

Resolução dos exercícios de Árvore de Refutação

1. $\exists xF(x) \vdash F(a)$

- | | |
|-------------------|--------------|
| 1 $\exists xF(x)$ | Premissa |
| 2 $\sim F(a)$ | Hipótese |
| 3 $F(b)$ | 1, \exists |

É inválida.

2. $\forall xF(x) \vdash F(a)$

- | | |
|-------------------|--------------|
| 1 $\forall xF(x)$ | Premissa |
| 2 $\sim F(a)$ | Hipótese |
| 3 $F(a)$ | 1, \forall |
| X 2,3 \sim | |

É válida.

3. $F(a) \vdash \exists xF(x)$

- | | |
|-------------------------|-------------------|
| 1 $F(a)$ | Premissa |
| 2 $\sim \exists xF(x)$ | Hipótese |
| 3 $\forall x \sim F(x)$ | 2, $\sim \exists$ |
| 4 $\sim F(a)$ | 3, \forall |
| X 1,4 \sim | |

É válida.

4. $F(a) \vdash \forall xF(x)$

- | | |
|-------------------------|-------------------|
| 1 $F(a)$ | Premissa |
| 2 $\sim \forall xF(x)$ | Hipótese |
| 3 $\exists x \sim F(x)$ | 2, $\sim \forall$ |
| 4 $\sim F(b)$ | 3, \exists |

É inválida.

5. $\forall xF(x) \vdash \sim \exists x \sim F(x)$

- | | |
|-----------------------------------|----------------|
| 1 $\forall xF(x)$ | Premissa |
| 2 $\sim \sim \exists x \sim F(x)$ | Hipótese |
| 3 $\exists x \sim F(x)$ | 2, $\sim \sim$ |
| 4 $\sim F(a)$ | 3, \exists |
| 5 $F(a)$ | 1, \forall |
| X 4,5 \sim | |

É válida.

6. $\sim \exists x \sim F(x) \vdash \forall x F(x)$

1	$\sim \exists x \sim F(x)$	Premissa
2	$\sim \forall x F(x)$	Hipótese
3	$\forall x \sim \sim F(x)$	1, $\sim \exists$
4	$\exists x \sim F(x)$	2, $\sim \forall$
5	$\sim F(a)$	4, \exists
6	$\sim \sim F(a)$	3, \forall
7	$F(a)$	6, $\sim \sim$
	\times 5,7 \sim	

É válida.

7. $\forall x \sim F(x) \vdash \sim \forall x F(x)$

1	$\forall x \sim F(x)$	Premissa
2	$\sim \sim \forall x F(x)$	Hipótese
3	$\forall x F(x)$	2, $\sim \sim$
4	$\sim F(a)$	1, \forall
5	$F(a)$	3, \forall
	\times 4,5 \sim	

É válida.

8. $\sim \forall x F(x) \vdash \forall x \sim F(x)$

1	$\sim \forall x F(x)$	Premissa
2	$\sim \forall x \sim F(x)$	Hipótese
3	$\exists x \sim F(x)$	1, $\sim \forall$
4	$\exists x \sim \sim F(x)$	2, $\sim \forall$
5	$\sim F(a)$	3, \exists
6	$\sim \sim F(b)$	4, \exists
7	$F(b)$	6, $\sim \sim$

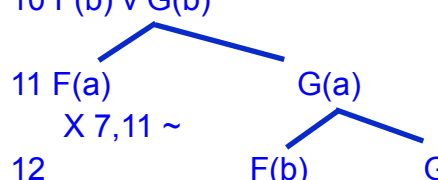
É inválida.

