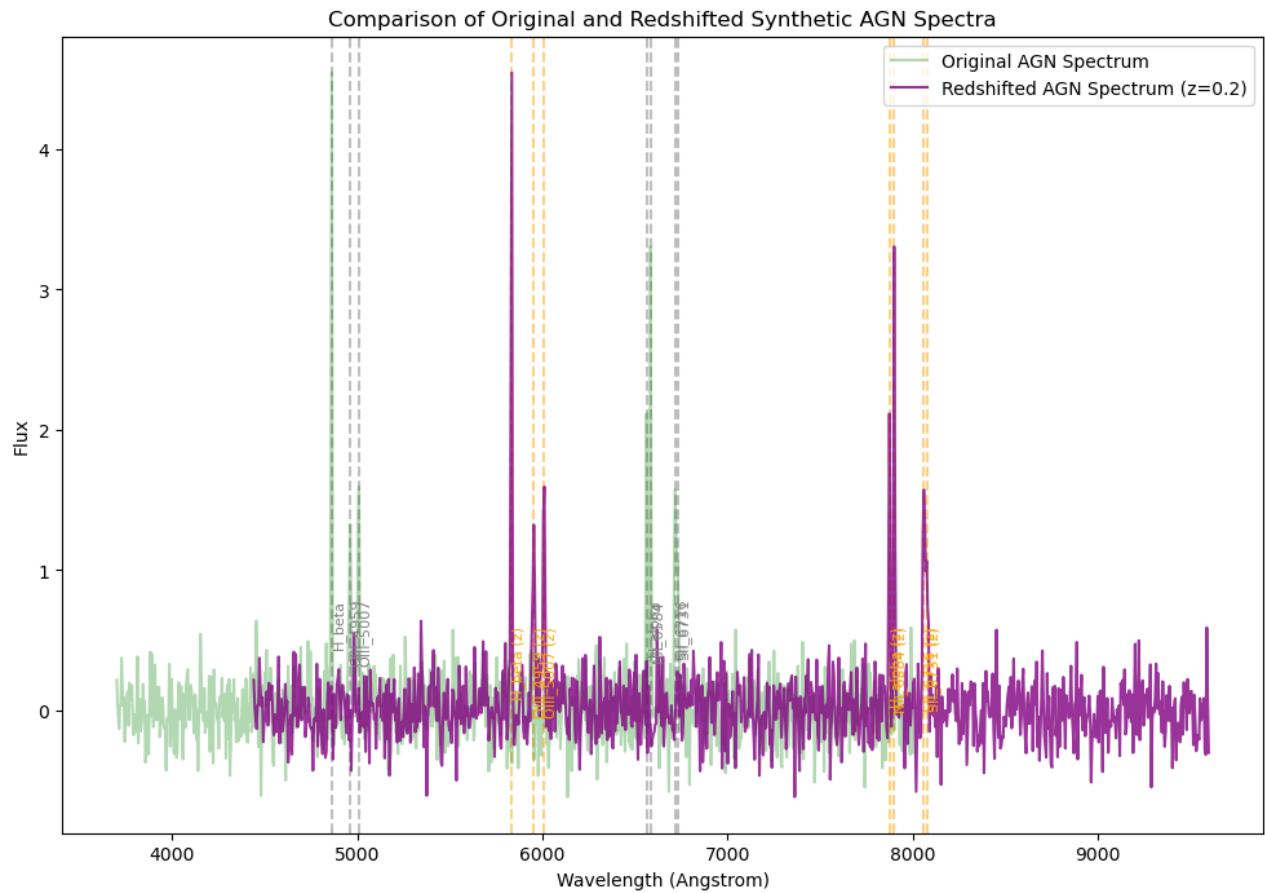
noise_level = 0.2
noise = np.random.normal(0, noise_level, wavelengths.shape)
flux_continuum += noise

original_flux_continuum = flux_continuum.copy()
redshifted_wavelengths = wavelengths * (1 + redshift)
redshifted_flux_continuum = original_flux_continuum

plt.figure(figsize=(12, 8))
for line_name, line in emission_lines.items():
    original_center = line["center"]
    redshifted_center = original_center * (1 + redshift)
    plt.axvline(x=original_center, color='gray', linestyle='--', alpha=0.5)
    plt.axvline(x=redshifted_center, color='orange', linestyle='--', alpha=0.5)
    plt.text(original_center, plt.ylim()[1]*0.8, line_name, rotation=90, color='gray')
    plt.text(redshifted_center, plt.ylim()[1]*0.6, f'{line_name} (z)', color='orange')

plt.plot(wavelengths, original_flux_continuum, label='Original AGN Spectrum')
plt.plot(redshifted_wavelengths, redshifted_flux_continuum, label=f'Redsifted {line_name} Spectrum')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('Flux')
plt.title('Comparison of Original and Redshifted Synthetic AGN Spectra')
plt.legend()
plt.show()
return redshifted_wavelengths, flux_continuum
```

In [30]: # example  
generate\_redshifted\_spectrum(0.2)



```
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 -9.75946451e-03, 8.46109611e-02, 1.25271155e-01, 2.56525656e-01,  
 6.23779734e-02, 1.42279916e-01, -2.34649104e-02, -7.41829568e-02,  
 2.98739203e-01, -3.57278262e-02, -2.38486883e-01, -8.10319581e-02,  
 -2.09036804e-01, 1.94801084e-01, -1.67381902e-01, 4.40053726e-02,  
 1.38095426e-01, 1.15783986e-01, -3.86664286e-01, -1.25496008e-01,  
 1.33470987e-01, 3.69569011e-02, 2.06720105e-01, 1.98734590e-01,  
 1.70460210e-01, -2.50814148e-01, 1.93462949e-01, -8.36076827e-03,  
 -8.53990179e-02, -2.91598874e-01, -1.79787919e-01, -2.52030957e-01,  
 -1.30209217e-01, 1.50455081e-03, -2.30189168e-01, 1.95369039e-01,  
 -4.44338764e-01, 1.88291366e-02, 2.81222909e-01, 6.91391845e-02,  
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 4.78786922e-02, 2.62844511e-01, -1.18828212e-01, -3.41750399e-01,  
 1.45612929e-01, 5.75128597e-01, 4.50407921e-02, -1.93141420e-01,  
 1.76400012e-01, -2.69952775e-01, -2.53821920e-01, -1.83662737e-01,  
 -1.95612829e-01, -4.39516079e-02, -9.99838647e-02, -1.10626567e-01,  
 4.26766155e-02, 1.17976096e-01, 1.68320546e-01, -3.85938625e-02,  
 2.38283100e-01, -1.24923745e-01, 6.41495923e-02, 3.69739862e-01,  
 -3.17075020e-02, 2.50074366e-02, 9.72335783e-03, 1.54447324e-01,

```

    2.12934329e-01,  1.03948726e-01, -1.73343706e-01, -5.37401543e-02,
    1.72128491e-01,  2.96468326e-01, -1.90294600e-02, -1.88028582e-01,
   -3.02961623e-01,  5.17590970e-02,  1.27681197e-01, -9.43564036e-02,
   -8.96284168e-02,  4.72994660e-03,  1.72362902e-01, -2.87490813e-01,
   3.56689618e-02,  2.13525997e-01,  4.24638383e-02,  3.48042175e-01,
  -2.58928062e-01, -4.75616430e-02, -3.57015803e-01, -1.56729981e-01,
   9.43768708e-02,  4.26144451e-01,  9.33714798e-02, -1.62515456e-01,
  -2.44379092e-01, -3.33107604e-01,  2.66189200e-01, -1.24631005e-01,
   1.01211452e-01, -1.45257932e-01, -3.78942405e-01, -4.04472810e-01,
   1.65080451e-01,  1.06944420e-01,  1.75579614e-01, -1.77670191e-01,
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   1.21228105e-01,  1.48528192e-01, -7.03231290e-02,  2.38417179e-01,
   8.15077575e-03, -2.69057425e-01,  1.08341680e-01,  2.67081350e-02,
   4.24267350e-02, -5.23549859e-02,  8.09309084e-02,  3.17922750e-01,
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   4.54589889e-02,  9.38341753e-02, -1.81441277e-01, -8.50300511e-02,
  -4.27041662e-02, -1.02888769e-02,  1.05808127e-01, -2.47687145e-02,
  -5.11370676e-01, -2.14097950e-01, -1.57757556e-01, -2.63902998e-02,
  -1.53463430e-01, -1.92711074e-01, -1.22746401e-01,  1.02159537e-01,
  -2.26267519e-01,  1.32578865e-01,  1.41437542e-01,  2.76331128e-02,
  -4.35941373e-01,  3.10335899e-01, -1.56419634e-01, -9.84582743e-03,
  -1.55307894e-01,  8.83870128e-02, -2.99400128e-01, -7.63911052e-02,
  -1.19783653e-01, -1.09748464e-01,  8.76607589e-02, -1.17738021e-02,
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   9.63587808e-02,  2.15791790e-01,  4.10898833e-02,  2.92664265e-01,
  -1.69333015e-01, -1.26905786e-01,  1.16476747e-01, -4.93162075e-02,
   1.43544121e-01,  2.01119914e-02, -8.19131896e-02,  1.14117493e-01,
  -3.01016758e-01,  1.82979765e-02,  4.74072987e-01, -2.39767641e-02,
  -1.72706556e-01, -7.66260870e-02,  4.99203685e-01, -2.09450335e-01,
  -1.71282676e-02,  1.64066573e-01,  2.14281759e-01, -1.05559490e-01,
  -3.71066933e-02,  1.13765204e-01,  7.61300519e-02,  1.54894384e-01,
  -5.01861499e-02,  3.39044305e-02,  3.28827518e-01, -5.45867246e-01,
  -5.83260228e-02,  2.03569053e-02, -1.20708898e-02,  9.60980830e-02,
   3.33183484e-01,  1.33007263e-02,  2.67571742e-01,  6.31606178e-02,
  -2.67598344e-02,  6.65253388e-02, -1.55726360e-01,  2.07648851e-01,
  -8.71996192e-02, -2.56724629e-01, -1.48292435e-01,  1.08600528e-01,
   1.11683426e-02, -7.13029099e-02,  1.75144304e-01, -7.27418388e-02,
   2.20124687e-02, -4.08824828e-01, -1.57281505e-01,  5.33036119e-02,
   8.61857304e-02, -3.52940666e-01, -9.02578388e-02, -1.13690382e-01,
  -8.67652548e-02, -2.24443420e-01,  3.93362159e-02,  9.71562689e-02,
  -1.71484387e-01,  2.00617587e-01,  2.92491388e-02,  1.87253955e-01,
  -1.37634579e-01,  2.23600788e-01,  2.57963986e-02, -1.79837833e-02,
   1.35056891e-01,  1.38971660e-01,  3.63906549e-02, -2.45788917e-01,
   1.11147303e-01, -7.69312816e-02, -2.87757861e-01, -2.09005930e-01,
  -1.76918854e-01, -2.07378483e-01,  8.65385764e-02, -6.98265887e-02,
   4.42561997e-02, -5.13436222e-02,  8.76158714e-02, -2.82627650e-01,
  -3.15084666e-01,  5.91144944e-01,  8.53821114e-02, -3.00324828e-01)
)

```

Add this into the class:

In [31]:

```

class Galaxy:
    '''initialize the galaxy'''
    def __init__(self, mass, position, velocity, black_hole_mass, initial_te
        self.mass = mass
        self.position = position
        self.velocity = velocity
        self.black_hole_mass = black_hole_mass

    '''black hole accretion model'''
    def calculate_accretion_rate(black_hole_mass,
                                  rho = np.random.uniform(1e-14, 1e-13),
                                  v = np.random.uniform(4.5e7, 5.5e7),
                                  c_s = np.random.uniform(1e6, 2e7)):
        # The input black hole mass should in the unit of solar mass
        G = 6.67430e-11 * u.m**3 / u.kg / u.s**2 # gravitational constant wi
        M = black_hole_mass * 1.98847e30 * u.kg # solar masses to kg
        rho = rho * u.kg / u.m**3 # density of the accreting material in kg
        v = v * u.m / u.s # relative velocity in m/s
        c_s = c_s * u.m / u.s # sound speed in the material in m/s
        # Bondi-Hoyle-Lyttleton accretion rate
        accretion_rate = (4 * np.pi * G**2 * M**2 * rho) / ((v**2 + c_s**2)*
        # Convert accretion rate to solar masses per year
        accretion_rate_solar_masses_per_year = accretion_rate.to(u.kg / u.s)
        return accretion_rate_solar_masses_per_year

    def calculate_radiation(accretion_rate, epsilon = 0.1):
        # The accretion rate is obtained from the previous function calculate_
        c = 299792458 * u.m / u.s
        # calculate the luminosity (or radiation)
        radiation = epsilon * accretion_rate * 1.98847e30 * c**2 # Convert
        return radiation

    def accretion_model(black_hole_mass, accretion_radius, other_parameters):
        accretion_rate = calculate_accretion_rate(black_hole_mass, accretion_
        radiation = calculate_radiation(accretion_rate, other_parameters)
        jets = determine_jet_production(accretion_rate, radiation, other_par
        return accretion_rate, radiation, jets

    '''cooling and heating process in different radius of the galaxy'''
    def heating_function(r,
                          time,
                          agn_luminosity_initial = 1e44,
                          star_formation_initial = 2.0,
                          rho_0 = 1e5,
                          r_0 = 1,
                          alpha = 1.5):
        decay_factor = 0.05
        star_formation_rate = star_formation_initial / (1 + decay_factor * t
        agn_luminosity = agn_luminosity_initial / (1 + decay_factor * time)
        density = density_radius(r, rho_0, r_0, alpha)
        new_temp = 1e-35 * (star_formation_rate + agn_luminosity) * density
        return new_temp

