Aula 23 – Solução dos Exercícios

Prove a validade das formas de argumento a seguir:

1. $\forall x(F(x) \vee G(x)) \vdash \exists xF(x) \vee \exists xG(x)$

1.	$\forall x(F(x) \vee G(x))$	P
2.	$F(a) \vee G(a)$	1 EU
3.	$\mid F(a)$	H (PC)
4.	$\mid \exists xF(x)$	3 IE
5.	$\mid \exists xF(x) \vee \exists xG(x)$	4 i$\vee$
6.	$F(a) \rightarrow (\exists xF(x) \vee \exists xG(x))$	3-5 PC
7.	$\mid G(a)$	H (PC)
8.	$\mid \exists xG(x)$	7 IE
9.	$\mid \exists xF(x) \vee \exists xG(x)$	8 i$\vee$
10.	$G(a) \rightarrow (\exists xF(x) \vee \exists xG(x))$	7-9 PC
11.	$\exists xF(x) \vee \exists xG(x)$	2,6,10 e$\vee$

2. $\sim\exists xF(x) \vdash \forall x\sim F(x)$

1.	$\sim\exists xF(x)$	P
2.	$\mid F(a)$	H (RAA)
3.	$\mid \exists xF(x)$	2 IE
4.	$\mid \perp$	1,3 e~
5.	$\sim F(a)$	2-4 RAA
6.	$\forall x\sim F(x)$	5 IU

3. $\sim\exists x(F(x) \wedge \sim G(x)) \vdash \forall x(F(x) \rightarrow G(x))$

1.	$\sim\exists x(F(x) \wedge \sim G(x))$	P
2.	$\mid F(a)$	H (PC)
3.	$\mid \mid \sim G(a)$	H (RAA)
4.	$\mid \mid F(a) \wedge \sim G(a)$	2,3 i$\wedge$
5.	$\mid \mid \exists x(F(x) \wedge \sim G(x))$	4 IE
6.	$\mid \mid \perp$	1,5 e~
7.	$\mid \sim\sim G(a)$	3-6 RAA
8.	$\mid G(a)$	7 e~~
9.	$F(a) \rightarrow G(a)$	2-8 PC
10.	$\forall x(F(x) \rightarrow G(x))$	9 IU

4. $\exists x(F(x) \vee G(x)) \vdash \exists xF(x) \vee \exists xG(x)$

1.	$\exists x(F(x) \vee G(x))$	P
2.	$\mid F(a) \vee G(a)$	H (EE)
3.	$\mid \mid F(a)$	H (PC)
4.	$\mid \mid \exists xF(x)$	3 IE

5.	| | ∃xF(x) ∨ ∃xG(x)	4 i∨
6.	| F(a) → (∃xF(x) ∨ ∃xG(x))	3-5 PC
7.	| | G(a)	H (PC)
8.	| | ∃xG(x)	7 IE
9.	| | ∃xF(x) ∨ ∃xG(x)	8 i∨
10.	| G(a) → (∃xF(x) ∨ ∃xG(x))	7-9 PC
11.	| ∃xF(x) ∨ ∃xG(x)	2,6,10 e∨
12.	∃xF(x) ∨ ∃xG(x)	1,2-11 EE

5. ∃xF(x) ∨ ∃xG(x) ⊢ ∃x(F(x) ∨ G(x))

1.	∃xF(x) ∨ ∃xG(x)	P
2.	| ∃xF(x)	H (PC)
3.	| | F(a)	H (EE)
4.	| | F(a) ∨ G(a)	3 i∨
5.	| | ∃x(F(x) ∨ G(x))	4 IE
6.	| ∃x(F(x) ∨ G(x))	2,3-5 EE
7.	∃xF(x) → ∃x(F(x) ∨ G(x))	2-6 PC
8.	| ∃xG(x)	H (PC)
9.	| | G(a)	H (EE)
10.	| | F(a) ∨ G(a)	9 i∨
11.	| | ∃x(F(x) ∨ G(x))	10 IE
12.	| ∃x(F(x) ∨ G(x))	8,9-11 EE
13.	∃xG(x) → ∃x(F(x) ∨ G(x))	8-12 PC
14.	∃x(F(x) ∨ G(x))	1,7,13 e∨

6. ∃x∀yL(x,y) ⊢ ∀x∃yL(y,x)

1.	∃x∀yL(x,y)	P
2.	| ∀yL(a,y)	H (EE)
3.	| L(a,b)	2 EU
4.	| ∃yL(y,b)	3 IE
5.	| ∀x∃yL(y,x)	4 IU
6.	∀x∃yL(y,x)	1,2-5 EE

7. ∀x(F(x) → ∃yL(x,y)), ∃x(F(x) ∧ G(x)) ⊢ ∃x∃y(G(x) ∧ L(x,y))

1.	∀x(F(x) → ∃yL(x,y))	P
2.	∃x(F(x) ∧ G(x))	P
3.	| F(a) ∧ G(a)	H (EE)
4.	| F(a) → ∃yL(a,y)	1 EU
5.	| F(a)	3 e∧
6.	| ∃yL(a,y)	4,5 MP

7.	| | L(a,b)	H (EE)
8.	| | G(a)	3 e∧
9.	| | G(a) ∧ L(a,b)	7,8 i∧
10.	| | ∃y(G(a) ∧ L(a,y))	9 IE
11.	| | ∃x∃y(G(x) ∧ L(x,y))	10 IE
12.	| ∃x∃y(G(x) ∧ L(x,y))	6,7-11 EE
13.	∃x∃y(G(x) ∧ L(x,y))	2,3-12 EE

8. ∀x(F(x) → ~G(x)) ⊢ ~∃x(F(x) ∧ G(x))

1.	∀x(F(x) → ~G(x))	P
2.	F(a) → ~G(a)	1 EU
3.	| ∃x(F(x) ∧ G(x))	H (RAA)
4.	| | F(a) ∧ G(a)	H (EE)
5.	| | G(a)	4 e∧
6.	| | ~F(a)	2,5 MT
7.	| | F(a)	4 e∧
8.	| | ⊥	6,7 e~
9.	| ⊥	3,4-8 EE
10.	~∃x(F(x) ∧ G(x))	3-9 RAA