

Name:

AMATH 515

Homework Set 0

**Due: Wednesday, January 11th, by 11 pm.**

The goal of this homework is to make sure you are comfortable with all prerequisites for this class, to set-up Python and Jupyter Notebook, and to try submitting your work to Gradescope Autograder. The theoretical portion of the homework will be graded based on completeness, and is intended as a primer on calculus and linear algebra.

1. THEORY

- (1) Submit your write-up to Gradescope. Look for the assignment "Homework 0 – theory".
- (2) **Calculus primer.** For a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ , we define the *gradient* to be the vector of partial derivatives:

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

and the *Hessian* to be the matrix of second partial derivatives:

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & & \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n \partial x_n} \end{bmatrix}$$

Compute the gradients and Hessians of the following functions, with  $x \in \mathbb{R}^4$  in all three examples.

- (a)  $f(x) = \sin(x_1 + x_2 + x_3 + x_4)$
- (b)  $f(x) = \|x\|^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$
- (c)  $f(x) = \ln(x_1 x_2 x_3 x_4)$ .

(3) **Linear algebra primer.**

(a) What are the eigenvalues of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \pi & 2 & 0 & 0 \\ 64 & -15 & 3 & 0 \\ 321 & 0 & 0 & 5 \end{bmatrix}$$

(b) Write down bases for the range and nullspace of the following matrix, written as the outer product of two vectors:

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(c) Let  $A$  be a  $10 \times 5$  matrix, and  $b$  a vector in  $\mathbb{R}^{10}$ . The notation  $A^T$  denotes the *transpose* of  $A$ , where the columns of  $A$  are rows of  $A^T$ .

- What is the size of  $A^T A$ ? What is the size of  $A^T b$ ?
- How many solutions might there be to the system  $Ax = b$ ?
- How many solutions might there be to the system  $A^T Ax = A^T b$ ?
- Suppose the columns of  $A$  are linearly independent. How many solutions might there be to the system  $Ax = b$ ? To the system  $A^T Ax = A^T b$ ?

## 2. PRACTICE

(1) Install Anaconda3 distribution. Instruction:

<https://www.anaconda.com/products/individual>

- If you've never used Python before – here is an excellent Python introduction: <https://www.learnpython.org>
- If you have experience with scientific computing in MATLAB, but you've never tried Python, here is a useful migration guide: <https://www.enthought.com/white-paper-matlab-to-python-a-migration-guide/>

(2) Download "Homework0.ipynb" from Canvas, open it as a Jupyter Notebook, and complete all the tasks there.

- If you've never used Jupyter Notebooks then take a look at this tutorial: <https://www.dataquest.io/blog/jupyter-notebook-tutorial/>

(3) Submit your Jupyter Notebook to Gradescope. Look for the assignment "Homework 0 – practice". There is no limit for the number of attempts for the coding part this time.