```

```
def cooling_function(temperature, r, rho_0 = 1e5, r_0 = 1, alpha = 1.5):
    density = density_radius(r, rho_0, r_0, alpha)
    if temperature < 1e4:
        new_temp = 1e-22 * temperature * density**2
    else:
        new_temp = 1e-24 * temperature**1.5 * density**2
    return new_temp

def cooling_rate(new_temp, density):
    cooling_rate = new_temp * density**2
    return cooling_rate

def ism_temperature_evolution(initial_temperature,
                               r,
                               metallicity,
                               starFormationRate,
                               agnLuminosity, timestep, totalTime):
    temperature = initial_temperature
    density = density_radius(r, rho_0, r_0, alpha)
    for time in range(totalTime):
        # cooling and heating rates
        cooling_rate = cooling_function(temperature, r, rho_0 = 1e5, r_0 = 1)
        heating_rate = heating_function(starFormationRate, agnLuminosity)
        temperature_change = (heating_rate - cooling_rate) * timestep
        temperature += temperature_change
        if temperature < 10: # 10 K is a reasonable lower limit for ISM
            temperature = 10
    return temperature

def ism_temperature_evolution_over_time(initialTemperature,
                                         r,
                                         rho_0=1e5,
                                         r_0=1,
                                         alpha=1.5,
                                         metallicity=1.0,
                                         starFormationInitial=2.0,
                                         agnLuminosityInitial=1e44,
                                         timestep=1e3,
                                         totalTime=10000):
    temperature = initialTemperature
    temperatures, coolingRates, heatingRates = [], [], []
    for time in range(totalTime):
        density = density_radius(r, rho_0, r_0, alpha)
        cooling_rate = cooling_function(temperature, r, rho_0, r_0, alpha)
        heating_rate = heating_function(starFormationInitial, agnLuminosityInitial)
        temperature_change = (heating_rate - cooling_rate) * timestep
        temperature += temperature_change
        if temperature < 10:
            temperature = 10
        temperatures.append(temperature)
        coolingRates.append(cooling_rate)
        heatingRates.append(heating_rate)
```

```

    return temperatures, cooling_rates, heating_rates

'''spectra modeling'''
def generate_redshifted_spectrum(redshift):
    wavelengths = np.linspace(3700, 8000, 1000)
    alpha = -15
    flux_continuum = wavelengths ** alpha

    emission_lines = {
        "H_alpha": {"center": 6563, "element": "H"},
        "H_beta": {"center": 4861, "element": "H"},
        "OIII_5007": {"center": 5007, "element": "O"}, 
        "OIII_4959": {"center": 4959, "element": "O"}, 
        "NII_6584": {"center": 6584, "element": "N"}, 
        "SII_6716": {"center": 6716, "element": "S"}, 
        "SII_6731": {"center": 6731, "element": "S"}
    }

    def add_emission_line(wavelength_center, width, height):
        return height * norm.pdf(wavelengths, wavelength_center, width)

    width_range = (2, 10)
    base_height_range = (10, 20)

    for line in emission_lines.values():
        width = random.uniform(*width_range)
        height = random.uniform(*base_height_range)
        if line["element"] in ["H", "O", "N"]:
            height *= 1.5
        flux_continuum += add_emission_line(line["center"], width, height)

    noise_level = 0.2
    noise = np.random.normal(0, noise_level, wavelengths.shape)
    flux_continuum += noise

    original_flux_continuum = flux_continuum.copy()
    redshifted_wavelengths = wavelengths * (1 + redshift)
    redshifted_flux_continuum = original_flux_continuum

    plt.figure(figsize=(12, 8))
    for line_name, line in emission_lines.items():
        original_center = line["center"]
        redshifted_center = original_center * (1 + redshift)
        plt.axvline(x=original_center, color='gray', linestyle='--', alpha=0.5)
        plt.axvline(x=redshifted_center, color='orange', linestyle='--', alpha=0.5)
        plt.text(original_center, plt.ylim()[1]*0.8, line_name, rotation=90)
        plt.text(redshifted_center, plt.ylim()[1]*0.6, f'{line_name} (z={redshift})', rotation=90)

    plt.plot(wavelengths, original_flux_continuum, label='Original AGN Spectrum')
    plt.plot(redshifted_wavelengths, redshifted_flux_continuum, label='Redshifted Spectrum')
    plt.xlabel('Wavelength (Angstrom)')
    plt.ylabel('Flux')
    plt.title('Comparison of Original and Redshifted Synthetic AGN Spectra')

```

```
plt.legend()
plt.show()
return redshifted_wavelengths, flux_continuum

# Gas Dynamics and Interstellar Medium
def gas_dynamics(galaxy):
    """
    This models the implement model of the dynamics of gas in and around
    The processes like cooling, heating, and interaction with AGN radiat
    """
    pass

# Galaxy Interaction and Dynamics
def update_galaxy_dynamics(galaxies):
    """
    Implement gravitational interactions between galaxies.
    Update positions and velocities based on gravitational forces.
    """
    pass
```

```
In [32]: observed_wavelengths, observed_flux = generate_redshifted_spectrum(0.3)
emission_line_centers = [6563, 4861, 5007, 4959, 6584, 6716, 6731]
line_width = 10

def model_spectrum(wavelengths, redshift, line_intensities):
    model_flux = wavelengths ** -1.5
    for i, line_center in enumerate(emission_line_centers):
        shifted_center = line_center * (1 + redshift)
        model_flux += line_intensities[i] * norm.pdf(wavelengths, shifted_center, line_width)
    return model_flux

def chi_squared(obs_flux, model_flux):
    return np.sum((obs_flux - model_flux) ** 2)

def fit_spectrum(line_intensities, redshift):
    model_flux = model_spectrum(observed_wavelengths, redshift, line_intensities)
    return chi_squared(observed_flux, model_flux)

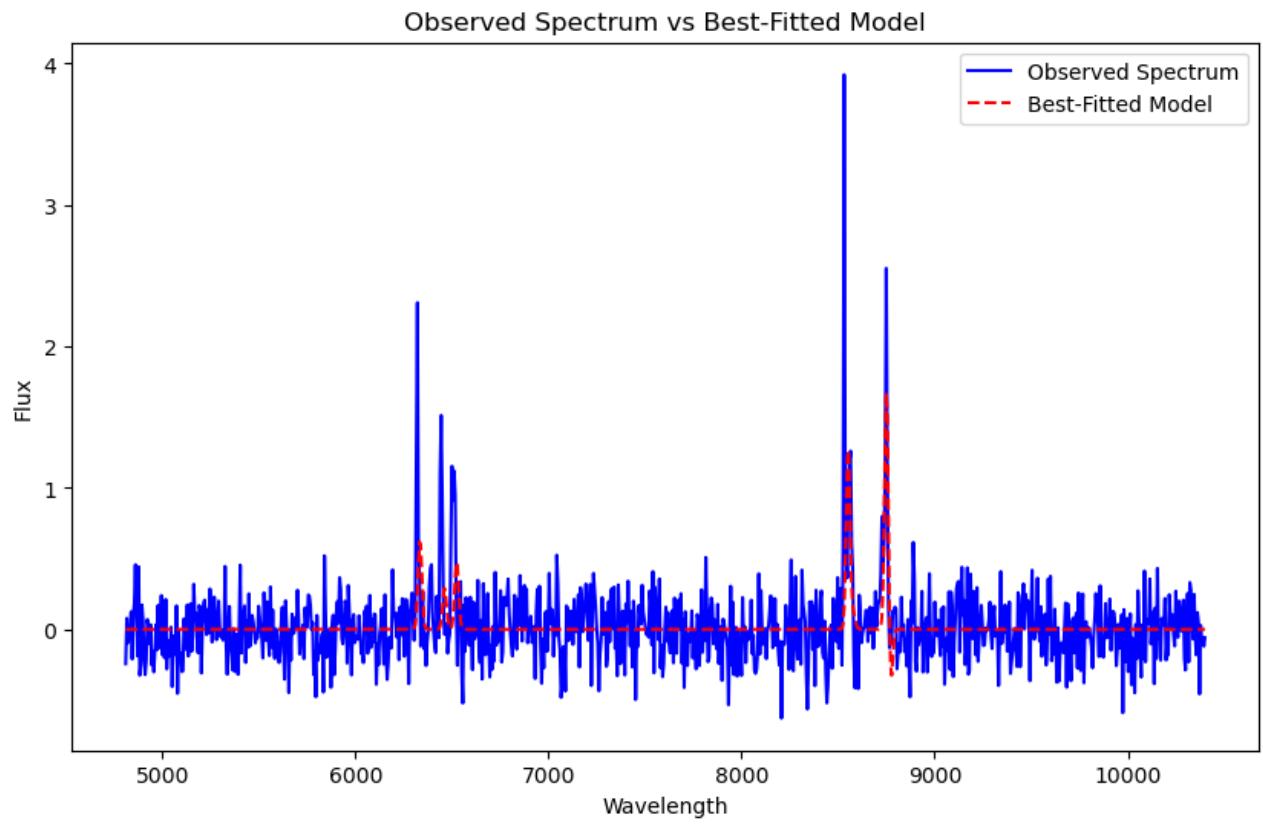
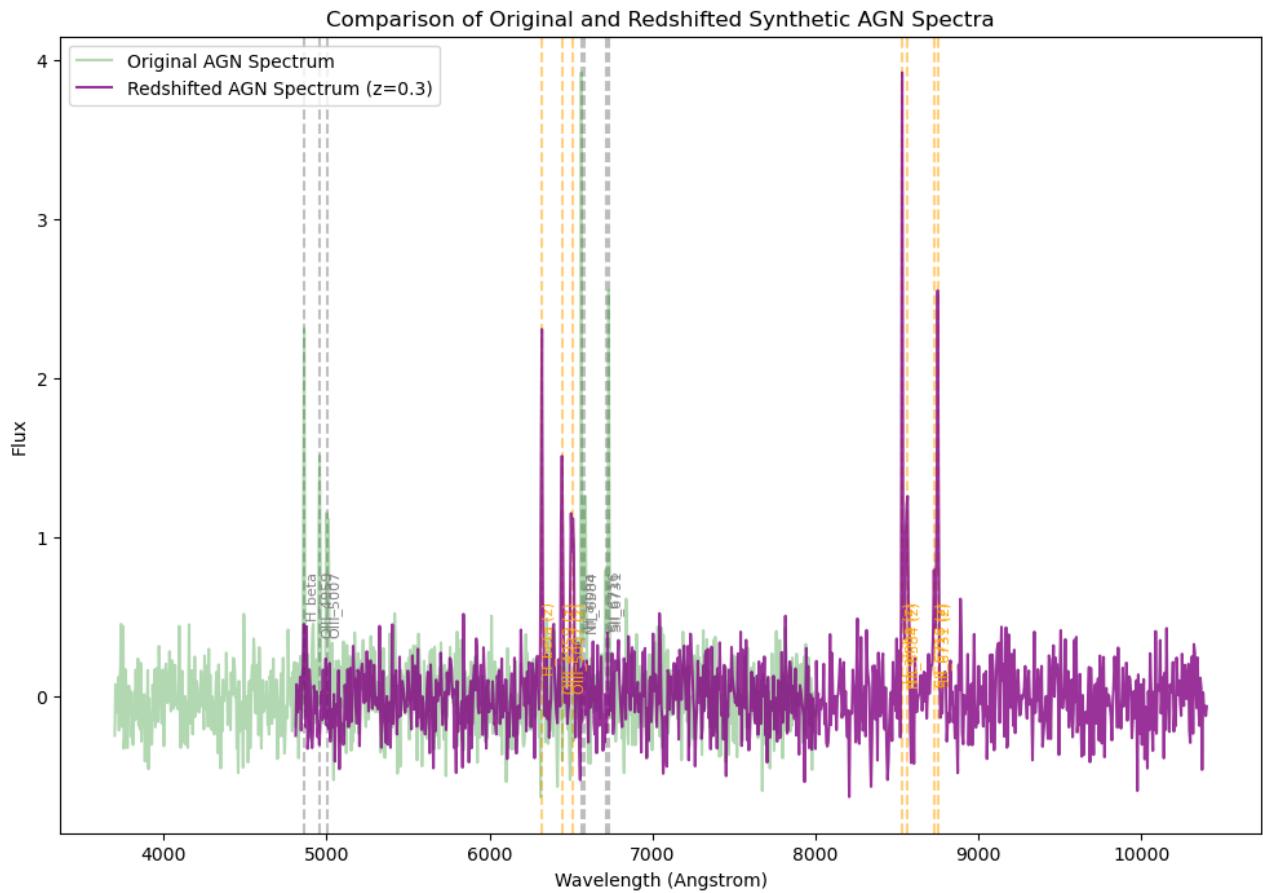
N = len(emission_line_centers) # number of emission lines
initial_line_intensities = [10] * N # Initial guess for each line intensity

# Optimization Model
best_fit = None
for redshift in np.linspace(0, 1, 100):
    result = minimize(fit_spectrum, initial_line_intensities, args=(redshift))
    if best_fit is None or result.fun < best_fit[1]:
        best_fit = (redshift, result.fun, result.x)

best_fit_flux = model_spectrum(observed_wavelengths, best_fit[0], best_fit[2])

plt.figure(figsize=(10, 6))
plt.plot(observed_wavelengths, observed_flux, label="Observed Spectrum", color='blue')
plt.plot(observed_wavelengths, best_fit_flux, label="Best-Fitted Model", color='red')
plt.xlabel("Wavelength")
plt.ylabel("Flux")
plt.title("Observed Spectrum vs Best-Fitted Model")
plt.legend()
plt.show()

print(f"Best fit redshift: {best_fit[0]}")
print(f"Best fit line intensities: {best_fit[2]})")
```



```
Best fit redshift: 0.30303030303030304
Best fit line intensities: [ 32.10930647  15.80354273  12.08655795   7.30040
496   2.64035827
        43.95218399 -11.92880199]
```

```
In [33]: # do another attempt
observed_wavelengths, observed_flux = generate_redshifted_spectrum(0.12)
emission_line_centers = [6563, 4861, 5007, 4959, 6584, 6716, 6731]
line_width = 10

def model_spectrum(wavelengths, redshift, line_intensities):
    model_flux = wavelengths ** -1.5
    for i, line_center in enumerate(emission_line_centers):
        shifted_center = line_center * (1 + redshift)
        model_flux += line_intensities[i] * norm.pdf(wavelengths, shifted_ce
    return model_flux

def chi_squared(obs_flux, model_flux):
    return np.sum((obs_flux - model_flux) ** 2)

def fit_spectrum(line_intensities, redshift):
    model_flux = model_spectrum(observed_wavelengths, redshift, line_intensi
    return chi_squared(observed_flux, model_flux)

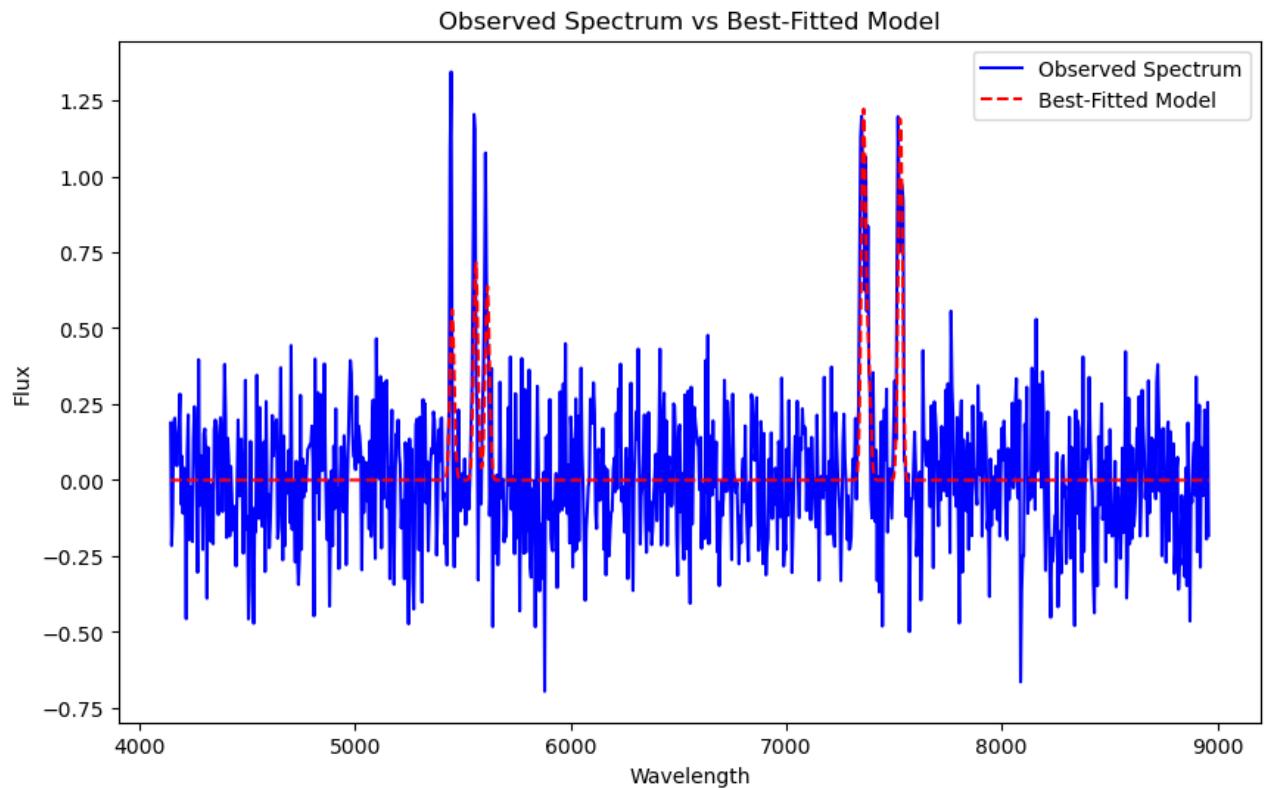
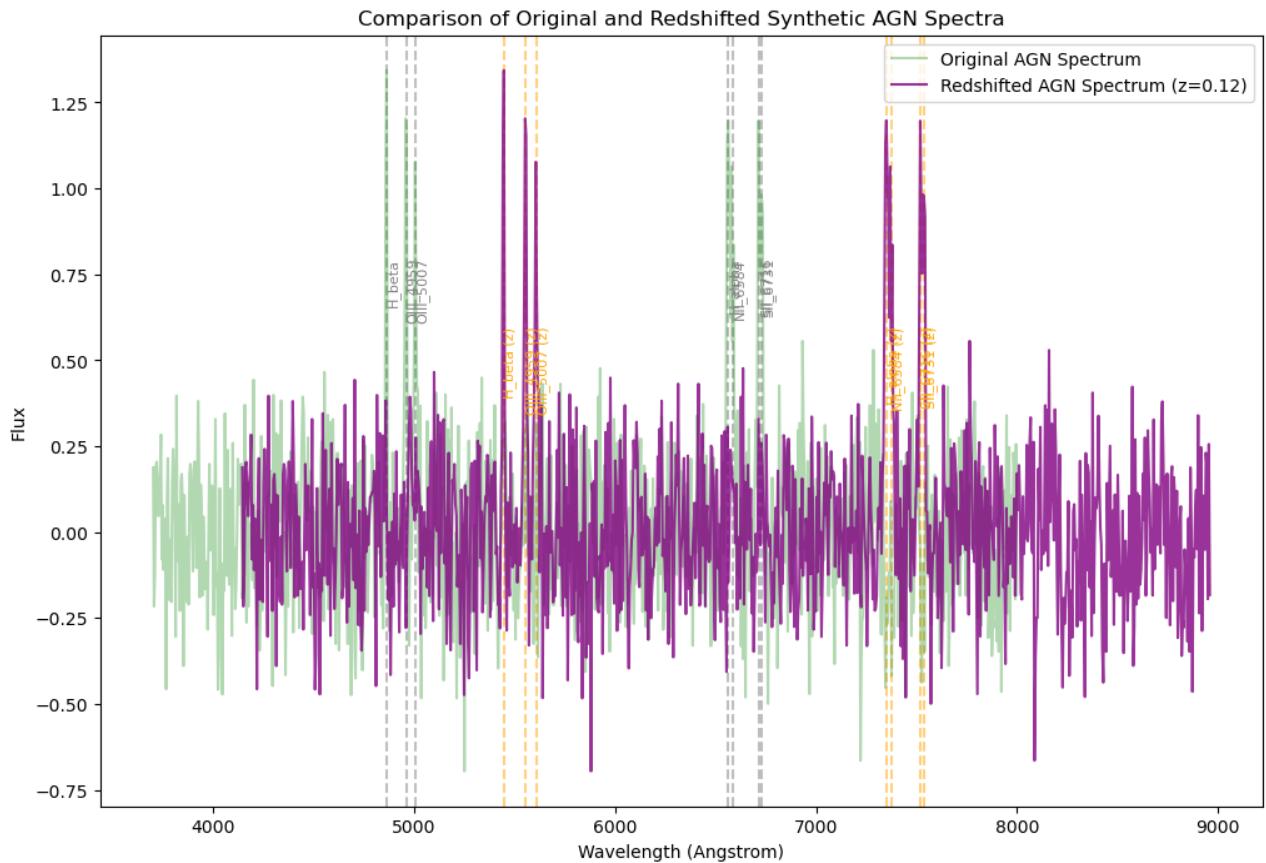
N = len(emission_line_centers) # number of emission lines
initial_line_intensities = [10] * N # Initial guess for each line intensity

# Optimization Model
best_fit = None
for redshift in np.linspace(0, 1, 100):
    result = minimize(fit_spectrum, initial_line_intensities, args=(redshift
    if best_fit is None or result.fun < best_fit[1]:
        best_fit = (redshift, result.fun, result.x)

best_fit_flux = model_spectrum(observed_wavelengths, best_fit[0], best_fit[2]

plt.figure(figsize=(10, 6))
plt.plot(observed_wavelengths, observed_flux, label="Observed Spectrum", col
plt.plot(observed_wavelengths, best_fit_flux, label="Best-Fitted Model", col
plt.xlabel("Wavelength")
plt.ylabel("Flux")
plt.title("Observed Spectrum vs Best-Fitted Model")
plt.legend()
plt.show()

print(f"Best fit redshift: {best_fit[0]}")
print(f"Best fit line intensities: {best_fit[2]}")
```