9. $\forall x F(x) \vee \forall x G(x) \vdash \forall x (F(x) \vee G(x))$

1	$\forall x F(x) \vee \forall x G(x)$	Premissa
2	$\sim \forall x (F(x) \vee G(x))$	Hipótese
3	$\exists x \sim (F(x) \vee G(x))$	2, $\sim \forall$
4	$\sim (F(a) \vee G(a))$	3, \exists
5	$\sim F(a)$	4, $\sim \vee$
6	$\sim G(a)$	4, $\sim \vee$
7	$\forall x F(x)$	$\forall x G(x)$ 1, \vee
8	$F(a)$	$G(a)$ 7, \forall
	\times 5,8 \sim	\times 6,8 \sim

É válida.

10. $\forall x(F(x) \vee G(x)) \vdash \forall xF(x) \vee \forall xG(x)$

1	$\forall x(F(x) \vee G(x))$	Premissa
2	$\sim(\forall xF(x) \vee \forall xG(x))$	Hipótese
3	$\sim\forall xF(x)$	2, $\sim\vee$
4	$\sim\forall xG(x)$	2, $\sim\vee$
5	$\exists x\sim F(x)$	3, $\sim\forall$
6	$\exists x\sim G(x)$	4, $\sim\forall$
7	$\sim F(a)$	5, \exists
8	$\sim G(b)$	6, \exists
9	$F(a) \vee G(a)$	1, \forall
10	$F(b) \vee G(b)$	1, \forall
		
11	$F(a)$	9, \vee
	$G(a)$	
12	$\text{X } 7, 11 \sim$	
	$F(b)$	
	$G(b)$	
	$\text{X } 8, 11 \sim$	
		10, \vee

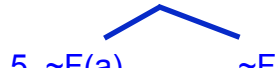
É inválida.

11. $\vdash \forall x(F(x) \vee \sim F(x))$

1	$\sim\forall x(F(x) \vee \sim F(x))$	Hipótese
2	$\exists x\sim(F(x) \vee \sim F(x))$	1, $\sim\forall$
3	$\sim(F(a) \vee \sim F(a))$	2, \exists
4	$\sim F(a)$	3, $\sim\vee$
5	$\sim\sim F(a)$	3, $\sim\vee$
6	$F(a)$	5, $\sim\sim$
	$\text{X } 4, 6 \sim$	

É válida.

12. $\vdash \forall x\sim(F(x) \rightarrow \sim F(x))$

1	$\sim\forall x\sim(F(x) \rightarrow \sim F(x))$	Hipótese
2	$\exists x\sim\sim(F(x) \rightarrow \sim F(x))$	1, $\sim\forall$
3	$\sim\sim(F(a) \rightarrow \sim F(a))$	2, \exists
4	$F(a) \rightarrow \sim F(a)$	3, $\sim\sim$
		
5	$\sim F(a)$	4, \rightarrow
	$\sim F(a)$	

É inválida.

13. $\vdash \exists x F(x) \leftrightarrow \sim \forall x \sim F(x)$

1 $\sim(\exists x F(x) \leftrightarrow \sim \forall x \sim F(x))$

Hipótese

2 $\sim \exists x F(x) \wedge \sim \forall x \sim F(x)$

$\exists x F(x) \wedge \sim \sim \forall x \sim F(x)$

1, $\sim \leftrightarrow$

3 $\sim \exists x F(x)$

$\exists x F(x)$

2, \wedge

4 $\sim \forall x \sim F(x)$

$\sim \sim \forall x \sim F(x)$

2, \wedge

5 $\forall x \sim F(x)$ 2, $\sim E$

$\forall x \sim F(x)$

3, $\sim \sim$

6 $\exists x \sim \sim F(x)$ 3, $\sim \forall$

$F(a)$

2, \exists

7 $\sim \sim F(b)$ 5, E

$\sim F(a)$

4, \forall

8 $F(b)$ 6, $\sim \sim$

\times 6,7 \sim

9 $\sim F(b)$ 4, \forall

\times 8,9 \sim

É válida.