Best fit redshift: 0.1212121212121212

Best fit line intensities: [30.027984 14.06055572 16.00710874 18.0805335

9.34198316 30.43715122

-0.68771816]

```
In [42]: # this does not work for extremely large z (example z = 2) It cannot be corr
observed_wavelengths, observed_flux = generate_redshifted_spectrum(2)
emission_line_centers = [6563, 4861, 5007, 4959, 6584, 6716, 6731]
line_width = 10

def model_spectrum(wavelengths, redshift, line_intensities):
    model_flux = wavelengths ** -1.5
    for i, line_center in enumerate(emission_line_centers):
        shifted_center = line_center * (1 + redshift)
        model_flux += line_intensities[i] * norm.pdf(wavelengths, shifted_ce
    return model_flux

def chi_squared(obs_flux, model_flux):
    return np.sum((obs_flux - model_flux) ** 2)

def fit_spectrum(line_intensities, redshift):
    model_flux = model_spectrum(observed_wavelengths, redshift, line_intensi
    return chi_squared(observed_flux, model_flux)

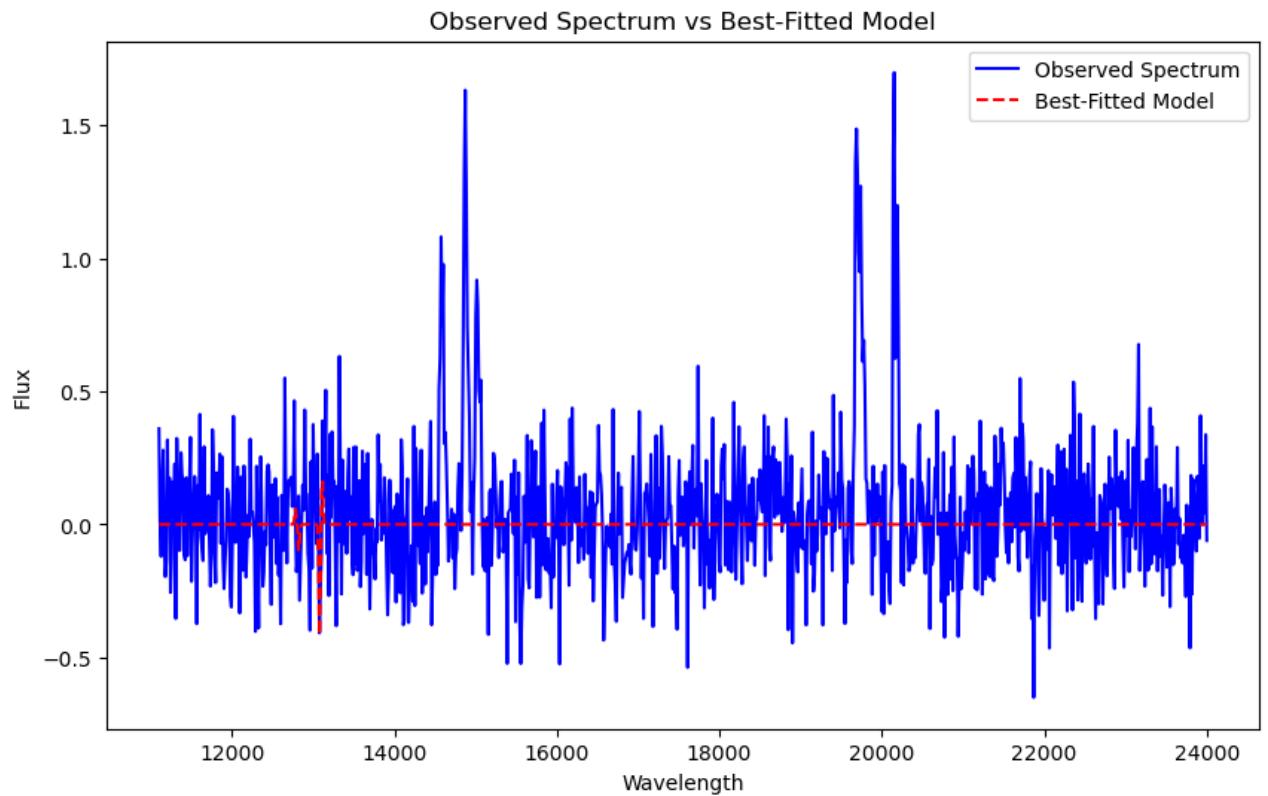
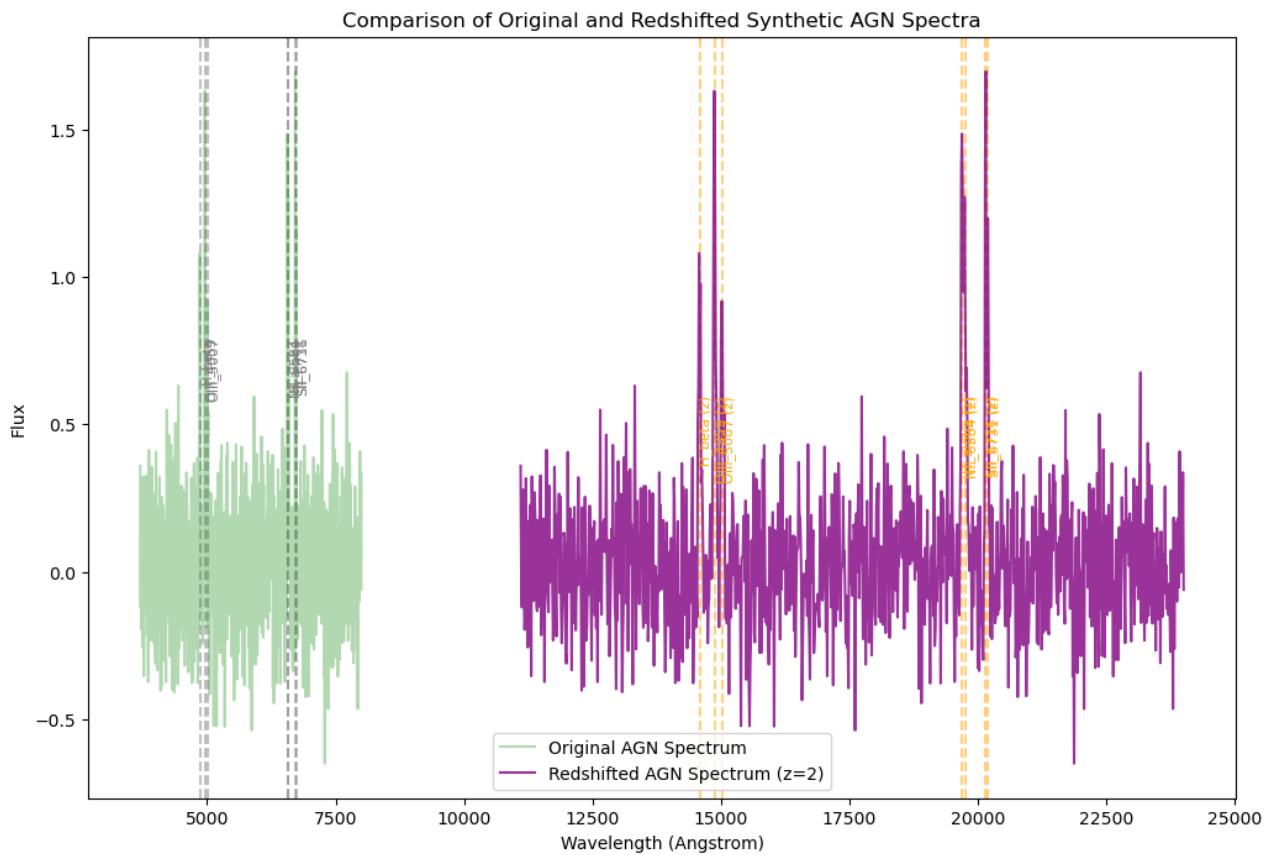
N = len(emission_line_centers) # number of emission lines
initial_line_intensities = [10] * N # Initial guess for each line intensity

# Optimization Model
best_fit = None
for redshift in np.linspace(0, 1, 20):
    result = minimize(fit_spectrum, initial_line_intensities, args=(redshift
    if best_fit is None or result.fun < best_fit[1]:
        best_fit = (redshift, result.fun, result.x)

best_fit_flux = model_spectrum(observed_wavelengths, best_fit[0], best_fit[2

plt.figure(figsize=(10, 6))
plt.plot(observed_wavelengths, observed_flux, label="Observed Spectrum", col
plt.plot(observed_wavelengths, best_fit_flux, label="Best-Fitted Model", col
plt.xlabel("Wavelength")
plt.ylabel("Flux")
plt.title("Observed Spectrum vs Best-Fitted Model")
plt.legend()
plt.show()

print(f"Best fit redshift: {best_fit[0]}")
print(f"Best fit line intensities: {best_fit[2]}")
```



Best fit redshift: 0.9473684210526315

Best fit line intensities: [ 1.49175322 10. 10. 10. -2.6171938  
-10.83645787 5.03885183 ]

# Week 3

Last time, I generated the spectrum for an AGN galaxy by specifying the emission lines and intensity. I then used optimization to find the best fit of the spectra and compute the redshift with a known "observed" spectra. However, as I discovered at the end of last time that the estimation becomes inaccurate at certain point. I here want to plot the error and the comparison between actual redshift and my estimated redshift out, to see where the accuracy "breaks".

```
In [41]: test_redshifts = np.linspace(0, 2, 5) #test redshifts with this range
estimated_redshifts = []
errors = []

N = len(emission_line_centers) # Number of emission lines

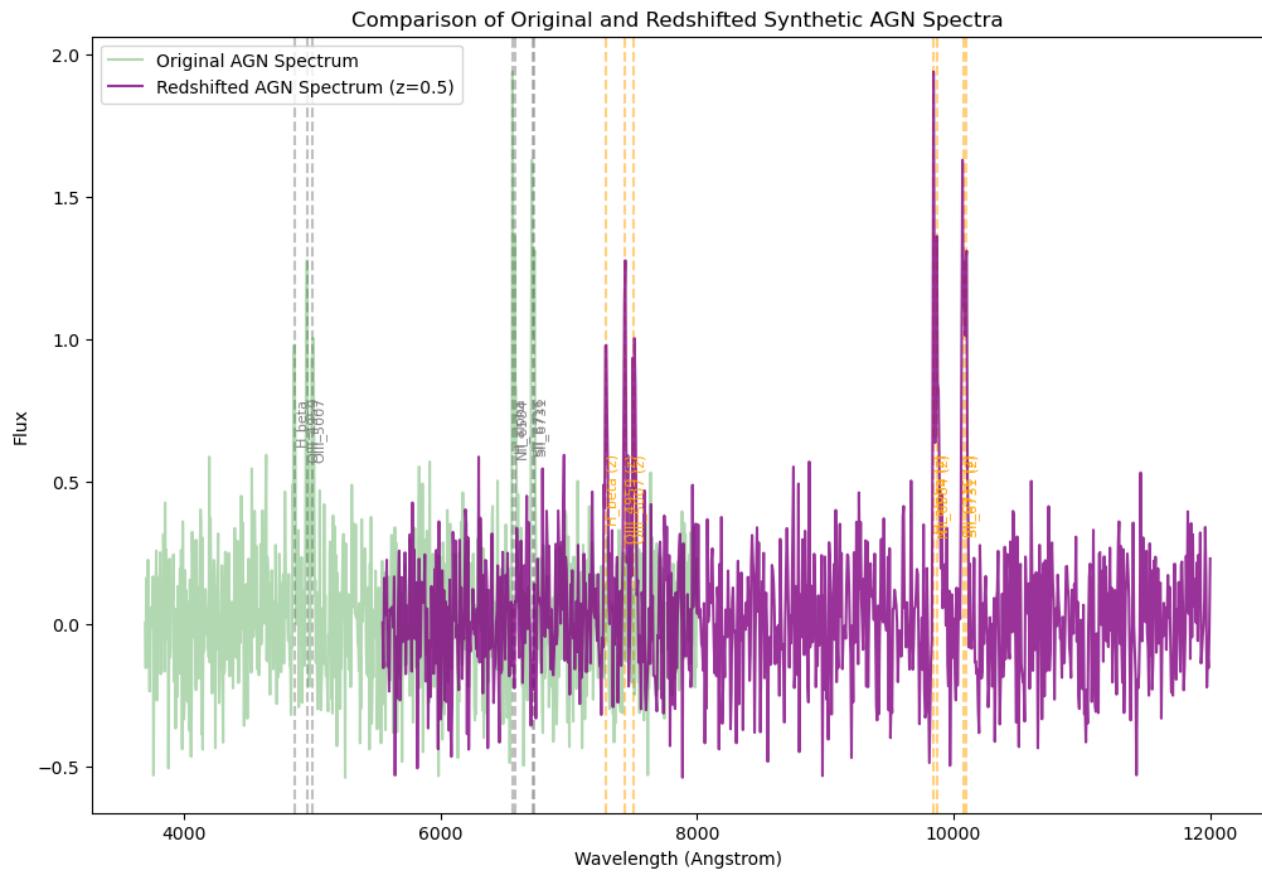
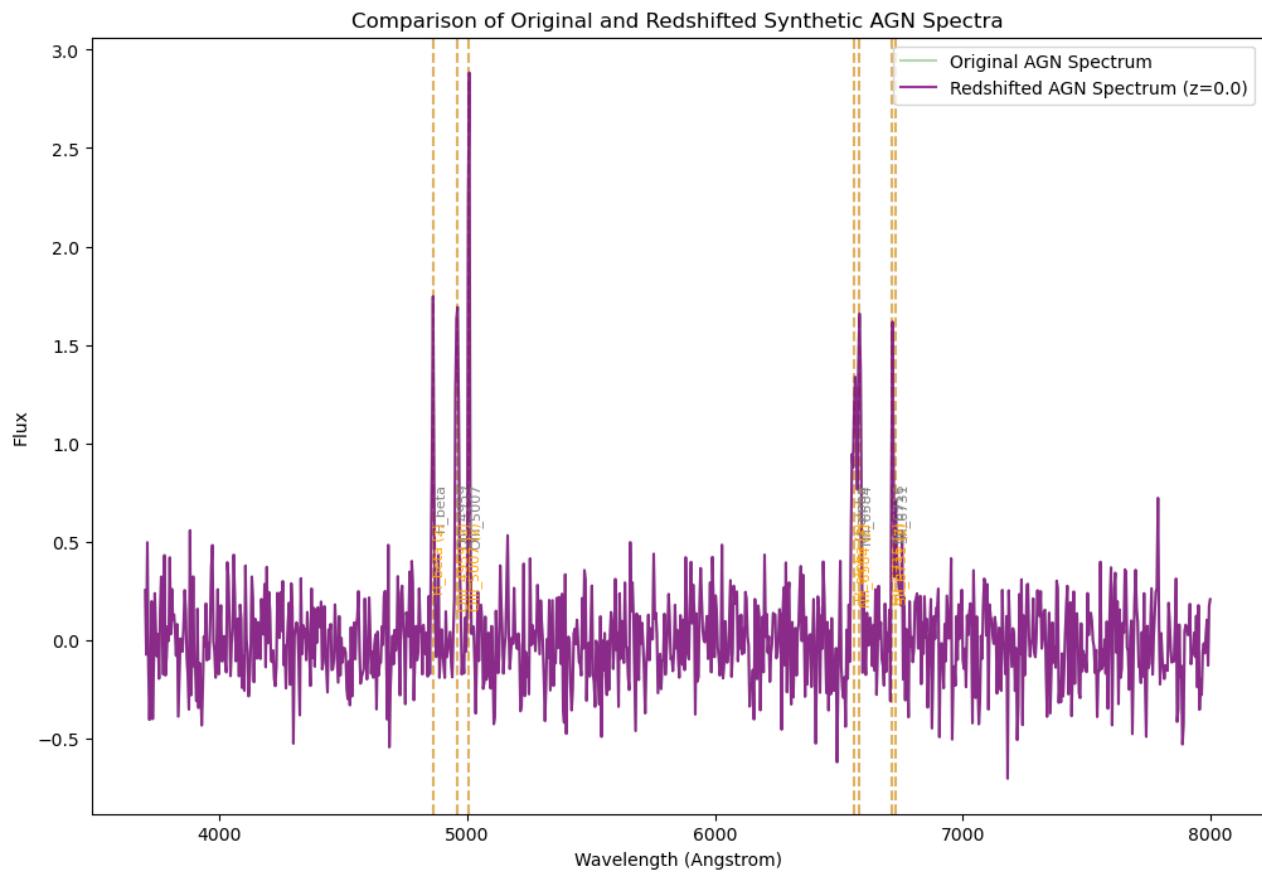
for test_redshift in test_redshifts:
    observed_wavelengths, observed_flux = generate_redshifted_spectrum(test_
    initial_line_intensities = [10] * N # Initial guess for line intensities

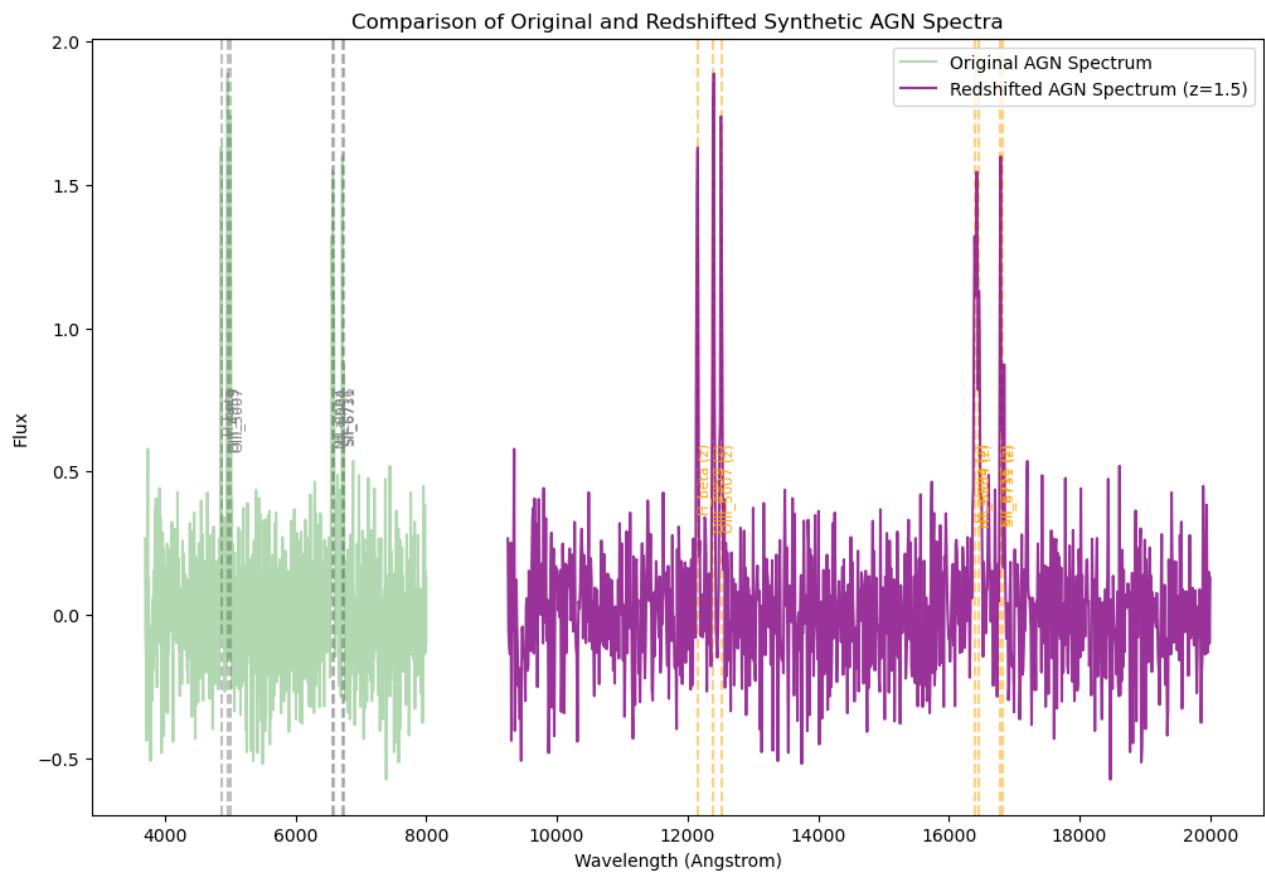
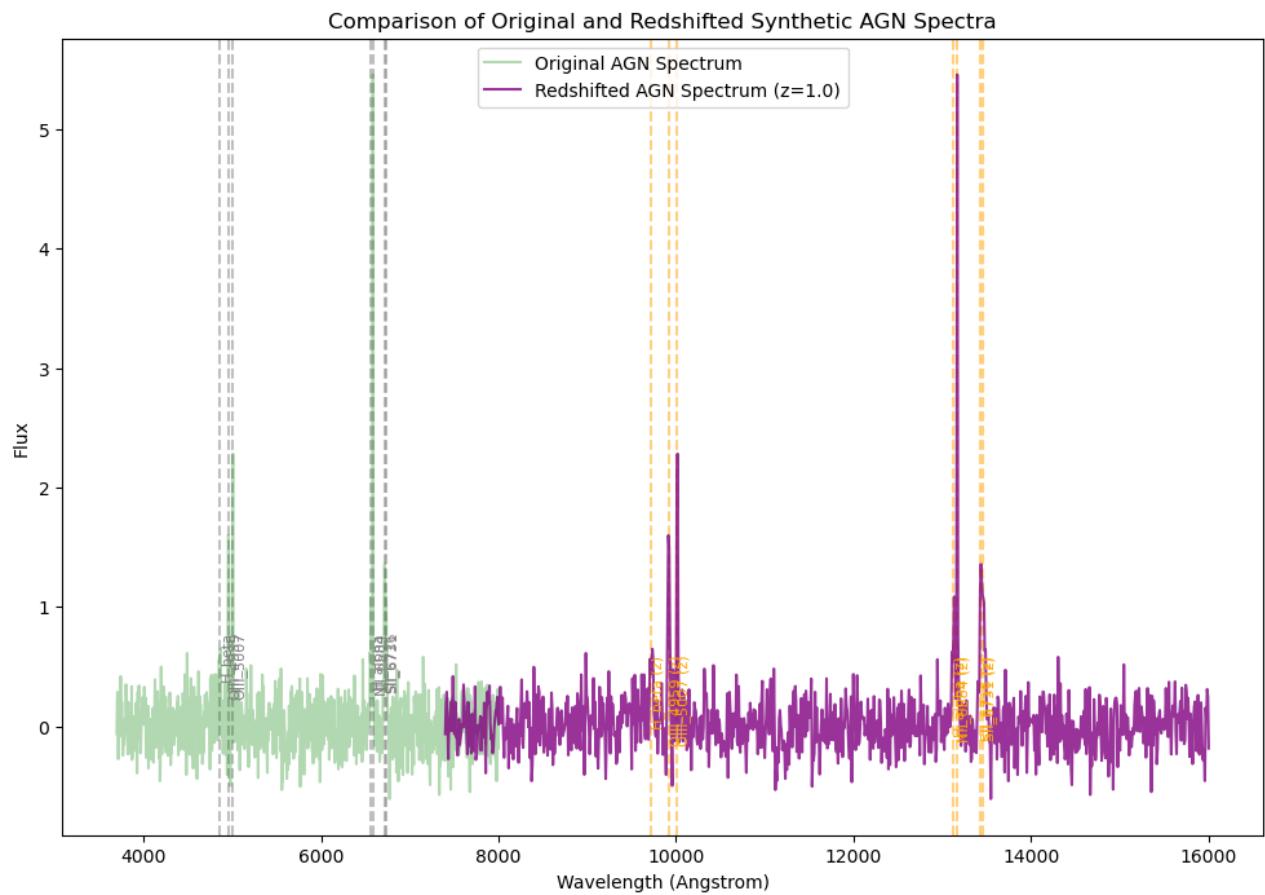
    # Optimization Model
    best_fit = None
    for redshift in np.linspace(0, 1, 20): # Fewer steps in the redshift range
        result = minimize(fit_spectrum, initial_line_intensities, args=(redshift))
        if best_fit is None or result.fun < best_fit[1]:
            best_fit = (redshift, result.fun, result.x)

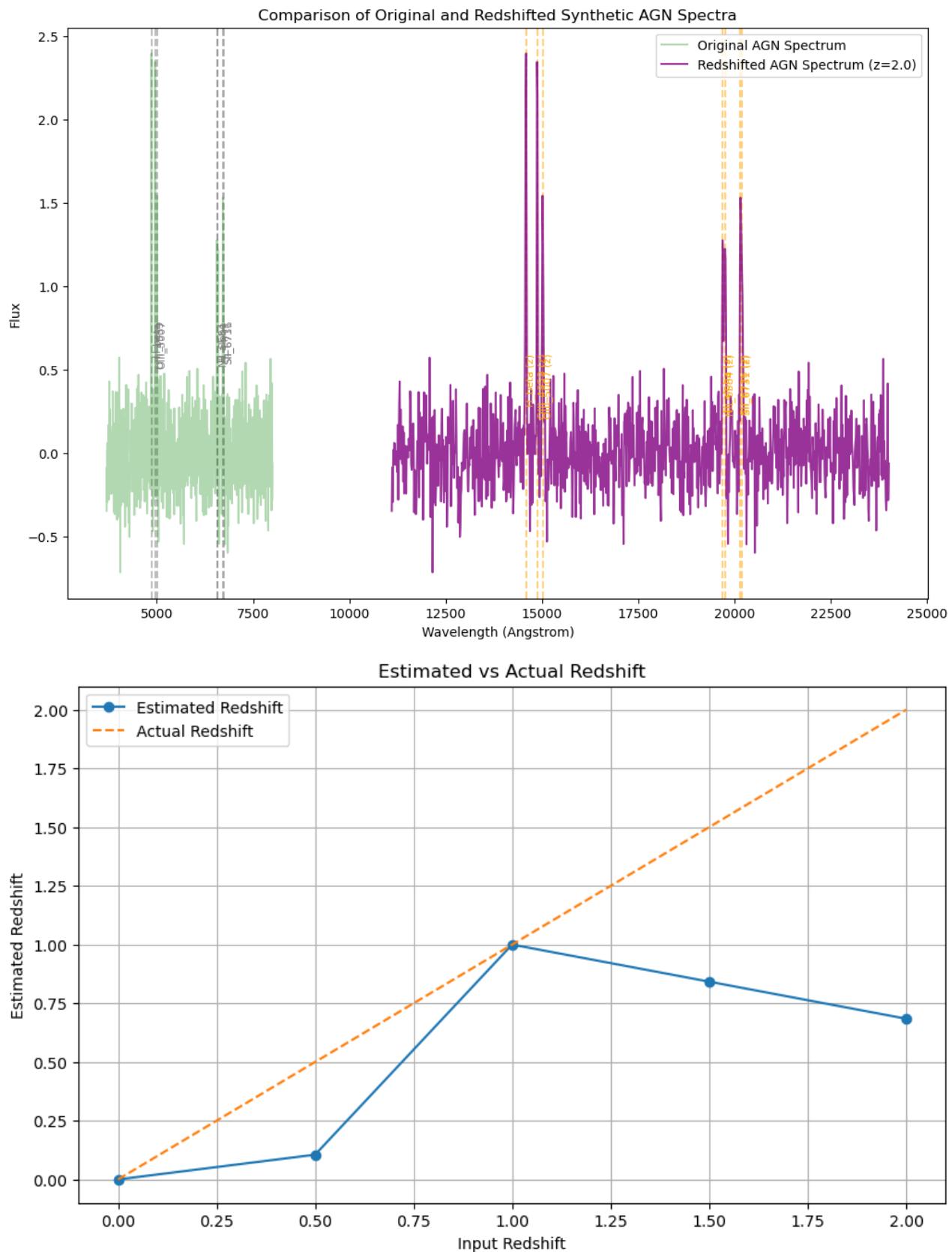
    estimated_redshifts.append(best_fit[0])
    errors.append(abs(best_fit[0] - test_redshift))

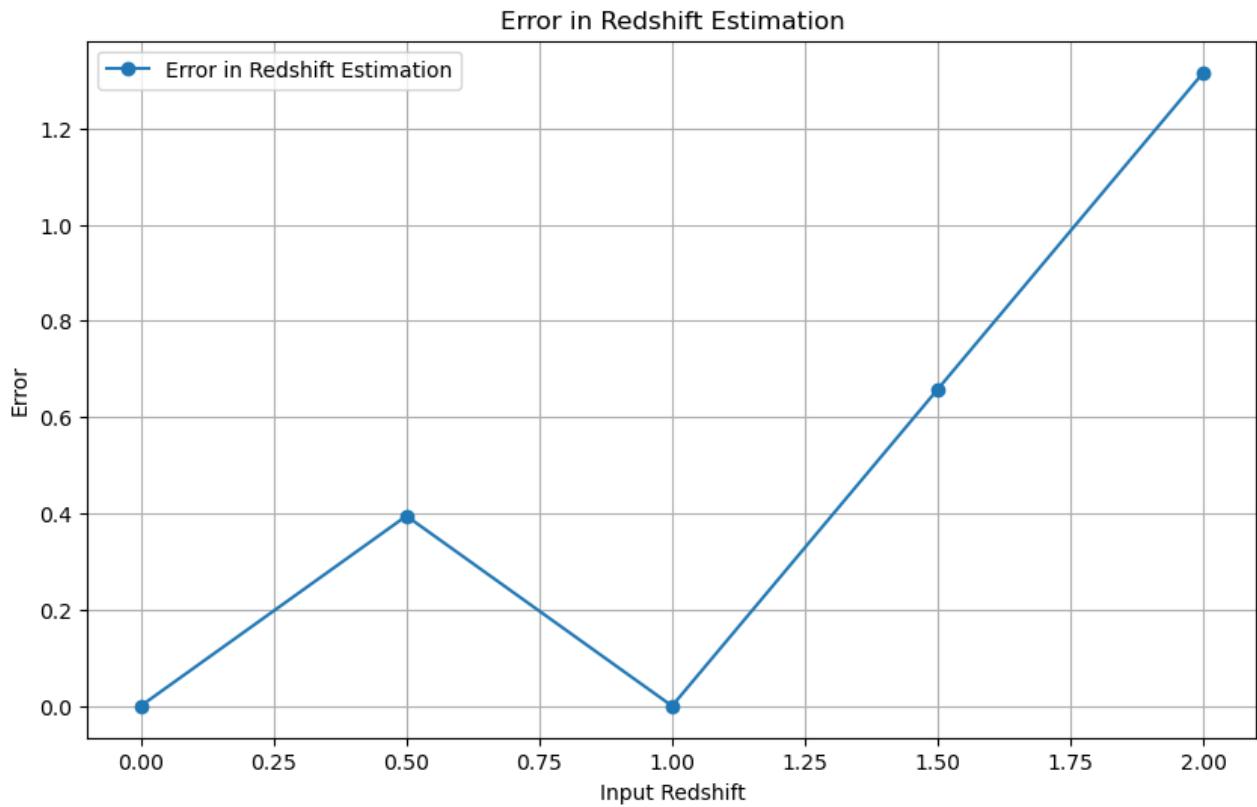
plt.figure(figsize=(10, 6))
plt.plot(test_redshifts, estimated_redshifts, label='Estimated Redshift', marker='o')
plt.plot(test_redshifts, test_redshifts, label='Actual Redshift', linestyle='--')
plt.xlabel('Input Redshift')
plt.ylabel('Estimated Redshift')
plt.title('Estimated vs Actual Redshift')
plt.legend()
plt.grid()
plt.show()

plt.figure(figsize=(10, 6))
plt.plot(test_redshifts, errors, label='Error in Redshift Estimation', marker='o')
plt.xlabel('Input Redshift')
plt.ylabel('Error')
plt.title('Error in Redshift Estimation')
plt.legend()
plt.grid()
plt.show()
```