14. $\exists x(F(x) \wedge \sim F(x)) \vdash P$

1 $\exists x(F(x) \wedge \sim F(x))$

Premissa

2 $\sim P$

Hipótese

3 $F(a) \wedge \sim F(a)$

1, \exists

4 $F(a)$

3, \wedge

5 $\sim F(a)$

3, \wedge

\times 4,5 \sim

É válida.

15. $\exists x F(x) \wedge \exists x \sim F(x) \vdash P$

1 $\exists x F(x) \wedge \exists x \sim F(x)$

Premissa

2 $\sim P$

Hipótese

3 $\exists x F(x)$

1, \wedge

4 $\exists x \sim F(x)$

1, \wedge

5 $F(a)$

3, \exists

6 $\sim F(b)$

4, \exists

É inválida.

16. $\sim \exists x F(x) \vdash \forall x(F(x) \rightarrow P)$

1 $\sim \exists x F(x)$

Premissa

2 $\sim \forall x(F(x) \rightarrow P)$

Hipótese

3 $\forall x \sim(F(x))$

1, $\sim \exists$

4 $\exists x \sim(F(x) \rightarrow P)$

2, $\sim \forall$

5 $\sim(F(a) \rightarrow P)$

4, \exists

6 $F(a)$	5, $\sim \rightarrow$
7 $\sim P$	5, $\sim \rightarrow$
8 $\sim F(a)$	3, \forall
\times 6,8 \sim	

É válida.

17. $\forall x \forall y (L(x,y) \rightarrow L(y,x)), \exists x L(a,x) \vdash \exists x L(x,a)$

1 $\forall x \forall y (L(x,y) \rightarrow L(y,x))$	Premissa
2 $\exists x L(a,x)$	Premissa
3 $\sim \exists x L(x,a)$	Hipótese
4 $\forall x \sim L(x,a)$	3, $\sim \exists$
5 $L(a,b)$	2, \exists
6 $\sim L(b,a)$	4, \forall
7 $\forall y (L(a,y) \rightarrow L(y,a))$	1, \forall
8 $L(a,b) \rightarrow L(b,a)$	7, \forall
9 $\sim L(a,b)$	8, \rightarrow
\times 5,9 \sim	
10 $L(b,a)$	
\times 6,10 \sim	

É válida.

18. $\exists x \exists y L(x,y) \vdash \exists x L(x,x)$

1 $\exists x \exists y L(x,y)$	Premissa
2 $\sim \exists x L(x,x)$	Hipótese
3 $\forall x \sim L(x,x)$	2, $\sim \exists$
4 $\exists y L(a,y)$	1, \exists
5 $L(a,b)$	4, \exists
6 $\sim L(a,a)$	3, \forall

É inválida.

19. $\forall x (F(x) \rightarrow \forall y G(y)) \vdash \forall x G(x)$

1 $\forall x (F(x) \rightarrow \forall y G(y))$	Premissa
2 $\sim \forall x G(x)$	Hipótese
3 $\exists x \sim G(x)$	2, $\sim \forall$
4 $\sim G(a)$	3, \exists
5 $F(a) \rightarrow \forall y G(y)$	1, \forall
6 $\sim F(a)$	5, \rightarrow
7 $\forall y G(y)$	6, \forall
\times 4,7 \sim	

É inválida.

20. $\forall x(F(x) \rightarrow \exists yG(y)) \vdash G(a)$

1	$\forall x(F(x) \rightarrow \exists yG(y))$	Premissa
2	$\sim G(a)$	Hipótese
3	$F(a) \rightarrow \exists yG(y)$	1, \forall
4	$\sim F(a)$	3, \rightarrow
5	$\exists yG(y)$	4, \exists
6	$G(b)$	1, \forall
7	$F(b) \rightarrow \exists yG(y)$	6, \rightarrow
8	$\sim F(b)$	7, \exists
9	$G(c)$	1, \forall
	$F(c) \rightarrow \exists yG(y)$	
	.	
	.	
	.	

É inválida.