I found that it's a mistake on the boundary condition (redshift range), since I set the estimated redshift to be 0 to 1, it only finds the best fit within this range. Also, the number of iteration is crucial as it gets more precise reading of the redshift. So I set the boundary from 0 to 10, which covers the most part of the redshift of AGN galaxy and changed the number of iteration to 1000. (This makes the code to run much longer, but gives much more accurate estimation).

```
In [43]: # with this new setting, I was able to predict redshift with a large number
observed_wavelengths, observed_flux = generate_redshifted_spectrum(5)
emission_line_centers = [6563, 4861, 5007, 4959, 6584, 6716, 6731]
line_width = 10

def model_spectrum(wavelengths, redshift, line_intensities):
    model_flux = wavelengths ** -1.5
    for i, line_center in enumerate(emission_line_centers):
        shifted_center = line_center * (1 + redshift)
        model_flux += line_intensities[i] * norm.pdf(wavelengths, shifted_ce
    return model_flux

def chi_squared(obs_flux, model_flux):
    return np.sum((obs_flux - model_flux) ** 2)

def fit_spectrum(line_intensities, redshift):
    model_flux = model_spectrum(observed_wavelengths, redshift, line_intensi
    return chi_squared(observed_flux, model_flux)

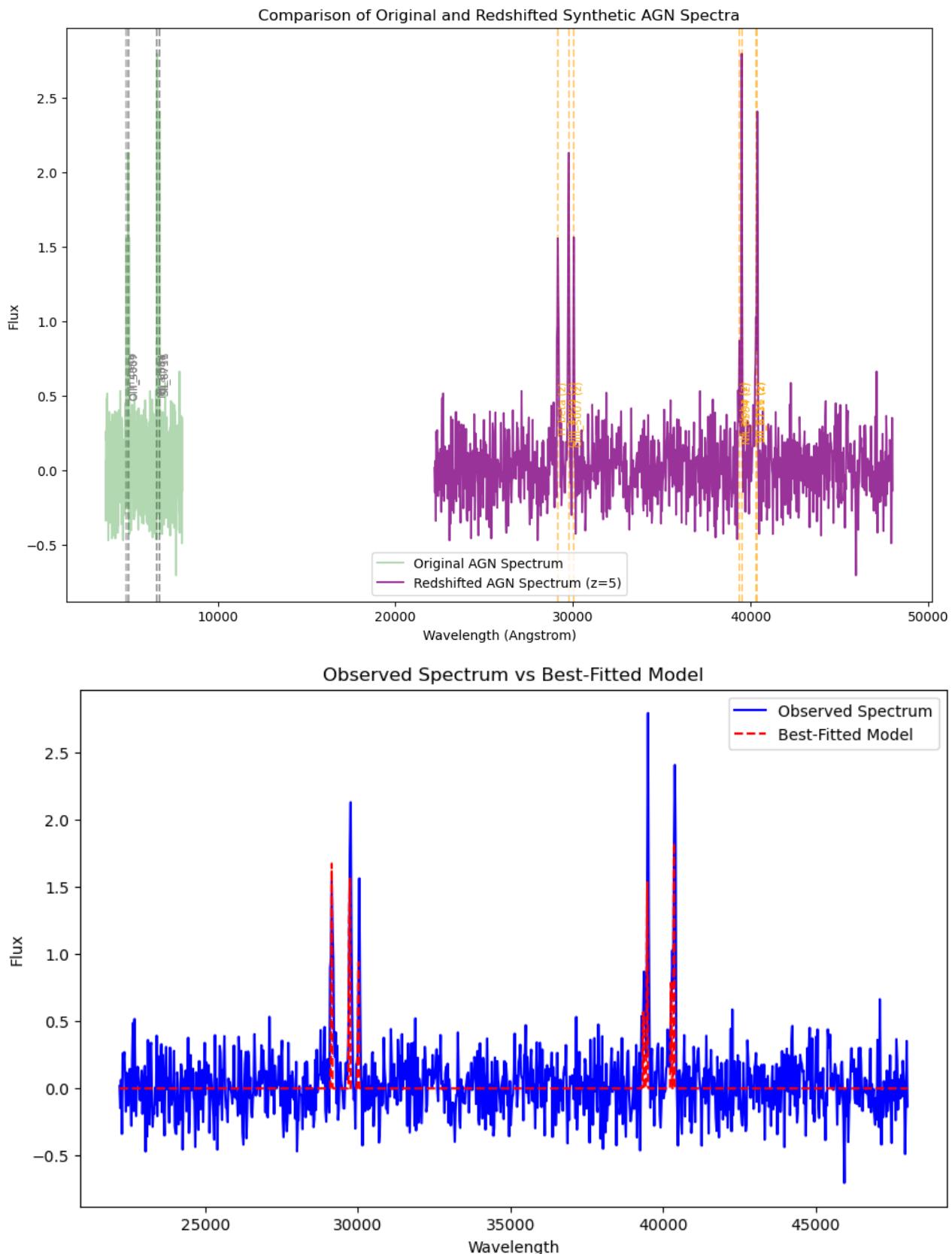
N = len(emission_line_centers) # number of emission lines
initial_line_intensities = [10] * N # Initial guess for each line intensity

# Optimization Model
best_fit = None
for redshift in np.linspace(0, 10, 1000):
    result = minimize(fit_spectrum, initial_line_intensities, args=(redshift
    if best_fit is None or result.fun < best_fit[1]:
        best_fit = (redshift, result.fun, result.x)

best_fit_flux = model_spectrum(observed_wavelengths, best_fit[0], best_fit[2]

plt.figure(figsize=(10, 6))
plt.plot(observed_wavelengths, observed_flux, label="Observed Spectrum", col
plt.plot(observed_wavelengths, best_fit_flux, label="Best-Fitted Model", col
plt.xlabel("Wavelength")
plt.ylabel("Flux")
plt.title("Observed Spectrum vs Best-Fitted Model")
plt.legend()
plt.show()

print(f"Best fit redshift: {best_fit[0]}")
print(f"Best fit line intensities: {best_fit[2]}")
```



Best fit redshift: 4.994994994994995

Best fit line intensities: [14.94898065 48.70501321 33.17850489 80.04403446  
47.2735389 32.88377089  
48.29315571]

This allows the predicted redshift to be as accurate as 0.01. Another important part of real spectrum is the noise, which is the error produced by the instrument or atmosphere, we can add that in our code to generate an error array.

```
In [47]: def generate_redshifted_spectrum(redshift):
    wavelengths = np.linspace(3700, 8000, 1000)
    alpha = -15
    flux_continuum = wavelengths ** alpha

    # emission lines
    emission_lines = {
        "H_alpha": {"center": 6563, "element": "H"},
        "H_beta": {"center": 4861, "element": "H"},
        "OIII_5007": {"center": 5007, "element": "O"},
        "OIII_4959": {"center": 4959, "element": "O"},
        "NII_6584": {"center": 6584, "element": "N"},
        "SII_6716": {"center": 6716, "element": "S"},
        "SII_6731": {"center": 6731, "element": "S"}
    }

    # Function to add emission line
    def add_emission_line(wavelength_center, width, height):
        return height * norm.pdf(wavelengths, wavelength_center, width)

    # Adding emission lines
    width_range = (2, 10)
    base_height_range = (10, 20)
    for line in emission_lines.values():
        width = random.uniform(*width_range)
        height = random.uniform(*base_height_range)
        if line["element"] in ["H", "O", "N"]:
            height *= 1.5
        flux_continuum += add_emission_line(line["center"], width, height)

    # Adding noise
    noise_level = 0.2
    noise = np.random.normal(0, noise_level, wavelengths.shape)
    flux_continuum += noise

    # Error array representing uncertainty in flux measurements (by instrument)
    error_array = np.random.normal(0.1, 0.02, wavelengths.shape) * flux_continuum

    # Signal to Noise Ratio (SNR)
    snr = flux_continuum / error_array

    original_flux_continuum = flux_continuum.copy()
    redshifted_wavelengths = wavelengths * (1 + redshift)
    redshifted_flux_continuum = original_flux_continuum

    plt.figure(figsize=(12, 10))
    plt.subplot(2, 1, 1) # Upper plot for spectra
```

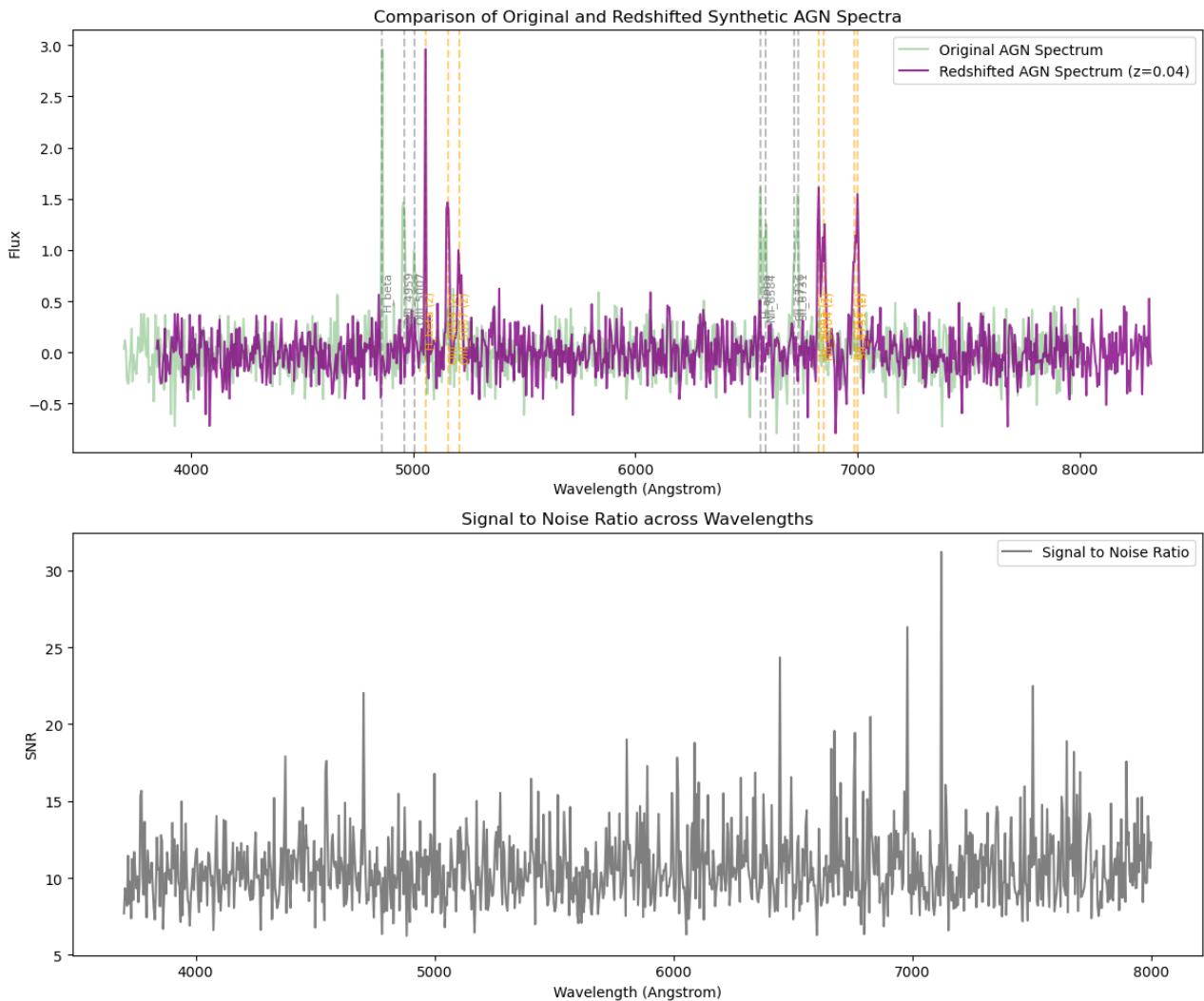
```
for line_name, line in emission_lines.items():
    original_center = line["center"]
    redshifted_center = original_center * (1 + redshift)
    plt.axvline(x=original_center, color='gray', linestyle='--', alpha=0.5)
    plt.axvline(x=redshifted_center, color='orange', linestyle='--', alpha=0.5)
    plt.text(original_center, plt.ylim()[1]*0.8, line_name, rotation=90, color='gray')
    plt.text(redshifted_center, plt.ylim()[1]*0.6, f'{line_name} (z)', color='orange')

plt.plot(wavelengths, original_flux_continuum, label='Original AGN Spectrum')
plt.plot(redshifted_wavelengths, redshifted_flux_continuum, label=f'Redsft {redshift} AGN Spectrum')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('Flux')
plt.title('Comparison of Original and Redshifted Synthetic AGN Spectra')
plt.legend()

plt.subplot(2, 1, 2) # Lower plot for SNR
plt.plot(wavelengths, snr, label='Signal to Noise Ratio', color='black')
plt.xlabel('Wavelength (Angstrom)')
plt.ylabel('SNR')
plt.title('Signal to Noise Ratio across Wavelengths')
plt.legend()

plt.tight_layout()
plt.show()

return redshifted_wavelengths, flux_continuum, error_array
observed_wavelength, observed_flux, error = generate_redshifted_spectrum(0.0)
```



```
In [80]: # Update the galaxy class, now I modified it according to what we covered in
class Galaxy:
    '''initialize the galaxy'''
    def __init__(self, mass, position, velocity, black_hole_mass, initial_temperature, redshift):
        self.mass = mass
        self.position = position
        self.velocity = velocity
        self.black_hole_mass = black_hole_mass
        self.initial_temperature = initial_temperature
        self.redshift = redshift

    '''black hole accretion model'''
    def calculate_accretion_rate(self, rho = np.random.uniform(1e-14, 1e-13)):
        black_hole_mass = self.black_hole_mass
        # The input black hole mass should be in the unit of solar mass
        G = 6.67430e-11 * u.m**3 / u.kg / u.s**2 # gravitational constant
        M = black_hole_mass * 1.98847e30 * u.kg # solar masses to kg
        rho = rho * u.kg / u.m**3 # density of the accreting material in kg
        v = v * u.m / u.s # relative velocity in m/s
        c_s = c_s * u.m / u.s # sound speed in the material in m/s
        # Bondi-Hoyle-Lyttleton accretion rate
```

```

accretion_rate = (4 * np.pi * G**2 * M**2 * rho) / ((v**2 + c_s**2)*
# Convert accretion rate to solar masses per year
accretion_rate_solar_masses_per_year = accretion_rate.to(u.kg / u.s)
return accretion_rate_solar_masses_per_year

def calculate_radiation(self, accretion_rate, epsilon = 0.1):
    # The accretion rate is obtained from the previous function calculate
    c = 299792458 * u.m / u.s
    # calculate the luminosity (or radiation)
    radiation = epsilon * accretion_rate * 1.98847e30 * c**2 # Convert
    return radiation

def accretion_model(self, accretion_radius, other_parameters):
    black_hole_mass = self.black_hole_mass
    accretion_rate = self.calculate_accretion_rate(black_hole_mass, accr
    radiation = self.calculate_radiation(accretion_rate, other_parameter
    jets = self.determine_jet_production(accretion_rate, radiation, othe
    return accretion_rate, radiation, jets

'''cooling and heating process in different radius of the galaxy'''
def heating_function(self, r, time, agn_luminosity_initial = 1e44, star_
    decay_factor = 0.05
    star_formation_rate = star_formation_initial / (1 + decay_factor * t
    agn_luminosity = agn_luminosity_initial / (1 + decay_factor * time)
    density = self.density_radius(r, rho_0, r_0, alpha)
    new_temp = 1e-35 * (star_formation_rate + agn_luminosity) * density
    return new_temp

def cooling_function(self, temperature, r, rho_0 = 1e5, r_0 = 1, alpha =
    density = self.density_radius(r, rho_0, r_0, alpha)
    if temperature < 1e4:
        new_temp = 1e-22 * temperature * density**2
    else:
        new_temp = 1e-24 * temperature**1.5 * density**2
    return new_temp

def cooling_rate(self, new_temp, density):
    cooling_rate = new_temp * density**2
    return cooling_rate

def ism_temperature_evolution(self, initial_temperature, r, metallicity,
    temperature = initial_temperature
    density = self.density_radius(r, rho_0, r_0, alpha)
    for time in range(total_time):
        # cooling and heating rates
        cooling_rate = self.cooling_function(temperature, r, rho_0 = 1e5
        heating_rate = self.heating_function(star_formation_rate, agn_lu
        temperature_change = (heating_rate - cooling_rate) * timestep
        temperature += temperature_change
        if temperature < 10: # 10 K is a reasonable lower limit for ISM
            temperature = 10
    return temperature

```

```
def ism_temperature_evolution_over_time(self, initial_temperature, r, rh
temperature = initial_temperature
temperatures, cooling_rates, heating_rates = [], [], []
for time in range(total_time):
    density = self.density_radius(r, rho_0, r_0, alpha)
    cooling_rate = self.cooling_function(temperature, r, rho_0, r_0,
heating_rate = self.heating_function(star_formation_initial, agn
temperature_change = (heating_rate - cooling_rate) * timestep
temperature += temperature_change
if temperature < 10:
    temperature = 10
temperatures.append(temperature)
cooling_rates.append(cooling_rate)
heating_rates.append(heating_rate)
return temperatures, cooling_rates, heating_rates

'''spectra modeling (with noise and measurement error included)'''
def generate_redshifted_spectrum(self, plot_spectrum = True):
    redshift = self.redshift
    wavelengths = np.linspace(3700, 8000, 1000)
    alpha = -15
    flux_continuum = wavelengths ** alpha

    # emission lines
    emission_lines = {
        "H_alpha": {"center": 6563, "element": "H"},
        "H_beta": {"center": 4861, "element": "H"},
        "OIII_5007": {"center": 5007, "element": "O"},
        "OIII_4959": {"center": 4959, "element": "O"},
        "NII_6584": {"center": 6584, "element": "N"},
        "SII_6716": {"center": 6716, "element": "S"},
        "SII_6731": {"center": 6731, "element": "S"}
    }

    # Function to add emission line
    def add_emission_line(wavelength_center, width, height):
        return height * norm.pdf(wavelengths, wavelength_center, width)

    # Adding emission lines
    width_range = (2, 10)
    base_height_range = (10, 20)
    for line in emission_lines.values():
        width = random.uniform(*width_range)
        height = random.uniform(*base_height_range)
        if line["element"] in ["H", "O", "N"]:
            height *= 1.5
        flux_continuum += add_emission_line(line["center"], width, height)

    # Adding noise
    noise_level = 0.2
    noise = np.random.normal(0, noise_level, wavelengths.shape)
    flux_continuum += noise
```

```

# Error array representing uncertainty in flux measurements (by instant)
error_array = np.random.normal(0.1, 0.02, wavelengths.shape) * flux_

# Signal to Noise Ratio (SNR)
snr = flux_continuum / error_array

original_flux_continuum = flux_continuum.copy()
redshifted_wavelengths = wavelengths * (1 + redshift)
redshifted_flux_continuum = original_flux_continuum

if plot_spectrum:
    plt.figure(figsize=(12, 10))
    plt.subplot(2, 1, 1) # Upper plot for spectra
    for line_name, line in emission_lines.items():
        original_center = line["center"]
        redshifted_center = original_center * (1 + redshift)
        plt.axvline(x=original_center, color='gray', linestyle='--',
                    label='Original')
        plt.axvline(x=redshifted_center, color='orange', linestyle='--',
                    label='Redshifted')
        plt.text(original_center, plt.ylim()[1]*0.8, line_name, rotation=90)
        plt.text(redshifted_center, plt.ylim()[1]*0.6, f'{line_name}'

    plt.plot(wavelengths, original_flux_continuum, label='Original AGN')
    plt.plot(redshifted_wavelengths, redshifted_flux_continuum, label='Redshifted AGN')
    plt.xlabel('Wavelength (Angstrom)')
    plt.ylabel('Flux')
    plt.title('Comparison of Original and Redshifted Synthetic AGN Spectra')
    plt.legend()

    plt.subplot(2, 1, 2) # Lower plot for SNR
    plt.plot(wavelengths, snr, label='Signal to Noise Ratio', color='blue')
    plt.xlabel('Wavelength (Angstrom)')
    plt.ylabel('SNR')
    plt.title('Signal to Noise Ratio across Wavelengths')
    plt.legend()

    plt.tight_layout()
    plt.show()

return redshifted_wavelengths, flux_continuum, error_array

```

In [81]:

```

# Now, I can randomly generate a AGN galaxy using the "Galaxy" Class
mass = 1e12 # Galaxy mass in solar masses
position = np.array([1e6, 2e6, 3e6]) # Galaxy position in light years (x, y, z)
velocity = np.array([1000, 2000, 1500]) # Galaxy velocity in km/s (velocity components)
black_hole_mass = 1e9
initial_temperature = 1e4
redshift = 0.03
AGN_gall = Galaxy(mass, position, velocity, black_hole_mass, initial_temperature)

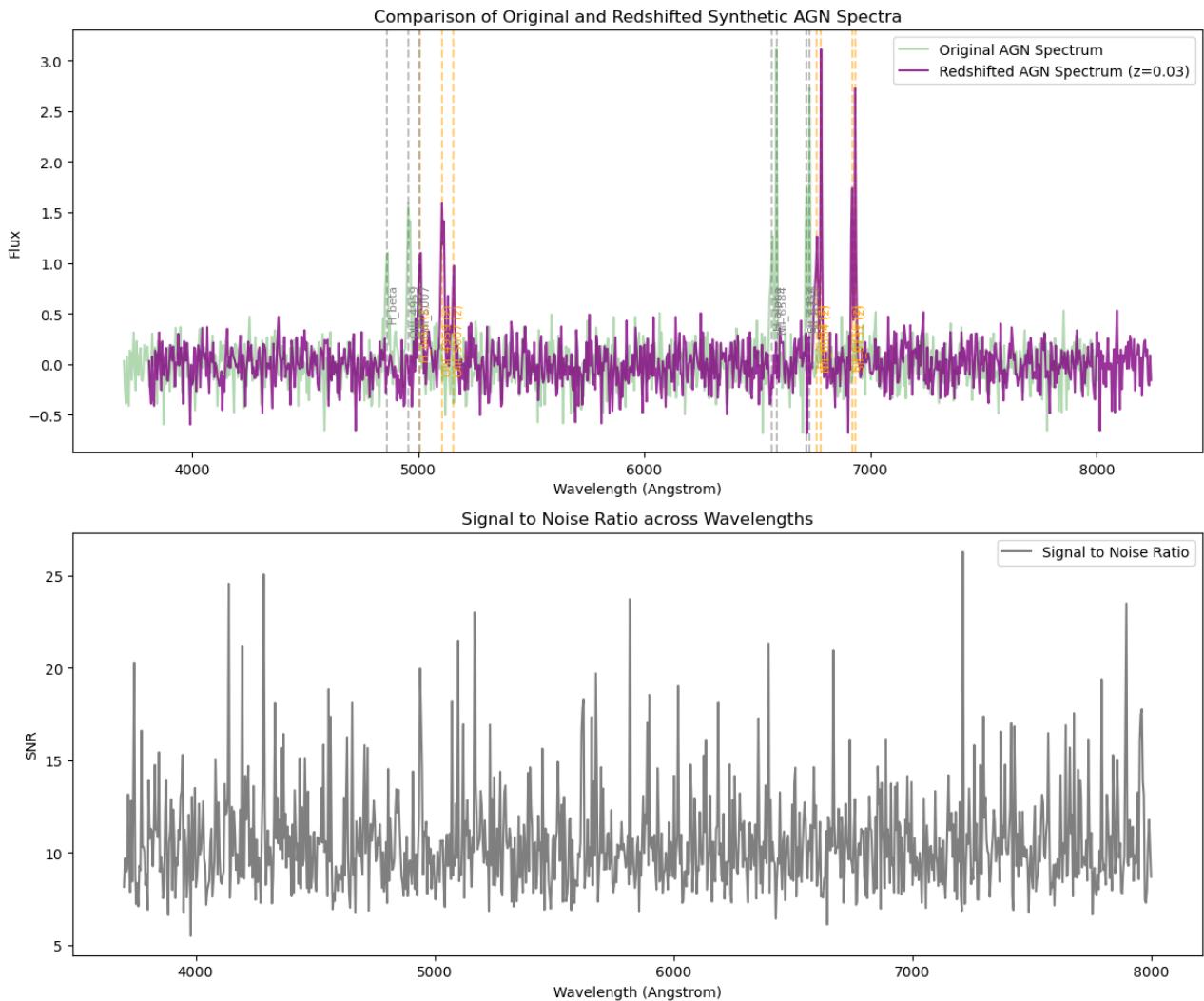
```

In [82]:

```

# I can plot the spectra of this galaxy and find the wavelength, flux, and error arrays
redshifted_wavelengths, flux_continuum, error_array = AGN_gall.generate_redshift()

```



```
In [83]: def calculate_line_ratio(intensity1, intensity2):
    return intensity1 / intensity2

def estimate_sfr_from_Halpha(L_Halpha):
    # Kennicutt (1998) relation: SFR = 7.9e-42 * L(H-alpha)
    return 7.9e-42 * L_Halpha
```

```
In [84]: # the intensity of H_alpha and H_beta lines can be calculated from:
H_alpha_flux = flux_continuum[np.argmin(np.abs(redshifted_wavelengths - 6563
H_beta_flux = flux_continuum[np.argmin(np.abs(redshifted_wavelengths - 4861
# then the H_alpha/H_beta ratio is
H_alpha_H_beta_ratio = calculate_line_ratio(H_alpha_flux, H_beta_flux)
print('H_alpha/H_beta =', H_alpha_H_beta_ratio )
# and calculate the star formation rate
SFR = estimate_sfr_from_Halpha(H_alpha_flux)
print("Star Formation Rate:", SFR, "Solar Masses per Year")
```

H\_alpha/H\_beta = 0.894918435893807  
 Star Formation Rate: 7.627137991139248e-42 Solar Masses per Year

```
In [107]: # To observe the correlation between H_alpha/H_beta ratio and Star formation
```

```
# I can model 100 such galaxies using the galaxy class, and plot the correlation
def simulate_galaxy_and_calculate_properties():
    redshift = np.random.uniform(0.01, 1)
    # initialize galaxy
    galaxy = Galaxy(mass=np.random.uniform(1e11, 1e13),
                    position=np.random.rand(3),
                    velocity=np.random.rand(3),
                    black_hole_mass=np.random.uniform(1e7, 1e9),
                    initial_temperature=np.random.uniform(1e4, 1e5),
                    redshift=redshift)
    r = redshift

    redshifted_wavelengths, flux_continuum, _ = galaxy.generate_redshifted_s

    # H_alpha and H_beta
    H_alpha_flux = flux_continuum[np.argmin(np.abs(redshifted_wavelengths - 6563))]
    H_beta_flux = flux_continuum[np.argmin(np.abs(redshifted_wavelengths - 4861))]

    # line ratio and SFR
    line_ratio = calculate_line_ratio(H_alpha_flux, H_beta_flux)
    sfr = estimate_sfr_from_Halpha(H_alpha_flux)

    return line_ratio, sfr, r

line_ratios = []
sfr_values = []
redshifts = []

for _ in range(1000):
    ratio, sfr, r = simulate_galaxy_and_calculate_properties()
    line_ratios.append(ratio)
    sfr_values.append(sfr)
    redshifts.append(r)

# print(redshifts)
line_ratios = np.array(line_ratios)
sfr_values = np.array(sfr_values)
redshifts = np.array(redshifts)

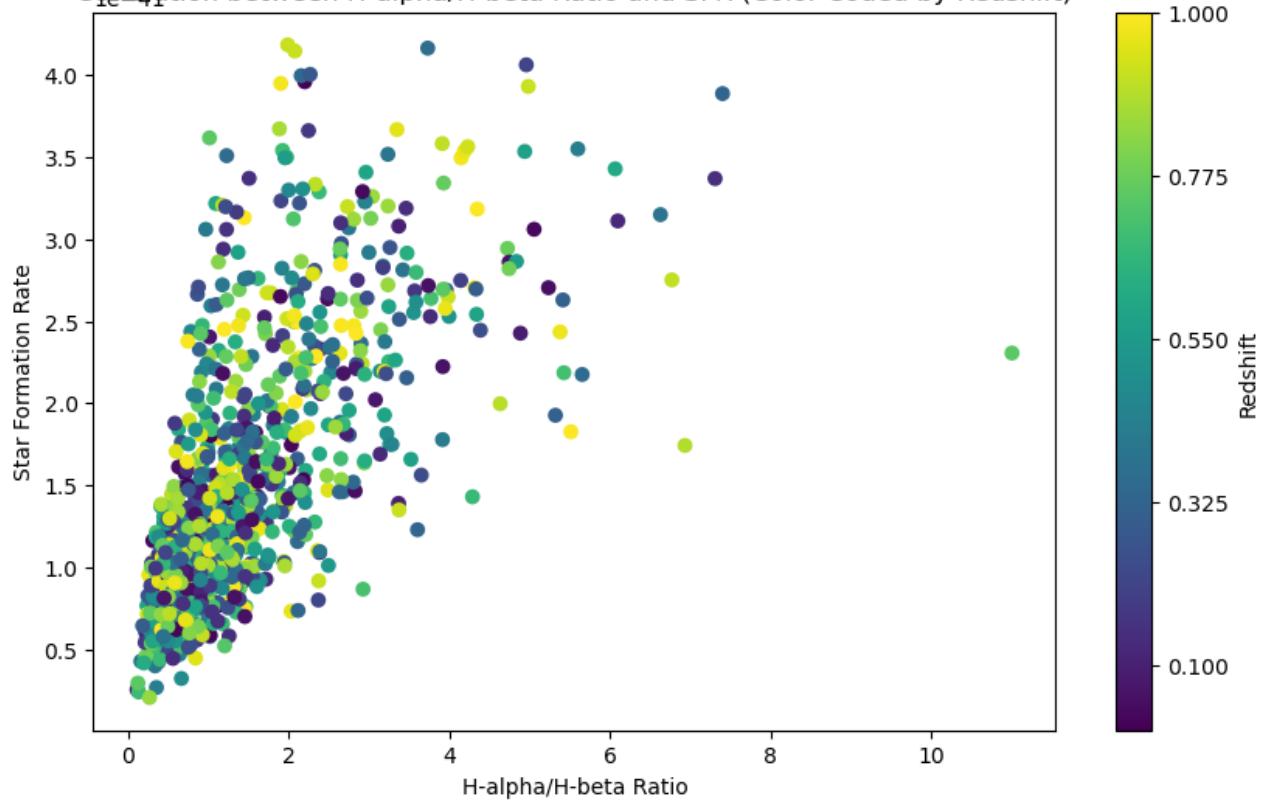
correlation = np.corrcoef(line_ratios, sfr_values)[0, 1]
print("Correlation between H-alpha/H-beta ratio and SFR:", correlation)
plt.figure(figsize=(10, 6))
scatter = plt.scatter(line_ratios, sfr_values, c=redshifts, cmap='viridis')
plt.xlabel('H-alpha/H-beta Ratio')
plt.ylabel('Star Formation Rate')
plt.title('Correlation between H-alpha/H-beta Ratio and SFR (Color-Coded by Redshift)')

# Adding a color bar
cbar = plt.colorbar(scatter)
cbar.set_label('Redshift')
cbar.set_ticks(np.linspace(0.1, 1, 5))
```

```
plt.show()
```

Correlation between H-alpha/H-beta ratio and SFR: 0.6704983723809521

Correlation between H-alpha/H-beta Ratio and SFR (Color-Coded by Redshift)



In [112...]

```
# I can use the KDE plot to present the density distribution of the H_alpha/
# 2D Gaussian KDE
xy = np.vstack([line_ratios, sfr_values])
z = gaussian_kde(xy)(xy)

# sort the points by density for the scatter plot
idx = z.argsort()
line_ratios, sfr_values, z, redshifts = line_ratios[idx], sfr_values[idx], z[idx]

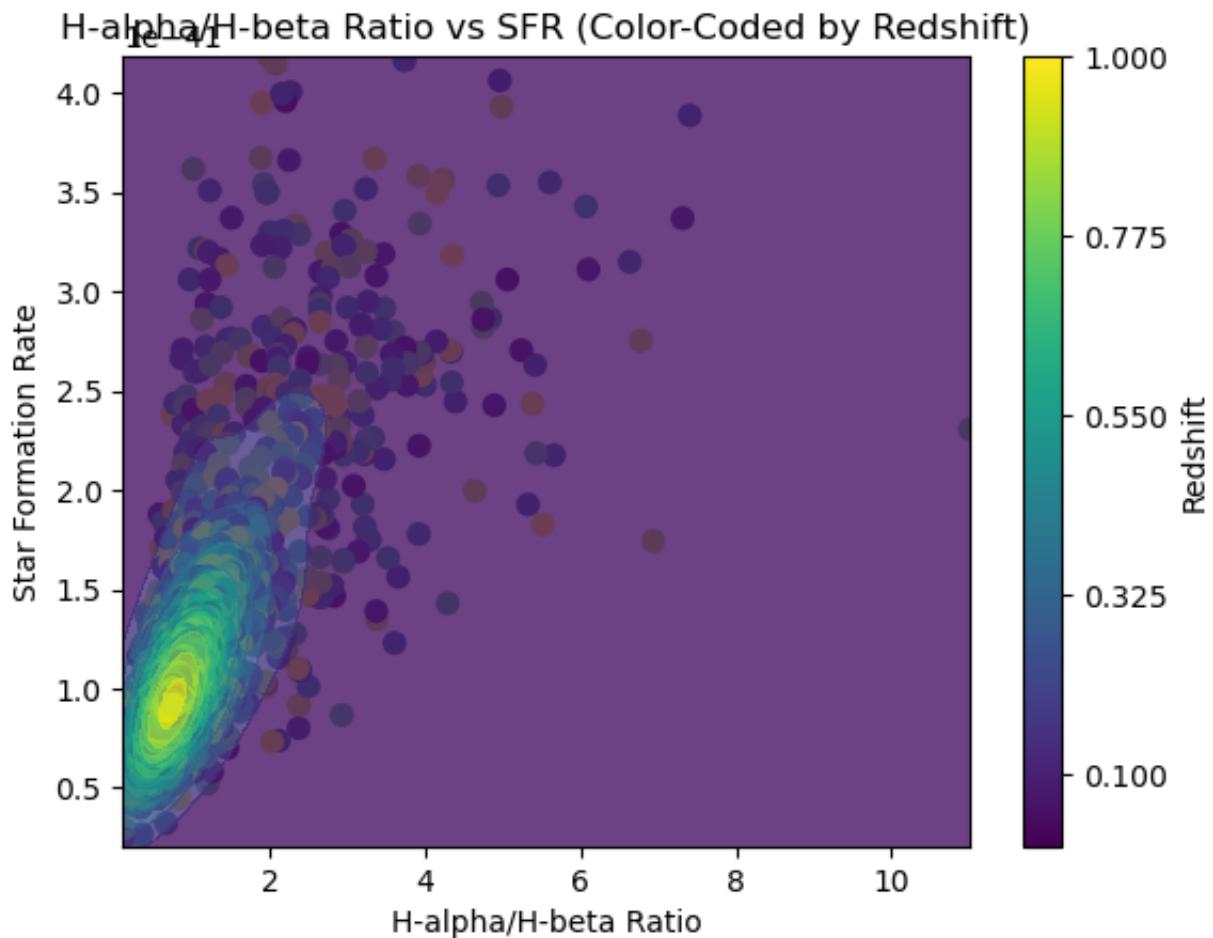
xmin, xmax = line_ratios.min(), line_ratios.max()
ymin, ymax = sfr_values.min(), sfr_values.max()
xx, yy = np.meshgrid(np.linspace(xmin, xmax, 100), np.linspace(ymin, ymax, 100))

# evaluate the KDE on the grid
zz = gaussian_kde(xy)(np.vstack([xx.ravel(), yy.ravel()])).reshape(xx.shape)

fig, ax = plt.subplots()
scatter = ax.scatter(line_ratios, sfr_values, c=redshifts, s=50, cmap='viridis')
ax.contourf(xx, yy, zz, levels=10, alpha=0.8, cmap='viridis') # Adjust levels

ax.set_xlabel('H-alpha/H-beta Ratio')
ax.set_ylabel('Star Formation Rate')
plt.title('H-alpha/H-beta Ratio vs SFR (Color-Coded by Redshift)')

cbar = plt.colorbar(scatter)
cbar.set_label('Redshift')
cbar.set_ticks(np.linspace(0.1, 1, 5))
plt.show()
```



This correlation plot and the calculated correlation above shows that the H\_alpha/H\_beta ratio and the star formation rate of the AGN galaxies are closely correlated (correlation 0.6 in astrophysics is already considered a good correlation).

Even though the redshift is randomly generated, we can see the points are scattered at low H\_alpha/H\_beta ratio, and low star formation rate. Also, star formation rate increases as the H\_alpha/H\_beta ratio increases, and the points distribute more scattered at high H\_alpha/H\_beta ratio and high star formation rate.

# Implications

There are much more to consider about the complete model of AGN galaxy, including things hard for me to analyze such as the fluid dynamics or each element's properties. Thus I can only model the most simple and straightforward spectra. However, other people have developed the computation codes that models the spectra very well over decades.

With the observed spectra, we are able to model the galaxy spectra, calculate its redshift, and find the correlation between parameters of a galaxy.

For example, ALF is a code developed by Conroy (Conroy 2014) that models a galaxy's spectrum with parameters single age, [Z/H], velocity, and velocity dispersion, and the abundances of the elements C, N, O, Mg, Si, Ca, Ti, and Na.

After putting in the observed spectrum and running the simulation on a HPC for Nburn = 20000 and Nwalker = 512, I got the resulting .bestspec, .mcmc, and .sum file.

The .bestspec contains the best fitted model, .mcmc is the monte carlo simulation result, and .sum contains the statistical result of the modeling.

I here define some functions to read and interpetate the data.

```
In [136]: # used the code I wrote during the summer
import sys
import warnings
from copy import deepcopy
import numpy as np
from numpy.polynomial.chebyshev import chebfit, chebval
from scipy import constants, interpolate
import matplotlib.pyplot as plt
from matplotlib.backends.backend_pdf import PdfPages
from astropy.io import ascii
from astropy.table import Table, Column, hstack
import os
import matplotlib.pyplot as plt
import numpy as np
from astropy.table import Table
from astropy.io import ascii
class analyze_spectra(object):
    def __init__(self, outfiles, read_mcmc=True, info=None, index=False):
        self.outfiles = outfiles
        self.nsampel = None
        self.spectra = None
        if read_mcmc:
```

```

        self.mcmc = np.loadtxt('{0}.mcmc'.format(self.outfiles))
results = ascii.read('{0}.sum'.format(self.outfiles))
self.labels = np.array([
    'chi2', 'velz', 'sigma', 'logage', 'zH', 'FeH', 'a',
    'C', 'N', 'Na', 'Mg', 'Si', 'K', 'Ca', 'Ti', 'V', 'Cr',
    'Mn', 'Co', 'Ni', 'Cu', 'Sr', 'Ba', 'Eu', 'Teff',
    'IMF1', 'IMF2', 'logfy', 'sigma2', 'velz2', 'logm7g',
    'hotteff', 'loghot', 'fy_logage', 'logemline_h',
    'logemline_oii', 'logemline_oiii', 'logemline_sii',
    'logemline_ni', 'logemline_nii', 'logtrans', 'jitter',
    'logsky', 'IMF3', 'IMF4', 'h3', 'h4',
    'ML_v', 'ML_i', 'ML_k', 'MW_v', 'MW_i', 'MW_k'
])
results = Table(results, names=self.labels)
types = Column(['mean', 'chi2', 'error',
                'c125', 'c116', 'c150',
                'c184', 'c198', 'lo_prior',
                'hi_prior'],
               name='Type')
results.add_column(types, index=0)
self.results = results[['Type', 'chi2', 'velz', 'sigma', 'logage', 'zH',
                       'C', 'N', 'Na', 'Mg', 'Si', 'K', 'Ca', 'Ti', 'V', 'Cr',
                       'Mn', 'Co', 'Ni', 'Cu', 'Sr', 'Ba', 'Eu', 'Teff',
                       'IMF1', 'IMF2', 'logfy', 'sigma2', 'velz2', 'logm7g',
                       'hotteff', 'loghot', 'fy_logage', 'logemline_h',
                       'logemline_oii', 'logemline_oiii', 'logemline_sii',
                       'logemline_ni', 'logemline_nii', 'logtrans', 'jitter',
                       'logsky', 'IMF3', 'IMF4', 'h3', 'h4',
                       'ML_v', 'ML_i', 'ML_k', 'MW_v', 'MW_i', 'MW_k']]
my_df = Table(self.mcmc, names = self.labels)
self.my_df = my_df[['chi2', 'velz', 'sigma', 'logage', 'zH', 'FeH',
                    'C', 'N', 'Na', 'Mg', 'Si', 'K', 'Ca', 'Ti', 'V', 'Cr',
                    'Mn', 'Co', 'Ni', 'Cu', 'Sr', 'Ba', 'Eu', 'Teff',
                    'IMF1', 'IMF2', 'logfy', 'sigma2', 'velz2', 'logm7g',
                    'hotteff', 'loghot', 'fy_logage', 'logemline_h',
                    'logemline_oii', 'logemline_oiii', 'logemline_sii',
                    'logemline_ni', 'logemline_nii', 'logtrans', 'jitter',
                    'logsky', 'IMF3', 'IMF4', 'h3', 'h4',
                    'ML_v', 'ML_i', 'ML_k', 'MW_v', 'MW_i', 'MW_k']]
with open('{0}.sum'.format(self.outfiles)) as f:
    for line in f:
        if line[0] == '#':
            if 'Nwalkers' in line:
                self.nwalkers = float(line.split('=')[1].strip())
            elif 'Nchain' in line:
                self.nchain = float(line.split('=')[1].strip())
            elif 'Nsample' in line:
                self.nsamp = float(line.split('=')[1].strip())
MCMC = ascii.read('{0}.mcmc'.format(self.outfiles))
m = np.loadtxt('{0}.bestspec'.format(self.outfiles))
data = {}
data['wave'] = m[:,0]/(1.+self.results['velz'][5]*1e3/constants.c)
data['m_flux'] = m[:,1] # Model spectrum, normalization applied

```

```

data['d_flux'] = m[:,2] # Data spectrum
data['snr'] = m[:,3] # Including jitter and inflated errors
data['unc'] = 1/m[:,3]
if not index:
    data['poly'] = m[:,4] # Polynomial used to create m_flux
data['residual'] = (m[:,1] - m[:,2])/m[:,1] * 1e2
self.spectra = data

def normalize_spectra(self):
    """
    Normalize the data and model spectra
    """
    self.spectra['m_flux_norm'] = deepcopy(self.spectra['m_flux'])
    self.spectra['d_flux_norm'] = deepcopy(self.spectra['d_flux'])
    self.spectra['unc_norm'] = deepcopy(self.spectra['unc'])

    chunks = 1000
    min_ = min(self.spectra['wave'])
    max_ = max(self.spectra['wave'])
    num = int((max_ - min_)/chunks) + 1

    for i in range(num):
        k = ((self.spectra['wave'] >= min_ + chunks*i) &
              (self.spectra['wave'] <= min_ + chunks*(i+1)))

        if len(self.spectra['d_flux_norm'][k]) < 10:
            continue

        coeffs = chebfit(self.spectra['wave'][k],
                          self.spectra['d_flux_norm'][k], 2)
        poly = chebval(self.spectra['wave'][k], coeffs)
        self.spectra['d_flux_norm'][k] = self.spectra['d_flux_norm'][k]/poly
        self.spectra['unc_norm'][k] = self.spectra['unc_norm'][k]/poly

        coeffs = chebfit(self.spectra['wave'][k],
                          self.spectra['m_flux_norm'][k], 2)
        poly = chebval(self.spectra['wave'][k], coeffs)
        self.spectra['m_flux_norm'][k] = self.spectra['m_flux_norm'][k]/poly

    def plot_model(self, fname):
        chunks = 5000
        min_ = min(self.spectra['wave'])
        max_ = max(self.spectra['wave'])
        num = int((max_ - min_)/chunks) + 1

        with PdfPages(fname) as pdf:
            for i in range(num):
                fig = plt.figure(figsize=(14,9), facecolor='white')
                ax1 = plt.subplot2grid((3,2), (0,0), rowspan=2, colspan=2)
                ax2 = plt.subplot2grid((3,2), (2,0), rowspan=1, colspan=2)

                j = ((self.spectra['wave'] >= min_ + chunks*i) &
                      (self.spectra['wave'] <= min_ + chunks*(i+1)))

```

```

        ax1.plot(self.spectra['wave'][j],
                  self.spectra['d_flux_norm'][j],
                  color='gray', lw=2, alpha = 0.7, label='Data')

        ax1.plot(self.spectra['wave'][j],
                  self.spectra['m_flux_norm'][j],
                  color='red', lw=2, alpha = 0.8, label='Model')
        ax1.legend(frameon=False)

        ax2.plot(self.spectra['wave'][j],
                  self.spectra['residual'][j],
                  color="#7156A5", lw=2, alpha=0.7)

        ax2.fill_between(self.spectra['wave'][j],
                         -(self.spectra['unc'][j])*1e2,
                         +(self.spectra['unc'][j])*1e2,
                         color='CCCCCC')

        ax2.set_ylim(-15.9, 15.9)
#ax2.set_ylim(-4.9, 4.9)

        ax1.set_ylabel(r'Flux (arbitrary units)',
                      fontsize=22)
        ax2.set_ylabel(r'Residual $\rm \times$',
                      fontsize=22)

        ax2.set_xlabel(r'Wavelength $(\AA)$',
                      fontsize=22, labelpad=10)

        ax1.set_xlim(0.5, 1.5)

        plt.savefig(fname + "_" + str(i) + ".jpg", format='jpg')

        plt.show()

def plot_corner_new(self, outname, save=True, xranges=None, plot_datapoints=False):
    import corner
    import numpy as np
    import matplotlib.pyplot as plt

    # Assuming you have a DataFrame named my_df
    data = np.array([self.my_df['sigma'], self.my_df['zH'], self.my_df['logage'],
                    self.my_df['MgFe']])

    # Create the corner plot without filled contours
    figure = corner.corner(data,
                           labels=['sigma', 'zH', 'logage', '[Mg/Fe]'],
                           show_titles=False,
                           color=plot_color,
                           fill_contours=True,
                           plot_datapoints=plot_datapoints
                           )

```

```
# Access the axes
axes = np.array(figure.axes).reshape((4, 4))

# Loop over the diagonal
for i in range(4):
    ax = axes[i, i]
    ax.axvline(np.median(data[:, i]), color="r")

# Loop over the histograms
for yi in range(4):
    for xi in range(yi):
        ax = axes[yi, xi]

    # Calculate medians
    median_x = np.median(data[:, xi])
    median_y = np.median(data[:, yi])

    # Calculate 1-sigma errors
    err_x = [median_x - np.percentile(data[:, xi], [16, 84]),
              np.percentile(data[:, xi], [16, 84])]

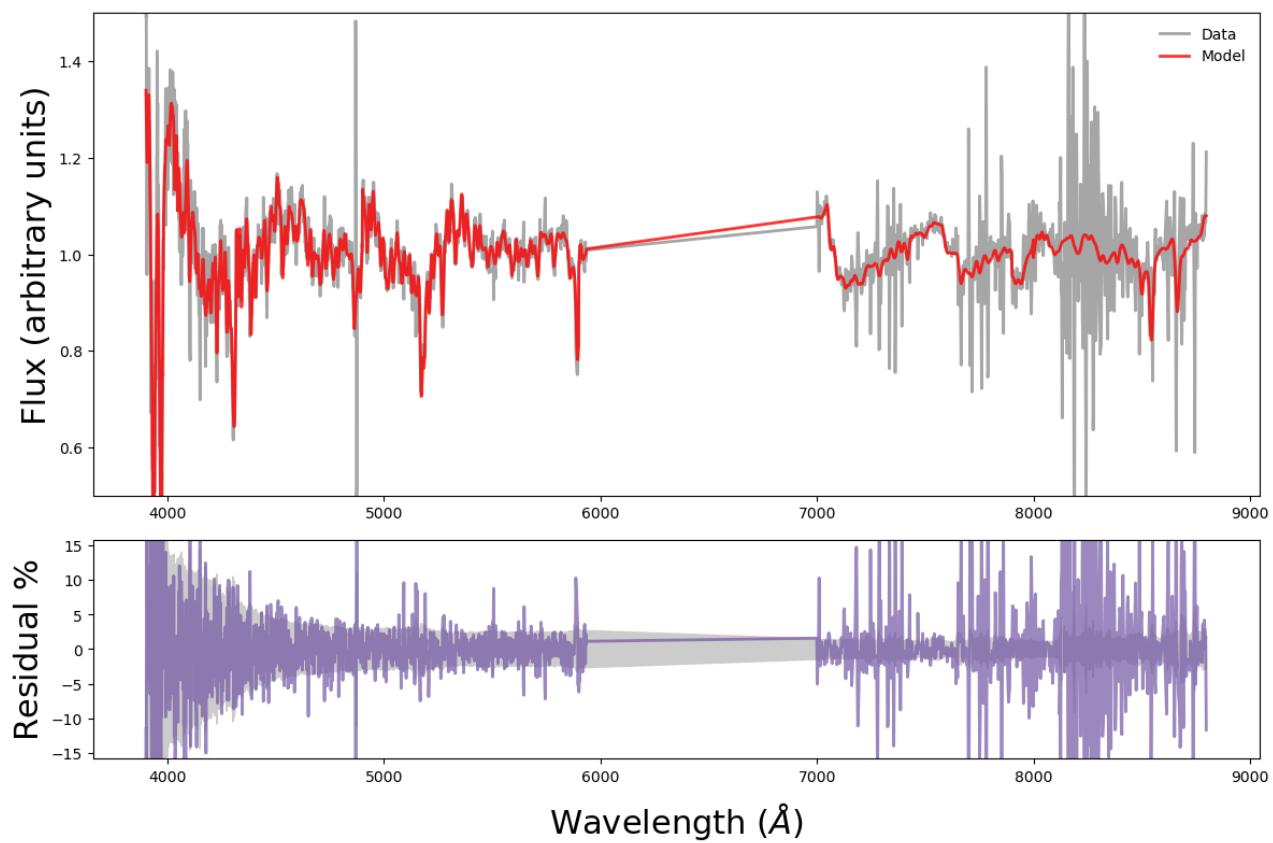
    err_y = [median_y - np.percentile(data[:, yi], [16, 84]),
              np.percentile(data[:, yi], [16, 84])]

    # Plot error bars
    ax.errorbar(median_x, median_y, xerr=err_x, yerr=err_y, color="black",
                ecolor="black", capsize=2)

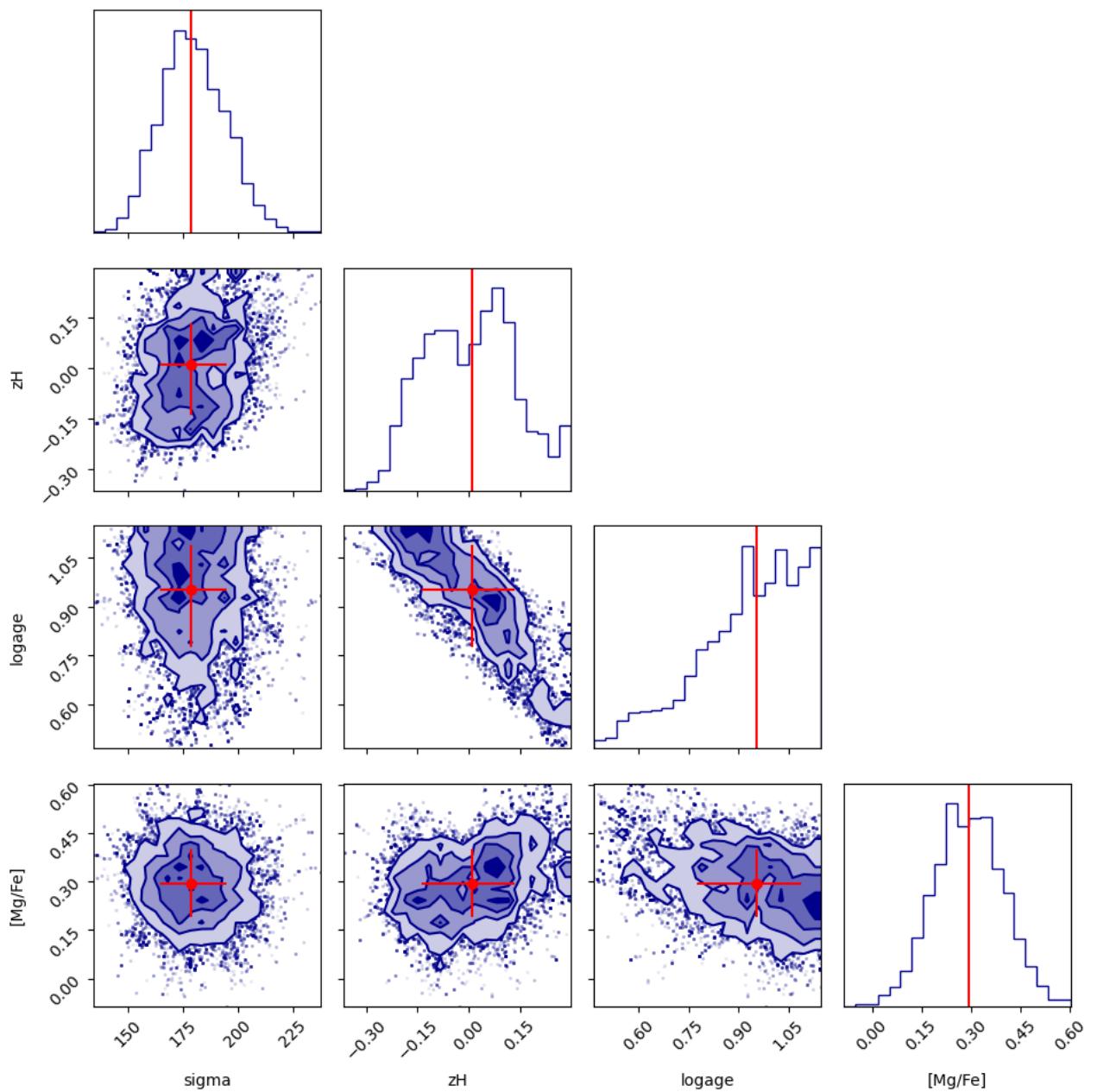
plt.tight_layout()
if save:
    plt.savefig(outname + '_corner')
```

```
In [137]: my_spec = analyze_spectra('shifted_combined_09000204_ob10_512_20k')
outfile = 'shifted_combined_09000204_ob10_512_20k'
```

```
In [138]: my_spec.normalize_spectra()
my_spec.plot_model(outfile)
```



```
In [139]: my_spec.plot_corner_new(outfile)
```



Here we can find the correlation between these stellar populations

In [ ]: